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Fundamentals of
Electricity & Magnetism

by

LEONARD B. LOEB

Professor of Physics, University of California

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PREFACE

This is the third edition of a textbook prepared in mimeograph form in 1927 and published in a first edition in 1931. It was written to fill a specific need at the University of California, which was one of the first universities to give basic physics for engineers, physicists, and chemists in *four* three-credit courses during the first two collegiate years of study. The first semester was devoted to mechanics; followed by heat and properties of matter in the second; electricity and magnetism in the third; and sound, light, and atomic structure in the fourth. This course in physics was closely coordinated with mathematics sequence, the first semester of which was analytical geometry, the second differential calculus, and the third and fourth integral calculus. For such a program the usual textbooks in beginning physics did not include a sufficiently advanced treatment of electricity and magnetism.

After World War II the basic course in physics was rearranged into *three* four-credit courses in a sequence starting in the second semester of the freshman year. Before starting physics the student takes a semester of analytical geometry and some differential calculus. In the next two semesters the concurrent courses in mathematics and physics are mechanics and properties of matter with differential calculus, and electricity, magnetism, and atomic structure with integral calculus. The second semester of the sophomore year is devoted to heat, sound, and light.

In the new plan, as in the previous plan, one credit is for laboratory work. The new plan allots fifty per cent more time for lectures on electricity, magnetism, and atomic structure. The textbook has been revised and enlarged to meet these requirements.

Since it has been written on a somewhat higher level than the conventional one-year, eight-semester-hour courses conventionally given in colleges and since it freely uses the calculus, it is possible that this textbook can and will be used in more advanced courses at colleges giving the single eight-semester-hour course in basic physics. In view of the demand for this book for such purposes, there has been included a considerable amount of additional material, comprising such topics as electromagnetic theory and the solution of the wave equation as well as a more comprehensive study of the fundamentals of electronics.

A brief historical outline replaces the historical survey to which a major portion of the first part of the text was devoted in previous editions. This has provided space for the major change, which is in the section on atomic structure, including conduction in gases, now

extended to give a knowledge of the fundamental principles of atomic behavior and organization. Besides this major change some new material resulting from recent advances has been added. This material includes the presentation of the ferromagnetic phenomena from the viewpoint of the theory of domains, extensions of the chapters on resistance, ionic conduction and its relation to atomic structure, thermoelectric effects, dielectric effects, paramagnetism and ferromagnetism, and the modern methods for measurement of magnetic fields and flux. In addition, there are sections dealing with the electrical principles underlying the achievement of high-energy electrical particles, including devices such as the betatron, synchrotron, synchrocyclotron, and linear accelerator as well as sections dealing with the principles of ultra-high-frequency generating devices such as the magnetron and Klystron. The revision adds some seventy new or improved figures.

The basic arrangement of the text, in which elementary magnetism and direct-current electricity is presented first, followed later by a study of static electricity, was dictated by conditions connected with the laboratory instruction. Although this sequence may not constitute an ideal treatment, it has given an approach that worked out well in practice.

In the belief that the empirical and somewhat dogmatic presentation of physics which characterized the textbooks in vogue in the earlier years of the century was basically wrong, the author has followed, so far as possible, the precepts of his teacher, Professor R. A. Millikan, by introducing the various concepts in terms of experimental observation and the logical deductions and inferences based thereon. This objective of presenting the subject as a consistent and related whole rather than a compilation of empirical, isolated, and dogmatically stated topics dominates the whole plan of the text.

Again, in developing the subject with the purpose of fixing in the minds of the students the fundamental elements of electricity and magnetism, the author has found that the greatest difficulty encountered by the students is the confusion in the definition and relation of the many electrical entities occurring in such a treatment. To systematize the teaching and to avoid vicious cycles the author has found it convenient to divide these entities, where they appear in the development of the subject, into two types, *fundamental* and *derived*. The entities chosen as *fundamental* are defined directly in terms of force or work. They are electrical current, quantity, and potential. The definitions are based on the classical experiments relating these concepts to force or work. The three *derived* entities chosen result from *ratios* of the fundamental entities and are generally properties of the shape, dimensions, and materials of the electrical system where they occur. They are resistance, capacity, and self-induction. The six entities, fundamental and derived, are for prac-

tical purposes of measurement and comparison expressed in the three systems electromagnetic, electrostatic, and practical, the origin and choice of the units being given wherever possible. This formulation of the fundamental notions of electricity leads to a summarizing table given on pages 412-413, which constitutes the kernel of the course. Experience has shown that this bird's-eye view of the interrelations of the electrical concepts has been exceedingly helpful to the students. It leads to a mnemonic system for remembering the units and their ratios which has many useful features. One of the interesting results from this choice of definitions leads to the observation that the *ratios* of the *fundamental* units in the *electromagnetic* and the *electrostatic* systems all involve the first power of the velocity of light. In contrast, the ratios of the *derived* units in the electromagnetic and the electrostatic systems involve the square of the velocity of light.

As a result of many years' experience in teaching electricity and magnetism to "run-of-the-mine" engineering students at the University of California, the author is impressed with the difficulty of introducing some of the more abstract concepts, such as field theory, potential theory, the line integral of the force, etc., in *generalized form*. If one recognizes that it was not until 1850 that the leading scientists of the world arrived at these concepts, perhaps this weakness on the part of the students may be condoned. The author thus found it quite unprofitable to derive important theorems in a *general* way, or to use the vector analytical mode of presentation. He has found it possible in many cases to derive such concepts as the energy per unit volume stored in electrical or magnetic fields by means of simple analyses based on special cases. With the results thus obtained he can indicate to the student the more general applications. This differentiates this textbook from some of the newer textbooks written purportedly for courses on the same level.

In common with most textbooks on electricity and magnetism the subject is consistently developed on the basis of the gaussian system of units, which appears to be the simplest for the beginning student. A discussion of these systems is introduced in Chapter XXVI.

Despite the feeling among certain groups of physics teachers and engineers that the m.k.s. system of units should be adopted, the author has not altered past procedure. The choice of units is a *purely aesthetic question*, since all our Newtonian-based systems are arbitrary ones. For gross mechanics and applied, or engineering, physics the m.k.s. system has advantages. For electricity, basically so closely associated with electronic and atomic phenomena in its fundamental presentation, the c.g.s. system, with its smaller units, is definitely to be preferred to the m.k.s. Thus for instruction in the *fundamentals of electricity and magnetism* this textbook will continue to utilize the c.g.s. system.

In an attempt to give the students some experience with the magni-

tudes of electrical quantities and to give them some drill in the applications of the principles a fairly complete set of problems has been developed over the course of the years. Failing to find more than just a drill in the substitution in formulas using the conventional problems usually assigned, the author has developed a large number of original problems, many of them involving the well-known principles but applied to more modern examples. These are included at the end of the text for those who wish to use them.

ACKNOWLEDGMENTS

In a book that has been through a period covering three editions the author must acknowledge valuable assistance, advice, and suggestions over the years from sources too numerous to list. It is hoped that failure to mention any but those few whose assistance contributed in a major degree to the development of the book will not cause disappointment. To the late Professor Elmer E. Hall who had organized the course before the author took it over the author must accord a great deal of credit and thanks. Thanks are also due to Professor J. C. Hubbard of Johns Hopkins University, who as reviewer for the publishers contributed many valuable suggestions and much constructive criticism for the first two editions. Professor David L. Webster of Stanford University made many useful suggestions for the first edition as did Professor Thomas Buck of the Mathematics Department of the University of California. To his colleague Professor Raymond T. Birge of the University of California the author owes a great deal for his valuable advice on the discussions of units, dimensions, the value of physical constants, and many other matters of similar nature. The author wishes to express his thanks to Professor Newell C. Page and his publishers, The Macmillan Company, for permission to use some of the problems in his excellent book, *Lessons and Problems in Electricity and Magnetism*. Thanks are due to Dr. O. E. Anderson of San Francisco Junior College for timely suggestions, corrections, and a few problems included in the third edition.

For the many new drawings in the second and the third editions the author is indebted to Mr. Foster Elton, Mrs. Virginia Stoltz, and Miss Colombe Winterhalder. He is indebted to Messrs. I. Tillin, Damon Beard, R. E. Worley, James Miller, and to Dr. A. F. Kip for solutions of the many problems supplied at the end of the text. He is especially indebted to his wife, Charlotte Pearson Loeb, for her help in the preparation of the manuscript for the third edition and for assistance in reading proof.

CONTENTS

CHAPTER	PAGE
I. HISTORICAL	1
1. General Development of Physical Science	1
2. The Development of Electricity and Magnetism	8
<i>a.</i> Static Period	8
<i>b.</i> Current Period	8
<i>c.</i> Electrotechnical or Instrumental Period	8
<i>d.</i> Systematic Period	9
<i>e.</i> Atomic Period	9
<i>f.</i> Quantum Period	9
<i>g.</i> Wave Mechanics Period	9
<i>h.</i> Nuclear Period	10
II. THE PHENOMENA OF MAGNETISM	11
3. The Discovery of Magnetism	11
4. The Discovery of the Compass	12
5. The Qualitative Phenomena of Magnetism	14
III. THE QUANTITATIVE TREATMENT OF MAGNETIC FORCES	28
6. The Concept of the Magnetic Field	28
7. Definitions of Unit Pole and Pole Strength — Coulomb's Law	28
8. Units and Dimensions	30
9. Dimensions of Magnetic Pole Strength	34
10. Application of Law of Force to Special Cases	35
IV. THE ABSOLUTE DETERMINATION OF POLE STRENGTH AND MAGNETIC FIELDS. THE EARTH'S FIELD — FIELD CONVENTIONS	40
11. The Absolute Determination of Pole Strength and Magnetic Fields	40
12. Experiment I: M/H	41
13. Experiment II: MH	42
14. Note on the Use of Oscillating Systems in Measuring Fields	44
15. The Earth's Magnetic Field	45
16. Conventional Representation of Fields of Force	47
V. ELECTRIC CURRENTS	51
17. Historical Sketch	51
18. Magnetic Field of a Current	52
19. Ampère's Rule and Definition of Unit Current	53
20. Alternative Definition of Unit Current	55
21. Definition of Quantity in Electromagnetic System of Units	56
22. Applications of Ampère's Law	56
23. The Tangent Galvanometer	62
VI. POTENTIAL DIFFERENCE AND WORK IN AN ELECTRIC CIRCUIT	64
24. The Concept of Potential	64
25. The Absolute Electromagnetic Unit of Potential	69

CHAPTER	PAGE
26. The Definition of Electrical Power Consumption.....	72
27. The Concept and Definition of Electrical Resistance.....	73
28. Joule's Law of Heating.....	74
VII. ELECTRIC CURRENTS AND RESISTANCE.....	76
29. Nature of Resistance and Its Variation.....	76
30. Resistance and Temperature.....	80
31. Combination of Resistances in Series and in Parallel.....	82
32. Comparison of Potentials and the Determination of Electro- motive Forces by Means of the Potentiometer.....	84
33. Potential and Electromotive Forces in Circuits.....	86
VIII. GENERALIZATION OF OHM'S LAW.....	90
34. Kirchhoff's First Law of Divided Circuits.....	90
35. Kirchhoff's Second Law of Divided Circuits.....	91
36. The Use of the Wheatstone Bridge.....	94
37. Note on the Solution of Kirchhoff's Law Equations.....	95
38. Short-cut Methods for the Solutions of Complicated Circuits: Maxwell's Method and Method of the Equivalent Y.....	99
IX. ELECTRICAL MEASURING INSTRUMENTS.....	103
39. Moving-Coil or d'Arsonval Galvanometer.....	103
40. The Galvanometer Constant.....	105
41. Types of Galvanometers, Ammeters, and Voltmeters.....	108
42. Shunts for Ammeters, Series Resistance for Voltmeters.....	111
43. The Wattmeter.....	113
X. LIQUID CONDUCTORS I: ELECTROLYSIS.....	115
44. Types of Conduction and Faraday's Laws.....	115
45. Atomic Structure and Conduction in Solution.....	119
XI. LIQUID CONDUCTORS II: BATTERIES.....	137
46. Solution Tension.....	137
47. Solution Pressure.....	137
48. Equilibrium.....	137
49. The Concentration Cell.....	140
50. The Copper Cell or Daniell Cell.....	141
51. The Acid Cell.....	141
52. Polarization.....	143
53. Calculation of the Approximate E.M.F. of a Cell.....	143
54. Cautions in Electroplating.....	146
55. Storage Cells.....	146
56. Dry Cells.....	149
57. Standard Cell.....	149
58. Most Efficient Arrangement of Cells.....	150
XII. THERMOELECTRICITY.....	154
59. The Peltier and the Seebeck Effects.....	154
60. Thermoelectric Power and the Calculation of Thermal E.M.F.'s.....	157
61. Uses of Thermocouples.....	165

CONTENTS

xv

CHAPTER	PAGE
XIII. STATIC ELECTRICITY I: CHARGES AND FIELDS.....	169
62. Electrification and Its Detection.....	169
63. Electrostatic Induction.....	170
64. One- and Two-Fluid Theories of Electricity.....	171
65. Consequences of Induction: The Ice-Pail Experiment.....	172
66. Uses of Induction.....	173
67. Quantitative Treatment: Coulomb's Law.....	175
68. Definition of Current in the Electrostatic System.....	180
69. Definition of Electric Field Strength.....	181
XIV. STATIC ELECTRICITY II: POTENTIAL.....	183
70. The Concept of Potential Difference in the Electrostatic System.....	183
71. Definition of the Electrostatic Unit of Potential Difference..	184
72. Further Discussion of the Concept of Potential Difference...	184
73. Charge Density and Its Applications.....	191
74. The Absolute Electrometer.....	194
75. Gold-Leaf Electroscope.....	196
76. Quadrant Electrometer.....	197
XV. STATIC ELECTRICITY III: CAPACITY.....	199
77. Definition of Capacity and the Units of Capacity.....	199
78. Capacity of the Spherical Condenser.....	202
79. The Dielectric Constant, Dielectric Polarization, and Hysteresis.....	203
80. The Capacities of Parallel-Plate and Cylindrical Condensers	215
81. Uses of Condensers, Energy of Charge, Energy in the Electrostatic Field, Condensers in Series and in Parallel.....	218
XVI. ELECTROMAGNETICS.....	226
82. Definition of Current and Work Done in Moving a Conductor in a Field.....	226
83. Field in an Infinitely Long Solenoid.....	227
84. Interactions between Fields and Conductors Carrying Currents.....	230
XVII. MAGNETIC PROPERTIES OF MATERIALS.....	236
85. The Manner of Regarding a Piece of Iron in a Magnetic Field.....	236
86. The Curve of Magnetization and Hysteresis.....	238
87. The Magnetic Behavior of Atoms: Paramagnetism, Diamagnetism, and Ferromagnetism.....	245
88. Diamagnetic Action.....	248
89. The Existence of Ampère's Molecular Magnets, the Magnetron, the Quantization of Atomic Magnets, Paramagnetic and Ferromagnetic Behavior in Solids, Gases, and Metals	253
XVIII. THE MAGNETIC CIRCUIT.....	266
90. Concept of Magnetomotive Force and Definition of Reluctance.....	266

CHAPTER	PAGE
91. The Value of Reluctance.....	267
92. Demagnetization, Leakage, Stray Flux, Effect of Air Gaps, Electromagnets, Lifting Power of Magnets.....	273
XIX. INDUCED ELECTRIC CURRENTS.....	278
93. Description of Induced Currents, Lenz's Law, Direction of Induced E.M.F.....	278
94. Electromagnetic Induction as a Consequence of Electron Theory.....	279
95. Magnitude of Induced E.M.F.....	281
96. Definition of E.M.U. of Potential.....	282
97. E.M.F. Produced by Cutting Lines of Force.....	282
98. Principle of the Earth Inductor.....	283
99. Application of Electromagnetic Induction to a Secondary Coil about a Primary in Which the Current Changes....	285
100. Application of Electromagnetic Induction to Faraday's Disk	288
XX. SIMPLE DYNAMOS AND MOTORS.....	290
101. Derivation of Equation for Simple Dynamo.....	290
102. Dynamos, Alternating Current, and Direct Current.....	291
103. E.M.F. and P.D. from a Dynamo, Efficiency of a Dynamo..	293
104. Reciprocal Relations between Dynamos and Motors.....	295
105. Power Consumption, Efficiency, and Torque for Motors....	296
106. Starting Boxes.....	298
XXI. INDUCTION.....	299
107. Self-Induction.....	299
108. Units of Self-Induction.....	302
109. Nature of Self-Induction.....	303
110. Mutual Induction.....	304
111. Energy of the Electromagnetic Field of a Current.....	310
112. Self-induction and Energy Losses, Eddy Currents.....	314
113. The Induction Coil.....	316
114. The Transformer.....	318
XXII. ELECTRICAL AND MAGNETIC MEASUREMENTS.....	321
115. Introduction.....	321
116. Magnetic Measurements.....	322
117. Summary of Methods of Magnetic Measurement.....	325
118. Current Measurement.....	325
119. Summary of Current Measurement.....	328
120. Potential Measurement.....	328
121. Summary of Potential Measurements.....	332
122. Resistance Measurement.....	333
123. Summary of Resistance Measurements.....	335
124. Capacity Measurement.....	335
125. Summary of Capacity Measurement.....	337
126. Self-Induction Measurement.....	338
127. Summary of Measurements of Self-induction.....	339
128. Power Measurement.....	340

CHAPTER	PAGE
XXIII. THE BALLISTIC GALVANOMETER — TRANSIENT PHENOMENA	341
129. The Ballistic Galvanometer	341
130. Derivation of the Equation of the Ballistic Galvanometer	342
131. The Fluxmeter	343
132. Damping	346
133. Transient Phenomena	349
134. Measurement of High Resistances	362
XXIV. ALTERNATING CURRENTS I: DEFINITIONS, ROOT MEAN SQUARE AVERAGES, AND EFFECT OF SELF-INDUCTION	363
135. The Importance of Alternating Current	363
136. Definitions of Alternating-Current Terms	363
137. Values of Currents Given by Instruments, Root Mean Square Current	366
138. Effect of Self-Induction on an Alternating E.M.F.	370
XXV. ALTERNATING CURRENTS II: EFFECT OF CAPACITY ON ALTERNATING CURRENT AND THE EFFECT OF CAPACITY AND SELF-INDUC- TION COMBINED — ELECTRICAL OSCILLATIONS	375
139. Effect of Capacity on the Impedance of an Alternating Cur- rent	375
140. Where Self-Induction, Capacity, and Resistance Are Present	378
141. Power Factor	379
142. Electrical Oscillations	381
143. Derivation of the Equation for Oscillatory Discharge of a Condenser with Resistance Capacity and Self-Induction	382
XXVI. THE RATIO OF ELECTROMAGNETIC AND ELECTROSTATIC UNITS, MAX- WELL'S EQUATIONS AND ELECTROMAGNETIC WAVES, AND THE CHAR- ACTER OF ELECTRICAL UNITS AND THEIR DIMENSIONS	390
A. THE RATIO OF THE UNITS	390
144. Introduction	390
145. The Ratio of the Electrostatic and Electromagnetic Systems of Units	390
B. MAXWELL'S EQUATIONS AND ELECTROMAGNETIC WAVES	392
146. Introduction	392
147. The Equations of Continuity	393
148. The Generalization of Ampère's Law	395
149. Maxwell's Displacement Current	396
150. The Generalizations of Faraday's Laws of Electromagnetic Induction	398
151. Derivation of the Wave Equation	400
152. Solution of the Wave Equation	402
C. THE IMPORTANT ELECTRICAL CONCEPTS, THEIR UNITS AND INTER- RELATIONS, AND THE PROBLEM OF THEIR DIMENSIONS	410
153. Important Concepts	410
154. Table of Auxiliary Concepts	415
155. Systems of Electrical and Magnetic Units	417
XXVII. DISCHARGE THROUGH GASES AND ATOMIC STRUCTURE	420
156. Discovery of X-Rays	420
157. Cathode Rays	420

CHAPTER	PAGE
158. Electron Velocities — Electron Volts.....	424
159. Considerations of Relativity.....	425
160. The Charge of the Electron.....	427
161. Properties of Electrons.....	431
162. Positive Rays and Mass Spectrographs.....	434
163. The Thomson Atom and the Nuclear Atom.....	438
164. The Bohr Atom.....	442
165. Character of Excitation and Ionization in Atoms.....	446
166. X-Ray Levels and X-Rays.....	449
167. Approach to a Determination of Nuclear Constituents....	460
168. The Neutron.....	462
169. Nuclear Constitution.....	464
170. Artificial Radioactivity.....	466
171. Mass Energy Relations — Nuclear Stability.....	466
172. Nuclear Energy Sources and Fission.....	468
173. Character of Nuclear Reactions.....	470
174. Other Ultimate Particles.....	471
175. The Achievement of High-Energy Particles.....	472
176. The Linear Accelerator.....	473
177. The Cyclotron.....	475
178. The Synchrotron and the Frequency-Modulated Cyclotron.	479
179. The Betatron.....	482
180. Conduction in Gases.....	484
Gases as Insulators.....	484
Rendering Gases Conducting.....	484
Gaseous Ions.....	485
Ion Recombination.....	487
The Formation of Negative Ions.....	487
Electron Velocities in Fields in Gases.....	487
Ionization Currents in Gases.....	488
Ionization by Collision.....	489
Spark Breakdown.....	492
Forms of Gaseous Discharge.....	495
The Glow Discharge.....	495
The Arc Discharge.....	497
Corona Discharges.....	499
Vacuum Sparks.....	501
XXVIII. PHOTOELECTRIC AND THERMIONIC EFFECTS.....	503
A. THE PHOTOELECTRIC EFFECT.....	503
181. Early Discoveries.....	503
182. Liberation of Electrons by Light.....	505
183. The Einstein Photoelectric Law.....	507
184. The Intensity of Photoelectric Emission.....	509
185. The Sommerfeld-Pauli Concept of the Free Electrons in Metals.....	511
186. The Influence of the Sommerfeld-Pauli Theory on Photo- electric Emission.....	516
187. Further Facts Concerning Photoelectric Emission.....	518

CONTENTS

xix

CHAPTER	PAGE
B. THE THERMIONIC EFFECT	523
188. The Discovery of the Thermionic Effect	523
189. The Laws of Thermionic Emission	525
190. The Current-Potential Characteristic of Thermionic Emission	531
191. The Thermionic Rectifier	534
192. The Three-Electrode Tube	537
193. The Thermionic Oscillator	544
194. Piezoelectric and Magnetostrictive Oscillators	548
195. Tube Counters	550
196. Radar and Ultra-High-Frequency Electric-Wave Generation	553
PROBLEMS	561
NAME INDEX	637
SUBJECT INDEX	641

CHAPTER I

HISTORICAL

1. GENERAL DEVELOPMENT OF PHYSICAL SCIENCE

Physical science, including electricity and magnetism, started around the beginning of the seventeenth century, i.e., 1600, with the introduction of the method of *controlled quantitative investigation*. Preceding the introduction of this method, man's inquiry into Nature's ways had been either through the speculative Greek philosophical approach, or through the disputation based on authority which characterized mediaeval scholasticism. Thus, the seventeenth century ushered in a new practice of referring decision on natural phenomena to controlled quantitative experimental test, rather than to base it on authority. The method was initiated, primarily, by Galileo Galilei. Other scientists of the period, however, including William Gilbert, one of the pioneers in electricity and magnetism, contributed their share.

It was, and is even more today, the method of controlled quantitative investigation which has led mankind to the state of complex technical development exemplified by radar, television, aviation, submarines, jet propulsion, and nuclear energy.

This basic procedure in science is best illustrated by an example. Let us assume that a phenomenon such as the inclined plane has become of interest. The earliest practical experience with the plane had shown that the steeper the incline the more force was required to raise a body up it. It then became necessary to know how the force increases with steepness, and how much force will be required by a given load. In attacking this problem one proceeds as follows: First, to avoid argument, we shall agree that quantitative data must be had, for such data give irrefutable information as to magnitudes. As a measure of force one could choose the pull of gravity on known masses, i.e., the weights of bodies. The steepness of the inclined plane could be expressed by the ratio of the height of the plane relative to the length of the incline, both of which can be measured. The weights, heights, and lengths then give the quantitative data needed.

It is also common experience that the force required to move any object along a plane — inclined or not — depends on friction. Thus, since we want to study the plane effect alone, friction must be eliminated so far as possible. That is, we plan to *control* the measurement by eliminating the other variables which can complicate the result — in this case, friction.

Thus, a smooth, inclined plane is chosen. The weight to be moved along it is placed in a cart of known weight, provided with wheels.

A calibrated spring balance along the plane to determine the force on the car could be used. It is, however, more direct to balance the cart and its weights by the vertical force of gravity on a scalepan with weights. This is accomplished by suspending the pan by a string that passes over a frictionless pulley at the upper corner of the inclined plane. The other end of the string is then fastened to the car. The plane must be so arranged that the ratio of height h to length l , $\frac{h}{l}$, can be changed at will and can be measured.

The experimental procedure is, then, to choose a given ratio $\frac{h_1}{l}$ to begin with. Next weights are added to the car to a given value w_{c1} and the weights on the scalepan w_{s1} just needed to balance the car in place on the incline are determined. These are recorded. The weights w_{c1} are next changed to a value w_{c2} and the new value of w_{s2} required for balance is determined, and so on for a number of values. The results are then arranged in tabular form. Inspection of the tabulated values of w_s and w_c obtained for systematic regularities leads one to try taking the ratios $\frac{w_{s1}}{w_{c1}}, \frac{w_{s2}}{w_{c2}}, \frac{w_{s3}}{w_{c3}}$, etc. In doing so, it would be seen at once that for a given ratio, $\frac{h_1}{l}$, used, the ratios $\frac{w_{s1}}{w_{c1}} = \frac{w_{s2}}{w_{c2}} = \frac{w_{s3}}{w_{c3}} = A_1 = \text{constant}$. The experiment is then repeated with a new value of $\frac{h}{l}$, that is, for $\frac{h_2}{l}$, and a new value of A , A_2 is obtained for the ratio of $\frac{w_{s1}}{w_{c1}}$, etc. After several values of $\frac{h}{l}$, covering the range of values of which the apparatus is capable, are obtained, it can be observed that the values of A_1, A_2, A_3 are equal to the ratios $\frac{h_1}{l}, \frac{h_2}{l}, \frac{h_3}{l}$ used.

Thus, one can write quite generally that for the frictionless inclined plane as set up, $\frac{w_s}{w_c} = \frac{h}{l}$, or that $w_s = w_c \frac{h}{l}$. Since it is recognized that $\frac{h}{l} = \sin \theta$, where θ is the angle of the inclined plane with the horizontal, one may as well write $w_s = w_c \sin \theta$. The law obtained for the plane is now quite general. It replaces $\frac{h}{l}$ by a single measurable quantity, the angle of the inclined plane. It makes it possible, in the absence of friction, to evaluate the force w_s to move a given weight w_c up an incline of angle θ . θ could also be evaluated if w_s and w_c were measured.

Needless to say, such a law has countless practical applications in daily life.

The procedure outlined typifies a controlled quantitative investigation. It has led us to a single law describing the behavior of the inclined plane *within the limitations of the controlled experiment*. The law is most effectively written in symbolical form, $w_s = w_c \sin \theta$, where w_s is the force to balance the car on the plane, w_c is the weight of the car, and θ is the angle of inclination of the plane. The law could also be stated in words as follows: The force just necessary to hold a given weight on a frictionless plane, of inclination θ with the horizontal, is the weight multiplied by the sine of the angle θ . The economy of words in the use of a symbolical defining equation is quite obvious. The economy of space, the convenience, and the greater ease in memorizing such relations definitely prescribe the general use of symbolical relations or equations in the study of electricity, where very many such relations derived from controlled experiment will be encountered.

The development of this method of controlled quantitative investigation led, from 1600 until about 1700, to a rapid accumulation of knowledge of many isolated phenomena. It primarily established what might be called the period, or era, of measurement and instrumental development in the history of science. This was the period when thermometers for measuring heat were invented, when devices for the measurement of force were perfected, and when electrosopes for the measurement of electrification came into use. It was also the era in which scientists began to get together in groups to discuss their work and to facilitate publication and dissemination of their results. Probably one of the earliest organizations of this sort, still surviving, was the Royal Society of London, chartered by Charles the Second in 1662. These organizations played an important role in spreading knowledge of the discoveries throughout the scientific world, thus stimulating and accelerating further scientific advance.

In this era, however, any basis for unification and correlation of the isolated studies made was lacking. Thus, gravitational, electrical, and magnetic phenomena could not be compared because there were no unifying relationships among them. How heat and mechanical work were related could not have been determined, even had the relation between heat and work been observed. This deficiency was overcome by the publication of the *Principia* by Sir Isaac Newton in 1687. By means of his famous three laws of motion, which defined force and gave it a quantitative expression, the foundation was laid for the study of physics as a unified whole. With the guiding principles thus available, the next one hundred and fifty years, slowly at first but later more rapidly, saw the description of physical phenomena in terms of force, energy, and work accomplished. Thus, by 1850 most

of the basic quantitative data in the fields of mechanics, heat, light, and electricity had been accumulated in such a form that a number of important generalizations could be made.

These generalizations characterized what is termed the great systematic period of scientific development. They began with the studies of the mathematical physicists Fourier, Gauss, Laplace, Kirchhoff, Green, and Poisson in the earlier eighteen hundreds, leading to concepts of force and potential fields. The systematic period properly began with the enunciation of the systematic law of conservation of energy by Mayer and Helmholtz in 1847. This was followed by the discovery of the second law of thermodynamics by Clausius and Kelvin in 1850-1851 and by the development of the kinetic theory of gases by Clausius in 1857. These two lines of reasoning ultimately evolved into the beginnings of statistical mechanics with Boltzmann in 1880-1890, and were added to by Willard Gibbs with his phase rule. The generalized mechanics of Lagrange and Hamilton also developed in this period. Probably one of the most significant and important contributions to this field came with the electromagnetic theory of J. C. Maxwell. Maxwell took the experimental electrical and magnetic results and qualitative concepts of Faraday and others and formulated them into a system of mathematical equations. These relations remain the basic background of all electricity and magnetism today as much as they were in 1865, when Maxwell published his work.

The striking success of the applications of these laws and the apparent consistency of all natural phenomena within this framework by 1890 gave enormous confidence in the basic correctness of the underlying Newtonian mechanics. In other words, at the turn of the nineteenth century it was believed that within the framework set by Newtonian mechanics and the resulting generalizations all nature could be described and future events predicted from the past, given sufficient mathematical skill and patience. This confidence was due to be severely shaken in the subsequent quarter century of research.

It must, however, be noted that while some of the Newtonian concepts have been shown to be only first-order approximations to much more general laws, most of the great generalizations of physical science, such as the conservation laws, the second law of thermodynamics, and Maxwell's electrodynamics, still hold, though some of them in slightly modified forms. Thus it is not surprising that even today the classic aspects of physics developed prior to 1900 should furnish the sound basis of any fundamental course in physics, old or new. It will therefore not surprise the student to note that most of the content of this book will comprise the classic studies of electricity and magnetism antedating 1900. Wherever possible, however, the newer interpretations of classic physics, will be introduced if they do not tend to create confusion.

With the turn of the century, very close to 1900, the advances in

the field of atomic structure led to modifications of the rather simplified classic concepts and their underlying philosophy. The Michelson-Morley experiment in 1886, indicating the constancy of the velocity of light, led Einstein in 1905 to the special theory of relativity. This theory indicated that the Newtonian principles held within the realm of common observation, but that for higher speeds the principles were only first approximations to more general laws. With relativity the crudely conceived mechanical ether based on Newtonian explanations and Maxwell's theories vanished completely. What remained were the precise mathematical formulations of a general nature which devoid of any mechanical significance, more accurately represent nature than had Newton's concepts. Again, in 1900, Max Planck was forced to conclude that, contrary to the dictates of the classic mechanics of Newton and of Maxwell's laws, radiant energy was emitted and absorbed in chunks or quanta. The atomic investigations initiated by J. J. Thomson's discovery of the electron in 1896 led to further troubles. The photoelectric studies of Millikan in 1915, the inelastic electron impact experiments of Franck and Hertz in 1913, and the proof of the existence of the nuclear atom by Rutherford in 1913, together with Bohr's interpretation of the behavior of the nuclear atom, still further established the validity of quantum theory restrictions on classic mechanical and electrical behavior, and even threatened the validity of Maxwell's equations. By 1921 L. V. de Broglie had indicated that Maxwell's laws could still be retained if the electron in the atom ceased to be considered a charged mechanical particle and was represented by some sort of a complex wave motion. The actual wave character of the electron was experimentally established by Davisson and Germer and by G. P. Thomson in 1928. Still more confusion occurred when A. H. Compton in 1923 showed that electromagnetic waves such as x-rays were scattered by electrons as if these rays were corpuscles or material particles.

The confusion produced by these indications of the dual character of both light waves and electrons, inconceivable on Newtonian mechanics, was mathematically clarified and the phenomena unified by the development of the wave or, better, quantum mechanics. In the hands of Werner Heisenberg, Erwin Schroedinger, Wolfgang Pauli, and P. A. M. Dirac, the previous attempts at the analysis of electronic or particulate behavior, on the basis of forces and particles mechanically considered, were replaced by the description of the ultimate particles in terms of complex sets of mathematical equations, in the energy form, which varied with the state of the particle. Such equations at once give a complete description of the particulate behavior and are amenable to application either in the corpuscular or the wave-form aspect of behavior. If properly manipulated they lead to accurate numerical results. While such a description of physical behavior is not aesthetically satisfying, it is a useful and unifying expedient for

the present. So far the basic general laws of nature, such as the second law of thermodynamics, the conservation of momentum, and, as required by relativity, the conservation of mass and energy, have not been changed. Maxwell's *equations* as formulated still stand. Two new principles have been added, namely, the exclusion principle of Pauli and the uncertainty principle of Heisenberg, both of fundamental importance.

With the daily confirmation of the correctness of the principles of quantum mechanics and relativity, the physicist has been cured of his naive error in accepting a mechanical universe based on Newton's laws. Exactly what form the physics of the future will actually take no one can predict. All that can be said is that "explanation" in physics will no longer be limited by the Newtonian law, and that in its place the phenomena will be described as closely as possible by the accurate and involved expressions of wave mechanics, relativity, or similar formulations, with the probable added restriction that even these may prove to be only approximations to still more complete and accurate expressions yet to be discovered. What we can then hope to do is to measure and describe as best we can by means of the method of controlled quantitative investigation the phenomena of Nature and to formulate these in the most convenient general systems of representation which we have at hand. These will enable us to predict with some probability of success the behavior of our system under various conditions, and will enable us further to investigate and control it. They will also enable us to utilize the phenomena in one way or another to improve the condition of man on earth by furnishing new comforts, luxuries, and conveniences to material civilization.

Before turning to the detailed development of electricity and magnetism which can be outlined against this general background a few general comments might be made. It must be seen from what has gone before and what will follow that in each period the possible discoveries much depended on the general state of knowledge at that period. Thus it was to be expected that the discoveries of Coulomb and of Cavendish should have come within the period say 1730-1800, and that Maxwell's brilliant generalizations must have been due to arrive after the careful quantitative discoveries of Faraday and at about the same period as the generalizations of Helmholtz, Clausius, and Kelvin about energy. It is thus seen that a discovery is made at a time when conditions are ripe for the discovery, or, in other words, that a given discovery is "in the air" at the time when it is made. It is thus merely a chance that the discovery is made by a given man, and many a discovery is made independently and nearly simultaneously by several men. Cases are on record in which a discovery has been made by chance before the time is ripe: the discovery of absolute 0 of temperature by Amontons in 1700, for instance, or of the mechanical nature of heat by Davy, Rumford, and others in 1800, perhaps a

hundred or in the latter example thirty years before the accepted date of discovery. In general, however, the man making an important advance in a given field is the man who is most actively engaged in an intelligent research in the field at the time when the stage is set for the discovery. Excellent examples are the discoveries of x-rays by Roentgen, of electromagnetic induction by Faraday, and of the electromagnetic waves by Hertz. Sometimes the great discoveries are made by accident by men of relatively less genius. Thus the importance of a scientific discovery does not necessarily indicate genius in the discoverer. The test of greatness lies rather in a man's seeing a relation clearly long before it is seen by others, and working it out accurately and completely before it is handled by others. Perhaps the truest test of ability is the *successive* discovery of several important and far-reaching facts by a single man. On this criterion men like Galileo, Huygens, Newton, Ampère, Thomas Young, Fresnel, Faraday, Clausius, Maxwell, Millikan, Rutherford, Bohr, and Einstein could most easily be classed as the greatest leaders in physical thought.

It might be added that today with the remarkable interest, facilities, and number of workers in scientific fields, together with the rapid publication and dissemination of knowledge, the competition is so keen and the rate of advance is so rapid that it is almost impossible to credit great achievement to any one man. There is hardly an important problem attacked in which at least two men are not working along closely parallel lines and in which the results are not published nearly simultaneously. The recent theoretical and experimental proofs of wave mechanics furnish an excellent example of this, for it is almost impossible to find any unanimity of opinion among scientists as to who deserves the chief credit for these spectacular advances. In fact, the Nobel prize for this achievement was made in three awards to de Broglie, to Dirac and Schroedinger together, and to Heisenberg.

The appalling rate of increase of physical knowledge might lead one to ask whether this accelerative increase of scientific production will go on indefinitely. Because the more facts that are known, the more analogies there are to build on, and the more facts to correlate, the more rapid will be the acceleration of physical knowledge. It appears, however, that the time is approaching when the new facts found will accumulate so rapidly that the single human mind will not be able to hold enough facts to make use of the new correlations. The result will be that the rate of gain of knowledge will reach a steady state, the limitation being that produced by the incapability of the human mind of grasping more than a certain number of facts at once.

A study of the publication of scientific papers in chemistry and physics in the last hundred and fifty years indicates a nearly exponential rise for both sciences at the start. This, except for disturbances by war conditions, is now slightly slower than exponential in chemistry where many papers deal with new syntheses. In physics, where more

correlation between branches is needed, the advance has now reached the nearly linear stage characteristic of constant production. Fields are rapidly becoming more specialized and rapid advance is requiring larger cooperative groups of experts.

2. THE DEVELOPMENT OF ELECTRICITY AND MAGNETISM

With these remarks we may proceed to a more detailed study of the development of electricity and magnetism. To facilitate the study the development can be considered as being divided into eight general periods. These begin late in the great period of development of the scientific method starting with Galileo. Inasmuch as electrical and magnetic phenomena are more complicated to study and to observe than the simpler mechanical problems attacked by Galileo it is not strange that these fields should have developed somewhat more slowly. In fact, it is quite easy, neglecting the trivial development of electricity and magnetism in the Greek period, to start this history with the year 1600, when the first extensive qualitative investigation of magnetism took place. The eight periods into which electrical and magnetic advance may be divided are summarized as follows:

- a. Static Period, beginning in 1600 with William Gilbert's book on magnetism, *De Magnete*, and ending in 1799 with the discovery of the galvanic cell. It is divided into:
 1. A qualitative period containing the magnetic discoveries of Gilbert, the discoveries of two types of electrification by von Guericke and Du Fay, of conduction by Gray 1729, of the Leyden jar about 1746, and of Franklin's proof of the nature of lightning 1750.
 2. The quantitative period beginning with the measurement of magnetic poles and electrical quantity initiated by Charles A. de Coulomb and Henry Cavendish about 1785-1798.
- b. The Current Period, beginning in 1799 with the discovery of the voltaic pile and extending to Faraday's discovery of electromagnetic induction in 1831. It covers the discovery of the magnetic effect of a current by Oersted in 1819, the development of the laws of the magnetic fields produced by currents by Ampère in 1820 and later; the developments of the first motors by Faraday and Ampère as well as the carbon arc by Davy in 1822; and the discovery of the laws of resistance and Ohm's law by Ohm in 1827.
- c. The Electrotechnical or Instrumental Period, beginning in 1831 with Faraday's discovery of electromagnetic induction and ending with the development of the electromagnetic theory by Maxwell in 1865. It covers the great period of development of methods of quantitative measurement and the applications of electricity to practical problems. In this period lie the discovery of self-induction by Joseph Henry in 1832; Faraday's laws of electrolysis, 1834; Lenz's laws of induction, 1835-1839; the concept of specific inductive capacity and capacity by Faraday, 1838; Wheatstone's bridge, 1843; Kirchhoff's laws, 1846; the beginning of the unification of electrical phenomena through generalization of the concepts of capacity and potential by Gauss, Green, and Lord Kelvin, 1832-1850; and the beginning of the absolute measurements of the fundamental electrical units and the determination of the ratio between the electromagnetic and electrostatic systems by Wilhelm Weber, 1840-1852. In this period also began the evolution of the dynamo, from

the Faraday disk in 1831 and the earth inductor of Faraday in 1832, to the development of a practical dynamo by Werner von Siemens in 1866.

- d. The Systematic Period, beginning with the important mathematical generalizations of Maxwell's electromagnetic theory in 1865-1873 and terminating with the discovery of the electron by J. J. Thomson in 1896. It is named the "systematic period" because the notable advances lay in formulating the earlier experimental findings in terms of the new generalized laws of conservation of matter and of energy, the second law of thermodynamics, and Maxwell's electrodynamics. It covers the proofs of Maxwell's theory in relating light to electromagnetic waves, Heinrich Hertz's discovery in 1887 of the electromagnetic radio waves predicted by Maxwell, the establishment by the International Congress in 1881 of the international system of electrical units, and the Michelson-Morley experiment of 1886, which ultimately altered the Maxwellian concept of the ether and led Einstein to the theory of relativity.
- e. The Atomic Period, beginning in 1896 with J. J. Thomson's discovery of the electron and ending somewhat indefinitely by fusing into the beginning of the Quantum Period between 1913 and 1915. This period is characterized by the proofs of the granular structure of Nature, atoms of matter, atoms of electricity, atoms or quanta of radiant energy. In 1908 Perrin proved the reality of atoms postulated by the kinetic theory enunciated in 1857. Millikan in 1906-1911 evaluated the electron and showed its unitary character. The discovery of radioactivity by Becquerel in 1896 and its development, the mass spectrograph of Thomson in 1912, and the nuclear atom established by Rutherford in 1911-1913 added certainty to the previous discoveries. The proofs of the quantum character of radiation from radiation laws postulated by Planck in 1900 came from Einstein's work on specific heats in 1907, Millikan's proof of the Einstein photoelectric equation in 1915, the establishment of the Bohr atom in 1913, and demonstration of the elastic scattering of electrons by helium atoms by Franck and Hertz in 1913. In the proof by Laue of the wave character of x-rays and measurement of their wave lengths in 1913, the foundation was laid for a further proof of the quantum character of radiation. The development of the special and general theories of relativity by Einstein also occurred between 1905 and 1913.
- f. The Quantum Period, beginning between 1913 and 1915 with the establishment of the quantum *theory* and terminating with the development of wave mechanics starting from Schroedinger's wave equation in 1926. It covers essentially the accumulation of evidence proving the inadequacy of the classic wave theory of light and the particulate concept of the electron and atoms when considering the interaction between radiation and matter. The additional discoveries came from the more detailed application of the Bohr theory to studies of atomic magnetism, Zeeman effect and Stark effect studies, etc., culminating in the discovery of the Compton shift in wave length for x-rays scattered by electrons in 1923. In addition, Rutherford in 1919 achieved the first artificial transmutation of an atomic nucleus and proved that hydrogen nuclei or protons as well as helium nuclei and electrons were building stones of atomic nuclei.
- g. The Wave Mechanics Period. Beginning in 1926 with the Schroedinger wave equation, it carries on through the present day to what might possibly be termed a new period, the Nuclear Period. This latter period could begin in 1931 with the development of manmade sources of high-energy atomic projectiles capable of disrupting atomic nuclei. The wave-mechanical period covers the proposal of the wave electron by de Broglie in 1921, the development of the more universal Schroedinger wave equation in 1926, the later quantum mechanical developments by Heisenberg and Dirac in 1927 to 1928, the Sommerfeld theory of the metallic state in 1928, the experimental proof of the wave character of the electron by Davisson and Germer in 1927-1928 and by G. P. Thomson in 1928, and the

analogous proof for the atoms of helium and molecules of hydrogen by Stern in 1928-1929.

- h.* **The Nuclear Period.** This begins with the successful achievement of accelerators for nuclear projectiles of high energy in the Cockcroft and Walton accelerator, the Van de Graaff static machine, and the cyclotron of Lawrence in 1931, and extends into the future. Into this period came the discoveries of the positive electron, or positron by Carl D. Anderson in 1932; the neutron by James Chadwick in 1932; and artificial radioactivity by Irène and Frédéric Curie-Joliot in 1934. It covers also the development of the Condon and Guerney-Gamow theory of the nucleus, the study of nuclear and neutron reactions, the discovery of nuclear fission by Hahn and Strassmann, 1939; the creation of the transuranic elements, neptunium (93), plutonium (94), americium (95), and curium (96), by McMillan, Seaborg and Kennedy, and Seaborg, 1940 to 1945. In 1938 to 1940 the mesotron was discovered in cosmic rays. In 1938 the betatron was devised by Donald W. Kerst and by 1945 it had achieved electron energies on the order of 10^8 volts. The discovery of the principles of the synchrotron and synchrocyclotron by Veksler and McMillan in 1944-1945 promise to extend the high-energy range of nuclear particles into the cosmic-energy range. The period also witnessed the development of the high-energy ultra-high-frequency electrical oscillators the magnetron and the Klystron with the consequent high technical perfection of microwave techniques and radar. Its most spectacular engineering feat was the perfection of the nuclear chain reaction culminating in the atomic bomb in 1945.

CHAPTER II

THE PHENOMENA OF MAGNETISM

3. THE DISCOVERY OF MAGNETISM

Magnetism became known to the world through discovery of natural magnets or lodestones consisting of pieces of the magnetic oxide of iron, Fe_3O_4 , which occur in nature in the magnetized state. Whether they become magnetized through the action of the earth's field or possibly through the currents due to lightning discharges is not known, though the former origin is most likely. They are, however, frequently found in deposits of this ore. The property of lodestones of attracting pieces of iron was known to man in antiquity. It is likely that this attractive property was independently discovered wherever primitive man smelted iron in the neighborhood of deposits of Fe_3O_4 .

The date of the first production of iron by man is unknown. Finds of iron in early predynastic Egyptian tombs have been identified as meteoritic. As there is no iron in Egypt, Asia Minor, where deposits do exist, must have been the origin of the art. There can be no doubt but that the art of smelting ores reached Greece from Asia Minor and Phrygia. In Greece its advent is first noted in legends associated with the name of a cult of roving iron miners and smelters, the Dactyls, who came from Phrygia inland of Troy on the northeast border of the Aegean Sea. They migrated to Crete through Asia Minor and north and east to the islands of Lemnos, Imbros, and Samothrace into the Thracian Sea. Next came the Cabiri, a second and more skilled group of workers who also made their trade a cult. They spread north and east to Macedonia and south and west to Syria.

The dates for the movement of iron from Asia Minor and Phrygia to Greece are roughly as follows. In 1500–1400 B.C. iron was definitely in use in the regions mentioned above and perhaps earlier in Babylonia. By 1300 B.C. it was known in Crete, and it appears to have reached Greece in quantity between 1200 and 1100 B.C. coincident with the Dorian invasion from the Danube countries which terminated in forming the Spartan state. That a knowledge of magnetism reached Greece at this time if not before is certain, for in the Phrygian iron mines about Mount Ida, on the isles of Elba, Crete, and Samothrace, there occurred, with the regular iron ores (Fe_2O_3) that were smelted, deposits of what the Greeks called siderites or ironstone, Fe_3O_4 , or magnetite, the natural magnetic oxide of iron. Hence early in Greek legend and in religious cults there appeared the Samothracian rings, rings of iron magnetized by contact with the magnetite. These attracted each other and occupied an important place in the early religious cults, such as those of the Dactyls and the Cabiri. These are

clearly discussed by Plato (429–348 B.C.), who speculated on them on philosophical grounds. Earlier mention is made of magnetism in quotations from the works of Thales of Miletus in Asia Minor (585 B.C.). Thales also mentions electrification of amber. The name *magnet* for these magnetized objects later came into use, and the name is attributed by some to a legend quoted by Pliny in which a Greek shepherd Magnes found a stone that attracted his iron-tipped staff. Lucretius, 95 B.C., refers to the name *magnet* as derived from the origin of magnets in the province of Magnesia lying in Thessaly on the seacoast between Mounts Ossa and Pelion. However, there is no iron in that region. It is probable that the Magnetes of Thessaly, who because of overpopulation migrated east across the Aegean from Magnesia, founded in Ionia a city that is now lost, called Magnesia. Later the Magnetes were driven north and founded another city of Magnesia near Mount Sipylus in Lydia. It is assumed that it was this region's large deposits of magnetic oxides of iron, which must have been exported to Greece, which is responsible for the origin of the name. The city was destroyed by an earthquake in the time of Tiberius. The date of the supposed Magnesian migrations to Asia Minor lies between 700 and 1000 B.C. However, it is not certain whether the migrations mentioned here, which were described by Pliny, really took place. But the probability is great that the first commonly known magnets in Greece came from a town called Magnesia, which was probably in Asia Minor close to a large bed of magnetite where iron was also smelted.

4. THE DISCOVERY OF THE COMPASS

The discovery and use of the directive action of the earth's magnetic field on a floating or suspended magnetic needle, i.e., the discovery and use of the magnetic compass for direction-finding at sea or on land, is veiled in complete obscurity. The action being weak and subtle made its detection and use an event coming much later in the development of civilization than the discovery of magnetism itself. The uncertainty in the history of its discovery lies in the imperfect records coming down from antiquity. Large sections of naturally meager records were destroyed in the many political upheavals of the remote past. A further uncertainty lies in interpreting these ancient records that are couched in flowery and unfamiliar terms the present connotations of which must often differ from the original meaning. Park Benjamin makes out a case for the discovery of the compass in China around A.D. 100 to 400 and its subsequent use in travel by land. Later, he says, the compass was introduced into western Europe through Finland by the migration of the Finnish Mongols before A.D. 1100. Schück considers the possibility of its introduction to Europe from China by the passage of Hindus and Arabs into the Mediterranean. He inclines, however, to the belief that the compass was dis-

covered and used simultaneously and independently by the Chinese in the East and the Normans in Europe after A.D. 900 and in China perhaps later. A very careful recent survey by A. Crichton Mitchell published in *Terrestrial Magnetism*, June, 1932, gives the complete literature to that date and, together with other data, leads to the following conclusions. Iron was known in the Shensi province in China as early as 220 B.C. and probably much earlier. Legends concerning a "South-Pointing Chariot" used in land navigation date from at least 1000 B.C. These cannot be considered as more than legends since the burning of the books in A.D. 213 destroyed any definite early records, leaving only legend and hearsay. There is further no evidence that the "South-Seeking Chariot" was magnetically motivated, and later disclosures of attempts to reconstruct these at various times indicate a mechanical rather than magnetic contrivance. These statements do not preclude an early discovery of a magnetic compass and its use in land travel. If it was discovered and used, it was definitely lost as an art at least once and perhaps several times in the terrific political upheavals in early Chinese history. It did not carry through as a development from early to more recent times. In a dictionary the Chow-wen compiled in A.D. 121 the word *magnet* or *lodestone* is defined as "the name of a stone with which we give direction to the needle." While this passage might seem quite obviously to allude to a compass needle, it could as well merely indicate the obvious fact of induction and attraction, as no specific mention is made of geographical directions. Subsequent to this there is a gap in definite knowledge until nearly A.D. 1000. Alexander Wylie has stated that in a text on the life of a Buddhist astronomer, Yih-Hing, of A.D. 700, it is asserted that this astronomer had observed the *deviation of the compass from true north*. No one has been able to rediscover the text or the passage from which this statement came, so that the chance of its having been interpolated into the manuscript at a later date cannot be investigated and ruled out. If correct, it would definitely indicate a knowledge of the compass in one part of China by A.D. 700. Between 1086 and 1099 there is diverse, definite, and indisputable evidence of the use of the compass in Chinese sea navigation.

In Europe the compass was first clearly and definitely described by Alexander Neckham (1157-1217), and the date ascribed is A.D. 1187. A very complete description of a more sophisticated type of compass is given and its features are discussed by Petrus Peregrinus in a letter of 1209. Claims for earlier discoveries from other sources in western Europe have been definitely discredited. There is at present no evidence of its use much earlier than 1100 in either China or western Europe, although negative evidence can never be conclusive. That the compass came to western Europe from China via the Arabs and the Mediterranean is highly doubtful, as records of Arabian compass navigation *postdated* the other records. This leads to the conclusion

that although a discovery of the directive action of the earth's field on a suspended magnetized needle up to A.D. 1000 or 1100 may have come independently perhaps at different times in different countries, it was neither generally known nor widely used for purposes other than sorcery or amusement. It even may have been lost to posterity and rediscovered more than once. By A.D. 1000 to 1100 the demands of developing sea-borne commerce and voyaging for geographic discovery made its use imperative, and it appeared simultaneously as an instrument of navigation in diverse places, probably as an independent invention. Its influence on discovery, travel, and commerce, and hence on civilization, is obvious, and it remains today the most reliable course indicator at sea despite later more complicated devices, which are constantly checked against the standard magnetic compass.

5. THE QUALITATIVE PHENOMENA OF MAGNETISM

As stated in the historical introduction, the scientific discoveries in magnetism really date from the first investigations of Petrus Peregrinus in A.D. 1256 and more properly from William Gilbert, physician to Queen Elizabeth, as set forth in his book *De Magnete* in 1600. The facts concerning magnetism given in this section, except for those dealing with electrical currents and with magnetostriction are essentially the qualitative facts established by these early investigators. It must be noted at the start that magnetism evidenced itself only by its manifestation of forces (i.e., attractions, repulsions, or orientations). It is clear, therefore, that until a quantitative analysis of forces was at hand (i.e., an analysis possible only after the time of Newton, 1700) little progress in magnetism beyond that detailed here was to be expected.

It is observed that, if a magnetized piece of steel (a magnet) be pivoted at its center about a vertical axis, the magnet if left to itself always comes to rest with the same end pointing northward. This end of the magnet is thus endowed with a property which always makes it seek the approximate geographical north. It will also be noted that the opposite end of the magnet is endowed with a property which always makes it seek the approximate geographical south. Thus a magnetized piece of steel appears endowed at its ends by some power which makes the ends orient themselves in opposite directions which are related in a rough manner to the geographical poles of the earth. Any body which exhibits a tendency to orient itself in this fashion over the greater portions of the earth's surface, in the absence of other magnets or of iron, may be considered as exhibiting *magnetic* properties and will be called a *magnet*.* It is thus not unnatural to

* Exception might be made to a helical coil of wire carrying a current which also exhibits this property. However, the exception need not be made, for such a coil acts in other respects as a magnet and though strictly not a magnet certainly behaves like one and possesses a magnetic moment.

call the north-seeking end a *north magnetic pole* and the south-seeking end a *south magnetic pole*. Such a magnetic pole shows the power of attracting iron and unmagnetized steel in its immediate neighborhood, and this property, perhaps the first magnetic property noted, may also be used as a further criterion of magnetism.

Now if two magnets are tested out so that their north-seeking or north magnetic poles can be marked, it will be found if one magnet is placed on a pivot so that it is free to turn, that when the north poles of the magnets are brought into proximity of each other they repel one another with considerable force. Likewise a south magnetic pole will be repelled by a south magnetic pole, while a north magnetic pole will strongly attract a south magnetic pole, and vice versa. Interchange of the two magnets will show that the actions of attraction and repulsion are mutual and reciprocal, each magnet pole repelling and being repelled, or attracting and being attracted, as the case may be. These observations can be summed up in the terse statement that like magnetic poles repel and unlike magnetic poles attract, the first experiment having showed that there are two kinds or types of poles, or magnetism, termed north and south, a fact confirmed by the behavior exhibited in the second experiment.

Both Peregrinus and Gilbert had spheres turned out of magnetite which was magnetized. A study of the magnetism of these spheres by means of small magnets mounted on pivots showed that these spheres acted as if they had a bar magnet running through them in one direction. At one point on the sphere a north pole was repelled; at the other end of a diameter of the sphere, starting at the first pole, the south pole was repelled. At various points over the surface of the spheres the small magnets on pivots oriented themselves in definite positions so that their axes appeared to lie on surface lines over the sphere running from one of the ends of the diameter to the other. The analogy of this behavior to the behavior of magnets pivoted on the earth's surface was so striking that both observers decided that the earth's magnetic action on the pivoted magnets was the result of north and south magnetic poles in the earth. They then considered that the earth itself must be a large magnet with one pole in the north geographical regions and another pole in the south geographical regions. This action is illustrated in Fig. 1. Since a north-seeking magnet pole points northward, *the earth's magnetic pole in the north must have south magnetic polarity, and the earth's magnetic pole in the south must have north magnetic polarity*. Hence it can be said that the earth's north geographical pole appears to have south magnetic polarity, and vice versa. Actually, as will be seen, the earth's magnetic poles are oriented in this general fashion, except that the earth's south magnetic pole in the northern hemisphere is not exactly at the earth's north geographical pole, and the earth's north magnetic pole does not correspond in position exactly with the earth's south geographical pole

(in other words, that the poles about which the earth rotates do not coincide in position with the earth's magnetic poles).

The use of the pivoted horizontal magnet for finding directions on the surface of the earth, through the use of the magnetic compass in navigation, has given the name of *magnetic compass* to the device

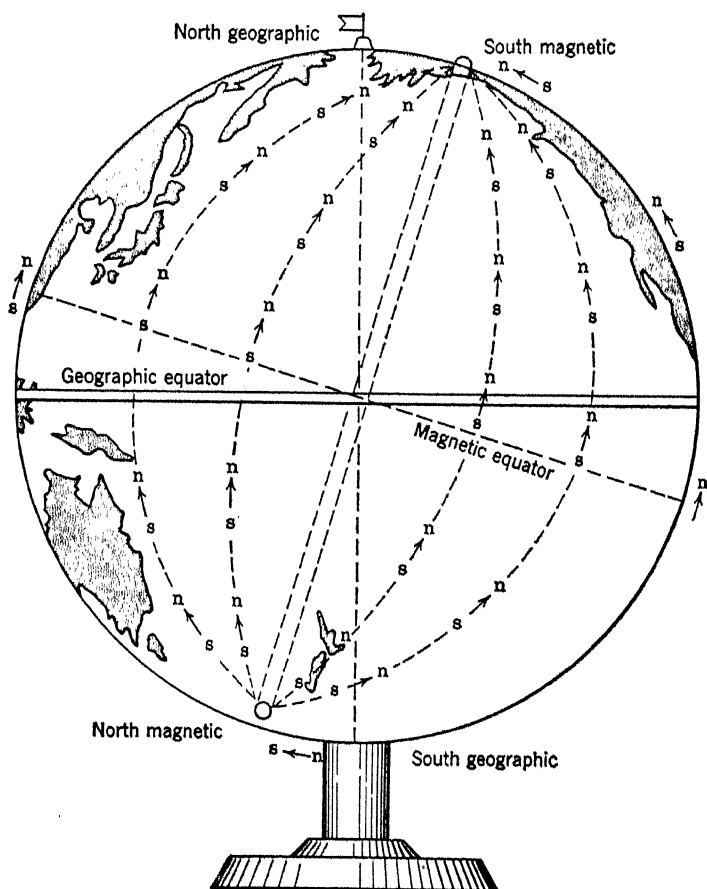


FIG. 1. Magnetic field of the earth as indicated by small horizontal compass needles ($s \rightarrow n$) placed on the surface of the globe. Note that dip needles with horizontal pivots placed along the circumference of the globe would be horizontal at the magnetic equator and vertical at the poles.

just referred to, a name which will be used hereafter. As was seen, compasses are very useful for studying conditions produced by magnetization and furnish the first method of analysis if a body is suspected of being magnetized. The compass is used continually by jewelers to test for the magnetization of watches which operate irregularly. The

action of a compass in studying the region about a magnetized body can be easily supplemented by the use of iron filings. These scattered over the region align themselves like compass needles and so a whole magnetic region can be mapped out at a single time.

The attractive and repulsive forces of magnets on other magnets led Gilbert to perform another experiment. He floated a compass needle or a magnet on a cork on water in the absence of other magnets. This floated magnet never *moved as a whole* to the north or the south, although if its axis was not parallel to the north and south magnetic line it *rotated* about its vertical axis so as to have its north pole pointing north and its south pole pointing south. This clearly indicated to Gilbert that the earth's magnetism exerted a directive force on a magnet, as differentiated from the attractive or repulsive force of another magnet in proximity to the floating magnet. The more modern implication of this discovery is that owing to some uniformity of the earth's magnetism, or, as we term it today, the earth's magnetic field, the magnet is not displaced north or south, as its north and south poles have equal amounts of magnetism. Thus it can be concluded that all magnets have equal amounts of north and south magnetism. That attractions and repulsions occur for magnet poles brought near the floating magnet is due to the fact that the action of the one pole of the magnet brought up is much greater on the one pole of the floating magnet than on the opposite pole because its distance from the first pole is small compared to its distance from the opposite pole. It will later be seen that this signifies that the force between two poles varies very rapidly with the distance. It can then be concluded that the magnetism of the two poles of a magnet is opposite in polarity and equal in intensity.

If a magnet be broken the poles at the ends are separated. However, at each of the broken ends new poles of opposite sign appear. This with the other observations of magnetism establishes the fact that magnetic poles always appear in opposite pairs and that there is no such thing as an *isolated pole*.* The only way in which a pole isolated from its opposite companion can be studied is to use a magnet so long that the second pole is ineffective in its action owing to the distance. Thus in some experiments magnets of magnetized steel tape a meter long are used. The fact that a magnet which is broken gives two new complete magnets no matter how often it is broken early led to the idea that the magnetism must be inherently lodged in the ultimate particles (the molecules or atoms of the iron).

It was later found that magnetism is not confined to iron and steel alone, but that the elements nickel and cobalt show it nearly as strongly.

* This follows from the fact that all magnetic action results from electricity in motion. Thus any circulating charge, spinning electron, or other creates a magnetic field that in one sense passes through the plane of the charge motion. This gives the appearance of two equal and opposite polarities on the two sides of the plane.

as does iron. In addition, certain alloys of copper, tin, and manganese known as the Heussler alloys show marked magnetic properties. The salts of iron, cobalt, and nickel show feeble magnetic properties (which will be discussed at another point). Outside of these the magnetic behavior of most bodies is feeble in the extreme, so that today the behavior is called *ferromagnetism* in contradistinction to the other weaker magnetic phenomena.

Magnetic forces will penetrate all nonmagnetic substances such as wood, ebonite, brass, glass, etc. The forces, however, are much attenuated in passing through thin sheets of iron, unmagnetized steel, or other magnetic substances. This property is often utilized to screen out magnetic effects. In certain sensitive galvanometers it requires some seven or eight sets of soft iron screens each made out of three or four sheets of soft iron 1 mm thick to cut out the earth's weak field fairly completely. The cause for the screening action of iron is easily seen when another feature of magnetism observed by Gilbert is studied. If a piece of soft iron be brought near a magnet it becomes a temporary magnet with its north pole opposite the activating magnet's south pole, or vice versa. This is what is called induced magnetism. The screening action of iron can at once be explained by the fact that a continuous strip of iron going from the north pole of a magnet around to the south pole becomes an induced magnet which acts to neutralize the magnetic poles by its opposite induced poles near by. Another explanation which could be given for this will be seen to lie in the fact that such a piece of iron acts as a magnetic conductor; see page 236.

The action of soft iron filings in adhering, like a mass of whiskers, to a magnet is owing to the inductive action of the strong magnet through a whole chain of filings which cohere due to the attractive forces.

In distinction to the action of soft iron, steel shows another type of action. If a magnet is brought near a piece of unmagnetized steel the steel shows the same inductive action as was exhibited by the soft iron. When the magnet is withdrawn, the steel, however, unlike the soft iron, *retains* at least a portion of its induced magnetism, that is, it has become a more or less permanent magnet. By more drastic treatment with a pole of polarity opposite to that of the first pole it can be caused to reverse its magnetism and again be attracted. Soft iron is always attracted and loses most of its magnetism as fast as the inducing magnet is removed. The difference in action of the two lies merely in the fact that *once magnetized*, steel retains its magnetism to a high degree whereas soft iron does so only feebly. All degrees of this property known as *retentivity* are encountered in different samples, in general the harder steel being the more retentive. The difference depends on the crystal form of the substance.

Gilbert found that if a magnet is heated its magnetism is weakened

or destroyed. The warmer a magnet the more rapidly it loses its magnetism. All magnets heated above a certain temperature (about bright red heat) cease to be magnetic. Jarring, shaking, or violent mechanical treatment causes magnets to lose their magnetism. Even the best steel magnets eventually weaken, a fact which is attributed to a very slow action of heat together with what is called demagnetization by the field discussed in Chapter XVIII. To prevent this demagnetization these magnets are provided with a soft iron keeper, or two bar magnets are kept together in pairs in a box with opposite poles adjacent and soft iron keepers between (Fig. 10).

Magnets can be made by the following procedures, all except the last one having been observed by Gilbert.

(a) By the action of a magnet or a lodestone on steel, through contact, or by stroking the steel with the magnet poles.

(b) By pounding a piece of iron or steel held in the direction of the earth's magnetic field. It is by the mechanical jarring in the earth's field that tools become magnetized, according to Gilbert.

(c) By heating steel or iron in a magnetic field and letting it cool.

(d) By the action of an electric current in a solenoid on steel or iron. This was discovered by Gay-Lussac and Arago in 1820, shortly after Oersted observed the magnetic effect of a current.

It was early observed in magnetizing a piece of steel that the amount of magnetizing treatment increased the strength of the magnet at first rather rapidly, and then more slowly. In all instances a state was finally reached beyond which the strength of the magnet could not be increased. The magnet was then said to be *saturated*. The physical aspects of saturation will become clearer when the magnetic properties of materials are discussed. The interpretation of the meaning of the phenomenon in its elementary form came however much earlier.

It was also later observed that bodies being magnetized undergo a series of changes of length in the process. If a bar of soft iron is placed in a solenoid and a current turned on the bar will appear to be elongated if the currents are weak enough so as not to produce saturation. The elongation can be shown by fastening the lower end of the bar rigidly and leaving the upper end of the bar free to move so that in moving it rotates a small mirror by means of a lever multiplying device. As the current is increased and the current is switched on and off the movement of the spot of light from the mirror indicating a lengthening of the bar decreases in amplitude, becomes zero and reverses, indicating a contraction of the iron bar as fields producing saturation are reached. Hence weak magnetization causes an elongation while saturation causes a contraction. The phenomenon is called *magnetostriction*.

To get a better picture of the conditions surrounding a magnet the use of iron filings representing a mass of small compass needles about a magnet can be resorted to. If a piece of Celophane or other trans-

parent substance is placed over a magnet lying in a beam of light, and if light iron filings be dusted over the Celophane and the latter jarred, the filings will arrange themselves in a set of regular patterns which can be projected on a screen. The patterns indicate the setting which compass needles would take in the regions about the magnet. It will be seen that the whole region around the magnet appears to

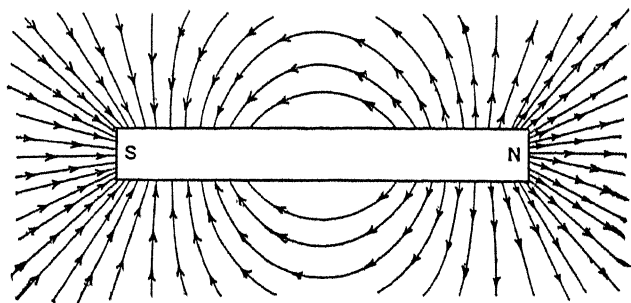


FIG. 2. Idealized field about an isolated magnet.

exert a directive action on the filings. Such a directive action implies the existence of magnetic forces at all points in the space about the magnet. Hence we can say that the region about a magnet represents a *field* of magnetic force, the direction of this field in each place being the direction represented by the long axis of the small piece of filing. The soft iron filings are temporary magnets or compass needles due to the inductive action of the field. An idealized picture of the field about an isolated magnet is shown in Fig. 2.

It is instructive to observe such filing patterns as displayed by different arrangements of poles and magnets. The attractive forces between N and S poles are clearly illustrated by the continuity of lines of filings running from the N pole to the S pole; see Fig. 3. The repulsive forces between the two N or two S poles can be seen by the recurving of the filing lines leaving one pole as they approach the other similar pole, leaving a *neutral point* between the two sets of recurving lines as seen in Fig. 4. The neutral points of a magnet placed in the earth's field are shown in Fig. 5.

A more important observation is the fact that in all these fields due to a single magnet the lines emerge not as if emanating from one point in the iron or steel but as if they came out all over the ends of the bar, as seen in Fig. 2. Thus the thing termed a pole is not a definitely located point of magnetism, but a general spread of magnetization all over the iron or steel. Hence the pole is no definitely situated point. This fact is of great importance when it comes to the quantitative study of magnetic forces due to poles. A similar situation arises when the attraction of the earth's gravitational field on a body is considered.

Each point of the body is attracted in proportion to its mass. This complex attractive action can be replaced in its behavior by a single force applied at one point, the *center of gravity*. In a similar fashion

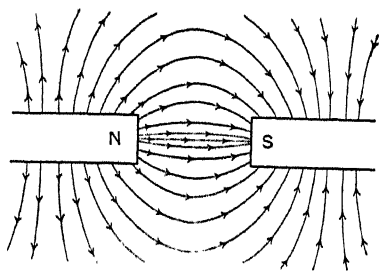


FIG. 3. Idealized field between unlike poles.

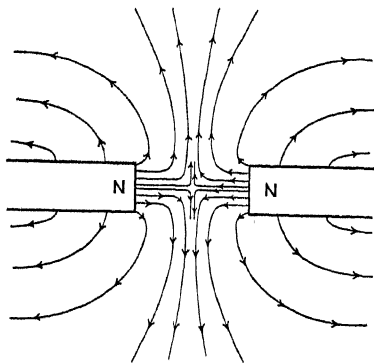


FIG. 4. Idealized field between like poles.

the diffusely distributed magnetism in the end of a magnet might conceivably be considered to act as if concentrated at a single point in its action with other magnets. Thus we can *idealize* a magnet as having two point magnetic poles located at a distance of l units from each other. The length of the magnet would then be spoken of as l units. Now for some simple geometrical forms it is easy to locate the *center of gravity* by calculation.* But locating the center of gravity in magnets is practically impossible. Thus the exact location of magnet poles and the length of a magnet is a matter of great uncertainty. This fact complicates the quantitative study of magnets and, were it not for a fortunate circumstance discussed on pages 39 and 40, the quantitative treatment of magnetism would be exceedingly onerous.

Perhaps one more interesting phenomenon may be investigated by the use of iron filings. A steel knitting needle is found to be magnetized on approaching one end to a compass needle in this specific

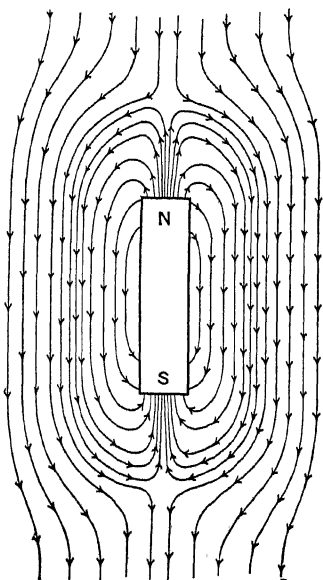


FIG. 5. Isolated magnet placed in a uniform field.

* A uniformly magnetized ellipsoid of revolution has its centers of magnetization located at the two foci of the ellipse along its axis.

instance with north magnetic polarity. If the other end of this specific needle be tested it will also be found to exhibit north magnetic polarity. According to the general law of magnetization, however, there should be just one north pole and one south pole in a magnet. If the needle is placed on the projection screen and dusted with iron filings as for other magnets the apparent paradox is at once solved. For it appears that

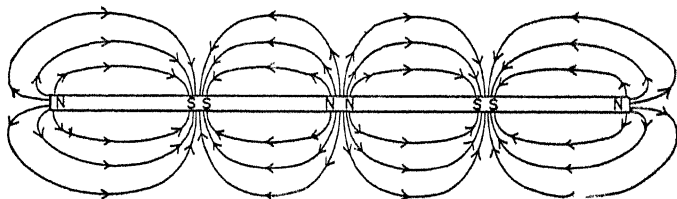


FIG. 6. Single magnetized bar showing consequent poles.

there are several poles in the needle as indicated by the iron filings. In fact, the pattern of filings appears, as shown in Fig. 6, as if there were in the one needle four separate magnets so arranged that the south poles are opposite the south poles and the north poles are opposite the north poles. It accordingly happens in this one situation that with four magnets thus arranged there is a north pole at each end of the rod. The needle does not therefore violate any principles but



FIG. 7. Windings for producing consequent poles.

represents what are termed *consequent* poles. Such a condition is created in a simple fashion by winding a coil about the wire with the sense of winding in the first quarter in one direction (say counter-clockwise as viewed from the end), in the next quarter the sense of the winding is in the opposite sense (i.e., clockwise when viewed from the end), at the third quarter the winding is reversed again, and the same way for the fourth quarter, as shown in Fig. 7. Thus when a current is passed through the wire the consecutive or opposing poles appear.

The experiments outlined give one a chance to form a fairly good qualitative picture of ferromagnetic behavior. On the basis of recent knowledge it will be assumed that all ferromagnetic substances consist of small *spontaneously magnetized* magnetic elements, or regions, called *domains*. The size of the *domains* varies with conditions, but they appear to be filamentary with lengths of perhaps 1 mm to some centimeters and volumes of the order of 10^{-9} cm.³ In the usual nonmagnetized state, these domains are oriented in such a way that the adjacent domains neutralize each other, i.e., the N poles of one domain are opposite the S poles of its neighbors. (See Fig. 8a.)

Furthermore, in well-formed single crystals these domains are oriented along *directions of easy magnetization* in the crystal, which vary with the crystal type. In Co it lies along the hexagonal axis, in Ni it is

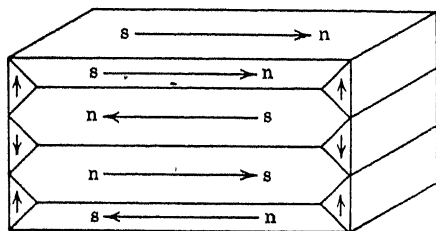


FIG. 8a. Domains in an unmagnetized single crystal of ferromagnetic substance.

along the 110 direction in the crystal, while in Fe it is along the 100 axis in the face-centered cubic lattice. Directions of easy magnetization can also be induced or altered in a crystal by putting it under tension. As the crystal is placed in a region of magnetic force with the direction of easy magnetization in line with that force then from time to time some of those domains of magnetization opposed to the field will become unstable and reorient in the direction of the impressed

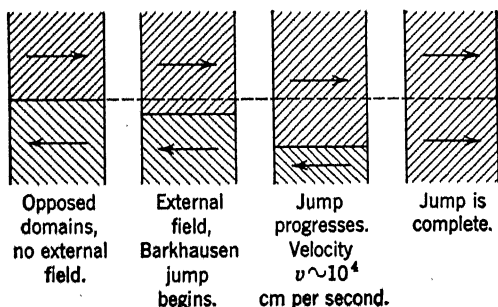


FIG. 8b. Progress of a Barkhausen jump.

force. The reorientation begins at the boundary of the adjacent domain in the field direction and propagates across the domain with a speed of the order of 10^4 cm per second. (See Fig. 8b.) As the magnetizing force increases, the domains realign more and more readily and at a higher and higher frequency. Eventually all domains are aligned in the *sense* indicated by the magnetizing force but along the axis of easy magnetization (Fig. 8c). Where this axis coincides with the direction of the applied magnetizing force the specimen is saturated. If the *direction of easy magnetization* is at an angle with the magnetizing force the domains can by further increasing the magnetizing force be forced to align parallel to that force. This requires higher and higher fields and also places the crystal under mechanical stress. When

alignment with the impressed field is complete the specimen has reached magnetic saturation. The region of alignment of all domains along the axis of easy magnetization and the region of bringing the domains into line with the force give the characteristic S shape to the magnetization curve. The effect on the magnetization curve produced by orientation for a single crystal of Fe is shown in Fig. 8d.

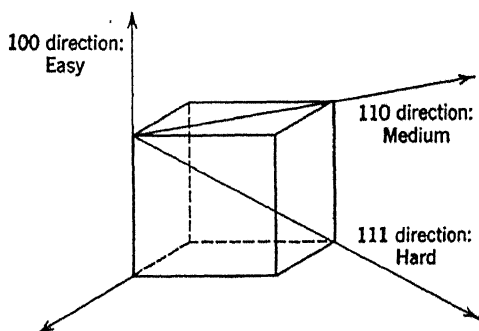


FIG. 8c. Single cubic crystal of Fe. Domains aligned along 100 axis of easy magnetization.

Usually, magnetic substances are not single crystals, but are microcrystalline aggregates, so that the axes of easy magnetization take on all positions relative to the magnetizing force. Thus, in general, substances show a rather greater reluctance to saturation and require larger magnetizing forces than for ideal single crystals. This is illustrated in the curve of Fig. 8e. The magnetic stresses introduced by alignment of domains, which are of considerable strength, together with the reorientation of the domains into the direction of the magnetizing force, produce mechanical stresses in the specimen that give the changes of length observed in magnetostriction. It is clear that the crystalline and the mechanical states of the material will give wide ranges of ease of magnetization and resistance to magnetization. Thus, a single tempered crystal oriented with the direction of easy magnetization parallel to the magnetizing force will reorient its domains with a minimum of difficulty; see Fig. 8d. Saturation in such cases has been obtained with fields of the order of magnitude of an oersted and less, giving permeabilities of the order of 10^6 . With hard polycrystalline specimens such as silicon steel quite the contrary must be expected, and high fields are required to orient the domains. Once oriented they remain and show great resistance to reorientation; see Fig. 8e. The flopping over of domains in the process of magnetization has been experimentally observed. The domains are large and powerful enough magnetically such that when one reverses it can cause sufficient of a change in magnetism as to induce an appreciable electromotive force in a secondary coil of many turns. With very powerful amplifiers

these electromotive force changes can be caused to register on a string galvanometer and can thus be recorded. This phenomenon was discovered by Barkhausen and the observed magnetic discontinuities

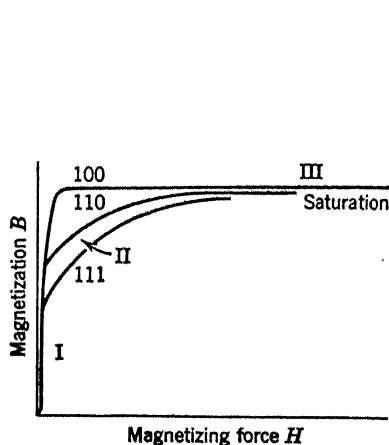


FIG. 8d. Magnetization curve of single Fe crystals with the magnetizing force along 100, 110, and 111 axes. Note the steep rise in all cases in part I of the curve—the Barkhausen region. Part II of the curve, missing for 100 direction, gets saturation part III directly. Note Part II is the region of alignment of the domains with the magnetizing force.

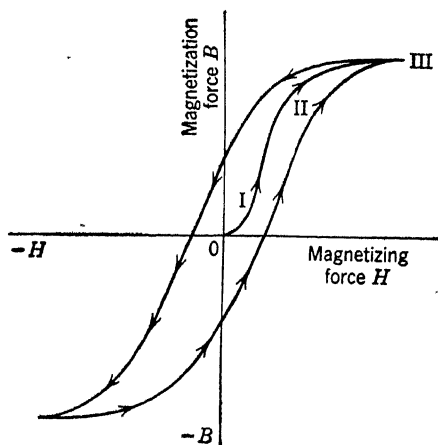


FIG. 8e. Magnetization curve of a hard polycrystalline ferromagnetic material. Starts at 0, part I drawn out, part II more extended, part III hard to reach. Note that the curve does not retrace as the force is reduced. Note, also, the remaining magnetization when force is 0. This is permanent magnetization as a result of retentive power.

called the *Barkhausen jumps* are illustrated in Fig. 8f. By polishing a single crystal of ferromagnetic material, on a face normal to the ends of the domains, Francis Bitter has visually demonstrated the domains. He did this through the alternate dark and light patches seen under high magnification when finely powdered Fe_3O_4 is suspended in oil and painted over the face of the crystal. The movement of the patches as the specimen is magnetized indicates realignment of domains.

The effect of mechanical jarring in changing the state of magnetization of a specimen is clear. In the absence of the field the mechanical working helps the domains to realign in the nonmagnetized state. In the presence of a magnetizing field it assists in aligning the directions of easy magnetization with the force. Heat will act in two ways. First, it reacts essentially in the same fashion as mechanical jarring on a more microscopic scale. Second, it acts to disturb the elementary free electrons, the interactions of which cause the magnetic spins of the electrons to align to give the spontaneous magnetization characterizing the domains. If the temperatures are too high then the

electron interactions can no longer give spontaneous magnetization, domains disappear, and ferromagnetism ceases.

A bar magnet if left to itself will tend to lose its magnetism, since the direction of the magnetic force outside the magnet is contrary to

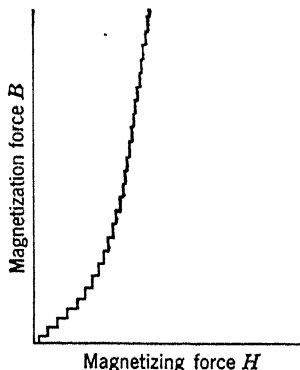


FIG. 8f. Much magnified section of the start of a magnetization curve, showing Barkhausen jumps.

the direction of the aligned domains in the magnet. (See Fig. 9.) Thus, the outer surface of the bar magnet is bathed in its own external field, which acts to demagnetize the surface. Thus, domains on the outer surface will flop and demagnetize. Gradually this will permeate

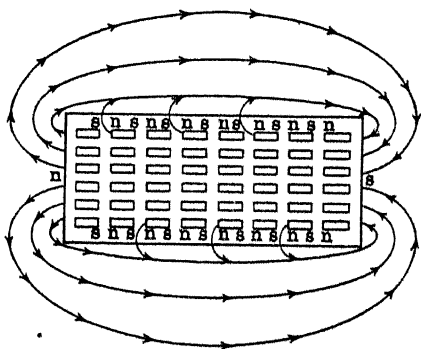


FIG. 9. Demagnetizing action of the external field of a bar magnet.

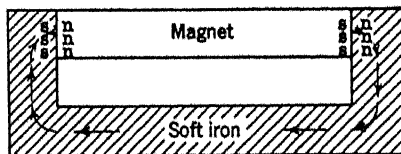


FIG. 10. Action of a keeper in diverting the field from the outside of a magnet. Contrast with Fig. 9.

the magnet and under the action of heat, time, and its own external opposing field it will lose its magnetism. The effect is greatest for short thick magnets. It is less with long thin ones. This can be prevented by a keeper (Fig. 10), which neutralizes the poles of the magnet by creating near-by opposing induced poles and thus diverting the

return flux from these poles from the sides of the magnet. The horse-shoe form given earlier magnets was primarily to reduce the demagnetizing field along the magnet, and simplify the form of the keeper. (See Fig. 11.)

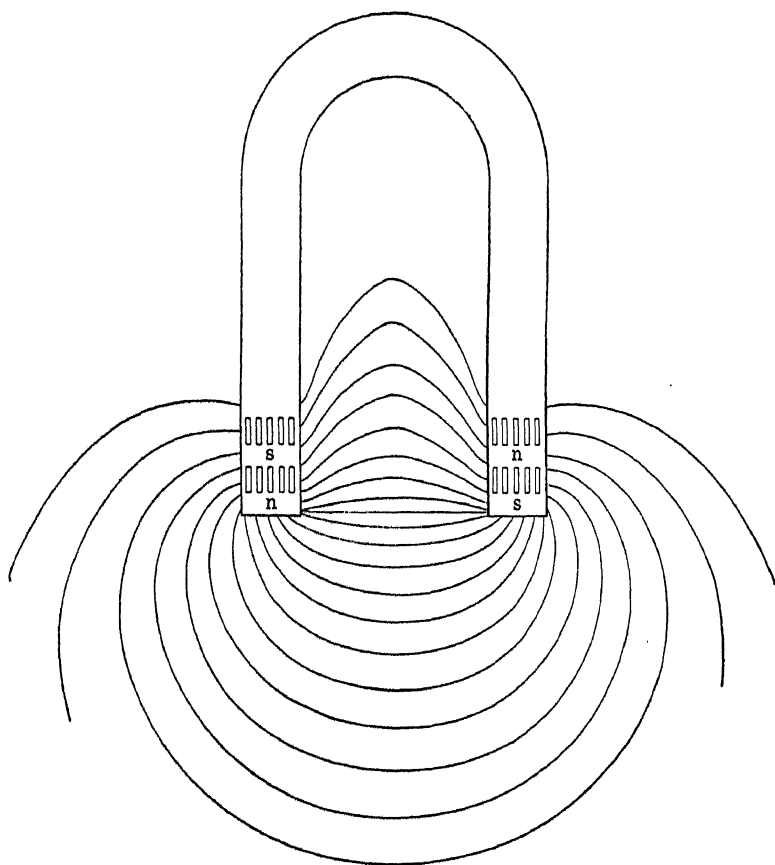


FIG. 11. Bar magnet bent into the shape of a horseshoe. Note the concentration of forces at the ends and the very weak or absent demagnetizing fields along the sides in contrast to Fig. 9.

The character of the fields about real magnets where magnetism is not localized in two poles is now clear. With the polycrystalline character of most worked metals and the lack of homogeneity, the alignment of domains is never perfect, so that the magnetism coming from the ends of the domain is distributed pretty much all over the bar with the maximum concentration near the ends where the maximum effects accumulate.

CHAPTER III

THE QUANTITATIVE TREATMENT OF MAGNETIC FORCES

6. THE CONCEPT OF THE MAGNETIC FIELD

It was observed from the interactions of magnets that forces must exist between the magnets. Therefore, in the region around a magnetic pole there exists a condition such that another magnetic pole, brought into this region, experiences a force at each point in the region. The direction of the force at any point may be determined by the orientation of a minute compass needle placed at that point, that is, the compass needle so orients itself that it is parallel to the resultant force. It is therefore possible to map the field of force about a magnet by finding the direction which the compass needle would take at all points of the space considered. The field can also be shown by shaking light iron filings over a paper giving the plane in which a study of the field of force is sought. The iron filings in this case constitute minute magnets which align themselves in the direction of the field. *A more rigorous definition of the direction of a field of force would be given by taking the path described by a hypothetical isolated north pole in the field.* It is found that the force at each point, in addition to having a direction, has a definite magnitude, and like all force is therefore a vector quantity. This force varies with the distance from the various magnetic elements which cause the field.

7. DEFINITIONS OF UNIT POLE AND POLE STRENGTH— COULOMB'S LAW

In order to determine the magnitude of the field, one must have a unit of measure. The existence of fields of magnetic force is manifested by the forces exerted on magnetic poles. It should therefore be possible to express the magnitude of a pole strength or of a field strength by the force exerted on some other pole. To define this force, a standard must be chosen. The choice of a standard can come only from an experimental study of the law of force between poles.

Coulomb was the first to study the forces experimentally. He initially studied the forces of attraction between poles using the torsion balance. In this study he suspended a magnet from a fiber to which was attached a mirror so that the deflection of the mirror enabled the deflection of the magnet produced by a given force to be measured. The force necessary to produce a given deflection on the balance was known, and hence, by measuring the angle of deflection, the force

produced by another pole on the suspended magnet could be measured. In his early experiments Coulomb used long magnets so as to study the effect of poles as nearly isolated from their accompanying opposite poles as possible. By this means he found that the force was proportional to what he defined as *the magnetic strength of the pole*, and inversely proportional to the *square of the distance* between the poles. If the one pole is designated as having a strength m and the one brought r cm from it as having a strength m' , the force between them can be written $f = \frac{mm'}{r^2} \frac{1}{\mu}$. Here $\frac{1}{\mu}$ is a constant of proportionality and depends on the medium and the units chosen.

The establishment of this law leads at once to a definition of the unit of pole strength. In any system of units, *a unit pole can be arbitrarily defined as one which repels an exactly equal pole at unit distance with unit force when μ is unity*. For if $f = 1$, $r = 1$, and $\mu = 1$, then $mm' = m^2 = 1$, and $m = 1$. In physics the c.g.s. system of units is generally used. In this system the unit of force is the dyne, and the unit of distance the centimeter. Hence the *unit pole in the c.g.s. system is the pole which repels an exactly equal pole at the distance of 1 cm with the force of the 1 dyne (in vacuum)*.

Practically it is difficult to obtain exactly equal poles. It is furthermore difficult to locate accurately the *center of magnetism* in a pole so as to measure distances accurately. Thus the measurement of pole strengths must be left to indirect methods. We shall see later that this is accomplished by performing two experiments which give a field strength and a pole strength multiplied by the distance between the poles from the equations involved. (See Chapter IV.) In static electricity one has an easier time in measuring the electrical quantity q , the defining equation $f = \frac{qq'}{r^2} \frac{1}{D}$ of which is very similar to the magnetic equation, as it is easy to realize equal quantities.

Concerning the quantity μ , this is a constant involving the nature of the materials in which the two poles are immersed. It is assumed to have the value unity for empty space. For air it is so nearly 1 (1.0000004) that it can be called unity. For iron it can be high (about 1000). μ is called the magnetic permeability. (See Chapter XVII.)

Definitions of Field Strength and Unit Field. The magnetic field in a region can be studied by means of the forces exerted on a test pole. As the force is dependent on the pole strength of the pole used for studying the field, it is convenient to express the field in terms of the force on a standard pole. To this end *field strength at a point is defined as the force exerted on a unit north pole at that point*, that is, the field strength H is given by $H = \frac{f}{m}$, where f is the force in dynes on a pole of strength m . If $f = 1$ dyne, and m is a unit pole, H is a unit field

strength. *Unit magnetic field in the c.g.s. system is therefore the field that exerts 1 dyne of force on a unit pole placed in it.* This unit of field strength has been named the *oersted*.* Thus a field of 1 oersted exerts 1 dyne of force on unit pole, and m dynes of force on a pole of strength m . The value of the concept of field strength lies in the fact that if H , the field strength, is known, the force on any pole m is at once given by the value of the product Hm .

8. UNITS AND DIMENSIONS

The physical units and their applications are organized on the basis of the mechanical system of Newton. All gross mechanical occurrences in nature are related by mathematical relations of a more or less simple nature to the system of definitions or axioms contained in Newton's three laws of motion. The quantitative expression of these laws in a restricted form is the expression defining force, $f = ma$, where m is the mass or inertia of the body and a its acceleration.

Acceleration is rate of change of velocity, $\frac{dv}{dt}$, and velocity is defined

as the space or distance covered in unit time, or $\frac{ds}{dt}$; so that $a = \frac{d^2s}{dt^2}$,

and $f = m \frac{d^2s}{dt^2}$. Thus the fundamental measurable quantities entering

into this basic equation descriptive of all physical occurrences are length or distance, time, and inertia or mass. All change in the condition of a mechanical body is accordingly expressible by appropriate combinations of these quantities.

Now the units employed for measurement in terms of these three fundamental quantities are defined in terms of simple convenient arbitrary standards: the second, the centimeter, and the gram. The second is merely a convenient fraction of the mean solar day; it is $\frac{1}{86400} \times \frac{1}{60} \times \frac{1}{24}$ of the mean time that elapses between successive transits of the sun across the meridian. Why this fraction is used instead of a fraction expressed in the more convenient decimal system is hard to say. The antiquity of this time system, going back to the ancients, and the fact that the custom has been to use it in tables for so many years, are the probable causes of its continuation. The unit of length is also a purely arbitrary, convenient standard. When the standard meter bar was made, it was supposed to be one forty millionth the circumference of the earth through the poles. However, later measure-

* The oersted was substituted in 1932 for the gauss to represent the c.g.s. unit of field intensity. The gauss is now the c.g.s. electromagnetic unit of induction B ; see Chapter XVII. The term *gauss* may still be used if magnetic induction and magnetic field strength have the same dimensions. This is the case when the Gaussian system of units is used, in which μ and D both are dimensionless constants; see Chapter XXVI.

ments showed this value to have determined the standard meter is now merely the length of a platinum bar kept at the International Bureau of Weights and Measures at Sèvres, France. Michelson later determined the length of the standard meter in terms of the wave length of three lines in the spectrum of cadmium so that, should the standard vary, a means of checking on it is available. The centimeter is chosen as 10^{-2} of a meter. Finally, the unit of mass was for convenience chosen in terms of the inertia of a cubic centimeter of water at 4°C , the temperature of its greatest density. Actually as this is hard to determine the unit of mass, the gram, is 10^{-3} of the standard kilogram, a mass of *Pt* kept at the International Bureau at Sèvres, France. The actual gram is very close to the inertia of the cubic centimeter of water.*

As is seen, the choice of our so-called fundamental units indicates that they are in *no sense fundamental*, but are merely convenient or even chance arbitrary standards to use with our Newtonian system of units. It is not surprising then to find that our real and probably truly fundamental units, such as the charge of the electron, h , the Planck action quantum, the mass of the electron, and the velocity of light are expressed as peculiar odd ratios of the chosen units in the fields where they are measured. We are accordingly committed to expressing all phenomena in physics in terms of Newtonian mechanics and ultimately in terms of these three arbitrarily chosen fundamental units in the c.g.s. system.*

Leaving the *units* aside, it is clear that, in general, it is possible to express by Newtonian mechanics all phenomena in terms of powers of length, mass, and time. Better to understand the foregoing statement, the process of obtaining a new entity or quantity in nature may be discussed. When a new phenomenon of nature is observed the practice is to deduce quantitatively the behavior manifested by means of controlled, quantitative investigation and to formulate behavior in terms of a mathematically expressed law. For example, Coulomb in deriving the law of electrostatic force proceeded essentially as follows. He took two charges and actually studied how the force varied for the same two charges as distance alone was varied. Then keeping distance constant he varied first the state of charge on one body and then the

* There is at present a movement by some teachers of physics toward abandonment of the historic c.g.s. system of units and toward the use of what is termed the m.k.s. system. This system has adopted the standard meter as a unit of length, the standard kilogram as a unit of mass, and the second as a unit of time. It has the advantage that the units are somewhat larger. Thus the m.k.s. unit of force is the newton, which is 10^5 dynes. For many gross mechanical problems it thus has the advantage of using smaller multiples of the units. When it comes to expressing ratios with molecular and atomic magnitudes, i.e., as in the world of modern physics, it is at a greater disadvantage. Since for general purposes the m.k.s. system has no definite advantage over the more universally used basic c.g.s. system, it will *not* be used in this book in order to avoid adding confusion to an already complicated array of units.

state of charge on the other.^{1d} *in the electrical* results or data were set down in tabular form and by analysis the mathematical laws controlling them were deduced and expressed in the generalized form of an equation. This procedure led a once to the idea that the force between two electrified bodies depended inversely on the square of the distance and on the product of two terms that varied with the electrical state of excitation of the two bodies. Either one of the two quantities q and q' describing the two electrical states of the bodies in the law deduced, $f = \frac{qq'}{r^2}$, he defined as a new quantity in nature, a property of electrification, and called it the *quantity of electricity*.

The law governing a phenomenon being once formulated, and the new quantity in nature defined in terms of things measurable in the mechanical terminology based on Newton, the unitary value of this quantity may easily be defined. For assuming as *fundamental for practical purposes* the c.g.s. system of units just defined, all that is needed is to solve the equation discovered for the quantities under simplifying assumptions, and to set the quantity as *unity when each of the items in the equation is taken as unity on the c.g.s. system*. Thus, one would write $qq' = fr^2$, and simplify it by letting $q = q'$; then $q^2 = fr^2$, and $q = \sqrt{fr^2}$. Unit electrostatic quantity was therefore taken as that quantity for which $\sqrt{fr^2}$ equaled unity as applied to the phenomena investigated. Put into words the law then is the formal definition so often learned in a parrot-like fashion by most students without an understanding of its meaning. This process of becoming familiar with a new concept of physics by means of the defining equation, and defining the unit in the manner outlined in previous paragraphs, should give a far clearer idea of the concept and the unit, as it presents in concise mathematical form the relations involved. This method will constitute the procedure to be followed throughout the book in defining the many new units to be encountered.

Now in developing a field of science such as electricity numerous new quantities are found. It becomes essential to relate and correlate them with one another in order that they may be used to the best advantage. It is further useful, so to speak, to "keep books" when new quantities are found to make sure that things equated are really legitimately equated. Thus writing an expression equivalent to an energy equal to something that is not energy would introduce obvious errors into the results. Finally some new quantities are derived under conditions where their nature is not obvious and it pays to establish their nature in order to make the best use of the newly gained knowledge.

To avoid possible mistakes, and to work to the best advantage, equations can be checked by analyzing them into the component three fundamental elements underlying all Newtonian mechanics, that is,

the *dimensions* of a quantity are determined in terms of length written L , mass written M , and time written T . If the quantities on the two sides of an equation have, outside of numbers or numerical ratios, which are ignored, the same powers of L , M , and T , the equations are dimensionally correct. By the same process the dimensions of a new quantity can be determined in terms of known ones.

The dimensions of a few important physical quantities are as follows: L = length, M = mass, T = time; velocity, $v = \frac{s}{t} = LT^{-1}$,

acceleration, $a = \frac{v}{t} = LT^{-2}$, force, $f = ma = MLT^{-2}$; work, $w = fs$

$= ML^2T^{-2}$, power, $p = \frac{w}{t} = ML^2T^{-3}$. Now two equations fre-

quently written are that impulse, ft , = momentum, mv , and that kinetic energy, $\frac{1}{2}mv^2$, = work, fs . To test the correctness of these assertions we can set $ft = (MLT^{-2})T = M(LT^{-1}) = mv$, which we see is an identity, and $\frac{1}{2}mv^2 = \frac{1}{2}M(LT^{-1})^2 = (MLT^{-2})L = fs$, which, neglecting a numerical constant, is also seen to be an identity. Another example of the use of dimensions is from modern physics. It was found that a body can be set into vibration at its natural frequency ν only if it receives an energy $\frac{1}{2}mv^2$ given by $h\nu$, where h is a new universal constant. It might be asked what units h is to be expressed in, or, in other words, what h is dimensionally. $\frac{1}{2}mv^2 = ML^2T^{-2} = h\nu$, now ν = frequency = number per unit time = T^{-1} . Thus $ML^2T^{-2} = hT^{-1}$, or $h = ML^2T^{-1}$. Now ML^2T^{-2} is energy so that h has the dimensions of energy times time and h is evaluated as 6.55×10^{-27} erg \times seconds. However, moment of momentum $mv r = ML^2T^{-1} = h$, so that h has simultaneously the properties of $mv r$ and wt , a conclusion of far-reaching importance for atomic structure.

Again, temperature can be considered as an admirable illustration. Temperature begins to acquire a physical significance with the adoption of the ideal gas law relation $pV = RT$, where T is absolute temperature. Now $p = \frac{f}{A} = ML^{-1}T^{-2}$, $V = L^3$ and $pV = ML^2T^{-2}$, or

energy. Hence, RT is work or energy, and in fact the later development of the kinetic theory showed that RT is $\frac{2}{3}$ the total kinetic energy of the molecules of a gas. Thus we gain an actual understanding of the nature of RT and hence of temperature, for T multiplied by R gives the energy in the gas at a temperature T .

It should be noted here that as long as product RT has the dimensions of energy, R and T may be assigned any dimensions we choose. Obviously, such dimensions must be consistent with other equations involving R and T separately, if such appear, but otherwise they can be assigned dimensions as desired. It is usually the custom here to assign to R the dimensions of energy per degree, and to make T a

numerical scale factor which, multiplied into R , gives two-thirds the total kinetic energy of the gas. The freedom of assigning dimensions to quantities which the theory of dimensions sometimes give is one that gives wide latitude to aesthetic taste. This freedom must, however, be used with caution. In all instances great care must be used in seeing that the assignments made are generally consistent. Thus, in *most* texts in electricity and magnetism the quantities μ and D are chosen as dimensionless quantities in the defining equations $f = \frac{qq}{Dr^2}$

and $f = \frac{mm'}{\mu r^2}$. This practice leads to what is known as the Gaussian system of units (see sections 145 and 155). However, when these quantities appear in the electromagnetic equations of Maxwell's theory, or in the evaluation of the ratio of the two systems of units, they require the introduction of a quantity η , which has the dimensions of a velocity such that $\frac{\eta}{\sqrt{\mu D}} = LT^{-1}$. Other systems of units set D as dimensionless, 0 for empty space, and $\eta = 0$. Then μ has the dimensions $\frac{1}{L^2 T^{-2}}$ or $L^{-2} T^2$. This is called the electrostatic system. Analogously, μ could be set as dimensionless and 0 for empty space with $\eta = 0$. This is called the electromagnetic system, and D has the dimensions of $\frac{1}{L^2 T^{-2}}$, or $L^{-2} T^2$. Which system of units is chosen makes little difference except for aesthetic taste or convenience, as long as it is consistently used. Probably if work is done largely in magnetics the electromagnetic system has advantages. For elementary instruction the Gaussian system with its symmetry has its advantages.

9. DIMENSIONS OF MAGNETIC POLE STRENGTH

Before proceeding further, one may digress to apply the procedure of section 8 to determine the dimensions of the new quantity magnetic pole strength. In the equation above, let the two poles be equal. That is, $m = m'$. The equation then becomes

$$f = \frac{m^2}{r^2},$$

whence

$$m = \sqrt{r^2 f},$$

if the dimensions of μ are disregarded as will be done throughout this book.

Force has the dimensions given by: $f = MLT^{-2}$.

Also

$$r^2 = L^2.$$

Therefore

$$m = \frac{M^{1/2}L^{3/2}}{T}, \text{ in the Gaussian system.}$$

Since field strength H is the force per unit pole, $H = \frac{f}{m}$, whence

$$H = \frac{M^{1/2}}{L^{1/2}T}.$$

10. APPLICATION OF LAW OF FORCE TO SPECIAL CASES

Now Coulomb's early measurements were inexact because of the complications of other poles, and the exact study of the forces between poles and their measurements must be achieved in a more indirect manner. Since pole strength can be determined by forces the methods applicable to forces can be used to study poles. The magnetic force has magnitude and direction. It is, therefore, a vector. The study of the resultant force at any point caused by a number of magnetic poles in its neighborhood is accomplished through the vectorial addition of the separate forces due to the separate poles. This treatment *assumes that the force due to each pole acts independently of the presence of other poles*. Using this assumption, the forces due to certain arrangements of poles can be studied at chosen points. These have a practical bearing on problems to come. Reduced to forces on unit pole this assumption gives the magnetic fields at the points in question.

Case 1. The field of an isolated north or south pole of strength $\pm m$ in air at a point r cm distant from the pole is, by definition,

$$\frac{f}{m'} = H = \frac{mm'}{m'r^2} = \pm \frac{m}{r^2} \text{ in the direction of } r.$$

Therefore the force f on a pole m' is $m'H = \pm \frac{mm'}{r^2}$.

Case 2. The field at a point A distant d cm from the center C , Fig. 12, of a bar magnet the length of which is l cm, and the pole strength of which is m units, may be found as follows. Because of the north pole the force is one of repulsion at A and the field from Case 1 is expressed by

$$H_1 = + \frac{m}{\left(d - \frac{l}{2}\right)^2},$$

and the force due to the south pole is one of attraction and the field H_2 is expressed by:

$$H_2 = - \frac{m}{\left(d + \frac{l}{2}\right)^2}.$$

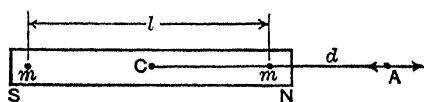


FIG. 12.

The resultant field H_r is given by:

$$H_r = + \frac{m}{\left(d - \frac{l}{2}\right)^2} - \frac{m}{\left(d + \frac{l}{2}\right)^2},$$

as a result of adding H_1 and H_2 , which act along the same line.

Reducing this by algebraic manipulation one arrives at the final expression

$$H_r = \frac{2 l d m}{\left(d^2 - \frac{l^2}{4}\right)^2} \text{ at } A.$$

If l is small, l^2 may be neglected compared to d^2 . The force on unit pole at A , i.e., the field H_r , then becomes $H_r = \frac{2 m l}{d^3}$, which suffices for the solution of many simple problems.

It is seen from this that the field falls off rapidly with the distance from a magnet, and this rapid fall makes the detection of the magnet at

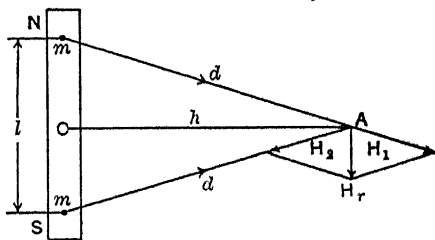


FIG. 13.

a large distance very difficult. Thus, although it was suggested that the magnetization exerted by the electrical equipment of a submarine might be used for detecting its presence, the very weakness of the magnetic forces at distances of approach which made the submarine dangerous made detection impossible.

Case 3. Consider the force at a point A , Fig. 13, distant h from the center C of a bar magnet of a length l , where h is taken in a direction perpendicular to the bar magnet of pole strength m at its center. Call d the distance from the poles of the magnet to the point at which the force is to be determined. It will be noticed that the distances d are equal. In this situation the force exerted on unit north pole by the north pole (i.e., the field) will be away from the latter and will have a magnitude H_1 given by

$$H_1 = \frac{m}{d^2} = \frac{m}{h^2 + \frac{l^2}{4}}.$$

The force on unit north pole due to the south pole (i.e., the field) will be directed toward that pole and will have the magnitude H_2 given by

$$H_2 = \frac{m}{d^2} = \frac{m}{h^2 + \frac{l^2}{4}}.$$

Here the resultant field H_r is obtained by the vectorial addition of H_1 and H_2 . It is represented by the small vector H_r , and since H_1 and H_2 are equal in magnitude, H_r will be parallel to the axis of the magnet. From similar triangles it can be seen that

$$\frac{H_1}{H_r} = \frac{d}{l}, H_r = H_1 \frac{l}{d} = H_2 \frac{l}{d},$$

whence

$$H_r = \left(\frac{m}{h^2 + \frac{l^2}{4}} \right) \left(\frac{l}{\sqrt{h^2 + \frac{l^2}{4}}} \right) = \frac{ml}{\left(h^2 + \frac{l^2}{4} \right)^{3/2}} \text{ at } A.$$

If l is small compared to h , $H_r = \frac{ml}{h^3}$ at A .

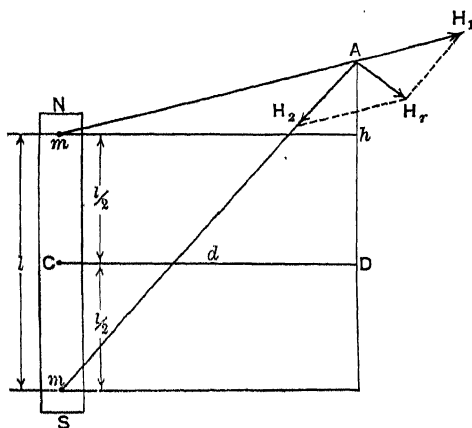


FIG. 14.

Case 4. For any other point A around the bar magnet the resultant force is easily determined by the following method. From A , Fig. 14, draw a line AD parallel to the axis of the magnet, join A to the north and the south poles, call the force of repulsion due to the north pole on unit pole H_1 , and that of attraction due to the south pole on unit pole H_2 . Call l the length of the magnet, and draw a perpendicular D to the center C of the magnet intersecting AD at D .

If d is called the distance CD from the center of the magnet along the normal to the point where it intersects the line from A parallel to the axis, and if h is called the distance AD from this point of intersection to the point A , the fields H_1 and H_2 at A can be expressed in terms of the pole strength m and the distances h and d by the equations:

$$H_1 = \frac{m}{d^2 + \left(h - \frac{l}{2}\right)^2}$$

$$H_2 = \frac{m}{d^2 + \left(h + \frac{l}{2}\right)^2}.$$

The resultant field H_r at A may be at once found from the relation

$$H_r^2 = H_1^2 + H_2^2 + 2 H_1 H_2 \cos \theta$$

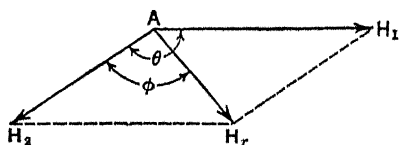


FIG. 15.

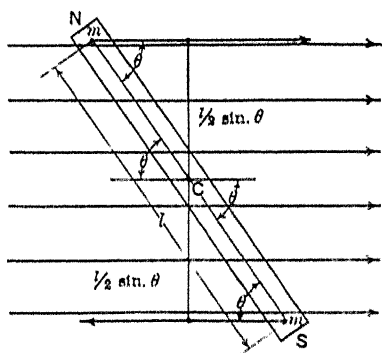


FIG. 16. Forces on a magnet in a uniform field.

where θ is the angle between H_1 and H_2 . It will be noted that if θ is less than 90° the cosine term is positive, and if θ is greater than 90° it is negative. The angle φ of the resultant H_r with one of the original forces H_2 , Fig. 15, can be found, as in the solution of all force triangles,

from the well-known relation that $\frac{H_1}{H_r} = \frac{\sin \varphi}{\sin \theta}$, whence $\sin \varphi = \frac{H_1}{H_r} \sin \theta$ and the value of φ can be found from the tables.

Case 5. For any number of magnets the problem resolves itself into calculating the fields for each pole and composing the fields to a single resultant, using the laws of composition of forces.

Case 6. The next problem which is of importance is the interaction between a field and a magnet oriented in any position in that field. Assume a uniform field of strength H represented by the parallel lines of Fig. 16. Assume a magnet of pole strength m lying in the field in

such a way that its axis makes an angle θ with the field. The north pole is urged in the direction of the arrow at N . The south pole is urged in the direction of the arrow at S . The result of the action of these two forces, since they must be equal, the field being assumed uniform and the poles equal, will be to exert a simple torque on the magnet, for the equality of the forces in opposite directions on the magnet as a whole precludes translatory motion. This torque would cause the magnet to turn so that its north pole is towards the south pole creating the field, and its south pole is towards the north pole creating the field. It is important to be able to calculate the magnitude of this torque. The force on the north pole is by definition Hm .

If l is the length of the magnet, it acts on a lever arm $\frac{l}{2} \sin \theta$. The south pole is urged in the opposite direction with a force Hm and the force moment which would cause rotation in the same sense as that acting by means of the north pole will be $Hm \frac{l}{2} \sin \theta$. The resulting force moment symbolically expressed by the quantity G is given by

$$G = 2 Hm \frac{l}{2} \sin \theta = Hml \sin \theta.$$

It is seen that the torque depends on the field strength, on the angle θ , and on the product of the pole strength by the length of the magnet. This quantity ml is a constant of the magnet as long as its pole strength is unchanged.

It will be observed that in all cases evaluated, i.e., 2, 3, and 6, the magnetic field or torque was characterized by the product ml . This is a constant of the magnet and represents its action completely. This is fortunate as the accurate location of poles and evaluation of l in real magnets is very difficult. Therefore this product ml is defined as the magnetic moment of the magnet and denoted by the symbol M . The magnetism at the end of a magnetic bar of iron is distributed in a complex manner. The determination of the location of the center of magnetism and hence determination of the distance between the poles composing the magnet is practically impossible with such a distribution. The magnetic moment defines the product of the pole strength by the length of the magnet, which is independent of any knowledge of the distribution of magnetism, but characterizes the action of the magnet on other magnets and in fields. Thus all our equations involving the study of magnets involve simply the evaluation of this convenient constant which sufficiently defines a magnet for practical purposes.

The torque on the magnet in the field is accordingly written as $G = HM \sin \theta$. This equation is very important and will be used freely in what follows.

CHAPTER IV

THE ABSOLUTE DETERMINATION OF POLE STRENGTH AND MAGNETIC FIELDS. THE EARTH'S FIELD — FIELD CONVENTIONS

11. THE ABSOLUTE DETERMINATION OF POLE STRENGTH AND MAGNETIC FIELDS

We now turn to an important question: the method of measuring pole strength. As was stated at the beginning, it is impossible to make direct use of the definition of pole strength in order to measure that quantity. It is essential, however, for us to be able to determine the pole strength m , or the magnetic moment M , in absolute units. If we could determine either the pole strength of a magnet or the magnitude of a uniform field H in absolute units it would be possible thereafter to determine the strength of all other magnets by means of these, using the equation for the torque on a magnet in a uniform field deduced in the last chapter, or the tangent law to be deduced in Chapter V.

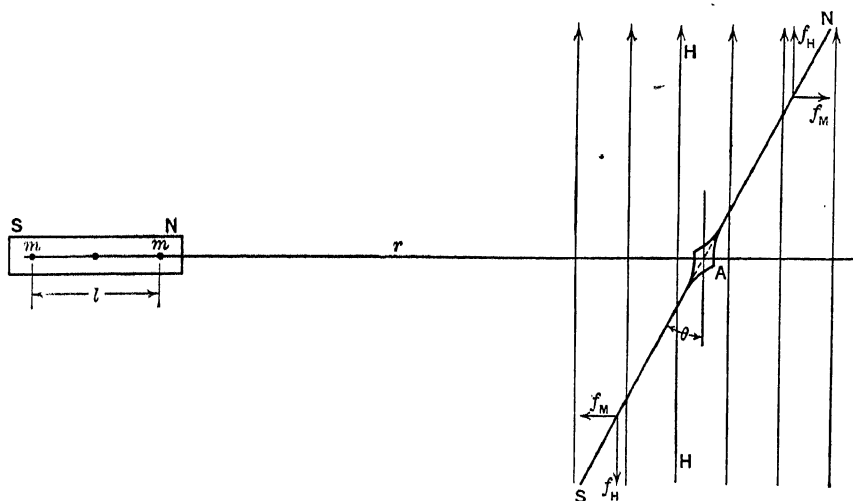
As it originally was impossible to obtain a magnetic field of known value or a magnet of known pole strength it became necessary to measure these quantities indirectly and thus establish standards. The method to be outlined makes use of the fact that the earth in the absence of magnetic materials gives a practically uniform magnetic field. By studying the forces acting on a magnet in such a uniform magnetic field it is possible to determine both the value of the magnetic moment M of the standard magnet and the absolute value of the earth's field H .

Because there are two unknowns, M and H , to be determined, it is necessary to set up two simultaneous equations containing M and H , solution of which will give M and H . This in turn requires two experiments which give two relationships between M and H . The first experiment gives the ratio of the magnetic moment M of the magnet to the strength of the earth's field H , or the quantity $\frac{M}{H}$.

The second experiment gives the product of the magnetic field H and the magnetic moment M of the magnet. From the value of these two ratios, M and H may be solved for, as we have two simultaneous equations with only two unknowns.

12. EXPERIMENT I: M/H

The standard magnet of pole strength m , length l , and moment M , Fig. 17, is placed with its axis at right angles to the earth's magnetic field H . At a distance r from the center of the magnet there is placed a small compass needle represented in the figure by magnet A . As a result of the field due to the bar magnet, the small compass needle suffers a torque tending to pull its south pole toward the magnet and to repel the north pole. At the same time, in the earth's field the small compass needle suffers a pull on its south pole urging it downward in the figure, and on its north pole urging it upward in the figure.

FIG. 17. The magnetometer, Experiment I: M/H .

The force f_M acting on the north pole of this needle due to the bar magnet and the force f_H acting on the same pole tend to cause a rotation of the compass needle in opposite senses. Since the torques vary with the angle θ , the needle of the small compass will come to rest at an angle θ with the field such that the torques due to the earth's field and to the bar magnet are equal.

The conditions for equilibrium may then be determined. G_M represents the torque on the compass needle due to the field H_M produced by the bar magnet at the point A along its axis. This torque acts to rotate the south pole of the compass needle toward the bar magnet. The value of the torque due to the field H_M on the small compass needle of moment M' is, from the equation at the bottom of page 39, $G_M = H_M M' \sin (90 - \theta) = H_M M' \cos \theta$, where θ is the angle of the compass needle with the earth's field H . The torque exerted by the earth's field is $G_H = H M' \sin \theta$ and acts to rotate the

south pole of the compass needle away from the bar magnet. At equilibrium the angle θ is such that $G_H = G_M$, whence $II_M M' \cos \theta = HM' \sin \theta$, so that $II_M/II = \tan \theta$. But by the equation on page 36, $H_M = 2M/r^3$ approximately. Hence one has sensibly

$$\frac{M}{II} = \frac{1}{2} r^3 \tan \theta,$$

if r is large compared to the length of the magnet l . For accurate work obviously the accurate expression for II , on page 36 must be used.

13. EXPERIMENT II: MH^*

In order to get a value of MII the bar magnet referred to in Experiment I is suspended in a stirrup in the earth's field by a fine fiber so that the bar magnet rotates about an axis that is vertical through its center, and perpendicular to its length. The fiber must be so fine that its torsional constant is negligible compared to the torques acting. If such a magnet is twisted from its position of rest parallel to the earth's field a torque is set up that tends to urge it to return to its position of rest. In the last chapter this torque is given by

$$G = IIM \sin \theta.$$

For small values of θ (less than 10°), $\sin \theta$ approximates θ , whence one has

$$MII = \frac{G}{\theta}.$$

Now if $\frac{G}{\theta}$ is known we can get the product MII . $\frac{G}{\theta}$ is nothing else than the torque constant of the system for oscillatory motion. If the fiber suspending it exerts no influence on the period of oscillation, a well-known theorem in mechanics relates this force constant for oscillatory motion to the period of oscillation T of the magnet and its moment of inertia I . This is

$$T = 2\pi \sqrt{\frac{I}{G/\theta}}$$

* An alternative way of determining the product MH is to suspend the bar magnet of moment M perpendicular to the earth's field by a bifilar suspension and to observe the angle through which the magnet is deflected from the rest position by means of a mirror and scale. The determination of the rest position of the magnet and the orientation of the axis of the magnet perpendicular to the earth's field with a bifilar suspension is achieved by replacing the magnet by a dummy magnet made of a non-magnetic substance of the same dimensions. The product MH is then evaluated from the angle of deflection and the characteristic equation for bifilar orientation. This method is, however, much less accurate than the method of oscillations, which is the one in standard use.

Therefore

$$T = 2\pi \sqrt{\frac{I}{MH}}$$

and

$$MH = \frac{4\pi^2 I}{T^2}.$$

By Experiment I, however, we had

$$\frac{M}{H} = \frac{r^3 \tan \theta}{2}.$$

Solving the two equations we obtain

$$H^2 = \frac{8\pi^2 I}{T^2 r^3 \tan \theta}$$

and

$$M^2 = \frac{2\pi^2 I}{T^2} r^3 \tan \theta.$$

Thus by the two measurements the value of M , the magnetic moment of the standard bar magnet, and the value of H , the earth's field, have been obtained. This is the fundamental measurement of the magnetic quantities in terms of the absolute c.g.s. system of units. The method is therefore the basis of all our quantitative knowledge of magnetic phenomena and as such is of prime importance. Either M or H having been determined, the evaluation of any other magnetic poles or fields is a simple matter by use of the tangent law, or the torsion balance. (See the end of Chapter V.) The instrument used in making the determination of MH is known as the *magnetometer** and it is used quite widely in studying the earth's magnetic field. The magnetic standards having once been established by magnetometer

* World War II with its magnetic mines required very sensitive, rapid measurement of magnetic fields or their components under water. These needs led to the development of the fluxgate magnetometers described more fully on page 323. These complicated but sensitive devices operate by means of the effect of the impressed fields on a nearly saturated Permalloy core. This acts as the core of a 500-cycle transformer which produces unsymmetrical oscillations in a secondary. These when rectified and amplified produce currents read on microammeters. The instruments are used as null instruments by having a compensating coil with a known field produced by a known current. Whenever the magnetometer is to measure a field its galvanometer readings are reduced to zero in a known field by the compensating coil current. In its new environment it then requires a readjustment of current to read zero. The compensating currents at once give the change in field. It can read to milli oersteds, i.e., 10^{-3} oersted, in fields of the order of 1 oersted. With automatic recording, a hundred such instruments can be read in 5 minutes. Properly calibrated, fluxgate magnetometers will be of great use in survey work in the future.

methods it is possible to calibrate magnetic fields produced by currents so that in terms of the current standards it is now possible to produce magnetic fields of known value without recourse to the investigations outlined.

14. NOTE ON THE USE OF OSCILLATING SYSTEMS IN MEASURING FIELDS

In section 13 it was seen that II and M could be evaluated by observing the period of oscillation of a suspended magnet and knowing its moment of inertia. It is of interest to consider the practical measurement. If a standard bar magnet is placed on a jeweled pivot and then placed in an evacuated space, it will oscillate freely with little damping if disturbed from its N and S position by a magnet temporarily brought up. By counting successive swings to one side over an accurate time interval, which could be done by a mirror, a spot of light, and a self-recording photocell, T can be determined with great precision.

As usually, I , the moment of inertia of standard magnets with mirrors and pivots cannot accurately be computed, it can be evaluated as follows. The bar magnet M is so arranged that it can be loaded by placing on it an accurately manufactured and centered nonmagnetic metal ring of known moment of inertia I' . T can then be measured for the unloaded magnet with unknown moment I in the field H . Then adding I' so that the total moment is $I + I'$ the new period T' can be observed in the same field H . Thus $T = 2\pi\sqrt{\frac{I}{MH}}$ and $T' = 2\pi\sqrt{\frac{I + I'}{MH}}$ can be written. From these two relations are obtained $\frac{T}{T'} = \sqrt{\frac{I}{I + I'}}$, whence I is determined.

Again the periods can be used to measure or compare fields. If the same standard magnet M , with constant I , is used and its periods T and T' measured in two fields II and H' , one known, the other unknown, we can write:

$$T = 2\pi\sqrt{\frac{I}{MH}}, \quad T' = 2\pi\sqrt{\frac{I}{MH'}}, \quad \text{whence} \quad \frac{T}{T'} = \sqrt{\frac{H'}{H}}.$$

This device is frequently used in the comparison of fields.

Where a fiber suspension is used the torque produced by the twisted suspension must be considered. In such instances the fiber is adjusted to be at rest with the magnet axis initially normal to the field H . When the magnet is left free to move it will have its axis twisted through an angle θ such that the torque $G = T_0\theta = MH \cos \theta$. By definition, the torque is $G = T_0\theta$, where T_0 , the torsional constant of

the fiber, can be computed from the characteristics of the fiber through the relation, $T_0 = \frac{\pi r^4 \eta}{2 l}$. Here r is its radius, l is its length, and η is the coefficient of rigidity found in tables for any one substance. η is usually given in absolute units. If r and l are in centimeters, T_0 is dynes \times cm, per radian. G is in dynes \times cm when θ is in radians.

15. THE EARTH'S MAGNETIC FIELD

The knowledge of the earth's magnetic field which can be obtained by the magnetometer is of considerable importance in many domains. The magnetic field of the earth has been measured over a greater part of the earth's surface and is constantly being measured at certain fixed stations.* This knowledge of the earth's magnetic field, in addition to helping to answer questions of theoretical interest such as the origin of the earth's magnetic field, has been an aid to navigation. Practically all merchant ships still use the magnetic compass, and accurate knowledge of the magnetic field of the earth is therefore essential in the making of marine charts. To some extent the so-called gyrostatic compasses are replacing the magnetic. They are mechanically complicated and subject to mechanical failure. A *failure of the gyrostatic compass* leaves a ship without means for proper navigation. Therefore, there is no ship in general service that is not equipped with an auxiliary magnetic compass. When a ship is under weigh the gyrocompasses are checked hourly and even more frequently against the standard magnetic compasses because the mechanically complex gyrocompass has a tendency to get out of order and undergo a precessional motion which over a course of time causes a progressively greater error in its indication.†

The direction of the magnetic field of the earth with respect to the

* The introduction of magnetic mines of great sensitivity into warfare in World War II necessitated in the years 1942 to 1945 a complete and careful resurvey of the earth's magnetic field, for the mines operated on changes of the vertical intensity of the earth's magnetic field which required determinations of the vertical component.

† The importance attached to the magnetic compass was further emphasized by the conditions encountered in World War II. The demagnetization of ships by use of compensating coils carrying currents, which were wound around the hull for protection against magnetic mines, i.e., *degaussing*, destroyed or altered the directive force of the earth's field at the compasses of the steel ships. To counteract this effect, counter-degaussing coils were wound around the binnacle housing the compass and were shunted across the degaussing coils of the ship. Thus, wherever degaussing was put on, the counteracting coils about the compass compensated for this and the compass responded normally to the earth's field. The effect of the removal of the permanent longitudinal magnetism of ships by various cycles, known as *deperming*, also altered the magnetism of the ship's hull so that compasses had to be recompensated at frequent intervals. The governments of Great Britain and the United States spent millions of dollars during the war in installing and adjusting the compass compensating coils as well as in recompensating compasses. These procedures were carried out both for merchant and combatant vessels, gyro or no gyro.

earth's surface varies with the point on the earth's surface chosen. At or near the equator the lines of force run parallel to the earth's surface. This, however, is not the case at the earth's poles. In fact, even in Berkeley, the compass needle if suspended on a horizontal axis would dip downward at an angle of about 70° with the horizontal. Furthermore, the magnetic needle does not point true north, that is, the magnetic axis of the earth is not the same as the axis of rotation. The location of the magnetic north pole is given by

$$N = 70^\circ 5' \text{ N lat, } 96^\circ 46' \text{ W longitude}$$

and the location of the south magnetic pole is given by

$$S = 72^\circ 25' \text{ S lat, } 155^\circ 16' \text{ E longitude}$$

Thus it is seen that not only does the magnetic axis of the earth deviate materially from its axis of rotation but the magnetic axis does not even pass through the center of the earth. Furthermore, the angle between the true north and the magnetic needle is not constant from year to year. For instance, in 1894, it was $17^\circ 0'$ west of true north at London. In 1910, it was $16^\circ 5'$ west of true north at London. In Berkeley, the magnetic north is $18^\circ 20'$ east of true north.

The deviation of the magnetic needle from the true north is called the *declination*.* The inclination of the needle, mounted on a horizontal axis, with the horizontal is in Berkeley about 70° and is called the *dip* or *magnetic inclination*. As stated before, the dip is 0 at the magnetic equator and approaches 90° near the poles (see Fig. 1). The intensity of the field H , as defined and measured above, acting on our compass needles which are mounted horizontally is therefore not the total intensity of the earth's field, for the field acts at an angle θ with the horizontal. The H mentioned in the discussion early in the chapter is the *horizontal component of the earth's field*. If the horizontal component H and the angle of dip, θ , are known the total magnetic intensity of the earth's field H_m may be obtained by the relation

$$H_m = \frac{H}{\cos \theta}.$$

For the use of mariners and of students of the earth's magnetic field maps have been constructed to show:

1. Lines of equal declination called *isogonic* lines.
2. Lines of equal dip called *isoclinic* lines.
3. Lines of equal intensity known as *isodynamic* lines.
4. Lines of equal vertical intensity.

Despite the fact that the knowledge of the earth's magnetic field

* In *maritime* terminology the declination is called the *variation*. The error in compass reading, the shifting from the correct magnetic indication that results from a ship's magnetism, is called the *deviation*. *Compass error* is the resultant combined effect of deviation and variation relative to true geographical direction.

dates back to A.D. 1100, the mechanisms giving rise to it are not known. The earth's field is seriously disturbed by magnetic storms on the sun, and by showers of charged particles, electrons and protons, erupted by solar explosions.

16. CONVENTIONAL REPRESENTATION OF FIELDS OF FORCE

The representation of the fields of force in terms of such maps is more useful for the mariner than for the physicist. It is often convenient for the physicist to represent a magnetic field diagrammatically or visually. In fact, both Faraday and Maxwell pictured forces between magnetic or electrified bodies as represented by hypothetical *lines of force*. Between an N and an S magnetic pole, lines of force emerged from the north pole and ended on the south pole. The attractive force, then, was caused by a tension or contractive force along the line. The more lines of force between any two poles the stronger the force. These lines of force also had the property of repelling *each other* by forces perpendicular to their length. Thus, the lines of force moving outward from two adjacent north poles would push as far into the space between the poles as they could until the repulsive forces between the lines caused the lines to bend laterally (see Fig. 4). The repulsion between lines of force also explains the bulging of the crowded lines between a north and a south pole (Fig. 3).

It is clear that the greater the magnetic force and hence the field strength, the more lines there would be threading through the space. Thus it is not surprising to note that this convenient hypothetical picture, represented accurately mathematically by Maxwell's equations but otherwise only a fiction, should have led to the convention for depicting a magnetic or an electrical force field.

A unit magnetic field, i.e., a field of 1 oersted, is depicted by drawing one line per square centimeter of area taken normal to the line representing the direction of the force. Thus, 1 oersted of magnetic field is represented by 1 line of force per square centimeter of area taken normal to the field direction. A field intensity of H oersteds will have H lines of force per square centimeter normal to the field direction.

Thus, if it is wished to represent a field of 4 oersteds 4 lines of force per square centimeter would be drawn as shown in Fig. 18a and for the local earth's horizontal component of 0.25 oersted 1 line would be drawn to every 4 cm², as in Fig. 18b. If it is desired to plot the field on a plane \sqrt{H} lines per linear centimeter measured perpendicular to the lines would be drawn as in Fig. 19a and Fig. 19b for 4.0 and 0.25 oersteds, respectively. The distance between lines is then $\sqrt{\frac{1}{H}}$ cm.

In employing the *description* by means of lines of force and in many problems encountered in electromagnetics, physicists and engineers are not as much concerned with the field strength or magnetic intensity

as in the total number of lines of force threading through a given area, irrespective of the density of lines of force. We then speak of the flux of lines of force or simply the *magnetic flux*. The unit of flux is 1 line of force, and to shorten description it is called the *maxwell*. Thus, it

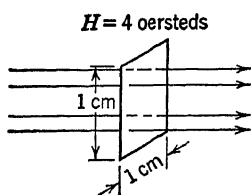


FIG. 18a.

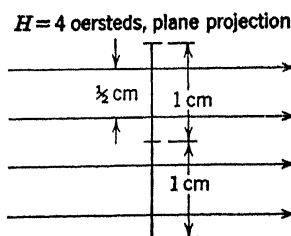


FIG. 19a.

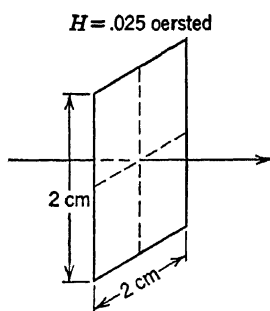


FIG. 18b.

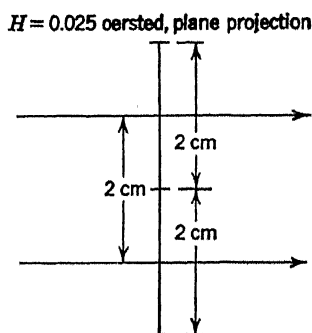


FIG. 19b.

Depicting the magnetic fields by lines of force. 18a shows a field of 4 oersteds. 18b shows a field of 0.25 oersted.

Depicting the magnetic fields on a plane. 19a shows a field of 4 oersteds. 19b shows a field of 0.25 oersted.

is easier to say 10^8 maxwells than to say 10^8 lines of force. An oersted of field intensity, or strength, is thus 1 maxwell per square centimeter. Similarly, the flux, represented by a symbol, ϕ , is the area A normal to the flux times the field strength H . Hence, $\phi = AH$. In tracing the lines of magnetic force from one magnetic region to another a region with a certain number of maxwells emerging from one pole can be chosen and, retaining the same lines, the same flux can be followed to its other pole. The surface enveloping the constant flux is called a *tube of force*. Since ϕ is known for such a tube the field H at any point is inversely proportional to A , i.e., $H = \frac{\phi}{A}$. This has certain useful applications in practice.

There is a very important and direct consequence of the assignment

of 1 line per square centimeter as unit field intensity. Consider an hypothetical isolated N pole of strength m at a point in space. There emerge from it lines of force radiating in all directions. Let us draw a sphere of radius $r = 1$ cm about the pole m (see Fig. 20). At the surface of the sphere it is known that $H = \frac{f}{m'} = \frac{mm'}{m'r^2} = \frac{m}{r^2}$, or since $r = 1$ cm, $H = m$. Thus, there are m lines of force emerging radially perpendicular to the sphere per square centimeter. But the surface area of a sphere is $4\pi r^2$ and the area when $r = 1$ is 4π cm². Thus,

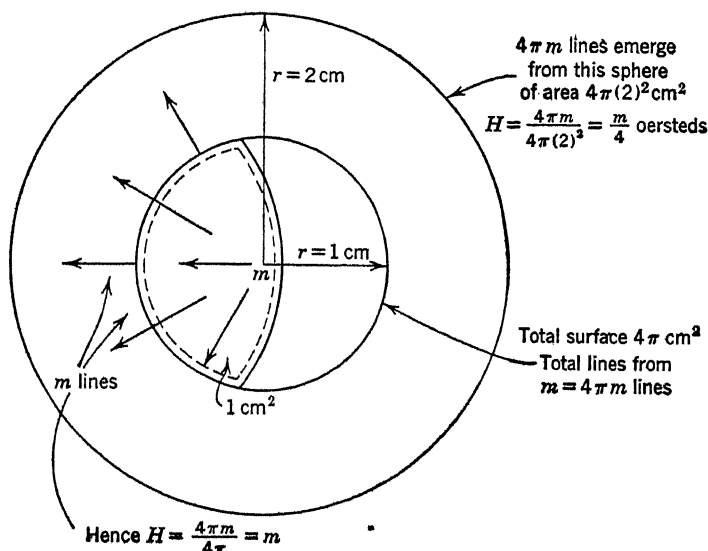


FIG. 20. The unit sphere about a pole m in which m lines emerge per square centimeter of surface and $4\pi m$ lines emerge from the whole unit sphere and thus from m .

from the surface of the unit sphere there emerge $4\pi m$ lines of force. Hence, in consequence of the definition of unit field given here it is seen that from a pole of strength m there will be a total of $4\pi m$ lines emerging. Thus it can at once be said that the total flux from a pole of strength m is $4\pi m$ lines or, better, $4\pi m$ maxwells.*

This is a most useful relation for it allows the calculation of the number of unit poles, or pole strength, from a flux, and vice versa. Furthermore, where the areas of a magnetized surface are known, since the flux is always perpendicular to a magnetic surface, the field

* A flux of 10^8 maxwells has significance, since if 10^8 maxwells are cut by one conductor a second the conductor gets 1 volt of e.m.f. generated. There is an absolute unit of pole strength that is being revived with the m.k.s. system, in some text-books called the *weber*. A *weber* is a pole strength that gives 10^8 maxwells, and therefore 1 *weber* = $10^8/4\pi$ units of pole strength. The *weber* is not frequently encountered and its use is not encouraged.

strengths can be calculated from the number of poles per square centimeter of surface, an exceedingly useful procedure.

Since the number of lines emerging from a pole of strength m , is $4\pi m$ and constant, in receding from the pole to greater and greater distances r the surface area increases as r^2 and thus the number of lines per square centimeter, or the field intensity, decreases as $\frac{1}{r^2}$.

CHAPTER V

ELECTRIC CURRENTS

17. HISTORICAL SKETCH

The discovery of electrical currents dates from Gray and Du Fay in 1729, who showed that a static electrical charge was carried from one body to another by means of conducting wires. Before 1750 it was known that the velocity of transport of electricity was very high. No further progress could be made in the study of currents owing to the fact that the currents from static charges flowed over such short intervals and were so weak that experimentation was impossible. In the period 1786 to 1799 the discovery of the means of producing larger currents was made. It came as a result of the investigations stimulated by the physiological researches of Galvani. He had observed that a static machine on a table made frogs' muscles twitch. Franklin's experiment with the kite had shown the identity of static electricity and lightning. Galvani therefore hung muscles on wires in the air. As might have been expected from Franklin's work, Galvani observed twitches. However, twitches occurred in the absence of thunder showers. Galvani found that a single wire in some cases sufficed to cause the twitching. He explained the effect as coming from the muscle nerve system, as such effects had been observed in the electric fish. Volta was more discerning. He ascribed the effect to the metal. He found that the effect was strong and reproducible when wires from two metals in contact were touched to two parts of the nerve. He found that one metal sufficed if there were a temperature gradient in the metal. The effect was made still more pronounced if the two metals were separated by a damp cloth containing an electrolyte. This was the origin of the voltaic pile, or electric cell. Volta showed by means of an electroscope and multiplier (see page 224) that the effect was produced by an electrical charge. He found that if he took a series of disks of two different metals separated alternately by wet cloths he obtained an additive effect so that the electrical effect of one element was multiplied by the number of elements used. Such sources of electrical current enabled currents to be investigated. Today, there are five sources of electrical currents:

1. The flow of static electricity.
2. The voltaic pile.
3. Heating of a junction of two metals.

4. The cutting of magnetic lines of force by a conductor (Faraday, 1831).

5. Animal electrification, which is merely a manifestation of voltaic pile activities in organic tissues.

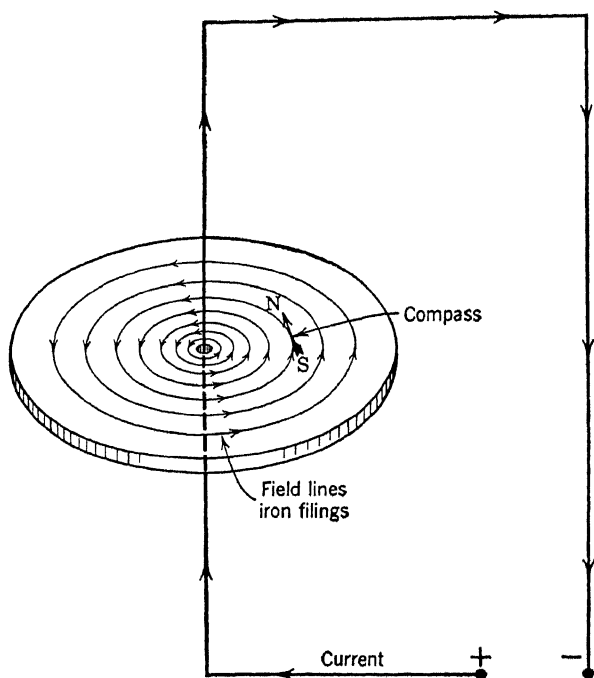


FIG. 21. Magnetic field in a plane normal to a vertical wire carrying a current. Note the decreased intensity as one recedes from the wire, shown by wider spacing between lines.

18. MAGNETIC FIELD OF A CURRENT

Until 1819, no quantitative measurements of electric currents were possible. Oersted was the discoverer of a phenomenon which led to a means of measuring the currents. Oersted, having noticed the electrical polarity of the voltaic pile, looked for a magnetic effect because magnets were known to have polarity. In experimenting with the circuits of electricity he observed that when an electrical circuit from a cell was closed a magnetic needle near one of the conducting wires was deflected. The investigation showed that there was a magnetic field about a conductor carrying a current. The character of the field around a straight conductor carrying a current is shown in Fig. 21.

The nature of the field produced was discovered by Ampère.

It is best grasped by memorizing a simple rule known as the *right-hand rule*. This rule says that if the thumb of the *right hand* indicates the direction in which the current is flowing the lines of magnetic force circle the conductor in the direction of the fingers of the closed hand. That is, if a wire carries a current as indicated in Fig. 22, an isolated north magnetic pole would move about the conductor in the direction

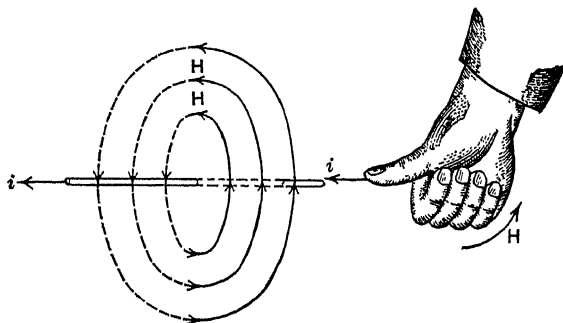


FIG. 22. The right-hand rule.

of the arrows. Although on the whole it is a poor policy to memorize several arbitrary rules such as the right-hand rule, the dynamo rule, and the motor rule, an exception may be made for the right-hand rule. It will be found that if this rule be remembered the interactions of wires carrying currents in magnetic fields of any sort can be predicted. (See Chapter XVI.)

19. AMPERE'S RULE AND DEFINITION OF UNIT CURRENT

In 1820 a brilliant experimentalist investigated the laws of currents. This was A. M. Ampère, and his study, which is the basis of all modern current measurements and is the essential to most current calculations, led to a general formulation of the field produced by the current. From these investigations have followed the definition of the unit of electrical current and the foundation of the so-called *electromagnetic system of units*. Ampère showed that the force on a magnet pole in the neighborhood of a wire carrying a current is proportional to the length of the wire *taken perpendicular to the line joining the element to the point considered*, is proportional to the current, and is inversely proportional to the square of the distance from that current. The law may thus be expressed in the following equation: The force f on a magnetic pole m , at a distance r , from an element of conduction of length, ds , perpendicular to r , carrying a current i_a , is given by

$$f = \frac{mi_a ds}{r^2}$$

If the conductor carrying a current be bent in the form of a circle of radius r , Fig. 23a, the center of the circle will everywhere be equally distant from the wire, and the line from the center of the circle to the conductor will be everywhere perpendicular to the current. Following the direction of the current in the figure, the field due to the

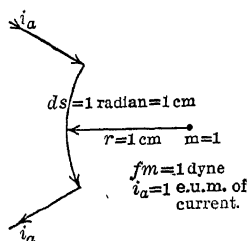


FIG. 23a. Definition of unit current.

current will urge a north pole at m out of the paper, and if the pole at m were held in position the wire would by reaction be urged downward into the paper. Thus the force f of the current on the pole m and reciprocally the force f of the pole on the current is given as above under conditions fulfilling the experimental law. By modifying this scheme slightly a device for defining unit current can be arrived at. For let r be made 1 cm, let the wire be bent so that it makes an arc of radius r , of length $r = 1 \text{ cm}$, i.e., that the wire subtends an angle of 1 radian at its center m . Then if m is unit pole and $f = 1 \text{ dyne}$ of force, $i_a = \frac{r^2 f}{m ds} = 1$ and i_a is 1 *absolute* electromagnetic unit of current.

This leads to the famous definition learned parrot-like by every high school physics student, which reads: "Unit current is that current which, flowing in unit length of conductor bent into a circle of unit radius, acts with a force of 1 dyne on unit magnetic pole placed at the center." Actually it is far simpler to remember the *form* of the equation and write $i_a = \frac{r^2 f}{m ds}$ such that i_a will be 1 absolute electromagnetic unit if $r = 1 \text{ cm}$, $f = 1 \text{ dyne}$, $m = \text{unit pole}$, and $ds = 1 \text{ cm}$ perpendicular to r . It is this system of definition of units by means of defining equations which will uniformly be adhered to in this book.

The Absolute Electromagnetic System. Returning to the new concept which we have just defined, it must be noted that as a result of an experimental observation it is found that a voltaic pile attached to a wire *produces a condition in that wire* which is manifested by a magnetic field. From the fact that a voltaic pile is a source of electrification and that from static experiments electrification is known to flow along a wire, this magnetic force was considered as being a manifestation of a current of electricity. Much later, with better instruments it was actually shown that a current of static electricity produces a magnetic effect. Experimental study in 1820 revealed the way in which the force f varies with r , m , ds , and a state of affairs in the wire, i.e., the assumed current. This state or current depends on the number of piles, the area, length, and material of the wire, etc. Thus we are led to define the unit of measure of this current by making the factors influencing its magnetic effect each unity in the c.g.s.

system of units. We have thus accomplished an interesting thing in that we have by means of a measurable magnetic effect obtained a means of defining the unit and hence of measuring an *electric* current. This establishes the unit of current as an *electromagnetic* one in view of the manner of its detection. When static electricity is studied, it will be found that there is another way of measuring current, and of defining it. Hence it is seen that in the magnetic definition there is the beginning of a system of measuring and describing electrical events called the electromagnetic system.

20. ALTERNATIVE DEFINITION OF UNIT CURRENT

The definition just given, which comes directly from the experimental law, is awkward when applied to a practical study of forces between electrical currents and fields. A corollary to that formulation is much more practical and useful. It was indicated that the current in Fig. 23a was unity when it acted with a force f of 1 dyne on a unit pole m , i.e., when it produced a field of 1 oersted at the center. Conversely at 1 cm from a unit pole m the field H is unity and is everywhere perpendicular to the wire. Thus a unit current could be called a current which in unit length of conductor everywhere perpendicular to unit magnetic field experiences a force of 1 dyne. This definition says nothing of the form of the wire. Hence if we could imagine a uniform field as shown by the horizontal lines in Fig. 23b perpendicular to a wire 1 cm long, then unit current will cause the force on the wire to be 1 dyne. Thus unit current can be defined as that current unit length of which flowing perpendicular to a uniform magnetic field of unit strength suffers a force of 1 dyne. This rule at once permits us to state that if the current is i_a and the length of the wire perpendicular to the uniform field of strength H is l centimeters, the force will be given at once by $f = i_a l H$. Thus the defining equation for unit current can be written $i_a = \frac{f}{Hl}$, where H is uniform and

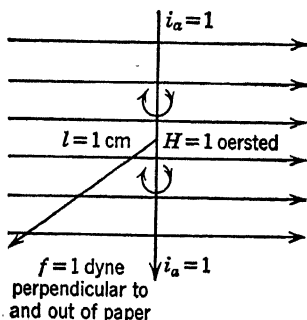


FIG. 23b. Alternative definition of unit current.

l perpendicular to H . $i_a = 1$ if $f = 1$ dyne, $H = 1$ oersted, and $l = 1$ cm. This equation will be applied as the basis of the study of many kinds of the action of electromagnetic forces, and in addition it furnishes the principle underlying the achievement of most of the present current-measuring devices.

Practical Unit of Current. In practice, the unit electrical current here defined is larger than the currents commonly dealt with. In

order to have a *practical* unit which is convenient in magnitude (i.e., a unit of such magnitude that values of the current must not always be written as a fractional part of the unit), *the practical unit of current is taken as one-tenth the absolute electromagnetic unit. It is called the ampere.* It is to be noted that in this situation, as in many others, the absolute units are by their magnitudes not convenient for practical use. Hence, aside from a set of absolute electromagnetically defined units, a parallel set of conveniently chosen *practical* units will be continually in use. These have been named in honor of the scientists whose studies did most to establish their respective concepts. Most absolute units have no names. In this book the practical units will be referred to by name and for current the symbol i will be applied for the ampere and i_a for the absolute electromagnetic unit.

21. DEFINITION OF QUANTITY IN ELECTROMAGNETIC SYSTEM OF UNITS

The definition of current of electricity leads at once to a definition of quantity of electricity in the electromagnetic system. The definition is based on an analogy between the flow of water and the flow of electrical current. The quantity of water which is flowing through a given pipe is defined in terms of the current multiplied by the time of flow. Similarly, the quantity of electricity is the current multiplied by the time during which it flows. *Thus the unit of quantity in the absolute electromagnetic system is that quantity which is represented by a current of 1 absolute electromagnetic unit flowing for 1 sec.* Symbolically, this may be represented by writing quantity q is equal to current i_a times time t , or $q = i_a t$. The practical unit of quantity is again one-tenth the absolute unit of quantity. *It is represented by the flow of 1 ampere of current for 1 second.* It is named the *coulomb* and will be represented by the symbol Q .

22. APPLICATIONS OF AMPÈRE'S LAW

We now turn to applications of Ampère's rule to several simple cases which have practical uses.

1. The Law of Biot and Savart. This law gives the field produced at any distance r from an infinitely long straight conductor carrying a current i_a . For practical purposes all that is needed is that the wire be straight and long compared to the distance r . The nature of this field is shown in Fig. 21.

Let AB , Fig. 24, be a long straight wire. Let P be a point distant r cm from it. Let any element of the wire dl be chosen, the lower end of which when joined to P gives a line Pdl which makes an angle θ with the line r . Call $d\theta$ the angle subtended by dl at P . Let ds be the projection of dl on the normal to the line joining dl and P , and let h represent the length of the line from P to dl .

Ampère's law says that contribution dH to the field at P by the projection of the element of wire dl on the normal to the line joining dl and the point P at which the field is required is given by

$$dH = \frac{i_a dl}{h^2} \cos \theta = \frac{i_a ds}{h^2},$$

for it is only the component of dl normal to r that by Ampère's law contributes to the field. Now

$$ds = h d\theta,$$

hence

$$dH = \frac{i_a d\theta}{h}.$$

As

$$h = \frac{r}{\cos \theta},$$

therefore

$$dH = \frac{i_a \cos \theta}{r} d\theta.$$

To get the total field at P due to all the little elements, dl or their projections ds , the contributions dH to this field must be integrated for all the little elements dl , that is, dH must be integrated for elements dl running along the wire, from minus infinity to plus infinity distant from the perpendicular r . For the positive value of infinity, θ has the value $\frac{\pi}{2}$. For the negative value it has the value $-\frac{\pi}{2}$. Hence,

$$H = \int_{dH \text{ at } -\infty}^{dH \text{ at } +\infty} dH = \frac{i_a}{r} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos \theta d\theta = \frac{2 i_a}{r}.$$

The field, therefore, at any distance r from the long straight wire is merely twice the current in *absolute units* divided by the distance r . The field strength thus obtained will be expressed in oersteds, or dynes of force per unit pole. This is called Biot and Savart's Law after its discoverers.

There is a simple experimental proof of the correctness of this law. Let us place a bar magnet B on a frame F so that it is rigidly fixed (i.e., cannot rotate about its pivot) perpendicular to a long wire W carrying a current, as in Fig. 25. The frame F is so mounted that the frame and bar magnet are free to rotate about the long wire following the lines of force. Now suppose the north pole of the bar magnet to be nearer the wire than the south pole. If the current i_a is flowing upward in the wire, the north pole will be urged in the direction of the

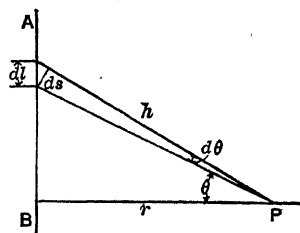


FIG. 24. Field at a point due to an infinitely long straight conductor.

arrows and the south pole will be urged in the opposite direction. The torque on the north pole will be $G_N = H_N m r_N$, where H_N is the field at r_N centimeters from the wire acting on the N pole of strength m . Likewise the torque on the south pole is $G_S = H_S m r_S$, where the subscripts s denote the same quantities for the south pole. On closing

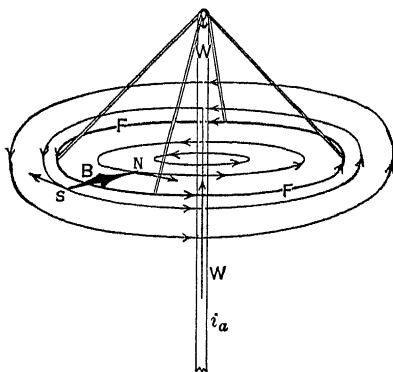


FIG. 25. Experimental proof of Biot and Savart's law.

the switch no movement is noted. Hence $G_S = G_N$, and hence $H_S m r_S = H_N m r_N$, and therefore $\frac{H_N}{H_S} = \frac{r_S}{r_N}$. This is exactly what

Biot and Savart's law, which says that $H_N = \frac{2 i_a}{r_N}$ and $H_S = \frac{2 i_a}{r_S}$, leads one to conclude, for $\frac{H_N}{H_S} = \frac{2 i_a}{r_N} \bigg/ \frac{2 i_a}{r_S} = \frac{r_S}{r_N}$. It is of interest to note that the discovery of this law gave one of the first proofs that the magnetic field due to a current varied as $\frac{1}{r^2}$.

2. The Field at the Center of a Plane Circular Coil of Radius r . Ampère's rule says that the force on a unit pole by a current in a wire is given by

$$f = \frac{i_a ds}{r^2}.$$

In this case r is the radius of the coil, and ds is the length of the arc normal to the line from the center of the circle to the coil. As the radius of the circular coil is always perpendicular to the arc, the expression for the *magnetic field* becomes

$$H = \int_0^H dH = \frac{i_a}{r^2} \int_0^{2\pi} ds = \frac{i_a 2 \pi r}{r^2} = \frac{2 \pi i_a}{r}.$$

For n turns of wire in the coil, this will be n times as great, or $H_n = \frac{2 \pi n i_a}{r}$.

3. The Field Produced by a Plane Circular Coil at a Point Distant d Centimeters from the Plane of the Coil, Along the Normal to the Coil, at Its Center. Let the coil be represented by the circle drawn in perspective in Fig. 26. Call O the center of the coil, and let its axis be the line OB . A current i_a is flowing around this coil, the radius of which is r . It is desired to know the magnetic field in magnitude and direction at a point P , at a distance x from the center of the coil. Take any element ds of the coil and draw a line joining ds to P , and r

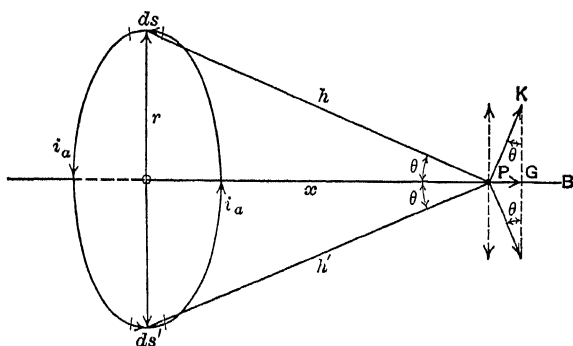


FIG. 26. Field on the axis of a circular coil.

joining it to O . According to the right-hand rule the current flowing in the coil, as indicated by the arrows, produces a field at P at right angles to h , the line joining ds and P , as indicated by PK in the diagram. Let the length of the line from ds to P be h , and let θ be the angle between h and x . Now, it is obvious that if all the elements ds in the circumference of the circular coil are regarded, there will be for each element an equal element ds' , at the opposite end of the diameter of the circle. The fields produced by two such elements will be of the same magnitude but in directions such that they will in part neutralize each other, that is, their components normal to the axis will cancel each other. The resultant field will be the sum of the components of the fields produced by the elements ds which are not annihilated by the fact that each element ds is paired against a similar element at the end of its diameter. Therefore, the component of the field due to the element ds , which is of importance, will be the projection of the vector represented by PK on the line OP ; that is, it will be the force due to the element ds multiplied by $\sin \theta$ represented by PG . One is now in a position to apply Ampère's rule. This says that $dH = \frac{i_a ds}{r^2}$. In

this case, the element of field in which we are interested is the component parallel to the axis of the coil and is consequently called:

$$dH_1 = dH \sin \theta.$$

Furthermore, r in Ampère's equation is the length of the line from ds to P , or h . If there are n turns of wire in the coil the expression for dH_1 must be multiplied by n .

Hence,

$$dH_1 = \frac{ni_a ds \sin \theta}{h^2}.$$

As

$$h = \frac{r}{\sin \theta}, \quad dH_1 = \frac{ni_a r ds}{h^3}.$$

But,

$$h^2 = r^2 + x^2,$$

and

$$H_1 = \int_0^{H_1} dH_1 = \frac{ni_a r}{(\sqrt{r^2 + x^2})^3} \int_0^{2\pi r} ds = \frac{2 \pi ni_a r^2}{(r^2 + x^2)^{3/2}}.$$

At a point at the center of the coil $x = 0$, H_1 becomes equal to $H_1 = \frac{2 \pi ni_a}{r}$, in agreement with Case 2. It is seen that H_1 varies with x and decreases from $x = 0$ to $x = \infty$ along a curve such as shown for H_A in Fig. 27b. The rate of change of H_1 with x is the derivative of H_1 with respect to x and is $y = \frac{dH_1}{dx}$. Now this rate of change of H_1 with distance will become constant at some point along the axis for there is a point of inflection. The point can be found by taking $\frac{dy}{dx}$ and setting it equal to 0. Thus if the quantity $\frac{d^2 H_1}{dx^2}$ be found by twice differentiating H_1 as given above with respect to x and if we set $\frac{d^2 H_1}{dx^2} = 0$, solution of the resulting equation shows that at $x = \frac{r}{2}$ the rate of change of H_1 with x is constant. Hence if we have two coils that are identical and place them on the same axis each $\frac{r}{2}$ cm distant from a point P on the axis midway between them, H_1 is the sum of the value of H_1 from the two coils at P , and is quite constant for some distance on each side of P on the axis. The arrangement of a pair of such coils is shown in Fig. 27a. The fields produced by the pair of coils are shown in Fig. 27b. Such a pair of coils are called Helm-

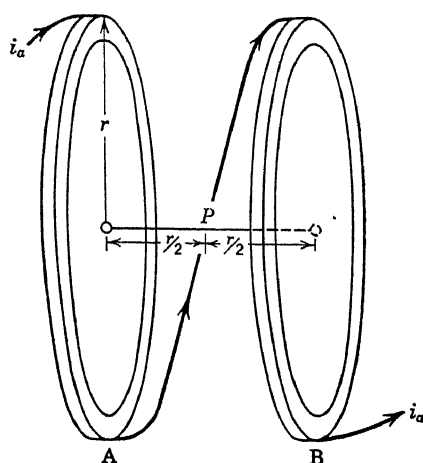
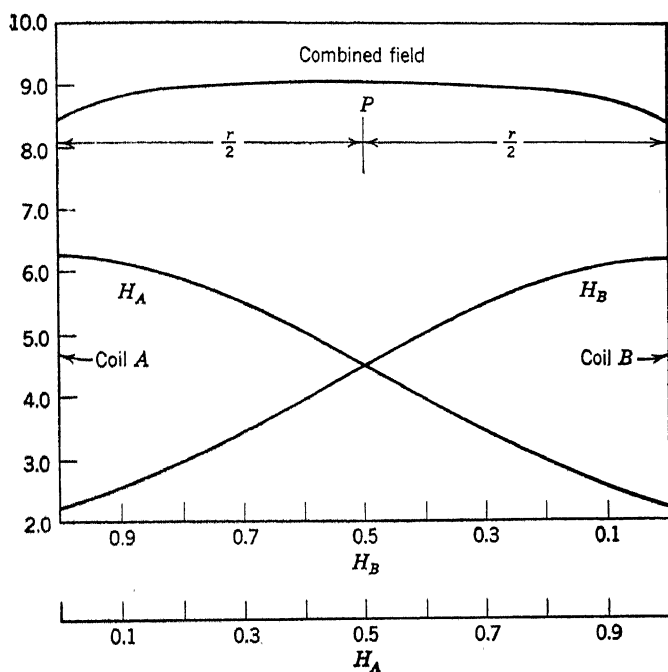


FIG. 27a. Arrangement of Helmholtz coils.

FIG. 27b. Separate and combined fields of Helmholtz coils. Note the point of inflection where H_A and H_B cross. Note, also, the region of constant resultant field at P.

holtz coils; they are of great practical value in the achievement of constant fields.* The value of H_1 in this case is twice the field that would exist at $\frac{r}{2}$ for 1 coil, for both coils have H_1 in the same direction, i.e.,

$$H_1 = \frac{4 \pi n i_a r^2}{\left(r^2 + \frac{r^2}{4}\right)^{3/2}} = \frac{32 \pi n i_a}{(5)^{3/2} r} = \frac{2.86 \pi n i_a}{r}.$$

23. THE TANGENT GALVANOMETER

In practice, i_a is measured by means of the circular coil discussed in Case 2. The instrument formerly used for this is known as the *tangent galvanometer*. The term galvanometer indicates measurer of galvanic currents. The type of galvanometer receives its designation from the essential feature of the measuring device to be discussed. The *tangent galvanometer* makes use of the comparison of the field produced by the current, and the field of the earth. For this purpose it is necessary to know the earth's field accurately — hence the necessity for the careful magnetometric measurement of the earth's field discussed in Chapter IV.

Call the field of the earth H , and represent it by the parallel lines of Fig. 28. Set the coil of the tangent galvanometer with its plane *parallel to the earth's field* as seen in the diagram. The field produced by this coil will then be at right angles to the earth's field. Besides the coil the tangent galvanometer has a compass needle mounted on an axis at the center of the coil. It is mounted to swing in a horizontal plane, and its angle with the plane of the coil can be read on a graduated scale. The magnetic needle of moment M indicated in the diagram will thus be subjected to torques from two sources. The field of the coil will cause it to try to set itself at right angles to the plane of the coil. The earth's field will cause it to try to set itself in the plane of the coil. As a result the needle will come to rest at an angle θ with the direction of the earth's field and the plane of the coil. This angle θ will be determined by the relative strengths of the earth's field H and the field H' of the current in the coil, as the following consideration will show. The torque due to the field H is given by $G_H = HM \sin \theta$, where M is the moment of the needle (see page 39). The torque due to

* The necessity for relatively weak uniform magnetic fields of large volume for testing purposes as a result of magnetic studies necessitated by magnetic mines and other problems in World War II has resulted in a considerable extension of coil design for the achievement of uniform fields. A study by the Naval Ordnance Laboratory has resulted in a system of five rectangular coils with proper spacing and turn distribution which gives volumes with fields uniform to some tenths of a per cent over meters of length. See S. M. Rubens, *Review of Scientific Instruments*, Vol. 16, page 243, 1945.

the current in the coil is G_H' , and is given by $G_H' = H' M \cos \theta$. At equilibrium $G_H = G_H'$, hence we can write $HM \sin \theta = H' M \cos \theta$.

Thus,

$$\frac{H'}{H} = \tan \theta.$$

Now H' is from Case 2 given by

$$H' = \frac{2 \pi n i_a}{r}.$$

Hence $\frac{2 \pi n i_a}{r H} = \tan \theta$, and $i_a = \frac{r H}{2 \pi n} \tan \theta$, where n is the number of turns of wire in the coil.

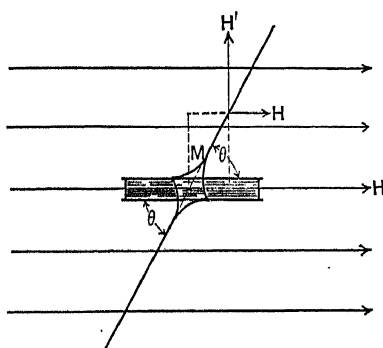


FIG. 28. Principle of the tangent galvanometer.

Thus as the earth's field is known and the constants of the tangent galvanometer are known, all that is necessary is to pass a current through the instrument, measure the angle θ , and i_a will be given in absolute units if the earth's field H is given in oersteds. The quantity

$\frac{r}{2 \pi n}$ is known as the constant of the galvanometer, for it is characteristic of the particular instrument. To get the current in amperes, the current as measured by the tangent galvanometer must be multiplied by the factor 10, so that $i = \frac{10 r H}{2 \pi n} \tan \theta$.

This instrument is the one which first gave the values of currents in absolute units. It is thus the basis of all our early absolute standards. Today as a result of later theoretical developments the absolute values are determined with greater precision by the current balance; see pages 313 and 325.

CHAPTER VI

POTENTIAL DIFFERENCE AND WORK IN AN ELECTRIC CIRCUIT

24. THE CONCEPT OF POTENTIAL

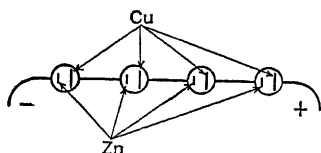
As early as 1734 the fact that like kinds of electricity repel each other and unlike kinds attract each other, as well as that electricity could travel along a wire, had led to the notion that electricity was a fluid. In fact, Benjamin Franklin had proposed the theory that there was a weightless electrical fluid, or positive electricity, which repelled itself and could flow especially well over metals. A neutral body with an *excess* of positive electricity on this concept is *positively charged*. A neutral body having some of its electrical fluid removed has *lost its positive electricity*, or has become *negatively charged*. The positive electricity was identified with the so-called vitreous electricity obtained

by rubbing glass or glass-like bodies with silk. Thus, as a result of attractions between unlike charges and repulsions between like charges; accumulations of positive electricity will always, if given a chance, flow to places where the density of positive electricity becomes less.

FIG. 29. Voltaic piles in series.

In other words, if electricity is likened to a fluid it will flow until the electrical pressure due to self-repulsion is everywhere equal.

The knowledge of the tendency of static electricity to flow and the treatment of this flow as resulting from some sort of electrical pressure (self-repulsion) was thus available when the discoveries of the galvanic pile in 1800 and the magnetic detection of the presence of a current in 1819 were made. Thus in considering the flow of electricity manifested by the magnetic effects it may be assumed that there is justification in treating these currents as the flow of some sort of electrical fluid conditioned by an electrical pressure due to self-repulsion. That such a concept is justified is at once borne out by the observation that the current in a wire (magnetically measured) is nearly in proportion to the number of voltaic piles placed in series, i.e., with the Cu of one pile connected to the Zn of the next, etc. (see Fig. 29), for the additive character of the sources of voltaic currents with proportional magnetic disturbances in the wire strongly indicates some sort of an increased driving force or pressure due to the connecting of cells in series. Thus



it is our task in some way to define and thus to measure this electrical pressure. Now while measurement of fluid pressure is possible by many devices, how the pressure of an invisible and intangible electrical fluid, detected magnetically, can be measured is not so obvious.

To solve the problem it is possible to turn to an analogy in the flow of water. In order that a flow of water continue down any section of a long pipe, there must be a pressure difference to maintain the flow. This can be furnished by a large tank or a turbine, etc. This pressure difference can readily be demonstrated by static pressure

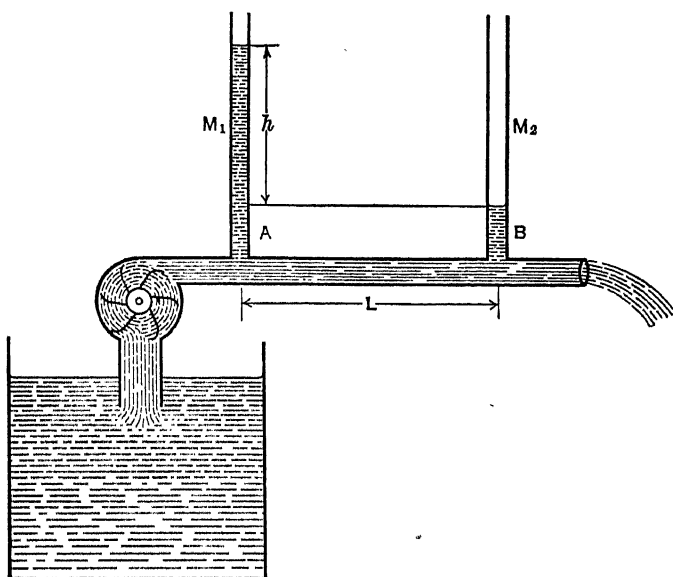


FIG. 30. Hydrostatic analogy of flow in a tube and the pressure difference maintaining it.

manometers placed at any two points, as in Fig. 30. Now assume that there are no such static manometers at hand, but that the volume outflow of water could be measured. Is there a way of measuring or determining the pressure, *utilizing flow* and not static effects? To see whether it is possible some of the fundamental equations of mechanics must be resorted to. Pressure P is equal to the force exerted on unit area. Symbolically, we would write $P = \frac{f}{A}$, where f is the force on

an area A . If the pressure is static and is caused by a stand of water in a pipe or a tank in the earth's gravitational field, $f = mg$, where m is the mass of water over a given area A and g is the constant of gravitational acceleration. Thus, $P = \frac{f}{A} = \frac{mg}{A}$. In the tank, however,

$m = hA\rho$, where h is the height of the water, A the area, and ρ the density. Then $P = \frac{\rho g A h}{A} = \rho g h$. Now ρg is the force of gravity per unit volume, for $\frac{mg}{V} = \rho g$, where V is the volume. Thus $\frac{mgh}{V}$ is the work $mgh = W$ to move a volume V a distance h . Hence $P = \frac{W}{V}$, and it is seen that pressure can be expressed as the work done to move unit volume, or unit quantity of water, to a height h .

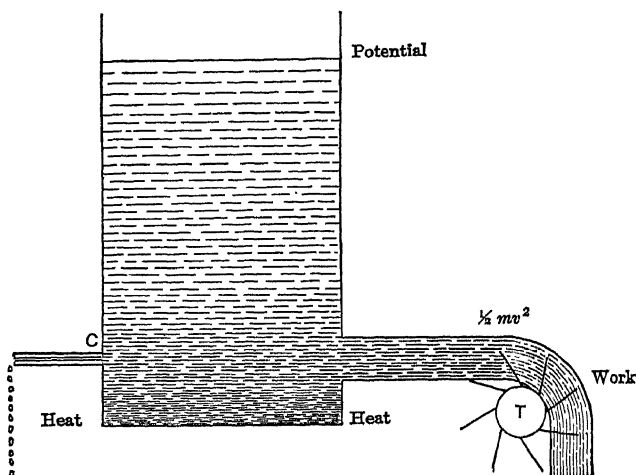


FIG. 31. Hydrostatic analogy of the production of work and heat at the expense of the fall of water (fall of potential) in a tank.

Now it behooves one to regard more closely the work W . Suppose that, in a tank furnishing a stream of water in a tube, unit volume of water moved downward a distance h in the gravitational field. Work $\rho g h$ has been done by the unit volume. This work must produce some effect. Similarly if a water turbine is used to cause flow, as in Fig. 30, the turbine has done work on unit volume in setting it in motion. Were there no friction in the tube this work $\rho g h$ would have given the unit volume a velocity v and an energy of motion $\frac{1}{2} \rho v^2$. Thus the work expended by the fall of the volumes of water in the tank or the work done by the turbine would produce a flow of the volumes of water down the tube, as shown in Figs. 31 and 32 with a velocity v . Were there no friction in the pipe, the velocity v from $\rho g h = \frac{1}{2} \rho v^2$ would remain constant. Actually the pipe offers a resistance to the flow and the water loses velocity, and hence energy, as it flows down the pipe. This is clearly illustrated by using a large-diameter tube, when it is seen that the water shoots out of the tank with considerable

velocity, so that the emergent velocity is nearly the velocity v given above. It can also do work as shown by the turbine wheel T of Fig. 31. If a narrow capillary tube C had been used, the water would have trickled out with all its energy lost to friction. Thus there is *loss of energy in the tube to overcome the frictional resistance*. As a result the kinetic energy of the water falls in proceeding downstream by the energy loss to friction. As shown by the static manometers of Fig. 30 there is also a fall of pressure which is equal to $\Delta P = \frac{\rho g A h}{A} = \rho g h$, where h is the difference in level in the manometers M_1 and M_2 ,

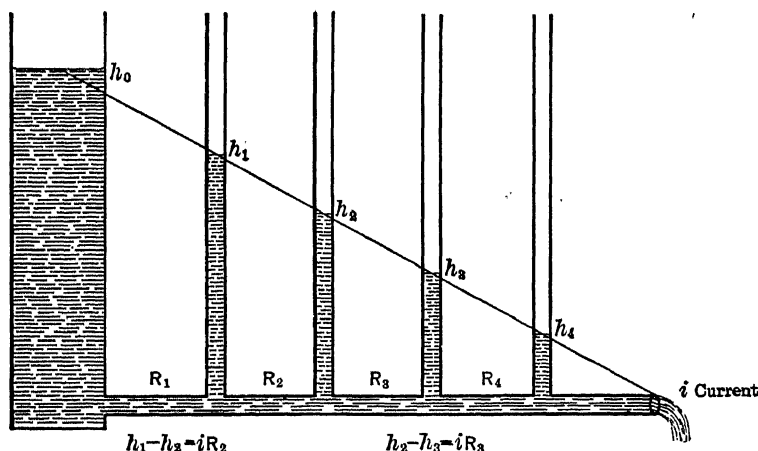


FIG. 32. Hydrostatic analogy of the fall of potential (pressure) down a tube carrying a current.

separated by the distance $AB = L$. But this pressure difference is by the equations expressible as $\Delta P = \frac{W}{V}$, or the work to move unit volume of water over a height h , and it must also represent the loss of kinetic energy $\frac{1}{2} \rho (v_A^2 - v_B^2)$ which went to overcome friction, for that is just where this energy loss in maintaining the current went.

Thus since the friction uses up energy which is just equal to the work necessary to move unit volume of water through h , ΔP can be measured by measuring h from $\Delta P = \rho g h$, or it can be measured by measuring the work-per-unit volume expended against friction in the section AB , from $\Delta P = \frac{W}{V}$.

Now if the student is interested in current measurements and is enjoined from using the value of h , as this is given by a *static* meter, he would have to contrive to use the expression $\Delta P = \frac{W}{V}$ for the measure

of pressure difference. It now happens that the energy lost in friction goes to heat energy according to Joule's famous equation, $H = \frac{W}{J}$, where H = heat in calories and $J = 4.18 \times 10^7$ ergs, the mechanical equivalent of heat. Hence by putting the pipe between AB inside a calorimeter and measuring H , he could at once obtain $W = JH$ for a volume flow V and thus get ΔP , from $\Delta P = \frac{W}{V} = \frac{JH}{V}$.

Now this happens to be precisely the sort of a situation which is encountered in the study of electrical pressure, for the picture of Fig. 33 is an electrical analogue of Fig. 31. It is true that it is possible to measure accurately the absolute value of the electrical pressure difference (called electrical potential difference) by means of a *static*

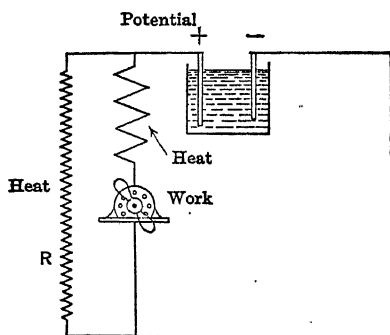


FIG. 33. The production of work and heat by a battery. Compare with Fig. 31.

instrument known as the absolute electrometer. This, however, is an instrument which depends entirely on electrical charges at rest and therefore cannot be of great value to electricity as studied by the magnetic fields of currents. Hence this description of electrical pressure difference cannot be made use of. On the other hand, electrical current i can be measured, and hence quantity $q = it$, where t is the time of flow. Again it is common experience that an electrical current due to friction in the wire causes a heating of the wire through which the work W

can be determined from a calorimetric measurement of the heating and the mechanical equivalent.

Hence in strict analogy with the case of fluid flow from the equation pressure difference, $\Delta P = \frac{W}{V} = \frac{W}{it} = \frac{JH}{it}$, where i is the fluid current, it is possible to write for the electrical case that electrical pressure difference or, better, *electrical potential difference* P.D. = $\frac{W}{q} = \frac{W}{it}$, where q is the quantity of electricity, i is current, and t is the time. This equation defines a new electrical concept — that of electrical pressure difference (electrical potential difference) in absolute fashion in terms of work and electromagnetic quantities of electricity.

The term "electrical potential difference," used to describe the electrical pressure causing a current flow, derives its name from its analogy to fluid flow. It came into being through the application of the potential theory to electrical phenomena by K. F. Gauss, the con-

cept of potential being borrowed from the generalized concept of mechanical potential theory. In the expression $\Delta P = \rho gh$, it is seen that, irrespective of the density ρ of a specific fluid, the pressure ΔP is proportional to the quantity gh . In mechanics, if $W = mgh$, $\frac{W}{m} = gh$ where gh is the work to move unit mass in a force field. It is a constant of the field depending only on the height difference and the value of the gravitational field, which is a constant for a given region of the earth. This constant, which is convenient for the treatment of many problems, is called the difference of gravitational potential. Hence the electrical concept analogously was termed the electrical potential difference, P.D.

Thus it is seen that, owing to the impossibility of *directly* evaluating an electrical pressure maintaining a current flow in terms of the current flow, we have been led by mechanical analogy to an indirect concept or definition of electrical pressure difference, called electrical potential difference P.D., which says that $\text{P.D.} = \frac{W}{q}$, where W is the work to move a quantity q .

25. THE ABSOLUTE ELECTROMAGNETIC UNIT OF POTENTIAL

Potential difference has now been defined by the relation $\text{P.D.} = \frac{W}{q}$, which says in words that the potential difference between two points is the work to move unit quantity of electricity between those points. It is at once seen that $\text{P.D.} = 1$ absolute e.m.u. of potential difference if $W = 1$ erg and $q = 1$ absolute electromagnetic unit of quantity. Thus the absolute electromagnetic unit is the potential difference which yields 1 erg of work when unit electromagnetic quantity is carried from one point to the other of the circuit.

Hence $\text{P.D.} = \frac{W}{q} = \frac{W}{i \Delta t}$, and $\text{P.D.} = 1$ e.m.u. if $W = 1$ erg and $q = 1$ e.m.u. of quantity, i.e., $q = 1$ e.m.u. of current for 1 second.

The Sign Conventions of Potential Difference. Long ago (before electrons were known) a convention was adopted for which Benjamin Franklin is responsible. In his convention the electrical fluid is the electrification obtained when glass or a glassy substance is rubbed with silk. This was then called vitreous electrification. Accumulations of this fluid build up a pressure. The flow of electricity on this basis will be from higher accumulations of this assumed fluid to lower accumulations. The vitreous fluid was therefore called *positive* electricity and the convention of flow was assumed to be from positive to less positive or to an actual *dearth* of positive fluid. By "dearth of fluid" is meant that on Franklin's theory each neutral or uncharged body has just

enough positive electricity. If it has more it is positively charged. If it has less it is resinously or negatively charged; i.e., it lacks enough positive electricity to make it neutral. The convention once accepted has remained, and it seems useless to attempt to change it even though it is known that both negative and positive *particles* (not fluids) exist and that in metals it is the negative electron that moves.* Thus when the subject of potential difference is reached it will be discussed in terms of a flow of electricity from a positive point (marked +) to one less positive, zero or negative (marked -). In instances of flow the concern is only with the relative value of the potential between the points in question. To give a standard or basis of comparison it is *assumed* that the surface of the earth as a whole is neutral or has zero potential. Potential differences are thus always relative to the earth or else represent the potential difference between the more vitreously charged point (+) and the less in a wire, the vitreous electrification today being identical with that on the copper or carbon of a copper-zinc or carbon-zinc cell. Frequently the negative side of a circuit is definitely connected to ground by means of a copper wire embedded in moist earth or connected to the local water table.

The Practical Unit of Potential. One can now return to the equation $P.D. = \frac{W}{i_a t} = \frac{HJ}{i_a t}$. In considering practical measurements

on the type of batteries or piles which are in daily use, it is found that when $i_a = 0.1$, i.e., when currents of 1 ampere or less are used, as in lighting a small flashlight bulb from an ordinary dry cell, the heat evolved appears of the order of 0.239 calorie per second. This means

that the value of the potential difference is $P.D. = \frac{0.239 \times 4.18 \times 10^7}{0.1 \times 1} =$

1×10^8 *absolute electromagnetic units*. Thus the potential difference given by an ordinary dry cell is 100 million times as large as the absolute electromagnetic unit of potential. Hence in practical work, where potential differences amounting frequently to those given by 100 dry cells are used, the nuisance of describing potentials in terms of hundreds of millions of absolute e.m.u. is obvious. By international

* Conventions adopted by International Commissions are binding to the subject. All specifications, tables, textbooks, and instructions are established on such a basis. To change entails very much effort, and much expense, and leads to confusion during transition. It appears that during World War II certain groups of radio and electronics engineers giving instruction in service schools set themselves up as an International Commission and adopted the direction of *electron* motion as the direction of the current. This has had a viciously confusing effect on service school graduates when they resumed their studies. It is immaterial, for example, whether one drives on the left- or right-hand side of the road. In this country we arbitrarily use the right. Consider the confusion, for example, if all red-haired drivers suddenly decided that they would drive on the left-hand side. It seems that until International Commissions decide otherwise all physicists and engineers will understand each other better if they follow the accepted conventions.

convention a practical unit of potential has been agreed on and named the volt in honor of Alessandro Volta, who discovered the voltaic pile. The volt is by definition equal to 10^8 or 100 million absolute e.m.u. Hence the volt is defined as 1 volt = 10^8 e.m.u. It represents the potential difference which will require 10^8 ergs of work to be done when 1 absolute electromagnetic unit of quantity is moved against it. As a result the following equations may be written:

$$H = \frac{\text{P.D.} \times i_a t}{4.181 \times 10^7} = 2.39 \times 10^{-8} \text{ P.D. } i_a t \text{ } 20^\circ \text{ calories,}$$

where P.D. is in absolute units, i_a is in absolute units of current, and t is in seconds.

$$H = \frac{10^8 \text{ P.D.} \times i_a t}{4.181 \times 10^7} = 2.39 V i_a t \text{ } 20^\circ \text{ calories,}$$

where V expressed in volts is 10^8 P.D.

$$H = 0.239 V i t \text{ } 20^\circ \text{ calories,}$$

where V is in volts, i is in amperes, and t is in seconds.

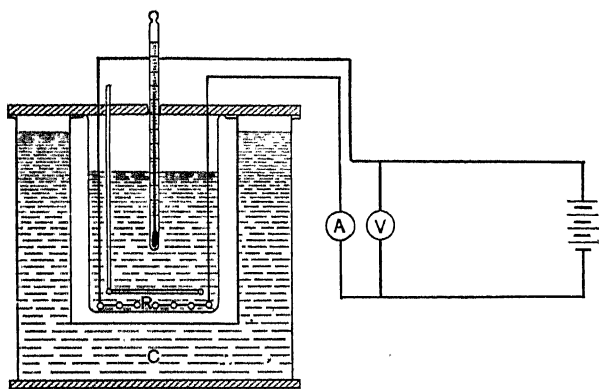


FIG. 34. Calorimetric measurement of potential.

If then a current i is measured in amperes, and if in t seconds it produces H in 20° calories of heat, the potential difference will be

$$V = \frac{H}{0.239 it} \text{ volts.}$$

The value of the potential difference as just defined can be measured by the arrangement shown in Fig. 34, which is the fundamental means of determining the value of the volt in the electromagnetic system. C is a calorimeter in which a wire coil of resistance R is placed. The coil is surrounded by water so that the weight of water plus the water equivalent of the calorimeter furnishes a convenient

mass for determining the heat liberated in the wire. The source of potential which is to be measured is connected to the two wires of low resistance of the coil R coming from the calorimeter C . In series with this coil is placed a galvanometer or ammeter A of low resistance for measuring the current. By taking the reading of the current on the ammeter with the potential turned on for a known time t , the potential can be evaluated at once by the equation above from the heat given to the calorimeter and the reading of the voltmeter V can be checked.

This method of establishing the absolute and practical standards of potential was the only method originally used. It is not much more accurate than 0.1 per cent, owing to the difficulties inherent in all calorimetry. Absolute electromagnetic potential could also be obtained with not much greater accuracy by measuring potential in the absolute electrostatic units with the attracted disk electrometer and dividing by the velocity of light, which is the accurately known ratio of the units in the two systems. The really accurate potential standards are established by calibrating the absolute ohm by means of an electromagnetic induction method and running a current evaluated in absolute electromagnetic units through the standard ohm. The older calorimetric experiment described above is now used to determine the mechanical equivalent of heat, which is given in 20° calories as $4.1813 \pm 0.0006 \times 10^7$ ergs.

26. THE DEFINITION OF ELECTRICAL POWER CONSUMPTION

From the equation developed that P.D. $i_a t = W$, is obtained the relation that

$$\text{P.D. } i_a = \frac{W}{t}.$$

$\frac{W}{t}$ is the time rate of doing work. That is, it gives the *power* developed.

Thus, P.D. (in absolute e.m.u.) \times current (in absolute e.m.u.) gives the power in ergs per second.

V (in volts) $\times i$ (in amperes) gives power in 10^7 ergs per second.

This work is a joule per second, where a joule equals 10^7 ergs, and the unit, joule per second, is called the watt.

Therefore one may write: volts times amperes equals watts.

The watt is the practical unit of power in the c.g.s. system and is of great importance in engineering practice. For large power output the kilowatt = 1000 watts is used.

The relation that volts times amperes times seconds equals 10^7 ergs leads to the interesting relation that volts times amperes times seconds equals volts times coulombs, or equals joules of energy. It is worth while to remember these relations, as they are of considerable importance in the solution of problems.

27. THE CONCEPT AND DEFINITION OF ELECTRICAL RESISTANCE

Two absolute units of electricity have now been defined in terms of the electrical current. These are the current i_a and the potential difference P.D. They have also been related to the practical units — the ampere and the volt. It is found *experimentally* that the current in a given circuit is proportional to the voltage or potential difference applied, that is, for a given circuit,

$$\frac{\text{P.D.}}{i_a} = \text{a constant.}$$

If P.D. is kept constant in going from one circuit to another, it is found that i_a varies with the form and dimensions of the circuit. That is, the ratio $\frac{\text{P.D.}}{i_a}$ depends on the form and dimensions of the circuit, and is a *constant of the circuit*. This constant is called the *resistance* of the circuit. Consequently the *resistance of a circuit*, or of a *portion of a circuit*, is defined as *the ratio of the potential difference to the current produced*. Thus, the unit of resistance in the absolute electromagnetic system is the resistance of a circuit which allows a P.D. of 1 absolute electromagnetic unit to maintain a current of 1 absolute electromagnetic unit in the circuit. Since the absolute unit of P.D. is small whereas the absolute unit of current is large, this absolute unit of resistance is *very small*. *In practice, the unit of resistance used is the resistance which permits a potential difference of 1 volt to maintain a current of 1 ampere through it. This unit is known as the ohm.* Symbolically, this may be represented by

$$\frac{V \text{ (in volts)}}{i \text{ (in amperes)}} = R \text{ (in ohms).}$$

Since the volt is 10^8 absolute e.m.u. and the ampere is 10^{-1} e.m.u., the absolute e.m.u. of resistance is 10^{-9} ohm.

This law is known as Ohm's law in honor of G. S. Ohm, who was the first to deduce this relation between potential difference and current, and to show the significance of potential difference. He derived the law using the analogy between the flow of heat in a circuit and the flow of electricity in a circuit. This was in 1827, shortly after the great mathematician Fourier worked out the laws of the flow of heat in 1822.

It is important to notice in this connection that now there is a ratio of two quantities defined in the absolute electromagnetic system, potential difference and current. From the ratio of these two it is found that a given circuit has a constant ratio, and *that this is characteristic of the form of the circuit. Therefore, the ratio of potential difference and the current is used to define a third quantity which is a characteristic of the circuit, called resistance.* The two quantities,

potential difference and current, are *fundamental quantities*, since they are directly derived by measurements given for quantities in terms of the absolute c.g.s. system. Resistance, however, is a *derived* unit, for it is derived from a ratio of two fundamental units.

It is essential to avoid the vicious cycle so often indulged in by students of elementary physics, of defining resistance in terms of current and potential and then turning around and defining potential in terms of current and resistance. However the quantities are defined, two of them must always be fundamental and one of them derived. It is more logical, in the treatment to follow, to define potential and current as fundamental units because of their relation to heat and magnetic fields and to treat resistance as a derived unit than any other arrangement. (See page 411.)

28. JOULE'S LAW OF HEATING

The relation of Ohm's law may now be applied briefly to the heating effect of an electric current. We wrote

$$H = 0.239 Vit.$$

Since

$$\frac{V}{i} = R,$$

we can write

$$H = 0.239 i^2 Rt.$$

This says that the heating effect is proportional to the square of the current and to the resistance of the circuit. The heating effect varies from circuit to circuit as the resistance varies. This law is known as Joule's law of heating.

This heat production in the circuit, it may be noted, is the result of the electrical friction in the wire which causes the potential to fall along the conductor. The greater the resistance between two points for a given current i , the greater the potential drop $V = iR$, and the greater the heat H dissipated.

A nice verification of Joule's law can be seen in the experiment to be described. Three coils, R_1 , R_2 , and R_3 , of exactly equal resistance R , are wound on insulating frames as shown in Fig. 35. The resistances R_1 and R_2 are connected in parallel and the leads from a potential main are connected across them directly, one of the wires from the main, however, first passing through the third coil R_3 . The current i from the main flows through R_3 and then splits, one-half going through R_1 and one-half going through R_2 . Now coils R_1 and R_2 are immersed in separate beakers with 200 cm³ of water in each of them while R_3 is in a beaker with 400 cm³ of water. The heat in the

beakers with R_1 and R_2 is $H_1 = 0.24 R \left(\frac{i}{2}\right)^2 t$ that in the beaker with R_3 is $H_3 = 0.24 R i^2 t$.* Since the heat, H_3 , in R_3 is four times H_1 ,

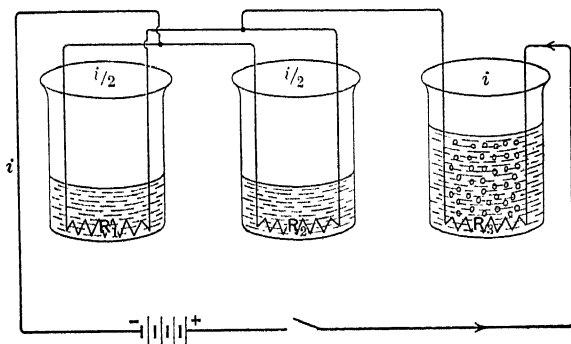


FIG. 35. Verification of Joule's law of heating.

that in R_1 and R_2 , then the 400 cm³ of water in the beaker with R_3 will boil long before that in the beakers R_1 and R_2 , with 200 cm³ of water each.

* For practical purposes 0.239 is sufficiently close to 0.24 so that hereafter the numerical factor 0.24 will be used.

CHAPTER VII

ELECTRIC CURRENTS AND RESISTANCE

In the last chapter it was shown that in each circuit there is a quantity defined by the ratio of potential difference to the current flowing which was called the resistance of the circuit. This *general law*,

$$\frac{V}{i} = R,$$

*holds for any part of the circuit as well as for the circuit as a whole, provided i is the total current flowing between the points across which the potential V exists.** If, by chance, R is known, and it is possible to measure i , the potential is given at once by the expression $V = iR$. This potential difference, defined by iR , is often spoken of colloquially as the iR drop in the circuit, as it measures the drop in potential between the two points delimiting R . Thus in virtue of this equation $V = iR$, the quantity iR acts as a potential difference and an iR drop can be so used. It can also be shown electrically by touching two wires to the extremities of a resistance R with a current flowing in it to the two plates of the gold-leaf electrostatic multiplier and separating the plates after breaking contact. The leaves of the multiplier will diverge, indicating the potential difference.

The potential difference between any point on a wire and the point of lowest potential on the wire drops continuously as the point of lowest potential is approached. An analogy to the example of flow of water in a long tube where the hydrostatic pressure, or the potential, at the orifice is 0, whereas at the other end of the tube it has the maximum potential applied, shows that if a series of manometers are placed at various points along the tube they indicate a progressive fall of potential, or pressure, down to the open orifice; see Fig. 32.

29. NATURE OF RESISTANCE AND ITS VARIATION

How the potential varies along a conductor can be calculated from the relation just given, $V = iR$, if i and the value of R over the portion of the circuit considered are known. It therefore becomes necessary to study the laws of resistance. *Experiment* has shown that, for a

* Note that this statement holds only for the case of the steady direct currents with which we are dealing at this point.

given uniform conductor, the resistance R is proportional to the length l , and is inversely proportional to the area of cross-section A . The relation between these three quantities is given by the expression

$$R = \frac{R_s l}{A}.$$

In this equation R_s is the constant of proportionability and is a *characteristic constant of the material of the conductor*. It is seen that $R = R_s$ when $l = 1$ cm length and $A = 1$ cm² area of cross section. Thus R_s is the resistance of a *unit cube* of the material. R_s is called the *specific resistance* of the material. Unlike the use of the term *specific* in connection with specific gravity, specific heat, or specific inductive capacity, *specific resistance*, R_s , is *not a dimensionless ratio*. Specific

resistance, $R_s = R \frac{A}{l} = \frac{RL^2}{L} = R \times L$. Thus specific resistance has dimensions of resistance times length, so that the specific resistance of copper is written as 2×10^{-6} ohm \times centimeters. In stating that specific resistance is the resistance of a geometrical unit cube of substance this term is meant *literally*. A cubic centimeter *volume* of copper could be spun into a very long fine wire, with a very high resistance, or into a thin flat plate with virtually negligible resistance. Thus it can be noted that the form of the material as well as its character influence its resistance, and that specific resistance R_s is the value characterizing the material. The values of specific resistance of all common substances can be found in tables.

In dealing with many forms of electrical connections and circuits the conductors of different resistances are frequently strung along continuously, one attached to the end of the next, as indicated in Fig. 36. These resistances are said to be in series, and, as will later be seen, their values are additive. The circuit calculations of resistances in series are thus facilitated by additive properties in calculating total *resistances*. Under these conditions resistance is found to be a useful concept. There are, however, other occasions where there are several resistances in parallel, that is, there are connections where the current divides and passes through several resistors, to be reunited later, as seen in Fig. 37. On these occasions the reciprocals of the resistances, i.e., $\frac{1}{R}$, add, which sometimes leads to awkward computations. At these times it is better to work directly with the reciprocals of resistances, i.e., $\frac{1}{R} = \sigma$. Here σ is called the *conductivity*. This is particularly convenient in the study of liquid conductors or in gaseous conduction. Thus, the *conductivity* is defined as $\sigma = \frac{1}{R}$. The unit of

conductivity is the reciprocal ohm, $\frac{1}{\text{ohms}}$, or the mho (i.e., ohm spelled backward). The *specific conductivity*, $\sigma_s = \frac{1}{R_s}$, is expressed

in mhos per centimeter, or in reciprocal ohms per centimeter. Under most conditions, however, the resistance is the term most encountered.

The specific resistance of substances varies through enormous ranges. Pure silver and copper have the lowest values of R_s , beginning about 1.4×10^{-6} to 1.7×10^{-6} ohm \times cm at 0° C, respectively, and increasing on up through the metals toward the right-hand side of Mendeleev's periodic table of the elements and into the heavier elements. The value for zinc is about 6; for iron about 10; for platinum, 10; lead, 22; antimony, 42; bismuth, 119 — all multiplied by 10^{-6} ohm \times centimeters at 20° C. Alloys of the metals range higher in resistance than the pure constituent metals, with values in the range from 30 to 100×10^{-6} ohm \times cm near room temperature. For the nonmetallic substances the specific resistance increases for the metalloids like graphite with $R_s = 800 \times 10^{-6}$ ohm, carbon 3700×10^{-6} ohm, tellurium 2×10^{-1} ohm, \times centimeters at about 0° , and for a large group of substances like cuprous oxide called semiconductors. There are relatively few such substances in use in circuits, so that there are not much data on them. Actually, leaving out this rather rarely encountered group of conductors, there is a wide gap between the specific resistances of the metallic conductors and of a general class called ionic conductors, comprising molten salts and aqueous or liquid ammonia solutions of electrolytes. The specific resistances of the best aqueous conductors such as HCl, HNO_3 , KOH, and NaCl at optimum concentration for high conductivity ranges from around 1.3 to 10 ohms \times centimeters at 20° C. From the liquid ionic conductors through the various solid ionic conductors specific resistance rises continuously. It progresses in value through this group of solid conductors to the liquid and solid substances of a nonionic character. Some very refractory ionic substances will also have very high resistivities. These substances, with cast sulfur, hard rubber, pure quartz, and special paraffins (as ceresin), reach values as high as 5×10^{18} ohms \times centimeters. The conductivity of the thin adsorbed surface films of moisture and dirt in such substances transcends their volume conductivity and care is required in measurements and use to prevent surface leakage. Substances with high specific resistances are termed *insulators*. Although this word is a relative term, it is possible, for ordinary circuits with a hundred volts of potential and where leakage currents of microamperes are of no consequence, to call substances with R_s greater than 10^{10} ohms \times centimeters insulators. For work with static electricity or currents of electronic magnitudes, values of R_s of the order of 10^{16} ohms \times centimeters or better are required before we can speak of insulators.

It might be indicated that the various classes of conductors just mentioned depend on different basic mechanisms of transport for their characteristic values. Metallic conductors are materials in which the valence electrons of the atoms are pooled between the various atoms in the block. These electrons move fairly freely among the positive atomic ions comprising the space lattice making up the solid. Of these free valence electrons a given fraction share in the heat motions of the lattice and are available to conduct both heat and electrical current. Ohm's law is strictly obeyed. The electrons, being small and light, move relatively easily through the channels in the regularly oriented ions. If violent heat vibration distorts the regularity of the lattice, or if the lattice is distorted by impurities or by alloying, the channels are less free and resistance increases. Thus, electronic conductors increase their resistance with increase in temperature. The resistance depends on the number of available electrons in the conduction level, on lattice constants, and on temperature.

The semiconductors are substances the electrons of which are not free in conducting levels, but are present in considerable numbers in energy levels, where, either by additional thermal agitation, by irradiation with infra-red light, or by application of electrical fields, they can be raised to the conduction levels. There is thus a voltage threshold for conductivity and Ohm's law is not strictly obeyed. In some other substances, for instance, in rock salt (an ionic lattice) metallic atoms, e.g., Na, can be liberated by means of x-rays. Light by photoelectric action is sufficient to free electrons from the Na atoms, and the rock salt becomes then an *electronic photoconductor*. The number of carriers liberated by light are few so that the resistivity of such substances is high.

For many other substances comprising salts, oxides, solid acids, bases, and minerals, matter is found to be composed of regularly spaced lattices of positively and negatively charged atomic ions. These materials are made up of combinations of metallic atoms which readily lose valence electrons, and of nonmetallic, electro-negative elements like Cl, Br, I, O, etc., which pick up electrons to form negative atomic ions. In combinations of these substances electrons cannot remain free, as they are taken up by the negative elements to give ions. In solution such substances dissociate fairly completely into a metallic positive atomic ion and nonmetallic negative ion or radical. These are usually fairly numerous and can move freely in the solution if a potential difference is applied. Thus, solutions and ionic conductors fairly closely obey Ohm's law. Since ions are much larger than electrons, collisions of ions with molecules even in solution are more frequent. Resistance is therefore nearly 10^5 -fold greater than for electronic conductors. All of this class of conductors depends on dissociation of the ions and mobility or freedom of the ions to move. Thus, the resistance of solids will be much higher than for liquids. Tempera-

ture increase, which increases dissociation and increases mobility by decreasing viscosity, will *decrease* resistance in ionic conductors. This is in marked contrast to metallic conduction.

Finally, the balance of combinations in nature are nonpolar aggregates of molecules where electrons are bound firmly in neutral atoms or molecules in liquid and solid states. These are substances like the liquid inert gases, He, Ne, A, etc., liquid and solid gases like H_2 , O_2 , and N_2 , and countless nonpolar organic liquids and solids like the hydrocarbons, paraffins, sulfur, etc. In these materials, in theory no conductivity should be observed, i.e., they are perfect insulators. However, impurities of an ionic sort, conducting surface contaminants, external ionizing radiations, etc., render them conducting in a small degree.

30. RESISTANCE AND TEMPERATURE

As indicated, for most substances resistance is affected by temperature. In the ionic type of conductors the resistance decreases as temperature increases through increased dissociation and decreased viscosity. The rise for solids is at first slow in ionic-type conductors, but as melting temperatures are reached, the increase is nearly exponential.

In the metallic substances heat disturbs the lattice and increases resistance as well as affecting the number of conducting electrons in the level. An interesting example of the two forms of conductivity is the element carbon. Carbon, as used in the old incandescent lamps and in arcs, has a specific resistance which *decreases* from 3500 to 900×10^{-6} ohm \times centimeter from 0° to 2500° C, whereas graphite has a specific resistance that *increases* from 800 to 1100×10^{-6} ohm \times centimeters from 0 to 2500° C. Diamond acts like a typical ionic lattice with R_S at 10^{14} ohms \times centimeter at 35° and 10^6 ohms \times centimeter at 2500° C.

Wave mechanical theory and experiment indicate that specific resistance for electronic conductors varies nearly linearly with the absolute temperature for pure metals. To a first-order approximation $R_S = R_o(1 + \alpha t)$, where t is the temperature in degrees centigrade and α is the temperature *coefficient* of resistance. This equation can be manipulated as follows:

$$R_S = R_o(1 + \alpha t) = \alpha R_o \left(\frac{1}{\alpha} + t \right) = \alpha R_o (T_R + t).$$

Here t is in degrees centigrade above the ice point and T_R is a characteristic temperature for the metal. For some metals T_R is *nearly* 273° as is the case of a similar equation for the expansion of gases with temperature. This would mean that at 0° absolute, i.e., 273° below the ice point, R_S was zero. Actually, T_R is always greater than 273.18° and we write, $T_R = 273.18 + B$. Hence, $R_S = \alpha R_o(B + T)$, where

T is temperature in degrees absolute and B is a constant of the order of some degrees to tens of degrees. Accordingly, by observation the specific resistance of metallic conductors in conformity with theory varies nearly linearly with absolute temperature, but has a finite low value at nearly 0° absolute.

There are, however, a certain number of substances which below a characteristic temperature in the neighborhood of absolute zero suddenly lose all resistance. These are known as *superconductors*, and at these temperatures they are said to be in the *superconducting state*. The first to be discovered by Heike Kamerlingh Onnes was lead, which becomes superconducting at 7.3° K. Other elements are Ta 4.5° , Hg 4.2° , Sn 3.7° , In 3.4° , Tl 2.5° , Th 1.4° , Au-Bi alloys 2.15° , all temperatures being in degree absolute. Very recently a type of superconductivity has been observed in Na-NH₃ amalgams at relatively high temperatures.

The phenomenon of superconductivity can be detected as follows. The metal is in the form of a helix of fine wire with the two end wires connected. The field due to a current in this wire can be detected by a small compass needle near it. The needle is placed near the coil before it is cooled down and its zero position noted. A bar magnet is also brought near the coil. If the coil is cooled near, but above, the superconducting temperature and the magnet is removed, an e.m.f. is induced in the lead coil shorted on itself. A *transient* current results and flows for in the order of 10^{-3} of a second in the coil. It damps out to zero by the i^2Rt loss in heat due to the resistance R of the coil. The compass needle temporarily deflects due to movement of the magnet and the transient current. Then it settles down to 0 deflection. If the coil is cooled below the superconducting point when the magnet is removed, the compass needle takes on a permanent deflection due to the magnetic field of the induced current in the coil. This deflection lasts for several days, but gradually decreases as the current dies down by i^2Rt loss due to residual resistance. This proves that R is reduced to a very minute fraction of its value below the superconducting temperature. What the situation is that causes superconductivity is not known. If it can be produced at higher temperatures and if it can be conveniently controlled, it will assume great industrial importance.

With the high degree of precision which can today be achieved in the comparison and measurement of resistances the temperature variation of resistance can be most accurately measured. In general, resistors which usually consist of alloys do not have a strictly linear temperature variation. The variation of alloy-constructed resistors can be represented by empirical equations in the form of a power series, such as,

$$R_T = R_s(1 + \alpha t + \beta t^2 + \gamma t^3 + \dots).$$

The constants b and c may have positive or negative values. These equations for certain metals of standard purity such as *Pt* are given in tables of constants. By making suitable resistance coils of such metals, these can be used to measure temperatures with great precision through changes of resistance. Thus, the platinum resistance thermometer is the standard precision instrument for temperature measurement and can give the most accurate temperatures of any thermometer.

The variation of resistance with temperature cannot ever be lost sight of. It is a constant source of error in electrical measurement unless allowed for. By judicious choice of alloying elements we can get wires with relatively low temperature coefficient of resistance. One of these is a commercial alloy known as constantan and another is manganin, an alloy of Cu, Mn, and Ni. Manganin has an exceptionally low value of α between 12° and 475° C.

31. COMBINATION OF RESISTANCES IN SERIES AND IN PARALLEL

Having defined specific resistance the question of the laws of resistances in circuits may now be turned to.

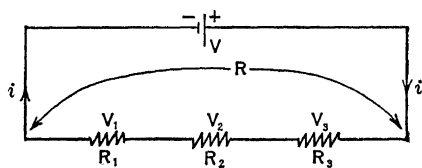


FIG. 36. Resistances in series.

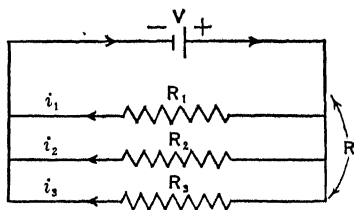


FIG. 37. Resistances in parallel.

Case 1. Law for Resistances in Series. Recalling Ohm's law, $V = iR$, and remembering that it applies to the parts of the circuit as well as to the whole circuit, we can write for the circuit pictured in Fig. 36 that

$$V = V_1 + V_2 + V_3,$$

and hence

$$iR = iR_1 + iR_2 + iR_3,$$

where R_1 refers to V_1 , R_2 to V_2 , and R_3 to V_3 . Therefore, as the same i flows through all coils we can write $R = R_1 + R_2 + R_3$. The resistances above are said to be *connected in series*.

Case 2. Law for Resistances in Parallel. Consider the voltage V of the battery placed across the resistances R_1 , R_2 , and R_3 , *connected in parallel* as shown in Fig. 37. The currents through R_1 , R_2 , and R_3 are, respectively, i_1 , i_2 , and i_3 . The potential across them is the same,

namely, V . From Ohm's law, we have

$$\frac{V}{R} = i, \frac{V_1}{R_1} = i_1, \frac{V_2}{R_2} = i_2, \frac{V_3}{R_3} = i_3.$$

Since

$$i = i_1 + i_2 + i_3,$$

therefore

$$\frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}.$$

But

$$V = V_1 = V_2 = V_3$$

whence

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

The resistances here are said to be *connected in parallel*.

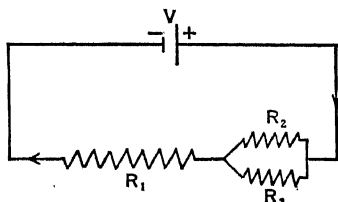


FIG. 38. Series-parallel combination of resistances.

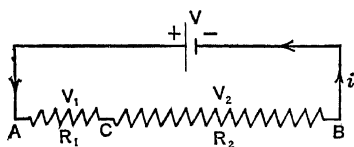


FIG. 39. Fall of potential along a wire.

As an example of the application of these laws the simple case depicted in Fig. 38, namely, the combination of the resistances R_1 , R_2 and R_3 , may be used. It is required to find the total resistance R . From the laws of resistance above we can write:

$$R = R_1 + R_{23} \text{ (resistance of } R_2 \text{ and } R_3 \text{ in combination).}$$

Since

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2 + R_3}{R_2 R_3},$$

therefore,

$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}.$$

Case 3. Fall of Potential along a Wire. Another case to which this analysis may be applied is *the fall of potential along a wire*. Given the wire represented in Fig. 39, the total resistance of which is R , at

assume that there is a potential difference V across it. There is flowing in the circuit a current i . Let us call the positive end of the wire A and the negative end B . It is required to determine the fall of potential between A and some point C , and between C and B . Call R_1 the resistance in the wire from A to C , and R_2 the resistance in the wire from C to B . Call the potential difference sought from A to C , V_1 , and that from C to B , V_2 . Now, by Ohm's law, V the total potential equals R times i . Since the same current flows in the parts AC and CB one has $V_1 = R_1i$, $V_2 = R_2i$, and $V = Ri$. Since

$$R = R_1 + R_2,$$

therefore

$$V = V_1 + V_2.$$

From this we get at once that

$$\frac{V_1}{V} = \frac{V_1}{V_1 + V_2} = \frac{iR_1}{iR_1 + iR_2} = \frac{R_1}{R}.$$

In a similar fashion,

$$\frac{V_2}{V} = \frac{V_2}{V_1 + V_2} = \frac{R_2i}{R_1i + R_2i} = \frac{R_2}{R}.$$

Finally,

$$\frac{V_1}{V_2} = \frac{R_1i}{R_2i} = \frac{R_1}{R_2}.$$

This simply states that the fall of potential across AC or CB is to the potential across AB as the resistance of AC or CB is to the total resistance, and further that the fall of potential across AC is to the fall of potential across CB as the resistance of AC is to the resistance of CB . Since it is possible accurately to compare resistances or to measure them, *it is at once possible by properly choosing the resistances to obtain any fraction of a given fall of potential.* This is the principle of the *potential divider*, or *potentiometer*. It underlies so many electrical measurements and comparisons that it is perhaps one of the most important principles of current electricity.

32. COMPARISON OF POTENTIALS AND THE DETERMINATION OF ELECTROMOTIVE FORCES BY MEANS OF THE POTENTIOMETER

The theory of the potentiometer as given in the last section enables the potential to be determined by comparison with a known potential.

For comparing electromotive forces the potentiometer schematically shown in Fig. 40 is made use of. Any battery of *constant* potential *larger* than the batteries the potential of which it is required to compare is used at the point V in the circuit. G is a galvanometer inserted in series with one of the batteries to be compared. These bat-

teries are labeled V_s for a standard cell or V_x unknown. Both V and V_s , or V_x , are connected with negative poles together. They are thus both forcing current through R_2 in the same direction. The arrow indicates a variable resistance contact which divides the resistance R_1R_2 into two parts, the ratios of the resistances R_1 and R_2 of which can be accurately determined. The law of fall of potential down a resistance wire states that if the potential at P due to the two cells is the same, the potential difference of the standard must be to the potential V as R_2 , the resistances across the standard, is to $R_1 + R_2$. If G shows no deflection, the condition of equal potential is fulfilled. If the ratio of the resistances is not properly adjusted the current will flow through the galvanometer G in one sense or the other. By careful adjustment of resistance until no current can be detected an accurate measurement of the potential in terms of the standard cell and the resistances can be obtained.

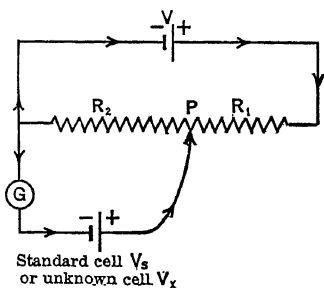


FIG. 40. Comparison of e.m.f.'s by use of the potentiometer.

$$\frac{V_s}{V} = \frac{R_2}{R_1 + R_2}.$$

In practice the standard cell is placed in position and the resistance ratio of R_1 to R_2 is changed until no current flows through the galvanometer. The standard cell V_s is then removed and the unknown cell V_x is put in its place. Adjustment is again made until the balance is obtained with no current in the galvanometer. In this case, the ratio of the resistances will be different and will be represented by resistances R_3 and R_4 , where R_3 replaces the value of R_1 , and R_4 replaces the value of R_2 obtained with the standard cell. If the potential difference produced by the unknown cell is designated by V_x then when no current flows in the galvanometer

$$\frac{V_x}{V} = \frac{R_4}{R_3 + R_4}.$$

If V was constant, the ratio of $\frac{V_x}{V_s}$ is given by

$$\frac{V_x}{V_s} = \frac{R_4(R_1 + R_2)}{R_2(R_3 + R_4)} = \frac{R_4}{R_2} \text{ for } R_1 + R_2 = R_3 + R_4.$$

Since the method does not involve the flow of current through V_x or V_s the potential difference read is really the *electromotive force*, and not the variable potential difference which a cell gives under any

conditions. Since this method makes it possible to compare the electromotive force of a standard cell with an unknown potential, the potential can be determined in terms of standard electromotive force. With the modern accurate potentiometers such comparisons can be made with a very high degree of precision (i.e., to one part in a million).*

33. POTENTIAL AND ELECTROMOTIVE FORCES IN CIRCUITS

The fact that Ohm's law holds for the whole of the circuit, as well as for parts of the circuit, leads to a discussion of considerable importance in the treatment of generators of electricity. Consider a battery, and let it be connected by wires to a galvanometer, as indicated in Fig. 41.

It will later be seen that any generator, either battery or dynamo, will produce a *certain maximum potential difference between its terminals*.

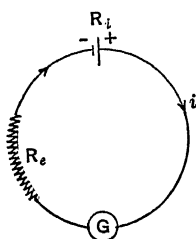


FIG. 41. External and internal resistance in a circuit.

Thus depending on the chemical nature of the constituents a battery will produce a given *electromotive force or maximum potential difference* the value of which can be calculated from the knowledge of the energy involved in the chemical transformations taking place. Again, a dynamo will also produce a certain maximum potential difference between its terminals, which depends purely on the design of the dynamo and the speed with which its armature is being rotated. *This maximum potential is called the electromotive force and is represented by the abbreviation e.m.f.* In both cases the e.m.f. obtained can be determined directly only by a static measurement or one involving no flow of current.

The reason for this will become evident immediately.

Every battery as well as every dynamo has a resistance within itself (i.e., resistance of the electrolyte or of the armature windings). This is called the *internal resistance*. If current is taken from a battery or from a dynamo, in order to maintain the potential difference at the terminals, the electricity which is being used must be supplied as rapidly as it is being drawn away. In order to accomplish this two conditions must be met: (1) The supply of the current to the external circuit must be kept constant. (2) The current through the

* It is often asked why either V_s or V_x could not replace V and thus the reading of the ratio V_s/V_x could be obtained from the ratios of R_2 and $R_1 + R_2$ directly without further measurement. However, the cell replacing V must deliver the current i through R_1 and R_2 to maintain the iR drop needed. Thus the cell acting for V would deliver a current and therefore *not* be giving its e.m.f. In addition, the resistance of the leads to the ratio boxes, or wire R_1R_2 introduces errors avoided by the procedure outlined. The cell V must give a constant potential difference and current throughout the two measurements.

generator to the terminal must be kept at the same value as in the external circuit. Thus a current must be forced through the internal portions of the generator to charge the plates at the same rate at which the plates are discharging to the external circuit. The current is being fed to the external circuit at a certain potential V which, by Ohm's law, is equal to the external circuit resistance R_e times i the external current. If R_i is called the internal resistance, the current i flowing through it to supply the external current i requires that a

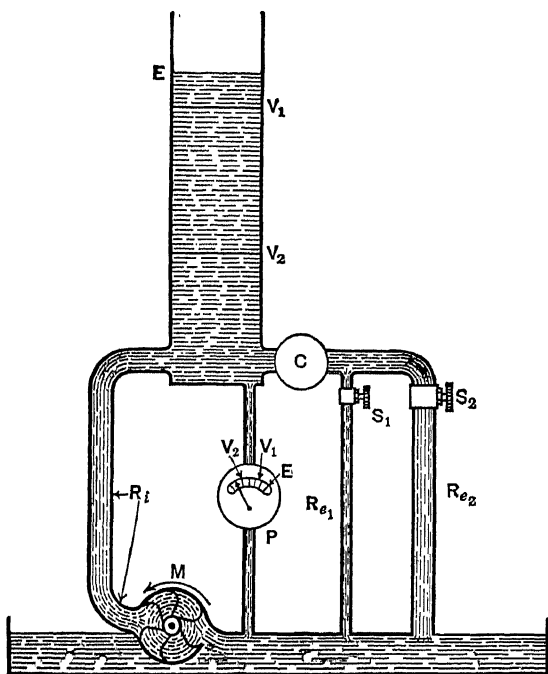


FIG. 42. Hydrostatic analogy representing electromotive force, E , potential, V , internal resistance, R_i , and external resistance, R_e , in a circuit. Compare with Fig. 43, which represents the electrical case.

fall of potential iR_i exist across the interior of the generator. Hence, if the current i is to be maintained the total potential, or the electromotive force (generally written e.m.f.), E , developed by the generator, must be equal to the sum of the iR_i drop inside the generator and the iR_e drop in the external circuit. Hence we must write

$$E = iR_i + iR_e = iR_i + V.$$

It is therefore seen that to maintain a potential V in an outside circuit the generator supplying the current must produce an electromotive force E which is greater than V by the iR_i drop in its internal resistance. This behavior is shown graphically in Figs. 42 and 43, which represent

hydrodynamical and electrical examples. M the water motor and B the battery are the sources of pressure. R_i represents the internal resistance. V_1 and V_2 are the potentials or pressure heads existing when valves S_1 and S_2 , respectively, are open. E represents the static heads when all valves are closed. A and C represent current meters. Only when $i = 0$, so that $iR_i = 0$, can the potential V , maintained by a generator in the external circuit, be equal to the electromotive force. This means, even if a high-resistance voltmeter is used with

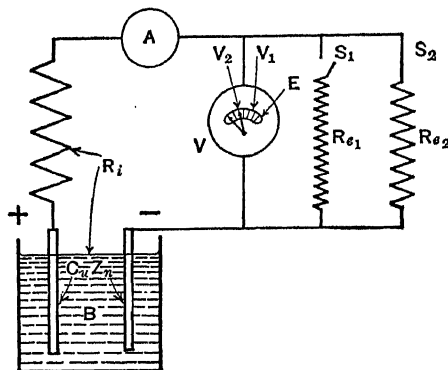


FIG. 43. Electromotive force, potential difference, internal resistance, and external resistance in a circuit. Compare with hydrostatic analogy in Fig. 42.

which to read the potential of a generator of some sort, that potential read will be the external potential drop through the coils of the voltmeter, and it will be less than the e.m.f. of the generator because the voltmeter draws a current even though it be a small one. It is seen that the difference between the electromotive force and the external potential maintained is merely the product of the current and the internal resistance or the internal drop of potential. If a method is capable of measuring potential with zero current the method will give the e.m.f. of a generator. Such measurements may be made by means of *static* voltmeters, quadrant electrometers, or the potentiometer.

From all this it may be seen that essentially the term potential drop or potential difference applies in the case of current flow to the difference of potential which maintains a current i through a resistance R in any portion of a circuit. *The electromotive force or e.m.f. is the total potential which is required to be generated by a source of current in order to maintain the current flow.* The e.m.f. is usually a function of the chemical constitution of the cell or of the magnetic conditions and speed of a dynamo. The electromotive force can be used to calculate the current through an external circuit at a given external potential provided the internal resistance is known, or vice versa. While the e.m.f., E , can be measured only by 0 current methods directly, it can

be calculated, and R_i can often be measured directly. If neither R_i or E is known, it is possible, by using two values of R_e and observing the two values of V maintained across R_e , to obtain two equations of the form below which may be solved simultaneously for R_i and E . If the equations $E = i(R_e + R_i)$ and $V = i(R_e)$ it can at once be seen that $\frac{E}{V} = \frac{R_e + R_i}{R_e}$. Accordingly, if by varying R_e two values of V (V and V' corresponding to R_e and R_e') are obtained, we have the relations

$$\frac{E}{V} = \frac{R_e + R_i}{R_e}, \quad \text{and} \quad \frac{E}{V'} = \frac{R_e' + R_i}{R_e'},$$

from which E and R_i can at once be found.*

That the phenomenon discussed above is an important one may be seen from the following example: Assume a battery having an internal resistance $R_i = 2$ ohms, and an external resistance $R_e = 4$ ohms, and assume that the electromotive force E is 6 volts; then the potential V across the 4-ohm resistance would be $\frac{V}{E} = \frac{4}{2 + 4}$. Thus $V = \frac{2}{3} E = 4$ volts. It is thus seen that in working with small resistances we must be careful to distinguish between the electromotive force of a cell and the potential difference given by it. An ordinary voltmeter has a resistance of the order of 1000 ohms. Consequently, if placed across five dry cells with perhaps a total internal resistance of 10 ohms, the potential read by the voltmeter is less than the electromotive force of the cell in the ratio of $\frac{10}{1010}$, that is, it is 1 per cent in error. For very high resistances where R_e is very much greater than R_i , V can be said to approach E .

* This method is not accurate because of change of resistance by heating when large currents flow and because of the resistance of the voltmeter. It is of considerable illustrative value.

CHAPTER VIII

GENERALIZATION OF OHM'S LAW

34. KIRCHHOFF'S FIRST LAW OF DIVIDED CIRCUITS

Ohm's law, though applicable to a great many of the simpler circuits, is not sufficient for determining the currents in complicated circuits. There are many problems, such as the problem of the Wheatstone bridge when it is not in balance, in which the existing currents and potential drops cannot be found by the simple Ohm's law. The reason is that Ohm's law is a special case of a far more general set of relations from which enough equations may be obtained to solve the problem. The method of treatment to be taken up here is that originated by Kirchhoff about 1842. It is indispensable in the practical study of any circuits, and electrical engineers cannot work conveniently unless they have a good command of these laws. The laws themselves are simple enough. The method of applying them, however, is more difficult. In what follows the laws will first be stated and they will then be applied to a simple problem, namely, the Wheatstone bridge. As an additional example a numerical problem will be worked out showing the actual method of approach to the study of any problem.

The first of Kirchhoff's laws says this: *The sum of all currents flowing into any point in a circuit must be zero if taken with due regard to sign.*

Regard, for example, the five wires radiating from the point O which may be a binding post in any portion of a complex circuit, Fig. 44. Flowing toward or from this point there are five currents, represented respectively by i_1 , i_2 , i_3 , i_4 , and i_5 . The arrows in the diagram indicate the direction in which the currents are assumed to flow. Now Kirchhoff's law merely states that, since the currents which are flowing towards this point O can flow only along the wires, and since there is no accumulation of electricity at the point, which will be the case *when a steady state exists*, the sum of the currents flowing toward the point must equal the currents flowing away from that point. Of course, during the first instant when the currents start to flow, as the point O is being raised to its equilibrium potential, there will be an accumulation of electricity.* This, however, is a

* Where this is occurring we can again set up equations, but the sum of the currents will then equal the rate of charging of the point; i.e., $\sum i = \frac{dq}{dt}$, where q is the quantity.

appear with a *negative* sign. This means that *the direction of the current flow is opposite to that assumed*. If this occurs it should cause no concern and *under no conditions* should diagrams be altered. The answer with the *negative* sign and the *chosen direction* tell the story. It should in practice be noted that the flow is reversed in connecting in a meter with marked terminals. The equations can be checked by taking any one of the combination equations for some of the meshes and substituting the values obtained.

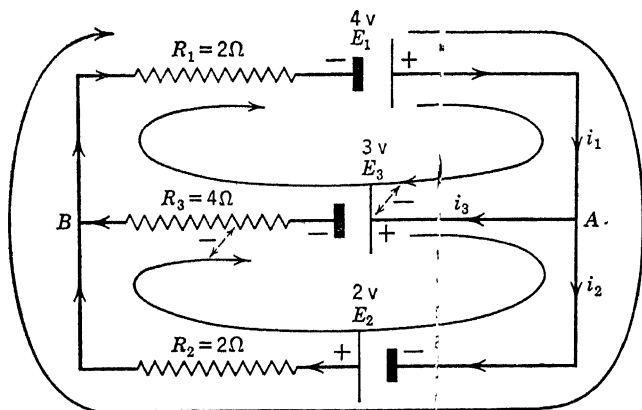


FIG. 47. A two-point circuit to illustrate the method of solving Kirchhoff's laws given in the text.

A practical application of Kirchhoff's laws may be made to the circuit shown in Fig. 47. Let E_1 , E_2 , and E_3 be batteries producing potentials of 4, 2, and 3 volts, respectively, while the resistances R_1 , R_2 , and R_3 have the values 2, 2, and 4 ohms, respectively. It is desired to know the currents i_1 , i_2 , and i_3 in the branches of the circuit corresponding to the resistances R_1 , R_2 , and R_3 . To solve this problem proceed as follows. Inspection of the circuit shows that E_1 has the highest potential, and the presumption is that the current from E_1 will flow in the direction of the small arrows indicated on the wires. At the point A the current will divide, one part going into R_3 and the other part going into R_2 . Were E_3 large and R_3 small it is possible that the current from E_2 would flow through R_3 in the opposite sense from that indicated. It is therefore not certain exactly how the current through R_3 is going to flow. We simply make the arbitrary *assumption* that it flows from A to B , basing our judgment on the fact that E_1 is greater than E_3 . If it should turn out that the current flows in the opposite sense a negative value for i_3 will be obtained in the result. Having made this assignment of current flow the ~~arrows~~ current at be placed on the wires in the diagram at once and the equati

for the currents at the junction points A and B .

$$\text{At} \quad \left. \begin{array}{l} A \quad i_1 = i_2 + i_3 \\ B \quad i_2 + i_3 = i_1 \end{array} \right\} \quad (1)$$

With this simple circuit it happens that the two current equations at the symmetrical junction points A and B are the same. Hence only one of these equations is necessary since either one contains all of the currents once. This is not usually true in more complicated circuits.

Turning now to the meshes, note first that E_1 and E_3 are opposed, while E_1 and E_2 are in series. It is noted that E_1 is greater than E_3 and hence the circular potential arrow in the mesh E_1AE_3B can indicate a clockwise fall of potential. Similarly, since the cells E_3 and E_2 are in series aiding each other, the circular potential arrow in the mesh E_3AE_2B will be in a clockwise sense. Similar reasoning applies to the mesh E_1AE_2B , since E_1 and E_2 are aiding and in series. Placing these potential arrows in the drawings it is noted that the arrow in E_1AE_3B opposes E_3 . This is indicated by the dotted arrow between E_3 and the potential arrow, with a negative sign alongside. Again, the potential arrow in E_3AE_2B is contrary, or opposed, to the assumed current flow in i_3 . This is indicated by the dotted arrow, with a negative sign between the two. The equations for the meshes are then written as:

EQUATION	MESH	
$E_1 - E_3 = i_1R_1 + i_3R_3$	E_1AE_3B	(2)

$E_3 + E_2 = i_2R_2 - i_3R_3$	E_3AE_2B	(3)
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$E_1 + E_2 = i_2R_2 + i_1R_1$	E_1AE_2B	(4)
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Note the negative signs on E_3 and i_3R_3 because they oppose the assumed potential fall.

Inspection shows that the mesh equation 4, E_1AE_2B , is the sum of equation 2, E_1AE_3B , and equation 3, E_3AE_2B . It can be set aside for check purposes since the one current equation and the two mesh equations contain each e.m.f., current and resistance in the circuit at least once. We write:

SUBSTITUTE NUMERICAL VALUES

$$i_1 = i_2 + i_3 \quad i_1 = i_2 + i_3 \quad (5)$$

$$E_1 - E_3 = i_1R_1 + i_3R_3 \quad 4 - 3 = 1 = 2i_1 + 4i_3 \quad (6)$$

$$E_3 + E_2 = i_2R_2 - i_3R_3 \quad 3 + 2 = 5 = 2i_2 - 4i_3 \quad (7)$$

quantity. Substitute equation 5 into equation 6 and subtract the resulting

expression from equation 7:

$$\begin{array}{rcl}
 1 & = & 2 i_1 + 4 i_3 = 2 i_2 + 6 i_3 \\
 & & 5 = 2 i_2 - 4 i_3 \\
 & - & (1 = 2 i_2 + 6 i_3) \\
 \hline
 & 4 = & -10 i_3
 \end{array}
 \qquad i_3 = -\frac{4}{10} = -0.4 \text{ amp}$$

$$1 = 2 i_2 - \frac{24}{10} \qquad i_2 = \frac{17}{10} = 1.7 \text{ amp}$$

$$i_1 = \frac{17}{10} - \frac{4}{10} = \frac{13}{10} \qquad i_1 = \frac{13}{10} = 1.3 \text{ amp}$$

Test equation 4:

$$\begin{aligned}
 4 + 2 &= 2 \left(\frac{13}{10} \right) + 2 \left(\frac{17}{10} \right) \\
 6 &= \frac{26 + 34}{10} = 6. \quad \text{Check.}
 \end{aligned}$$

Note that i_3 is -0.4 ampere in the result. This means that the current flows out of E_3 and into A instead of as indicated in the diagram. No further change is required; the answer is correct as given, and indicates that "guessing" about circuits does not give the correct solution.

38. SHORT-CUT METHODS FOR THE SOLUTIONS OF COMPLICATED CIRCUITS: MAXWELL'S METHOD AND METHOD OF THE EQUIVALENT Y

Maxwell's Method. J. C. Maxwell has shown that by a simple but clever artifice in first setting up the Kirchhoff's law equations the conditions imposed by Kirchhoff's first law can be automatically introduced into the mesh or second law equations. This gives distinctly fewer equations for solution and shortens and simplifies the work. Let us regard the Wheatstone's net depicted in Fig. 48. Call the resistances R_1, R_2, R_3, R_4, R_5 , and R_6 and the electromotive force or potential given by the cell E . Now the circuit can be divided into one enclosing three internal meshes, which will be indicated by the arrows and designated as u, v , and w . The currents in these meshes may arbitrarily be assumed to flow in the sense of the arrows around the encircling resistances. Looking at the circuit it will be noted that w represents the current in R_6 , u that in R_1 , v that in R_2 . In R_5 the current is represented by $u - v$. In R_3 it is $w - v$, in R_4 it is $w - u$. Thus all currents in the six conductors can be made up of combinations of the three assumed currents u, v , and w . On this basis it is necessary only to set up the three Kirchhoff's second law equations for meshes u, v , and w to have equations assembled containing each current at

least once. These three equations are easier to solve than the six independent equations in the formal treatment. These equations are as follows: In mesh w ,

$$E = (R_3 + R_4 + R_6)w - vR_3 - uR_4,$$

for w goes through R_3 , R_4 , and R_6 and in addition the currents flowing in meshes u and v are opposing w in sense, and flow respectively in

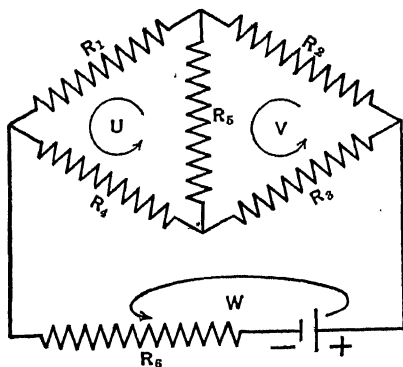


FIG. 48. Circuit designation for use in Maxwell's method of solution of Kirchhoff's laws.

R_3 and R_4 . Hence they take a negative sign and must be included in the equation for the mesh. Analogously we may write for meshes u and v :

$$0 = (R_2 + R_5 + R_3)v - uR_5 - wR_3;$$

$$0 = (R_1 + R_4 + R_5)u - vR_5 - wR_4.$$

Substitution of numerical values of R_1 , R_2 , R_3 , R_4 , R_5 , R_6 , and E into these equations makes it possible to solve at once for u , v , and w . If the value of only one current, say of the current i_5 in R_5 , is wanted, it is possible, since $v = u - i_5$, merely to substitute $u - i_5$ in the equation for v and get the three equations expressed in u , w , and i_5 and so solve for i_5 at once. This treatment is especially valuable where a-c circuits of a complicated character are to be solved.

Method of the Equivalent Y. It will have been noted that the really troublesome factor in the solution of equations for current flow resulted from the presence of R_3 in Fig. 49. It is seen that this breaks up the two parallel resistances R_1R_4 and R_2R_5 , the resistance of which can be calculated by the laws of series-parallel resistance into one triangular or delta (Δ) circuit $R_1R_2R_3$, and a Y circuit R_4R_5 , with the unknown current in R_3 . Now, could a delta circuit be replaced by a circuit with a resistance that is equivalent, such that in place of the structure of Fig. 49 the result is a circuit such as shown by Fig. 50, with a

resistance that is the same, it is seen that with this simplification we could break down many complicated circuits into combinations of series-parallel resistances. It happens that a very simple rule exists

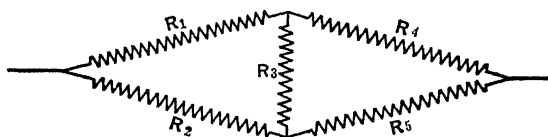


FIG. 49.

for calculating such Y circuits equivalent to any Δ circuit. Consider Fig. 51, where there is an unknown Δ circuit with resistances R_1 , R_2 , and R_3 . Since the Δ circuit leading out to A , B , and C presents currents which in effect flow out at these points, it should be possible to

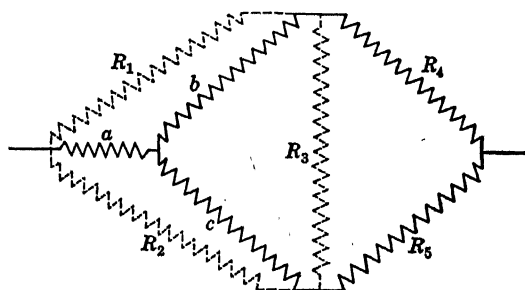


FIG. 50. A Y and a V circuit whose resistance can be made equivalent to the ghostlike Δ circuit (dotted) plus a V circuit.

get a Y circuit that has resistances a , b , and c and that gives the same effect. To calculate such a circuit, consider the resistances of the Y abc placed inside the Δ . It is clear that the resistances $a + b$ in series will now replace the parallel resistances R_1 and $R_2 + R_3$. Hence we can write

$$\frac{1}{a + b} = \frac{1}{R_1} + \frac{1}{R_2 + R_3}.$$

Analogously, $a + c$ replace R_2 and $R_1 + R_3$, and $b + c$ replace R_3 and $R_1 + R_2$ in parallel. Whence

$$\frac{1}{a + c} = \frac{1}{R_2} + \frac{1}{R_1 + R_3}, \quad \frac{1}{b + c} = \frac{1}{R_3} + \frac{1}{R_1 + R_2}.$$

Solution of these three equations for a , b , and c because of symmetry leads at once to the relations

$$a = \frac{R_1 R_2}{R_1 + R_2 + R_3}, \quad b = \frac{R_1 R_3}{R_1 + R_2 + R_3}, \quad c = \frac{R_2 R_3}{R_1 + R_2 + R_3}.$$

That is, the equivalent resistances of the branches of the Y are the product of the resistances of the two sides of the Δ bracketing the particular branch of the Y divided by the sum of the Δ resistances.

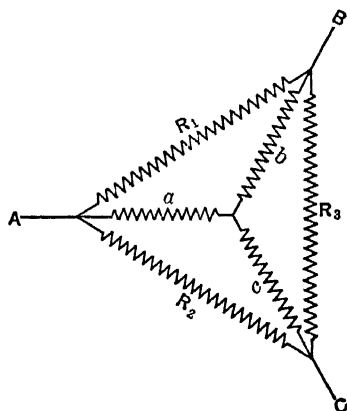


FIG. 51.

Hence the Δ of any Wheatstone's net can be broken down to an equivalent Y plus an equivalent V circuit which forms a simple series-parallel resistance. These can often be used to excellent advantage.

CHAPTER IX

ELECTRICAL MEASURING INSTRUMENTS

39. MOVING-COIL OR D'ARSONVAL GALVANOMETER

Of all the electrical measuring instruments, probably the most important one, and the one the theory of which applies to most of the current and potential measuring instruments now in use, is the moving-coil, or d'Arsonval, galvanometer. In Chapter V it was shown that from Ampère's rule unit current could be defined in two ways. The second definition was that *unit current is that current which, flowing in*

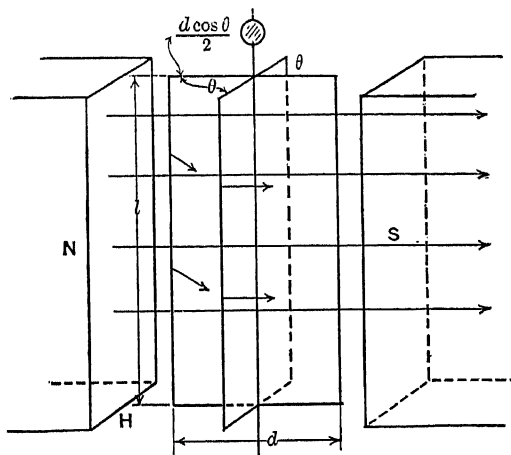


FIG. 52. Schematic diagram of a moving-coil galvanometer.

unit length of conductor perpendicular to a uniform magnetic field of unit strength, experiences a mechanical force of 1 dyne. This definition of unit current may be expressed in the statement $f = i_a H l$, where l is the length of conductor, H is the strength of the uniform field, i_a is the current, and f is the mechanical force in dynes. Assume a coil suspended in the uniform magnetic field, as shown in Fig. 52. The coil may be of rectangular cross-section and be so placed that its plane is initially parallel to the magnetic field, H . Assume that there are n turns of wire in the coil and that the length of the coil is l cm and its breadth is d cm. There is no force on the wires along the top and bottom of the coil as these are parallel to H . The force for the n

wires which run up one side of the coil when a current of i_a absolute units runs in the coil is given by

$$f = ni_a Hl.$$

When the plane of the coil lies parallel to the field, the force on the wires is perpendicular to the field and into or out of the plane of the paper in Fig. 52. Since the force on the vertical wires of the coil on one side is opposite in sense to that on the other the coil experiences a couple or torque of magnitude, $G = 2f \frac{d}{2} = ni_a Hld$. As the coil rotates through an angle θ with the field, the force is still at right angles to the wires and the field, but its full lever arm is no longer $\frac{d}{2}$. It is now the projection of $\frac{d}{2}$ onto the field direction, i.e., $\frac{d}{2} \cos \theta$. At 90° to the field the torque is zero. Hence the torque at an angle θ is,

$$G = ni_a Hld \cos \theta = A i_a H \cos \theta.$$

The quantity $A = nld$ is a constant of the coil, and is merely the total area enclosed by the n turns. If the coil had been of any other shape, the result would have been the same, as any area A can be broken up into rectangular elements of height dl and length d which now vary along the coil. The force on each element will again depend on $nldl$, and the total force will depend on $\int n d d l =$ the area of the coil.

Now the coil is suspended by a wire and the force moment on the coil due to the field acts to turn the coil at right angles to the field against the elastic forces of the suspension. For a deflection θ of the galvanometer, G is equal to $T_0 \theta$. (See section 14.) Thus the coil will be deflected through an angle θ with the field determined by the condition $T_0 \theta = A i_a H \cos \theta$. If small deflections are measured then $\cos \theta$ approaches unity, so that for deflections of less than 10° we can use the approximate equation

$$i_a = \frac{T_0 \theta}{A H}.$$

The use of small deflections is simplified by the fact that a mirror hung on the coil will give an appreciable deflection of a spot of light for a small angular deflection, if the distance of the source of light and the spot from the mirror is sufficiently great.

Actually at present soft iron cores of cylindrical shape are placed between the pole pieces having cylindrical notches concentric with these cut in them. The coil moves about the axis of the cylinders in the space between the core and the pole pieces. This field is thus

parallel to the plane of the coil, making $\cos \theta$ nearly 1 for deflections of from 45° to 90° .

40. THE GALVANOMETER CONSTANT

The equation for the current just deduced is the equation underlying all moving coil galvanometers. The ratio $\frac{T_0}{AH}$ is a constant of the apparatus as long as (1) the area of the coil remains constant, (2) the magnetic field due to the permanent magnets remains constant, and (3) the constant T_0 of the suspension is not changed. It is therefore called the galvanometer constant K' , and it can be written $i_a = K'\theta$, where θ is in radians. It is seen that to use such an instrument in measurement all that is needed is to measure θ , the deflection and, knowing K' , the value of i_a causing the deflection θ is obtained at once. Since K' is defined by $\frac{T_0}{AH}$, K' will be unity if the torsional constant T_0 is 1 dyne centimeter per radian, $A = 1 \text{ cm}^2$, and H is 1 oersted. Hence could these quantities be measured K' could at once be evaluated. Actually whereas a galvanometer with T_0 , A , and H can be built to *approximate* specifications, accurate evaluation of these items is a major task. What is then done is to observe θ for a known i_a and thus experimentally to *calibrate* the galvanometer. Hence all galvanometers, ammeters, and voltmeters operating on this principle, which are built to specifications, must be calibrated experimentally for use by observing θ for known currents.

In practical work it is more convenient to express currents in amperes. In this event the equation is written $i = K\theta$, where i is in amperes. Since in $i_a = K'\theta$, i_a is expressed in units ten times as large as i , to have i read in amperes, K must equal $10 K'$, i.e., $K = 10 K'$.

In practice again, galvanometers are either read by telescope and scale, or by having the image of a filament of a lamp reflected by the mirror and read on a translucent scale mounted above the lamp. These scales for small deflection ranges may be plane. For large deflections they are best bent to a circle of radius equal to the distance from the mirror to the scale. Now what is usually important in selecting a galvanometer is to get one such that the current measured will give deflections which can be read to the desired degree of accuracy. What is then demanded is an answer to the question, what is the characteristic of a galvanometer which will give a certain current to a certain degree of accuracy on a scale so many centimeters from the mirror? As it is convenient to use a scale divided into millimeters at about a meter from the mirror, a new description of the responsiveness or suitability of a galvanometer has been developed. This is the *figure of merit* of the galvanometer.

The figure of merit of a galvanometer is the value of the constant k so determined that it represents the current in amperes necessary to give 1-mm deflection on a scale 1 meter away. This gives the current through the equation $i = k\theta_m$. Here k is the figure of merit, i.e., it gives i when θ_m is a deflection of 1 mm on the scale at 1 meter scale distance. Now if the spot of light moves 1 mm at 1 meter, ϕ , the angle through which the light was deflected is given by $\tan \phi = \frac{1 \text{ mm}}{1000 \text{ mm}} = \frac{1}{1000}$. Since $\tan \phi$ effectively equals ϕ for $\frac{1}{1000}$ radian, then $\phi = \frac{1}{1000}$. Furthermore, the spot of light is deflected through twice the angle through which the mirror turns. Hence $2\phi = \theta$, $\phi = \frac{\theta}{2}$, and when a deflection of 1 mm at 1 meter's distance occurs, $\theta_m = \frac{1}{2000}$ of a radian movement of the mirror. Thus since as a unit of angle an angle $\frac{1}{2000}$ the angle designated by θ in the expression $i = K\theta$ is being used, then k will be $\frac{1}{2000}$ of K . Thus a current i will be given by multiplying the deflection θ_m read in millimeters at 1 meter's scale distance by the figure of merit $k = \frac{K}{2000}$.

Then for example, if, a galvanometer is desired which will give a current of 10^{-8} ampere to 1 per cent, it is noted that if $\theta_m = 10 \text{ cm} = 100 \text{ mm}$, 1 mm is 1 per cent. Hence for $i = 10^{-8}$, $\theta_m = 100$, and $k = \frac{i}{\theta_m} = \frac{10^{-8}}{100} = 10^{-10}$ is the desired figure of merit.

Manufacturer's catalogues also often list galvanometers in terms of their sensitivity, s . This is the resistance, expressed in millions of ohms (called megohms), which must be put in series with the galvanometer when 1 volt is placed across it to give 1-mm deflection at 1 meter's distance. Hence since $i = \frac{1 \text{ volt}}{\text{ohms}} = \frac{1}{s \times 10^6 \text{ ohms}} = k\theta_m$

with $\theta_m = 1$, $k = \frac{1}{s \times 10^6 \text{ ohms}}$, or $s = \frac{1}{k \times 10^6}$. A galvanometer is thus spoken of as having so many megohms sensitivity. For the galvanometer just referred to with $k = 10^{-10}$, $s = 10^4$ megohms.

The constants used in describing galvanometers may be summarized as follows:

$i_a = K'\theta$, K' in absolute units per radian, the angle θ being the deflection of the mirror.

$i = K\theta$, K in amperes per radian, the angle θ being the deflection of the mirror.

$i = k\theta_m$, k in amperes per millimeter deflection on a scale 1 meter away. This is the deflection of the spot of light.

$i = \frac{1}{s \times 10^6}$; s = resistance in 10^6 ohms (megohms) to permit 1 volt to cause 1-mm deflection on a scale 1 meter away. Hence, where $k = 10^{-10}$, $K = 2 \times 10^{-7}$, $K' = 2 \times 10^{-8}$, and $s = 10^4$ megohms.

In using these various units it must be noted that, in the defining equation, $i_a = \frac{T_0\theta}{AH}$, i_a is expressed in the torque $T_0\theta$ per unit area and unit field. $T_0\theta$ expresses the torque on the suspension. Hence the torsional constant T_0 , the torque per unit angle, must be expressed in such angular units that θ , the angle multiplied into it, gives torque. That is, T_0 must be in dyne centimeters per radian if θ is in radians, and dyne centimeters per millimeter deflection if $\theta_m = 1$ mm at 1 meter. This is obvious and would give no trouble if when using the constants K' , K , and k the constant T_0 had not been put into K' , K , or k and the θ left outside. Hence care must be used at all times to see that the deflection θ is expressed in the same units as the T_0 included in the constant. If this is done $K'\theta$ at all times will read absolute units of current and $K\theta$ and $k\theta_m$ will read amperes.

Concerning the values attainable for the figure of merit the following path of reasoning can be used. Since $i = k\theta_m$, or $k = \frac{i}{\theta_m}$, it is clear that the galvanometer will be the more sensitive the smaller the i required to give the same θ_m . Thus the smaller the i , and hence k the figure of merit, the more sensitive the galvanometer. But $k = \frac{K}{2000} = \frac{K'}{200} = \frac{T_0}{200 AH}$. Thus, to make k small, the suspension must be light (T_0 small), the coil area A must be large (as many turns over as large an area as possible), and the field H must be strong. It might be imagined that these quantities could be altered indefinitely so as to reduce k . However, H is not stable when it gets too large, and thus k would change. Again, a large number of turns increases the galvanometer resistance and its mass and moment of inertia. A large area increases the moment of inertia I of the coil. Finally, if T_0 is too small, while it can support the coil, the coil with a large I and small T_0 will have a long period of oscillation T , for $T = 2\pi\sqrt{\frac{I}{T_0}}$. If the period T exceeds much more than 60 seconds the galvanometer becomes unreliable and inconvenient. Hence at present with the best conditions it is possible to achieve a value of k only of the order of $k = 10^{-11}$.

41. TYPES OF GALVANOMETERS, AMMETERS, AND VOLTMETERS

For most current-measuring work today the galvanometers used are of the moving-coil type. Such galvanometers have been developed to a high degree of sensitivity and precision, and currents of the order of magnitude of 10^{-11} ampere may be measured with them. For still weaker currents the galvanometers used are generally of the suspended-needle type. In these the coil carrying the current generates the magnetic field and consists of many thousands of turns of fine wire. The magnets can be made very small and light so that the period is short for a low T_0 of the suspension.

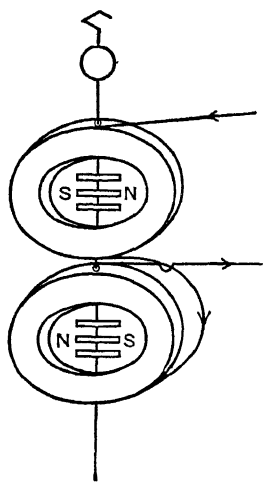


FIG. 53. Schematic diagram of the Kelvin astatic galvanometer.

Now the earth's field, weak though it is, begins to be commensurate with the fields produced by currents of 10^{-11} ampere. To cancel out the earth's field, Lord Kelvin developed the astatic type of galvanometer. In this, as shown in Fig. 53, the field of the wire is divided into two parts, an upper and a lower magnetic field. In this division the two halves are wound in a figure 8 so that the field of the current above is opposite to that in the lower loop of the 8. A small magnet is made as nearly equal to another one as possible. They are fixed to the single suspended rod carrying the mirror, and the one in the upper coil is placed with its N pole to the right and the lower one with the N pole to the left. Hence the earth's field will act on the rod only in the measure that the

upper and the lower magnet fail to be equally strong, while the coil acts on each magnet in the same sense.*

Galvanometers have been developed to a high degree of sensitivity for use with delicate thermocouples to measure the intensity of the heat radiation from the stars. They are also used in measuring the intensity of light in the various spectral lines. Such galvanometers must be protected from the earth's magnetic field, and this is accomplished by placing a series of soft iron shields around them. In one used by Professor Millikan, which was designed by Dr. Coblentz of the National Bureau of Standards, seven such shields of soft iron were required to cut out the earth's field. The limit of sensitivity in galvanometric measurements is about 10^{-12} ampere.

The galvanometer, as can be seen from this explanation, is used for measuring currents, and the deflection given is proportional to the

* Since it is very difficult to make magnets of the same strength the two small magnets referred to are made up of several carefully matched small magnets. This practice is fairly common where equality in magnets is required.

current. Since, however, the current through the galvanometer is proportional to the voltage across the galvanometer it is possible to calibrate it to read voltage. *In order that a galvanometer should read voltage all that is needed is that the current carried by the galvanometer should be so small compared to the current carried by the portion of the circuit across which the potential difference is to be measured that connecting the galvanometer shall not materially change the value of the current in the rest of the circuit, and thus alter the potential.*

Consider a wire of resistance R , through which a current i flows. The potential difference existing across this wire is sought. If a galvanometer of resistance R_g , which is very large compared to R , be placed across R , the current through the galvanometer will be so small as to leave the potential drop across R unchanged. The current through the galvanometer then will be practically proportional to the potential which existed across R before the galvanometer was connected in. If the deflection of the galvanometer in terms of the potential causing the deflection can be determined by calibration the galvanometer will read voltage.

Thus a galvanometer may be used in two ways. If it has a *low resistance* so that its *series resistance* does not materially increase the resistance of the circuit into which it is introduced, it will read the *current* in the circuit, that is, it acts as a galvanometer or an *ammeter*. If it has a *very high resistance* compared to the *resistance of a portion of the circuit across which it is introduced*, it will then read the *potential difference* of that part of the circuit across which it is connected.

In measuring potential and current for commercial purposes the galvanometers are modified in the following fashion. The coil is mounted on a vertical axis on jeweled bearings. The suspension is replaced by a small spiral spring. The coil has the same shape as before but the pole pieces are very ingeniously arranged as shown in Fig. 54 so that they give a uniform field perpendicular to the coil through a range of deflection of nearly 90° . The form into which the pole pieces must be fashioned to accomplish this is very simple and is shown in cross section in the diagram. The poles of the large permanent magnet are labeled N and S . They have a cylindrical hole drilled perpendicular to the plane of the paper. In the center of this hole is a soft iron lug I concentric with the axis of the cylinder. The

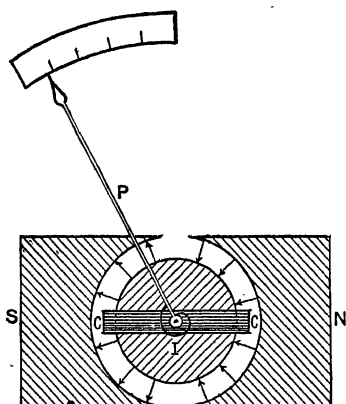


FIG. 54. Schematic diagram of a direct-current voltmeter or ammeter.

small rectangles labeled C and C constitute cross sections of the vertical portions of the coil. The arrow P is a pointer which moves over a uniformly divided scale on which volts or amperes are read directly. It is obvious that the soft iron lug causes a radial field to pass through it over a limited range of angles. Thus, as the coil rotates about its axis it stays in a nearly uniform field, so that the deflection is proportional to the current, for the restoring force of the spring is larger the greater the deflection. The instruments just described are called respectively ammeters or voltmeters, depending on their resistance, which determines their use, as has been stated.

It must be pointed out that such instruments, though they register current and voltage, cannot be used as *absolute* measures of current and voltage. Every voltmeter or ammeter put on the market *must be calibrated* in terms of the standard of current or potential.

In the electromagnetic system of units the standard of potentials was defined in terms of the heating effect of the current, that is, fixed points on the scale of the voltmeter can be determined as indicated on page 71 and as shown on pages 328–332. Again the ammeter must be calibrated to read amperes, and the calibration will be effected by means of a standard based ultimately on the measurements of the current balance.

It is to be noted that for galvanometers and voltmeters a given calibration holds good only as long as no change in the instrument has taken place. Although the field magnets on most galvanometers are so arranged as to maintain their magnetism for long periods of time the magnetism of such galvanometers must obviously change, and in general decrease in the course of years. No ammeter or voltmeter should be used the scale of which has not been checked against a standard within a year.

It is possible to arrange voltmeters or ammeters in which the current flows not only through the armature coil but also around coils which are wound on the soft iron which gives the field, as shown in Fig. 55. Thus both H , the field, and the torque, G , will be caused by the same current i . The deflection θ is therefore proportional to i^2 . Such instruments have the advantage that they are *independent of a variation in the magnetic field*. They have, however, a *scale in which the deflection varies as the square of the current*. Since the currents in the field coils and the armature or suspended coil are always in the same direction relative to each other the torque will always be in the same direction even if the current through the line changes in sense. Inasmuch as industrial development has led to the universal use of alternating currents which vary in direction with time, practical needs have demanded the widespread use of this type of ammeter and voltmeter. The interpretation of the meaning of the reading of such instruments must be reserved for Chapter XXIV, where alternating currents are discussed. Besides the use of such instruments for

alternating currents, the necessity of nonvarying standards makes these instruments of value, for the other type of instrument has permanent magnets the strength of which may vary with time.

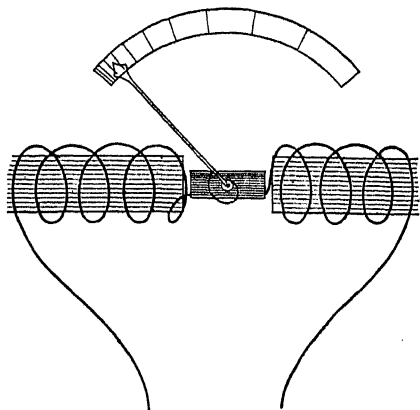


FIG. 55. Alternating-current voltmeter or ammeter.

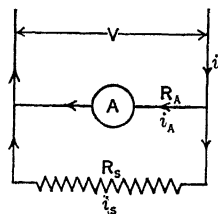


FIG. 56. Diagram of an ammeter shunt.

42. SHUNTS FOR AMMETERS, SERIES RESISTANCE FOR VOLTMETERS

The nonuniform scale divisions in ammeters and voltmeters which are used for standards, and which are based on the principle discussed at the end of the last section, where the field is excited by the current which flows through the armature, limit the range for accurate measurement on any given instrument. Such instruments often serve as standards, and to increase the range of instruments of this type a device known as the shunt, for ammeters, is utilized. For voltmeters, a device equivalent to the shunt is used. It is worth while to discuss the shunt briefly, the theory of which follows directly from Ohm's law. Assume an ammeter, represented by A in Fig. 56, which has a resistance R_A . Now place a resistance R_s in parallel with it. Then, as the potential difference is V , the current i_A flowing through the ammeter is given by

$$i_A = \frac{V}{R_A},$$

and the current i_s through the shunt is given by

$$i_s = \frac{V}{R_s}.$$

Finally, for the two combined, $\frac{V}{R} = i$, where i is the current through both ammeter and resistance, and R is the resistance of the combina-

tion of R_A and R_s . From the law of resistances in parallel,

$$\frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_s}.$$

Therefore,

$$R = \frac{R_A R_s}{R_A + R_s}.$$

Hence,

$$\frac{V}{\frac{R_A R_s}{R_A + R_s}} = i.$$

Thus

$$\frac{i_A}{i} = \frac{V}{R_A} \bigg/ \frac{V}{\frac{R_A R_s}{R_A + R_s}},$$

or

$$\frac{i_A}{i} = \frac{R_s}{R_A + R_s}.$$

This states that the current through the ammeter is to the total current through the circuit as the resistance of the shunt is to the sum of the resistance of ammeter and shunt. Thus, if the current given by the ammeter is read and that multiplied by the ratio of the sum of the resistance of ammeter and shunt to the resistance of the shunt, the answer is i , the current flowing through the whole circuit. If then an ammeter reads over a range of 1 ampere and the problem is to make it read over a range X times as great, all that is needed is to make a shunt of resistance such that the resistance R_A of the ammeter plus the resistance R_s of the shunt is equal to X times the resistance R_s of the shunt. Under these circumstances the ratio

$$\frac{R_A + R_s}{R_s} = X$$

$$i = X i_A.$$

This means merely that if the current through the ammeter as given by it is read, and this reading multiplied by the factor X , the answer is the current i flowing through the circuit.

To increase the range of a voltmeter a principle which follows simply from Ohm's law is made use of. Assume a voltmeter with an internal resistance R_V , and place in series with it a resistance R_s , as

shown in Fig. 57. Call R the combined resistance of R_V and R_s ; that is

$$R = R_V + R_s.$$

The current i , through the circuit, if a potential V is applied across the two resistances in series, is given by

$$i = \frac{V}{R} = \frac{V}{R_s + R_V}.$$

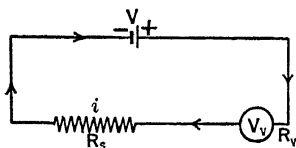


FIG. 57. Series resistance in a voltmeter circuit to extend its range.

The potential difference V_V across the voltmeter gives a current i , which is the same as the current flowing through the system as a whole. Thus

$$\frac{V_V}{R_V} = i.$$

From this equation one has

$$\frac{V_V}{R_V} = i = \frac{V}{R_s + R_V},$$

so that

$$\frac{V_V}{V} = \frac{R_V}{R_s + R_V}.$$

This says that the potential difference across the voltmeter, which is the potential difference registered by it, is to the total potential difference across the voltmeter and the resistance R_s , which was denoted by V , as the resistance of the voltmeter is to the sum of the resistances R_s and R_V . If a voltmeter which reads up to 150 volts is used to read 150 (X) volts all that is needed is to make the sum of the resistances R_s and R_V equal to X times the resistance R_V . Then

$$\frac{R_s + R_V}{R_V} = X$$

and

$$V = X \cdot V_V.$$

This states that, if we have a potential across the voltmeter of a given amount, the potential across the combined voltmeter and the resistance R_s is X times as great.

43. THE WATTMETER

The principle which underlies the voltmeter and the ammeter may be made use of to measure power directly. In Chapter VI the practical unit of power, the watt, was defined as given by multiplying

the amperes flowing through the circuit by the potential difference in volts which is maintained across the circuit. As was stated before, a moving-coil galvanometer with a high resistance acts like a voltmeter. If the magnetic field H of the galvanometer be produced by a coil of wire through which the main current in the circuit is flowing, the field H will then be proportional to the current i . The force on the moving coil will be the product of the field H and the current flowing through the high-resistance coil. But this current measures the potential, V , across the coil, and H depends on i , the current through the low-resistance system. Thus the torque produced will be proportional to

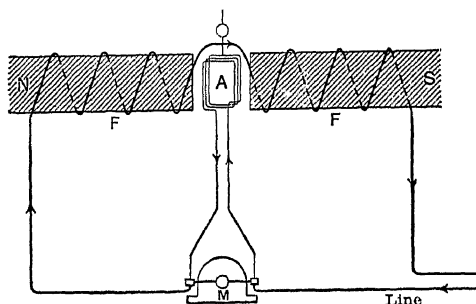


FIG. 58. Schematic diagram of a wattmeter.

the product of the current and the potential across the circuit, iV ; that is, the deflection will be proportional to the power in the circuit. Such an instrument is known as the wattmeter.

A schematic diagram of the instrument and its relation to the circuit is shown in Fig. 58. In the diagram the magnetic bars of soft iron N and S are magnetized by the coils F . The current which flows through these is the main current flowing through the line. M is a motor the power consumption of which is being measured. It is connected so that, as will be seen, M is in series with the coils. The small armature coil A of the wattmeter has a very high resistance. The current through it, therefore, does not materially affect the current through the motor or the rest of the line. Its terminals connect it to the two terminals of the motor. The current through it is therefore proportional to the potential across the motor. The deflection of A depends on the product of the potential across A and the current i through the coils F producing the fields. The instrument reads watts only if it is arbitrarily calibrated by comparison with standard instruments. Where the currents vary periodically with time, and potential difference and current in the motor are *not in phase* with each other (see Chapter XXV), the wattmeter described here gives the only means of measuring the true power consumption, for it automatically gives a deflection proportional to the average of the product of current and potential in the phase relations existing in the circuit.

CHAPTER X

LIQUID CONDUCTORS I: ELECTROLYSIS

44. TYPES OF CONDUCTION AND FARADAY'S LAWS

It was stated in Chapter VII in discussing conductivity that electrical conduction is of two types. The first type of conduction is metallic conduction, which depends on the transport of electricity by the very small and mobile electrons. Practically all the nonmetallic conductors depend on the transport of electricity by the atoms or the molecules of the substances themselves, the atoms and the molecules being charged. This constitutes the second type of conduction. The conductivity caused by carriers of this kind first came to light in a study of solutions. It was later extended to explain the conductivity of most nonmetallic solids. In certain crystals conductivity is partly electronic and partly molecular, and the remarkable work of Joffé and of Pohl has given a completely new insight into the problems of such conductivity.

These men have succeeded in measuring the very feeble conductivity of many nonmetallic crystals. In their studies they were able to demonstrate the electrolytic nature of the conductivity of some of the crystals, and to show that in certain crystals one ion is mobile while the other ions are fixed and in others that apparently both ions are mobile. Joffé further investigated the influence of temperature on this conductivity and showed that in some crystals temperature acted by decreasing the viscosity of the crystal and also by increasing the dissociation of the crystal lattice into ions, thus facilitating conduction. In certain crystals conductivity was found to be influenced by impurities, and, in fact, the higher conductivities of certain of the crystals were found to be entirely the result of the presence of impurities which could be electrolyzed out or separated by fractional crystallization. It was shown that in certain cases, particularly in crystals containing colloidal metals which had been separated by the action of x-rays, the ultra-violet light succeeded in producing a marked conductivity which resulted from the liberation of photoelectrons from the colloidal metal particles inside the crystal, the electrons then leading to an electronic conductivity in a dielectric crystal. The fascinating experiments of Joffé and of Pohl can be found described in Joffé's *Physics of Crystals* and in Gudden's book entitled *Lichtelektrische Erscheinungen*.

Most pure liquids, such as benzene, alcohol, and water as well as

certain solids such as porcelain and glass when cold, show little conductivity of either type. However, when salts, acids, or bases are present in water, the conductivity may become quite high. The same holds true for solutions of some substances in liquid NH_3 . High conductivity of a similar character is observed for molten salts and other solids such as porcelain or glass.

About the same year that Volta developed the voltaic pile it was discovered that an electric current flowing through water containing traces of dissolved inorganic material caused an evolution of gas at the metallic terminals in the liquid. These terminals, which from now on will be called *electrodes*, were covered with bubbles of gas. It was observed that at the negative electrode hydrogen was liberated, and at the positive electrode oxygen was liberated. It was also observed that twice as much hydrogen by volume was liberated as oxygen. Faraday investigated this phenomenon and showed that in solutions of other substances such as CuSO_4 , and AgNO_3 , the metal was liberated at the negative pole, and that oxygen was liberated at the positive pole, if platinum or noncorrosive electrodes were used. He investigated the deposition of these metals quantitatively and found that for a *given current flowing for a given time the weights of substances liberated were always the same, the weight for each substance being characteristic of that substance*. He further observed that the *weights of these substances liberated had very definite ratios to each other*. He also found that *these ratios were related by simple fractions to the atomic weights of the substances*. He named the phenomenon *electrolysis*, and he called the *negative electrode the cathode* and the *positive electrode the anode*.

Faraday noted that a quantity 96,500 coulombs (amperes \times seconds) of electricity liberated 1 gram atom of hydrogen, $\frac{1}{2}$ gram atom of oxygen, 1 gram atom of silver, and $\frac{1}{2}$ gram atom of copper. The same quantity has since then been found to liberate $\frac{1}{2}$ gram atom of zinc, 1 gram atom of chlorine, $\frac{1}{3}$ gram atom of bismuth, and $\frac{1}{4}$ gram atom of thorium. This fraction is called the *equivalent*, i.e., $\frac{1}{2}$ gram atom of copper is equivalent to 1 gram atom of H in carrying power for electricity. By the term *gram atom* is meant that *weight in grams corresponding to the atomic weight of the substance* as given in any table of atomic weights. Now since 1 gram atom of hydrogen, silver, or copper (and so forth) contains, according to Avogadro's rule, the same number of atoms, it follows that for 1 atom of hydrogen there is deposited $\frac{1}{2}$ atom of oxygen, 1 atom of chlorine, 1 atom of silver, $\frac{1}{2}$ atom of copper, $\frac{1}{2}$ atom of zinc, $\frac{1}{3}$ atom of bismuth, and $\frac{1}{4}$ atom of thorium. Thus, if an atom of hydrogen carries a unit charge an atom of oxygen carries two unit charges, that of chlorine one unit charge, that of silver one unit charge, that of copper two unit charges, that of zinc two unit charges, that of bismuth three unit charges, and that of thorium four unit charges.

This phenomenon is shown by the experiment illustrated in Fig. 59. In this figure four devices containing different solutions are seen in series with a battery *B* and an ammeter *A*. In the first one there is an H-shaped tube with an electrode in each of the vertical branches of the *H*. These branches are initially filled to the stopcock at the top with acidulated water. Down the sides of the two vertical branches there are graduations giving the volume, starting with zero at the stopcock. As the current passes through these cells the positive hydrogen ions go to the negative electrode, give up their charge, and

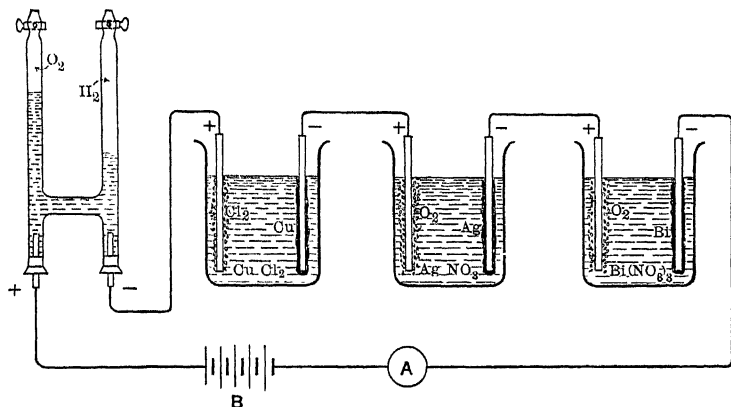


FIG. 59. Electrolysis.

are liberated as a gas which rises to the top of the tube. On the left-hand side, at the positive electrode, the OH ions which are negatively charged give up their charge to the anode. In doing this they react together to give an atom of oxygen and form water. The atoms of hydrogen and oxygen liberated at the cathode (−) and anode (+), respectively, shortly unite in pairs to form molecules. The molecules, since the water is saturated with these gases, are then liberated in the form of bubbles, and the water in both arms of *H* is displaced by the gas through an outlet tube not shown in the diagram. By adequately adjusting the pressures through this outlet tube the actual volume of gases of hydrogen and oxygen liberated can be estimated at standard temperature and pressure. Thus the weight of gas liberated may be computed and the flow of current determined from Faraday's laws. Experiment would show that the *volume* of oxygen liberated is one-half that of hydrogen. In the beaker labeled CuCl_2 there is a solution of cupric chloride, CuCl_2 . In this the copper is deposited at the negative pole, while chlorine gas is liberated at the positive pole. Some of the chlorine will react with water to form hydrochloric acid and liberate oxygen. Some of it will, if possible, react with the anode metal, forming a chlorine salt of the anode, and finally the balance of the chlorine

will be liberated as a gas. If the copper liberated at the cathode is weighed it will be seen that if 1 gram of hydrogen is evolved in the acid tube, 8 grams of oxygen will be liberated, and $\frac{8}{2}$ or 31.5 grams of copper deposited in the cell Cu. In the AgNO_3 beaker there is a solution of silver nitrate. In this beaker silver is deposited on the cathode while the nitrate ion gives up its charge to the anode, and thereupon acts with water to liberate oxygen and form nitric acid. In the silver cell the weight of silver deposited under the conditions that deposited 1 gram of hydrogen will be 107.9 grams. In the last beaker, labeled $\text{Bi}(\text{NO}_3)_3$, the current deposits bismuth from bismuth nitrate at the cathode, while again the nitrate ion liberates oxygen at the anode. The same current which deposited 1 gram of hydrogen will deposit $\frac{208}{3}$ or 69.3 grams of bismuth.

The *metallic* elements like silver, copper, and so forth, carry *positive charges* for they move to the negative pole, and the *nonmetallic* elements like chlorine carry *negative charges* for they move to the positive pole. This peculiar relationship of the charges carried by the various atoms has a definite relation to the combining ratios of the elements in the substances studied. Thus, one hydrogen atom combines with one chlorine atom in hydrochloric acid; two hydrogen atoms combine with one oxygen atom in H_2O ; two chlorine atoms combine with one copper atom in CuCl_2 ; three chlorine atoms combine with one bismuth atom, etc. In fact, the charge carried by the ion, that is, the charged atom of matter in electrolysis, is very closely related to the chemical *valency* of the substance, in each compound studied.

Thus, accepting Avogadro's hypothesis and the assumption of Faraday's law that 96,500 coulombs deposit a *gram atom equivalent* of the particular atom, it can be concluded that *electricity is atomic in nature*. That is to say, there is a *unit quantity of electricity*, for that is the assumption underlying the reasoning just preceding. At the time of Faraday, this idea was not ripe because people were thinking in terms of the one- and two-fluid theories of electricity. However, in 1867 Helmholtz stated that the only interpretation to be placed on Faraday's laws was that electricity exists in definite units. In other words, he stated that there was an atom of *electricity* and that it was associated with the *atoms of matter in electrolysis*. It was not until thirty years later that J. J. Thomson actually showed the existence of the negative atom of electricity, the so-called electron, and it was nearly ten years later that Millikan made the first accurate measurement of the electron, in terms of electromagnetic and electrostatic systems of units.

The best way of summing up Faraday's laws would be to state that *96,500 coulombs of electricity will liberate a gram atom equivalent of an element from solution*. The equivalent is the reciprocal of the valence of the element in the particular compound in question. Where unknown it can be determined by the comparison of the amount

liberated with the amount of an atom of which the electrochemical equivalent is known that is liberated by the same current. As an example, 96,500 coulombs of electricity liberate 1 gram of hydrogen, 8 grams of oxygen, 107.88 grams of silver, 31.78 grams of copper from CuCl_2 , 69.3 grams of bismuth from BiCl_3 , and 35.45 grams of chlorine. This fact is of great use in the measurement and comparison of currents. In fact, these laws furnish the basis of the *legal standard* of current, and the *instrument used in measuring currents by this method* is known as the *voltmeter*.

45. ATOMIC STRUCTURE AND CONDUCTION IN SOLUTION

The tendency for some compounds to show ionic conduction and for certain elements to show electronic conduction, while still other substances are insulators, is a direct consequence of systematic regularities in atomic structure among the elements which are expressed by the periodic table of the elements. Thus the electrical behavior of a substance will depend on the properties of its constituent atoms as related to their position in the periodic table of the elements.

In 1896 J. J. Thomson discovered the negative electron and showed it to be a universal constituent of all atoms. In 1913 Lord Rutherford showed that the atom was composed of a minute positively charged nucleus the size of which was comparable with that of the electron but which carried 99.98 per cent or more of the mass of the atom. The charge on the electron was shown by Millikan in 1906 to 1913 to represent the unit of electrical charge and was evaluated by him.

With the ratio of $\frac{e}{m}$ found by Thomson the electron mass was shown

to be $\frac{1}{1839}$ that of hydrogen, the lightest known atom. Nuclear and

electronic dimensions are of the order of 2×10^{-12} cm and less, whereas atomic dimensions were observed to be of the order of 10^{-8} cm. In 1914 Moseley proved that the charge Z on the atomic nucleus of a given element is the sequential number of the position of the element in the periodic table of elements. This table was first put out by Mendeleev about 1869 and was later modified by the chemists, after the discovery of the inert gases around 1900, to include the inert gas group elements He, Ne, Ar, etc., and the transition element triads Fe, Co, Ni; Ru, Rh, Pd; and Os, Ir, Pt. It was thus clear that the nuclear charge Z fixed the position of the element in the periodic table and determined its properties.

This is to be expected as atoms and molecules of substances are electrically neutral as observed. Thus there must be as many negative electrons in an atom as its $+Z$ nuclear charges call for, and the number of electrons should be related to its properties. The problem of the electron arrangement and the relation of electrons to the nucleus

was difficult. If electrons were particles of a mechanical nature with a negative charge they could remain about a massive minute nucleus of positive charge only if they were in constant orbital motion about that nucleus. Thus, at the start the physicists tried to picture the atom as consisting of a nuclear sun with planetary electrons in orbits about it. At first sight the task of organizing an atom of Z electrons with all possible radii of orbits, all possible shapes of orbits from eccentric ellipses to circles, and all inclinations of orbits appeared formidable. The concept of mechanical electron orbits, furthermore, ran into serious difficulties from the viewpoint of electromagnetic radiation theory. The problem was solved between 1913 and 1928 through the pioneer work of Niels Bohr, who suggested the existence of certain stable orbits and laid down the principles determining them. The work of the spectroscopists and studies of the magnetic properties of atoms finally led the countless physicists working on the problems to a proper solution by 1928. The solution began with the discovery of the spinning electron and the suggestion by L. V. de Broglie that the electron was some sort of a complex electromagnetic wave motion. The *wave mechanics* which then followed indicated that the electron is not simply a charged mechanical mass point. It is best represented by a complicated mathematical expression which accurately predicts its behavior but for which there is no adequate mechanical picture. It is, however, *possible* to a rough degree of approximation *to picture the electron as a negative charge moving in orbits about the nucleus*. Since these orbits precess about the atom the charge of the electron over some time is effectively smeared out about the nucleus in certain regions, or shells, with rough approximations to the charge density distributions indicated by the more correct wave mechanics.

In consequence, in this discussion the electron can be considered as executing orbits about the nucleus and, in addition, it can be assumed that the average radius, the shape, and the orientation of the orbit plane are restricted to certain chosen values by *quantum rules or restrictions* set by wave mechanics. If we regard these rules (which will be pointed out shortly) and add two further criteria it is possible to predict with uncanny precision the character of a given atom and its behavior. The other criteria are: (1) the Pauli exclusion principle, and (2) the tendency for an electron always to take the position of the lowest possible potential energy.

The restrictions concerning the allowed electron orbits are given by assigning certain *quantum numbers* influencing the various orbital properties. These quantum numbers take on integral numerical values only. For example, what is called the principal quantum number, n , can only have values 1, 2, 3, 4, 5, etc., while what is called the azimuthal quantum number, l , can have only values 0, 1, 2, 3, 4, etc. There are four such quantum numbers, as shown in the table.

PROPERTIES OF THE FOUR QUANTUM NUMBERS

Quantum Number	Function	Value
Principal, n	Fixes orbit radius.	It takes on values 1, 2, 3, 4.
Azimuthal, l	Fixes orbit shape.	It takes on n possible values 0, 1, 2, 3 for a given value of n . l has n values beginning with $l = 0$. $l = 0$ orbits are the most elliptical orbits.
Magnetic, m_l	Fixes the orientation of plane of the orbit. m_l is a number giving multiples in $h/2\pi$ units of the projection of the moment of momentum vector in the orbit on a given field axis.	It has values $-3, -2, -1, 0, 1, 2, 3$, and for a given l there are $2l + 1$ values.
Spin, m_s or s	Gives the spin of the electron relative to the orbit as $\pm \frac{1}{2} \left(\frac{h}{2\pi} \right)$.	It has only two values, $\pm \frac{1}{2}$.

With these numbers, an orbit and its properties can be described fairly well. To see this a few examples may be discussed.

Let us start with an orbit of $n = 2$. It has a certain radius r governed by $\frac{n^2}{Z} = \frac{r^2}{a_0}$. If $n = 2$, l can only be 0 or 1. If $l = 0$, m_l has only one value, 0. But in an orbit $n = 2$, $l = 0$, $m_l = 0$ there can be two electrons of $s = +\frac{1}{2}$ and $s = -\frac{1}{2}$. If $l = 1$, m_l has $2(1) + 1$ or three values: $-1, 0, +1$, and $s = \pm\frac{1}{2}$. It is seen that this takes a total of six electrons. Thus while for $n = 1$ there are only two electrons with $l = 0$ and $s = \pm\frac{1}{2}$ there are for $n = 2$ a total of eight electrons in two groups $l = 0$ and $l = 1$. In general, the $l = 0$ orbits are the very elliptic ones, while the highest-value l orbits in any class are circular.

The combination of n and Z in general pretty well fixes orbital radii. For $n = 1$ and $Z = 1$, as in the H atom, $\bar{r} = 0.528 \times 10^{-8}$ cm. For $n = 1$, $Z = 92$ as in the inner orbit in U, the orbit has $\bar{r} = 0.58 \times 10^{-10}$ cm. For $n = 2$, $Z = 1$ as in the outer orbit of the Li atom $\bar{r} = 2.32 \times 10^{-8}$ and for K with $Z = 1$ and $n = 4$, one would expect that $\bar{r} = 8.5 \times 10^{-8}$ cm. Actually, because of the ellipticity of orbits, the radii of outer orbits, such as for K, are somewhat less than the elementary theory gives. In any case it is seen that the value of n for any one class of orbit with a given Z will segregate that group of orbits about a mean value of \bar{r} with orbits of lower and higher n well inside or outside \bar{r} . As Z increases for a given set of orbits n , the orbit radii shrink in proportion. The orbit in H with $Z = 1$, $n = 1$ is known as the *unit Bohr orbit* and often acts as a measuring stick for atomic values.

It is seen that the character of orbits possible are very definitely set by the values of n , l , m_l , and s . If now we accept the Pauli exclusion principle, which says that *one and only one* electron in an atom can at one time occupy a given position set by n , l , m_l , and s , the organization of electrons in atoms as Z increases is pretty well fixed. If we also assume that each electron added to a structure will go to the position of lowest potential energy allowed by Pauli's principle we are in a position to arrange the electrons in their places in the atom. Because the $l = 0$ type elliptic orbits penetrate closer to the nucleus (greater average Z), they have a lower potential energy than the more circular ones.

This arrangement is best achieved by constructing a table of elements and indicating the number of electrons and their location in regard to n and l for each Z (see page 123). A study of the table shows that, beginning with He, the $n = 1$ group of 2 electrons is complete and that this group *persists as an inner shell in all atoms*. It is called the K shell, or level. As Z increases this shell has a smaller and smaller radius and is the innermost electron group. Having two electrons of opposite spin it is magnetically balanced and is a very stable configuration. It is next noted that in succeeding elements the L shell, or level, with $n = 2$ begins to fill in — first with two $l = 0$ electrons and later with the six $l = 1$ electrons. This group is complete with the inert gas neon, $Z = 10$. It is again noted that the outer octet, $n = 2$, structure of $2 + 6$ electrons complete in Ne remains as a separate inner shell, or zone, in all succeeding elements. Its radius is greater than that of the K shell but it lies inside the other shells.

Following Ne the next element Na starts to build on the $n = 3$ shell with one $l = 0$ electron in an elliptic orbit. This building of the beginning of the $n = 3$ shell, with two $l = 0$ and six $l = 1$ electrons, is complete with argon, another inert gas. With K, the element following Ar, the $n = 3$, $l = 2$ levels *do not begin to build out with their* $(2l + 1)2 = 10$ electrons. The reason is that the electron in the $n = 4$, $l = 0$ class, with its very elliptic orbit, is more tightly (lower potential energy) bound than is the circular orbit type $n = 3$, $l = 2$ electron. Beginning with Sc the $n = 3$, $l = 2$ shell builds up and is complete with Cu, $Z = 29$. In doing so, it passes through the triad Fe, Co, Ni, where the peculiar electronic conditions lead to ferromagnetism in the solid state. Following Cu the $2 + 6$ electrons completing the $n = 4$, $l = 0$, and $l = 1$ shells of Kr add on. Thus, with Cu the $n = 3$, or M shell, is complete and persists as the third shell in the interior of the atoms. From Kr on, the processes of filling in further shells as Z increases follows the same general pattern as before.

It will now be noted that in the periodic table the elements are grouped in vertical columns beginning with the alkali elements Li, Na, K, Cs, Rb, Cu, Ag, and Au in the first column, Be, Mg, Ca, etc., in the second column, etc., proceeding to the negative halogen column with F

BOHR-STONER TABLE OF ELECTRON ASSIGNMENT IN ATOMS

Element	Z	$n=1$ $l=0,s$		$n=2$ $l=0,s$ $l=1,p$		$n=3$ $l=0,s$ $l=1,p$ $l=2,d$			$n=4$ $l=0,s$ $l=1,p$ $l=2,d$ $l=3,f$				$n=5$ $l=0$
H	1	1	—	—	—	—	—	—	—	—	—	—	—
He	2	2	—	—	—	—	—	—	—	—	—	—	—
Li	3	2	1	—	—	—	—	—	—	—	—	—	—
Be	4	2	2	—	—	—	—	—	—	—	—	—	—
B	5	2	2	1	—	—	—	—	—	—	—	—	—
C	6	2	2	2	—	—	—	—	—	—	—	—	—
N	7	2	2	3	—	—	—	—	—	—	—	—	—
O	8	2	2	4	—	—	—	—	—	—	—	—	—
F	9	2	2	5	—	—	—	—	—	—	—	—	—
Ne	10	2	2	6	—	—	—	—	—	—	—	—	—
Na	11	2	2	6	1	—	—	—	—	—	—	—	—
Mg	12	2	2	6	2	—	—	—	—	—	—	—	—
Al	13	2	2	6	2	1	—	—	—	—	—	—	—
Si	14	2	2	6	2	2	—	—	—	—	—	—	—
P	15	2	2	6	2	3	—	—	—	—	—	—	—
S	16	2	2	6	2	4	—	—	—	—	—	—	—
Cl	17	2	2	6	2	5	—	—	—	—	—	—	—
A	18	2	2	6	2	6	—	—	—	—	—	—	—
K	19	2	2	6	2	6	—	1	—	—	—	—	—
Ca	20	2	2	6	2	6	—	2	—	—	—	—	—
Sc	21	2	2	6	2	6	1	2	—	—	—	—	—
Ti	22	2	2	6	2	6	2	2	—	—	—	—	—
V	23	2	2	6	2	6	3	2	—	—	—	—	—
Cr	24	2	2	6	2	6	5	1	—	—	—	—	—
Mn	25	2	2	6	2	6	5	2	—	—	—	—	—
Fe	26	2	2	6	2	6	6	2	—	—	—	—	—
Co	27	2	2	6	2	6	7	2	—	—	—	—	—
Ni	28	2	2	6	2	6	8	2	—	—	—	—	—
Cu	29	2	2	6	2	6	10	1	—	—	—	—	—
Zn	30	2	2	6	2	6	10	2	—	—	—	—	—
Ga	31	2	2	6	2	6	10	2	1	—	—	—	—
Ge	32	2	2	6	2	6	10	2	2	—	—	—	—
As	33	2	2	6	2	6	10	2	3	—	—	—	—
Se	34	2	2	6	2	6	10	2	4	—	—	—	—
Br	35	2	2	6	2	6	10	2	5	—	—	—	—
Kr	36	2	2	6	2	6	10	2	6	—	—	—	—
Rb	37	2	2	6	2	6	10	2	6	—	—	—	1
Sr	38	2	2	6	2	6	10	2	6	—	—	—	2

Element	Z	$n=1$ $l=0,s$		$n=2$ $l=0,s$ $l=1,p$		$n=3$ $l=0,s$ $l=1,p$ $l=2,d$			$n=4$ $l=0,s$ $l=1,p$ $l=2,d$ $l=3,f$				$n=5$ $l=0$

Cl, Br, I, in the seventh, and the inert gas group in the eighth column. It will be noted further that *the column number coincides with the number of electrons in the outermost portion of the atom*, i.e., in the shell that is filling in. Thus the first group elements have one, the second group two, the third group three, electrons in the outer shell, etc. These electrons are called *valence electrons*, as they correlate closely with the *principal valence* of the element in its chemical combinations.

It is next essential to note that the repetitive character of chemical behavior occurs primarily with the completion of the octet characterizing the inert gas group in the eighth column. These elements do not combine chemically. They are magnetically completely compensated. They have for their horizontal level in the periodic table the most tightly bound electrons. Thus every indication is that *in the helium pair, or in the completed octet, the atom has achieved its greatest outer dynamic stability*. It would thus be expected that such electronic arrangements represent ones which will form in chemical combination if they can. This tendency was first noted by G. N. Lewis and W. Kossel on chemical evidence in 1917. Thus, since any element or compound strives to reach maximum stability in its behavior, it is not surprising that wherever possible combinations will take place in such a way as to give pairs or octets. Accordingly, with atoms like H, N, O, or C, and H, it is not surprising to see molecules of the type of H_2 (electron pair); N_2 and CO with an outer octet and one inner electron pair; O_2 , with an outer octet and two inner electron pairs; and CH_4 , with an outer octet forming from the atoms where circumstances permit. All these are rather inert and stable configurations except at high temperatures. It is also possible in such combinations, through symmetry, to get these arrangements without causing serious disturbance of the electrical symmetries in the constituent atoms. Such combinations are known as *homopolar* or *nonpolar* compounds.

When, however, two such dissimilar elements are brought together as K and Cl, or Na and F, the situation alters. K and Na can by losing one electron get the *dynamical stability* of the nearest octet of A and Ne, even if they suffer loss of an electron and become electrically charged. That is, *electrical neutrality will be sacrificed for dynamic stability*, because of the stability of the octet. F and Cl can gain the dynamical stability of the octets of Ne and A by picking up an electron and becoming negatively charged. Thus, wherever atoms of a pronounced character difference, from two distant groups of the periodic table, are brought together the *octet formation will tend to take place about the individual atoms*, instead of about the combination as a whole. This leads to strongly electrically unsymmetrical molecules.

Thus, for instance, in NaCl the Na has lost its electron to Cl and the result is Na^+ with a Ne octet and Cl^- with an A octet *held together by electrostatic forces*. In the single molecule the A octet, being an $n = 3$ shell, probably surrounds the Na^+ and Cl^{7+} structures with their $n = 2$ shells. In HCl this is definitely the case, the H^+ lying inside the Cl octet but separated from the Cl^{7+} by a considerable distance, i.e., 0.9 of the diameter of the octet. In the solid crystalline state, however, NaCl arranges itself in a cubical space-lattice structure, where the Na^+ are each surrounded by eight Cl^- ions, and vice versa and the completed octets are quite separate. The separated ion octets such as Na^+ and Cl^- are inert chemically and, like the inert gas,

have no magnetic moments. They can only interact by virtue of their opposite charges to make compounds. Such compounds are called *polar compounds*, since they have such lack of electrical symmetry that there are effectively positive and negative charges separated by finite distances. Hence, in such molecules as H_2O , NaCl , HCl , etc., there are centers of positive and negative charge, e , separated by finite distances l . In analogy to magnetic dipoles which have a magnetic moment $ml = M$, two opposite electrical charges $+e$ and $-e$ separated by l have an effective *electrical dipole moment*, $\mu = el$. The character of the compound, however, as well as the nature of the reacting elements, determines whether it exhibits polar characteristics. Thus while in CCl_4 the carbon is C^{4+} and is surrounded by 4Cl^- , so that the structure is definitely polar, CCl_4 is spatially so symmetrical that it exhibits no dipole moment and is not strongly polar in behavior. Yet if one Cl^- is replaced by CH_3 the resulting compound is highly unsymmetrical and is strongly polar.

Thus chemical combinations exhibit all types of compounds, polar and nonpolar, with all the various intermediate degrees. These intermediate degrees of compounds are determined by the character and numbers of the atoms involved and by the structural principles indicated. If the polar compounds are placed under conditions in which the charged atoms such as Na^+ , H^+ , and Cl^- become separated, and are free to move, ionic carriers of electricity are the result. Such substances can then conduct the electric current through the movement of the separated charged atoms in the electrical field. The charged atoms are termed *positive ions* or *negative ions*, depending on the sign of their charge. Lattices composed of such ions in the solid state are called *ionic lattices*.

It will be noticed that the atoms in the Group I of the periodic table will lose one electron and thus will have one unit of positive charge, that is, they yield *monovalent* ions. The elements in group II will lose two units of charge and are thus *divalent*. Group III elements are *trivalent*. It is, however, to be noted that while it is relatively easy to remove one electron from an atom it becomes increasingly difficult to remove two and three. Thus the trivalent elements and carbon as a tetravalent element show increasingly great reluctance to go into the ionic state. Likewise, while group VII elements readily pick up the single electron to make their octet, groups VI and V elements show greater reluctance in forming divalent and trivalent negative ions.

It will also be noticed that in the upper portion of the periodic table the groups I to III elements have what are called distinctly metallic properties. In the solid state they are good reflectors of light, and good conductors of heat and of electricity. They readily give positive ions in solutions in polar liquids. Thus, metals are associated with atomic species that readily lose electrons. It will, how-

ever, be noted that in the transition elements, like Fe, Co, and Ni, there are metallic elements even in the fifth, sixth, and seventh groups. In fact, going on to elements of larger Z strong metallic tendencies are found in groups VI and VII elements even if the elements are not too distant from the heavy inert gases. Thus U in group VI is essentially a metallic element. This metallic tendency is related to the ability to lose electrons. It increases not only with elliptical orbit shapes but also as the nuclear charge increases, so that the outer electrons are more distant and screened from the nucleus by inner shells. Thus, despite octet-forming potentialities by taking up of electrons, the larger and heavier atoms tend toward metallic behavior in the solid state as their electrons are less tightly bound. Now, what constitutes the metallic state?

When an element like Li, Na, Ag, or any metal, in fact, condenses from the vapor state to the solid, the atoms are bound by very strong forces. These forces are so great that the average distance between adjacent atomic centers is less than the radii of the outer valence electron orbits in the free atoms, e.g., 3.3×10^{-8} cm in Na metal, while the outer valence electron on the average lies at about 3.7×10^{-8} cm from the nucleus in the free atom. As a result, the *valence electrons can no longer belong to the individual atoms of the metal*. Instead, the metal ions like Na^+ , Li^+ , Ag^+ , Cu^+ , Ca^{++} , and Al^{+++} form three-dimensional space lattices of characteristic unit-crystal form for each species of atom. Within the channels of this lattice the valence electrons are free and move quite at random. Thus, the valence electrons in a metal are free, and as such are in a position to produce the characteristic properties of metallic behavior. It may be added that of the total number of free electrons in any metal there are only a small fraction which are by quantum theory in a position to exchange energy with the lattice and thus to conduct heat and electricity. This number varies with temperature.

The situation in metals may be contrasted with that in solid compounds or pure substances which are combinations of, or contain, elements which have strong electron affinities, such as the groups VI and VII elements. Under these conditions the free electrons which come from the valence groups in metals no longer remain free, as they are immediately bound to the electronegative atoms to make negative ions. In such combinations there is no conduction of electricity unless the *ions* are set free so as to move about, or unless electrons can be liberated from the bound state by external agencies.

We are now in a position to consider conductivity in solutions. Acids, salts, and bases represent polar combinations of atoms in the ionic state held together in molecules largely by the attractions between opposite ionic charges. That is, the process of combination of sodium and chlorine, for example, to form a salt consists in the transfer of the extra electron of sodium to the chlorine atom. The two atoms

are therefore dynamically stable and inert chemically, but electrically they are bound together by their charges. One of the characteristics of water is its high dielectric constant (by a high dielectric constant, it will be seen in Chapter XV, is meant a substance which has the power of reducing the forces between electric charges). When a crystal of NaCl is placed in water, the molecules of the liquid immediately act to decrease the electric forces between the ions of the crystal. In the continuous heat motions which these ions undergo, the ions bound together by weakened electrical forces due to the dielectric action of water will be separated. Thus, when placed in water, NaCl will very quickly find itself broken up into sodium ions with a positive electrical charge and into chlorine ions with a negative electrical charge. In substances which are divalent in a compound, such as calcium in calcium chloride, this means, in terms of the picture discussed, that the calcium atom is deprived of two electrons, which makes it assume a configuration like argon, and that the two chlorine atoms have each picked up one electron. Accordingly, when calcium chloride is broken up in solution a double charged positive calcium ion and two negatively charged chlorine ions occur.

The term applied to this splitting up was called *electrolytic dissociation*. The theory of electrolytic dissociation was put forth by Arrhenius in 1887 to explain the conductivity of solutions. That NaCl in solution was dissociated in this fashion was at that time confirmed by the fact that in solution dissociated NaCl acts as if it had twice as many molecules or particles present as there are molecules of sodium chloride. In chemical terms this means that the osmotic pressure of NaCl solution is twice as great as the osmotic pressure to be expected from the number of sodium chloride molecules that would otherwise be present.

Obviously it can be expected that the positive ions and negative ions will not be equally tightly bound together in all the polar compounds. The secondary forces, the sizes of the next inner shells, and structural peculiarities must act in such a way as to cause a wide variation in the binding forces between the ions in the countless polar compounds existing. Thus many polar compounds are easily broken up or dissociated by thermal impacts in liquids of high dielectric constant at room temperature, so that the rate at which the substances go into solution as ions as the result of the thermal impacts of the surrounding molecules is greater than the rate at which the opposite ions can reunite to form molecules on impact. Thus even in strong solutions where encounters of ions are frequent the molecules are completely ionized. This is the case for the so-called *strong electrolytes* comprising most univalent and many bivalent salts and the so-called strong acids HCl and HNO₃, and strong bases NaOH, KOH, etc. Other substances, such as CH₃COOH, H₂CO₃, HCN, and ammonium hydroxide, have the ions bound together by much greater forces. Thus, in the more

concentrated solutions where ions of opposite sign meet quite frequently, the rate at which the molecules are struck by the few molecules of solvent having at room temperatures enough energy to knock them apart is of the same order as or somewhat less than the rate of recombination. Accordingly the *weak* electrolytes are only slightly dissociated in concentrated solutions. On dilution the weak electrolytes dissociate, the degree of dissociation varying with concentration exactly as theory indicates on the mechanism suggested. The details of this mechanism can be found in the *American Physics Teacher*, Vol. 5, p. 198, 1937.

A study of the variation of the apparent degree of dissociation of *strong* electrolytes with concentration in concentrated solutions by means of the conductivity of the solutions, or the effects on osmotic pressure and boiling points, does *not* follow this theory. In some cases the changes are so much at variance with theory that the degree of dissociation even *appears to increase with increasing concentration* in the neighborhood of 3-5 molar solutions (3-5 times the molecular weight of the substance in grams dissolved in 1 liter of water). In this condition it is hardly proper to speak of dissociation in the sense used here. Complete dissociation, as the expression is used in this section, means that ions instead of being associated in pairs in molecules are moving in a random fashion completely independently of each other. In the very concentrated solutions mentioned before, however, there are so many molecules per cubic centimeter in the solution initially and the ions are consequently so closely packed together that they continually move under the influence of one another's forces. Hence, though the ions are not united as molecules in pairs, they are no longer "free" in the sense originally meant. Their behavior is thus not open to interpretation by simple theory, since the process of recombination active in the dilute solutions is altered in the concentrated solutions. Since the nature of the deviation is understood in terms of the picture drawn, it becomes merely a question of terminology as to whether dissociation in such a solution is complete or not, a question which has agitated scientists for many years.

The existence of ions, however, as was stated earlier in the chapter, is not confined to solutions alone, although they show it most clearly on account of the fluid character of the medium and the number of ions present. In fact, many, if not most, heterogeneous substances have ions present in them. For instance, glass when it is molten or hot will conduct an electric current, and a very striking experiment can be made in electrolyzing metallic sodium from *sodium nitrate* at 350° through soda glass into an electric light globe.

In the incandescent light bulb *B* of Fig. 60 is an incandescent filament *F*. *F* emits electrons and is connected to the negative terminal of a 220-volt d-c supply main. As *E* the center of the main is at a 0 potential, the potential across the filament whose other end is grounded

is -110 volts, thus lighting the lamp and causing an emission of electrons from the filament. The lamp is well evacuated so that the electrons shoot out from F in all directions. The lower end B of the bulb is immersed in a molten mass of NaNO_3 at about 350°C in an iron crucible I . The positive terminal of the 220 -volt d-c line, which

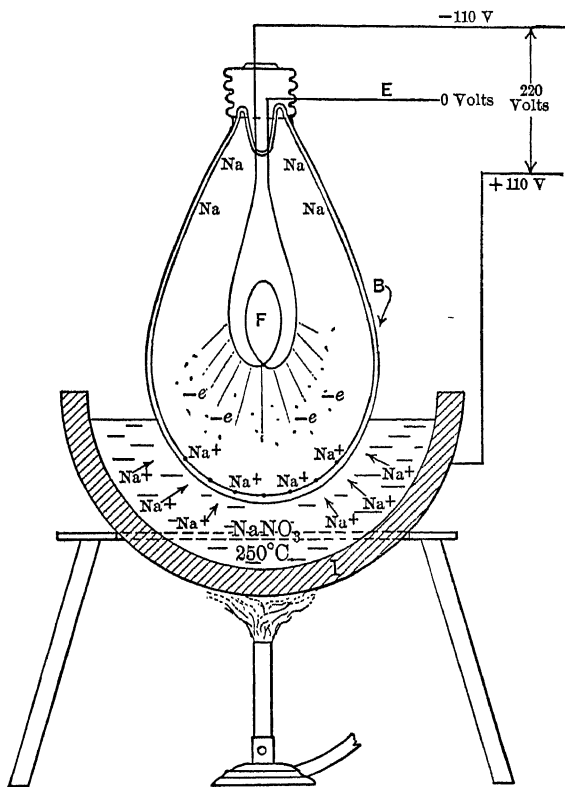


FIG. 60. Electrolysis of sodium through glass.

is 110 volts above the ground E , is connected to the crucible I . Na^+ ions are driven by the field from the crucible through the molten NaNO_3 to the surface of the glass. The warmed glass is also somewhat conducting owing to the Na^+ ions which are present because it is a soda glass in which Na^+ ions exist. It has the Na^+ ions driven from the outside surface to the inside surface of the bulb. The Na^+ ions arriving on the inside of the glass bulb pick up the electrons from the filament, which are driven toward the end of B in the NaNO_3 solution by the field. Now $\text{Na}^+ + e^- = \text{Na}$, where e is an electron and Na is the atom of sodium, so that the ions on arriving on the inside of the bulb are changed to neutral atoms. The neutral atom is volatilized in

the vacuum and deposits on the cold upper part of the tube. As Na^+ ions are removed from the glass by the process new Na^+ ions enter the glass from the NaNO_3 and the glass remains unchanged. The quantity of sodium electrolyzed through the glass in this fashion accurately obeys Faraday's laws of electrolysis, and the method has been proposed as affording an accurate voltameter. The NO_3^- ions go to the iron of the crucible and liberate oxygen or form oxides and nitrates of iron. Had K_2SO_4 been used in place of NaNO_3 the K^+ ions would have replaced the Na^+ which came from the glass. The K^+ which enters the glass, however, so changes its properties that the glass rapidly goes to pieces.

Another striking effect obtained in certain pure glasses which have a very small number of ions is the complete polarization of the glass in a constant electric field. For instance, Pyrex glass at 400°C will conduct an electric current. If a potential is applied to the glass for 20 minutes, the resulting initial current will rapidly fall and at the end of 20 minutes the material will act as a good insulator. On reversing the field the current will jump up to its initial value, falling to zero again in the course of time. The conducting ions result from impurities such as sodium or potassium in the borosilicate glass that move to the electrode and, owing to mechanical difficulties, are not deposited on the surface of the electrode. When these are removed to the negative pole and cannot reach an electrode to deliver their charge, they accumulate near the electrode and build up a space charge that annihilates the field. The current then ceases. On reversal of the field the ions again migrate through the glass and yield a temporary current.

The fact that the ions exist in solution has furthermore been verified by a study of the velocities with which the ions move in solution.

It is perhaps instructive to study the question of ionic velocity briefly and to relate it to the conductivity or resistance of the solution. Consider a pair of plane parallel electrodes of area A separated by a distance d filled with a solution of electrolyte, say NaCl in H_2O . A potential V placed across d produces an electrical field of strength closely $X = V/d$ (unless space charges render the field nonuniform). The positive ions carrying a quantity of electricity (i.e., charge) e experience a force in the field given by $F = Xe$ toward the negative electrode or the cathode. The negative ions are urged in the opposite sense. Owing to the resisting force of the medium the ions quickly achieve a steady velocity v in the electrical field. Since the force producing the motion F is proportional to the field X , it is not surprising that the velocity of the ions has been observed to be proportional to the field X also. Thus we may write $v = KX$, where K the constant of proportionality is the value of v when $X = 1$. X is expressed in terms of V/d as volts per centimeter, and v is in centimeters per second.

Hence the *mobility* K is the velocity of the ions in centimeters per second per volt per centimeter of field intensity.

This mobility, K , as will be seen, is a characteristic of the nature of the ion and the solution. It varies through considerable limits and is usually different for positive and negative ions. This difference in mobility leads to complications which the present analysis must, for simplicity, avoid. If the velocity of the positive ion is u cm and that of the negative ion is v cm per second, then in 1 second the total distance traveled by the two oppositely moving ions is $u + v = v'$ cm. To avoid the difficulties produced by differences in velocity of the ions

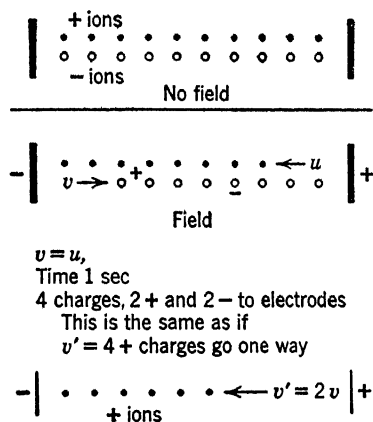


FIG. 61a. Positive and negative ions have the same velocity ($u = v$) and it is shown that their simultaneous movement is equivalent to one ion moving with $v = 2v$.

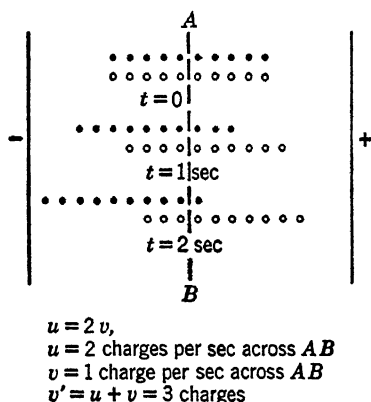


FIG. 61b. The ions have different velocities, complicating the flow.

FIG. 61. The motion of positive and negative ions in a solution in a field.

it will be assumed that $u = v$ and that the resultant velocity of the ions is v' . What this amounts to is saying that the positive ions of velocity u going to the negative electrode and the negative ions of velocity $v = u$ going to the positive electrode carry the same amount of electricity in 1 second as a single ion going from negative to positive, or vice versa, with a speed of v' . This is illustrated in Fig. 61a, which is self explanatory.

Thus, as a result of the motion of the ions all the n positive and n negative ions per cubic centimeter of the solution, with their respective velocities, will move as if n ions of one sign per cubic centimeter moved in one sense only with a velocity $v' = 2v$. Hence, across 1 cm^2 of area parallel to the electrodes and perpendicular to the field, $nv'e$ units of electricity will flow per second. That is, the current i measured

between plates of area A , divided by the area A , gives the current per square centimeter of area (called the *current density*) i_1 ; and i_1 is just $nv'e$. Hence we can write

$$i_1 = \frac{i}{A} = nv'e = KXne = \frac{V}{d} Kne,$$

which can be rewritten as

$$\frac{V}{i_1} = \frac{V}{i/A} = \frac{d}{Kne}.$$

Multiplying both sides of the denominator by A :

$$\frac{V}{i} = \frac{d}{A} \frac{1}{Kne} = R = R_s \frac{d}{A}.$$

Here R and R_s are the resistance and specific resistance of the solution, respectively, whence $R = \frac{d}{A} \frac{1}{Kne}$ and $R_s = \frac{1}{Kne}$. The conductivity, $\sigma = \frac{1}{R}$, and the specific conductivity, $\sigma_s = \frac{1}{R_s}$, are then $\sigma = \frac{A}{d} Kne$ and $\sigma_s = Kne$.

It is seen first that Ohm's law holds for such a conduction process as long as there are enough ions per cm^3 to keep n sensibly constant. It is also seen that by measuring R and R_s we get σ_s and thus Kne . The value of the ionic charge, e , for an ion of known valency is known. If the concentration of solute together with the degree of dissociation is known K can be evaluated. Now e for an ion of valence \bar{v} is given by $e = 4.8 \times 10^{-10} \bar{v}$ e.s.u. The number, n , of ions of a given type per cubic centimeter is given by multiplying Avogadro's number, 6.02×10^{23} , by the fractions of a mole of solute per cubic centimeter of solvent and by the number of dissociable atoms of that type per molecule of solute, if dissociation is complete. If the electrolyte is weak then the value of n as computed above must be multiplied by the fraction f , which determines the degree of dissociation, to get the true value of n . f will vary with concentration and can be determined from the effect on the osmotic pressure, from the lowering of the freezing point or the raising of the boiling point by the solute.

It is to be noted, however, that the K above, first measured in this fashion by Kohlrausch, is the sum of K_+ and K_- , for it comes from $v' = \frac{V}{d} K = (u + v) = \frac{V}{d} (K_+ + K_-)$. In this case we will have conditions as depicted in Fig. 61b, which is self explanatory. In actual practice however, since u is greater than v , it is evident that as the current carries on in time, positive ions are being lost from the neighborhood of the anode and negative ions from the neighborhood of the cathode. It can be shown that the ratio of the loss of concen-

tration at the anode to that at the cathode is equal to the velocity of the positive ions u relative to the negative ions v . By proper experimental arrangements to avoid convection and mixing, the changes in concentration can be obtained, and hence $\frac{u}{v}$ or $\frac{u}{u+v}$, called the *transport ratio* of Hittorf, can be obtained. Thus if K and $\frac{u}{v}$ are measured K_+ and K_- can also be obtained.

Direct observations of the velocity of migration of ions can be obtained by using different solutions at the two electrodes, so arranged that convective mixing is precluded. The solutions are so chosen that the one ion of interest reacts with the solution to cause a visible change. Thus the migration of the OH^- ion could be observed by the change in color of an indicator dye. The migration of a surface of transition by changes in optical refractive index as the one ion moves can also be used. As a result of these combined studies the values K_+ and K_- for different ions have been obtained. It must be noted that the values so obtained apply only to conditions of given concentration and that K can be changed by concentration, as we shall later see. The nature of values observed is indicated in the list.

MOBILITIES OF IONS IN DILUTE SOLUTION

Li^+	0.000347	Cl^-	0.000678
Na^+	0.000451	I^-	0.000685
K^+	0.000670	NO_3^-	0.000640
Ag^+	0.000570	OH^-	0.00178
H^+	0.00325		

A word must now be said about the effect of different velocities on the current. A current is registered in terms of the number of unit charges that have gone completely around the circuit in unit time. In a dynamo generator and an all-wire circuit this gives no trouble at all. The electron separated from one section of the dynamo metal conductor by action of the magnetic field and motion *effectively* travels by displacement around the whole circuit. In a solution positive and negative ions *created in the body of the solution* are being removed by the movement in opposite directions to the electrodes. If the concentrations of the ions as regards equivalent charge are equal and if $u = v$, then the picture would be relatively simple. For every $+$ ion liberated at the cathode a $-$ ion would arrive at the anode and a single charge would then travel around the circuit. The combined velocity v' and a single ion could then be used to describe events as was done earlier. When, however, the ions travel with different velocities and the changes in concentration observed by Hittorf occur, the situation becomes difficult to understand. For then more of the *assumed* faster positive ions would be expected to leave the solution at the cathode than do negative ions at the anode. This would produce

complications and spoil the symmetry of charge movement around the circuit. What actually happens, however, is that, owing to just these differences of velocity of migration and the changes of concentration near the cathode and the anode, the movement of ions is altered. Thus the crowding of positive ions around the cathode with few compensating negative ions and, vice versa, the crowding of the negative ions at the anode with few compensating positive ions alters the electrical field. We can no longer write that $X = \frac{V}{d}$ all the way across

the space between the plates. For near the cathode the accumulation of positive ions makes a *positive space charge* that acts to increase the field X acting on the positive ions to some value X_+' . Likewise, near the anode owing to the slower speed of negative ions there will be a much stronger negative charge *creating a space charge field* X_-' . This field will act to increase the motion of the negative ions. The relative magnitude of these fields adjusts itself in such a fashion that sufficiently near the electrodes there is effectively $u = K_+X_+' = v = K_-X_-'$, so that equal numbers of oppositely charged ions arrive at the electrodes and the continuity of charge transfer is maintained. This would not occur if the ions all came from one electrode and went straight across. Where ions are liberated at one electrode and cross and where some ions come from the body of the liquid, the problem is again complicated.

Such space charges play a very important role in all ionic conduction phenomena and even with electronic conduction both in gaseous or in liquid mediums. They cannot be ignored but are beyond the scope of this course. Layers of ions at electrodes are often called *polarization layers*. They markedly modify the character of the current and where they exist Ohm's law for the conductor as a whole may not be strictly obeyed. In addition, in electrolytic conduction studies the ions liberated at electrodes can produce chemical reactions of a secondary character. These must be avoided. Thus, most careful conductivity studies are carried out with high-frequency alternating current to avoid changing conditions by local accumulations of charge. Under these circumstances Ohm's law is normally observed to hold for solutions. In gaseous conduction it usually does not hold.

Before proceeding further in the discussion the expression $\sigma_s = Kne$ should be considered in its relation to the changes of concentration and dissociation. Let us first consider K constant and independent of concentration. Then for a given value of $e\tilde{v}$, σ_s depends directly on K and on n . If the electrolyte is strong, $f = 1$, and thus σ_s and n increase in direct proportion to concentration of solute. If the electrolyte is weak the situation is more complicated, as $\sigma_s = Kfne$, and while n increases f decreases. The ratio $\frac{\sigma_s}{Kne} = f$ provides one means of evaluating f . It is best, however, to check f by effects depending

on the number of ions or dissociated molecules present, such as osmotic pressure, or change in boiling or freezing points.

The situation in very concentrated solutions of strong electrolytes, is complicated by unforeseeable variations in K . Thus it is essential to inquire into the way in which K varies.

The character of the quantities determining the value of K_+ and K_- is next of interest. It was early suggested that ions could be considered as spheres of radius a and charge e moving in a continuous viscous medium. It had been theoretically deduced by Stokes and experimentally proven that in such a viscous medium of coefficient of viscosity η a sphere under the influence of a constant force F would be accelerated up to a constant equilibrium velocity v . This velocity v is related to the force F by Stokes' law, which reads $F = 6 \pi \eta a v$. Since the force F on the ions in solution is the field strength X acting on the charge e : $F = Xe = 6 \pi \eta a v = 6 \pi \eta a K X$. This gives the value of $K = \frac{e}{6 \pi \eta a}$.

It is seen that K depends on the charge e , the viscosity η , and radius a . From the value of e and η a fictitious "ionic radius" a was computed. Actually as Debye has shown and as is obvious from the behavior of gaseous ions the matter is much more complex. The solution is not a continuous resistant medium. It is composed of molecules of the same order of mass as the ions which are interacting with ions with various types of forces and are undergoing constant disturbance owing to heat motions. The resistance of the fluid to the motion of the ion, which determines K , depends on transfer to the solution molecules of momentum gained by the ion from Xe , the force resulting from the field. This depends on actual collision transfer, which is dependent on the area of cross section of the ion (this is quite well known from spectroscopic data), that of the solvent molecules, and the density of the solvent molecules. However, this action is complicated by the fact that the solvent molecules are so close together that they attract each other and hence momentum imparted to one is also imparted in part to the neighboring molecules. While the action would be hard to compute, it would all properly come under the old viscous drag theory and be included in the quantity η . But by virtue of its field the ion polarizes distant solvent molecules with which it does *not collide physically*, attracts them, and gives them momentum by indirect means. This is a drag proportional to e and is not viscous in the usual sense of the word. Finally, in solutions in which there is of the order of a mole per liter, the ions are present in the order of 6×10^{20} ions per cm^3 . The ions are so close together that the forces between the ions are considerable. Hence, the ions cease to move independently of one another. The momentum transfer to the other ions becomes exceedingly complicated, and the influence on the mobility is difficult to explain in a limited space.

Some attempts have been made to associate ions with *many permanently bound water molecules*, so that ions are larger than just the charged atoms. This is called hydration or solvation of ions. Such a situation is not borne out in the case of gaseous ions, though one or two molecules may unite in some instances to give a stable complex ion configuration with the gas molecules. Furthermore, under conditions where Na^+Cl^- is so loosely bound that it is completely dissociated by heat impacts, it does not seem likely that a still less electrically stable compound $\text{Na}^+(\text{H}_2\text{O})_n$ would remain intact. The observed transport of water or solvent by ions can be ascribed to the polarization forces and momentum transfer previously mentioned.

As noted above, K varies as the ionic charge, e , and inversely as the coefficient of viscosity, η , and a fictitious radius a . It would be expected that doubling e would double K . However, a , the fictitious radius, is increased partly by the forces of polarization caused by e on the water molecules. Thus a may vary with e in such a fashion as to make K vary much less with e than the simple equation above requires. K will vary as $\frac{1}{\eta}$. As heat decreases, η , partly by expanding the sub-

stance and decreasing the forces of attraction and partly by increasing heat motions which reduce the effective action of forces, K will increase as temperature increases. If, in addition, as in a solid, or in a molten liquid, heat also increases n , σ_s will increase very rapidly as T increases. The effect of solute ions and solvent molecules in the fictitious value of the radius a will make K a complicated function of concentration, especially for the higher values of concentrations of strong electrolytes. Thus σ_s will not vary with concentration according to $\sigma_s = Kfne$ with constant K . The theory for this behavior has been extensively developed by Debye, Hückel, Onsager, and many others.

Ionic conduction and the attendant electrolytic separation of various substances has enormous industrial applications. It is involved in, or used:

1. For making important industrial gases and chemicals including H_2 , Cl_2 , and HClO .
2. For making Na, Al, Be, Mg, etc.
3. For the purification of Cu, Fe, and other metals.
4. For electroplating with Au, Ag, Ni, Cr, etc.
5. For the accurate measurement of electrical currents. And:
6. Ionic conduction is of great importance industrially in the effects of parasitic and stray electrical currents from industrial sources which erode or corrode the frames of steel buildings and other steel structures embedded in the ground. Similar problems occur in relation to electrical machinery and various mechanical portions of ships as well as their hulls where subjected to sea water.

CHAPTER XI

LIQUID CONDUCTORS II: BATTERIES

A new problem in the topic of conductivity of solutions is now presented. The subject to be discussed will require the introduction of two new concepts, namely, *solution tension* and *solution pressure*.

46. SOLUTION TENSION

If a piece of metal like zinc or magnesium is placed in water, it has a tendency to go into solution. As was intimated in the last chapter, when an electropositive metal like zinc goes into solution it tends to do so in the form of an ion, that is, it tends to go into solution as an atom which has lost electrons in order to reach its most stable chemical or dynamical form. If then a piece of zinc is placed in water, zinc ions with a double positive charge go into solution. As they leave the metal with a positive charge the metal which was neutral must retain an equivalent negative charge (i.e., the valence electrons of the zinc which were sacrificed to gain dynamic stability). As solution continues the negative charge on the zinc accumulates. The negative charge, however, acts to draw back the positive ions of zinc and cause them to return to the metal. The tendency to go into solution as ions is called solution tension.

47. SOLUTION PRESSURE

The second concept which is necessary to an understanding of this chapter is that of solution pressure. If a substance like zinc chloride is in solution the zinc ions which are present will tend to condense out on any surface, owing to the continuous bombardment of the surface by the ions in their heat motion. This phenomenon is called solution pressure. As the ions start to condense they carry a positive charge to the surface and so very quickly charge it up to a potential such that no more positive zinc ions can reach the surface.

48. EQUILIBRIUM

Consequently there are two opposing tendencies when any metal is immersed in a solution of its salts. On the one hand, it sends ions into solution, and on the other hand ions try to precipitate out. These two processes lead to *equilibrium* with a certain number of ions in solution carrying their charge, while the metal has an equivalent number

of opposite charges. This equilibrium is determined by the nature of the metal and the form in which it ionizes, the charge on the metal which pulls the ions back, temperature, and the concentration of the ions in solution which also limits solution.

If then we place a piece of zinc into a solution of zinc chloride or zinc sulfate, the zinc will go into solution until the concentration of zinc ions produces equilibrium, that is, there will be a solution of the zinc until the zinc acquires a negative charge so that the number of zinc ions striking the electrode per second equals the number that leave the electrode. In this state the zinc electrode has a negative charge. Had magnesium been used instead of zinc the tendency to go into solution would be even greater, and the negative charge necessary to draw in enough magnesium ions to make the rate of precipitation equal the rate of solution would be higher than with the zinc.

In practice the equilibrium potential can be determined for any metal and the metals may then be arranged in the order of the potential which they give under standard conditions. Thus each metal would be characterized by a specific potential with regard to the solution it was placed in, and a series of the solution tensions of the various metals is possible. If the various metals in such a scheme were listed, it would be found that magnesium has a very high solution tension, zinc a slightly lower one, hydrogen still lower, and copper and platinum have still lower tensions. Such a table of elements is given here.

VOLTAIC OR REPLACEMENT SERIES OF THE IONS OF THE ELEMENTS DERIVED FROM THE HEATS OF FORMATION, AS GIVEN BY OSTWALD

Positive Ions					Negative Ions
Li +	Ca ++	Zn ++	Sn ++	Cu +	OH -
Rb +	Mg ++	Fe +++	Pb ++	Hg +	Cl -
K +	Al +++	Co ++	H +	Ag +	Br -
Na +	NH ₄ +	Cd ++	Fe ++		I -
Sr ++	Mn ++	Ni ++	Cu ++		S -

It must be noted, however, that this table is based on the relative heats of formation of the particular ions listed. It can be used only after a careful consideration of the conditions present in the solutions in which the metals are used to be sure that the reaction is such as is indicated by the table. If certain of the substances can as a result of the chemical reactions possible in a given solution react in a different manner from the one indicated in the table the whole order of the reaction may be modified. For example, Fe in slightly acid solution tends to go into solution replacing hydrogen, which, as will be seen later, is given off at the surface of any relatively inert conductor such as a carbon, copper, or platinum electrode. If the solution is neutral

or alkaline the tendency is much decreased on account of the formation of insoluble hydrated iron oxides. The iron electrode will therefore be negative while the copper electrode will be positive owing to the deposition of H^+ ions on its surface. If the acid solution is replaced by an alkaline solution of KCN the reaction is completely changed. Copper will dissolve, giving a complex copper cyanide ion, $Cu(CN)_2^-$, as copper in this solution is very soluble. On the other hand, in an alkaline solution the iron will show little tendency to replace hydrogen ions, i.e., it will be relatively inert while copper is actively dissolving. Thus the copper will go into solution as Cu^+ uniting with $2CN^-$ to give $Cu(CN)_2^-$ and will leave a negative charge on the copper electrode while the K^+ ions will give a positive charge to the iron electrode and subsequently react to form KOH liberating H_2 gas at the inert iron electrode. The relative potentials of the Cu and the iron electrodes will thus be reversed from those in the acid solution which will also be true for the course of the reaction. The reversal can be readily seen by the direction of the deflection of a galvanometer in a circuit in which the Cu and Fe electrodes are first placed in acid and then washed and placed in the cyanide solution.

Substances such as chlorine or oxygen also go into solution. They go into solution, however, taking on a negative charge, that is, they yield negative ions. This results from the fact that both the chlorine atom and the oxygen atom are more stable chemically and dynamically when they have gained enough electrons to fill out their shell of eight, even when in doing so they gain a negative charge. Experimentally, it is difficult to obtain electrodes of gases because gases are nonconductors. The way in which a chlorine gas electrode can be achieved is by bubbling the gas through meshes of a platinum gauze which constitutes the electrode.* The adsorbed monomolecular layer of chlorine then serves as the electrode surface. As it ionizes with a negative charge the electrode acquires a positive charge and this process goes on until the opposing tendencies of solution tension and solution pressure again establish equilibrium. Accordingly, referring to the negative ions again there is a series of elements which exhibit varying tendencies to go into solution. Those which give the electrodes the highest positive charge are the so-called most electronegative gases, that is, they have the highest electron affinity. Thus chlorine is far more electronegative than oxygen and oxygen very much more so than nitrogen, which really shows no tendency to go into solution in the form of ions.

These principles discussed in this section underlie the whole action of the chemical cells or batteries. Several different types of cells will be examined in the following section.

* Note that only metals which are inert both to gas and solution, and which adsorb the gases as monomolecular films, can be used.

49. THE CONCENTRATION CELL

Perhaps the simplest of all cells is the *concentration cell*. Suppose that we place a zinc electrode in a solution of concentrated zinc sulfate, place another electrode in a solution of very dilute zinc sulfate, and then connect the two solutions by means of a conducting bridge filled with zinc sulfate. A cell of this sort could perhaps most easily be achieved by placing the dilute zinc sulfate in a porous porcelain cup and placing this cup in a concentrated solution of zinc sulfate which contains the other electrode. The porous cup serves to connect the two solutions by small capillary channels which conduct current but retard mixing by decreasing the rate of diffusion. The zinc in the concentrated solution will tend to dissolve, charging this electrode negatively. It will do the same in the dilute solution, but owing to the lower concentration in the dilute solution the equilibrium charge of the zinc electrode in the dilute solution will be more highly negative than that in the concentrated solution. Thus there are present two electrodes of different negative potentials, the potential of the zinc in the concentrated solution being less negative, or consequently more positive, than the electrode in the dilute solution. There is accordingly a potential difference between the two electrodes.

If a galvanometer be connected to the two zinc terminals, current will be observed to flow from the zinc in the concentrated solution to the zinc in the dilute solution trying to equalize the potential of the two electrodes. If the electron is accepted as the carrier of the current, the superfluous electrons on the electrode in the dilute zinc solution would really flow to the zinc in the more concentrated solution. In any case, the equalization of charges would take place. As soon as the charge of the zinc in the dilute solution is lowered, more zinc will go into solution, and the flow of negative electricity to the zinc in the concentrated solution will increase the negative charge of that electrode. This increase immediately acts to draw in more zinc ions because the equilibrium potential has been destroyed. Thus, on the concentrated side zinc ions will go to the electrode and give up their positive charge, becoming zinc atoms, while on the negative side zinc will go into solution. In the meanwhile, current flows through the galvanometer. It is seen at once that the direction of the the processes at work is such as to reduce the concentration of the strong solution and increase the concentration of the weak solution, the net result being the equalization of the concentrations of the two solutions. Thus the electrical current is kept flowing as long as the concentrations are different. The energy which is used during this process corresponds to the heat of dilution of the zinc, for what has occurred is that the zinc which was in a concentrated solution is now occupying the total solution volume of both the concentrated and the dilute solution. In studying the energetics of such cells, the electro-

motive forces would at first sight appear to be computable from the heats of dilution alone. This is so in some cells, but not in others. In general, another term must be regarded, and that is the *entropy* of the reaction. A study of entropy lies beyond the scope of this course and belongs properly in an advanced course in physical chemistry.

50. THE COPPER CELL OR DANIELL CELL

A cell made of dissimilar metals may now be discussed. Place a zinc electrode in a *dilute* solution of zinc sulfate inside of a porous cup, and place the porous cup in a *concentrated* solution of copper sulfate. In this copper sulfate solution may be placed an electrode of copper. Now, as before, the zinc takes on a negative charge, zinc ions going into solution. Equilibrium in the case of copper is set up by a few copper ions going into solution or perhaps, if the solution is concentrated enough, a few copper ions precipitating out as copper is relatively inert. In any case the potential of the copper relative to the potential of the zinc is distinctly more positive. The instant a wire is attached to the two electrodes making a transfer of electricity from one to the other possible, the transfer occurs. The equilibrium which had existed is at once disturbed. The copper which was before in equilibrium, owing to the loss of the positive electricity which flowed to the very negative zinc electrode, now strongly attracts the copper ions. The copper is deposited rapidly, and the electrode is found to be coated with small nodules of new metallic copper. The zinc of the zinc electrode goes into solution as soon as the equilibrium is disturbed because through the advent of the positive electricity the potential of the zinc electrode is not sufficient to hold back the zinc ions. Thus the action of this cell consists in the solution of zinc and the precipitation of copper. The laws of electrolysis lead us to suppose that as many ions of bivalent zinc go into solution as ions of bivalent copper go out of solution. In this case, however, the chemical action would go on until all the zinc was dissolved or all the copper was liberated. The energy which is expended by the current then comes from the replacement in solution of copper ions by zinc ions, that is, it corresponds to the differences in heat of solution between zinc and copper, with, of course, a correction for the entropy term involved, as mentioned under the discussion of the concentration cell.

51. THE ACID CELL

In the first sections it was mentioned that the various elements could be placed in a series with regard to the potentials which they attained. This series corresponds to the electromotive forces series of the elements and was first discovered by Volta in his study of the voltaic pile. One way of investigating the replaceability would be to

place the metal to be tested into a solution of the metal which it is expected to replace. That is, if a piece of iron which has a higher solution tension than copper is put into a solution of copper it will be observed that the iron goes into solution and the copper goes out of solution. The action is very much like that of the Daniell cell, the precipitation of copper taking place at points where there are bits of inert impurity which decrease the solution tension of the iron. Again, if a piece of zinc be placed in the acid solution, hydrogen is liberated as a gas, that is, hydrogen has a lower solution tension than zinc and is thus replaced by it. Here again the hydrogen is liberated at relatively inert points on the zinc surface, for the active solution of the zinc electrode appears to prevent liberation of hydrogen where it occurs. In fact, really pure zinc will hardly dissolve in really pure sulfuric acid solution. The relative solution tension may be measured in three different ways: (1) by the method of replacement just discussed, (2) by the relative potentials acquired by the metals in solution,* and (3) by the relative energy liberated by the elements when they go into solution.

In discussing this table of solution tensions we find that hydrogen stands higher in the series than either platinum or copper. Thus, hydrogen replaces copper in solution, and tends to go into solution taking on a positive charge more readily than copper. It is well known that copper will not dissolve in HCl, i.e., it is relatively inert. This fact enables us to explain the action of the third type of cell, the *acid cell*.

Suppose that a zinc electrode and an electrode of copper are placed in a solution of hydrochloric acid. As before, the zinc atoms go into solution charging the zinc negatively. It is possible that a number of the copper ions may go into solution having a very low solution tension, but this number will not be great. In the solution besides the ions of zinc and the few ions of copper there are *many hydrogen ions*. These ions will attempt to precipitate out because hydrogen has a solution pressure. On immersion in the acid the copper electrode being the inert electrode, will come to equilibrium with the hydrogen ions as well as with the copper ions. If, now, the copper electrode and the zinc electrode are connected, the negative electricity on the zinc electrode will flow to the copper. The potential will then become too negative for the equilibrium of that electrode with the solution. At first, the copper having the lowest solution tension will precipitate out. The potential of the electrode, however, will be even more negative than will permit the existence of hydrogen ions in the solution in such a concentration, and the hydrogen ions will proceed to precipitate out. Meanwhile, the zinc ions continue to go into solution giving more negative charges to the copper electrode, and thus more hydrogen ions are

* These can be measured by potentiometer when appropriate electrode arrangements are used.

electrolyzed out. As soon as hydrogen ions reach the electrode, they give up their charge and go off as bubbles of hydrogen gas. Their positive charge draws more electrons from the zinc electrode and consequently more zinc can go into solution. Thus the zinc goes into solution and the hydrogen goes out of solution. Since the hydrogen has one positive charge, two hydrogen ions will go out of solution for every zinc ion that goes into solution. The current will flow until all the zinc has gone into solution or all the hydrogen ions of the acid have been replaced by zinc ions. The energy in this case is the difference in the solution energy of zinc ions and hydrogen ions, that is, the energy given by this cell is the energy of formation of zinc chloride ZnCl_2 less the energy of formation of hydrochloric acid HCl , with the added restriction caused by the neglect of the entropy term.

52. POLARIZATION

The acid cell is the one most frequently encountered in chemical reactions. Practically, however, it is not useful because of the phenomenon known as polarization. This phenomenon of polarization is a result of the liberation of hydrogen gas. It is obvious that as soon as the hydrogen is liberated it forms bubbles. These can make a more or less continuous coating on the electrode which does not conduct electricity. Such a coating reduces the area available for the passage of the current. It increases the *internal resistance* of the cell, and thus enables the discharge of the electrodes to take place more rapidly than the charge. The e.m.f. is unchanged. The potential difference that will be maintained between the two terminals as a result of the rapid discharging of the terminals and their slow rate of charging because of the high resistance caused by reduced surface area will therefore *not be the potential difference at which one could expect the cell to operate in the absence of polarization*. As a number of convenient cells used in commercial practice have a tendency to undergo polarization, features have been arranged to obviate this. All that is required for the removal of polarization is a removal of the gaseous film by: (1) the mechanical jarring or scraping of the electrode to remove gas bubbles, (2) a chemical method depending on the use of oxidizing agents such as K_2CrO_4 or MnO_2 , (3) the use of solutions which do not polarize. The Daniell cell already discussed is a nonpolarizing cell inasmuch as copper is deposited and this forms a *conducting* film. It was for this reason nearly universally used in the early telegraphy, where constant current was required. Electromagnetic generators and storage batteries have now replaced it.

53. CALCULATION OF THE APPROXIMATE E.M.F. OF A CELL

In the discussion of the mechanisms active in different types of cells, it was stated that the electrical energy given by the cell when a

current i is drawn for a time t , i.e., the work $W = EQ = Eit$, where E is the electromotive force, came from the heat of reaction of the substances dissolved and liberated. It is now profitable to study this question further and to derive from this relation the approximate e.m.f. of a cell. Let H_A be the heat of reaction in calories when the molecular weight in grams (a mole) of a substance giving ions A is consumed in a cell. Let H_B be the heat of reaction in calories when the appropriate amount (e.g., $\frac{1}{2}$ mole, 1 mole, 2 moles, etc.) of the substance which was in the form of ions B is liberated from solution through the replacing of ions B by ions A in the cell action. Then the total heat of a reaction in which a mole of ion A goes into solution replacing ion B will be $\Delta H_A = H_A - H_B$, as regarded from the point of view of substance A . As an example, H_A would be the heat of solution in HCl of 65.38 grams of zinc and could be found calorimetrically by dissolving zinc in a solution of HCl of appropriate concentration. H_B would be the heat of formation of the equivalent amount of HCl plus its heat of solution in water to give the original HCl solution. In this case the equivalent amount of HCl would be 2 moles, and 2 moles of H atoms or 1 mole of H_2 gas (2.016 grams) would be liberated.

The energy in ergs available to the cell if all the energy were available would be $J\Delta H_A$, where J is the mechanical equivalent of heat, i.e., 4.18×10^7 ergs. If then the statements made as to equivalence of heat and electrical energy are correct, $J\Delta H_A = Eit = EQ$, where E and Q are in absolute e.m.u. Hence the electromotive force $E = \frac{J\Delta H_A}{Q}$.

Now Q represents the quantity of electricity carried by 1 mole of the A ions in this process. Faraday's law, as was seen in the last chapter, says that 1 mole of univalent ions carries 9650 absolute e.m.u. of quantity. If the valence of the ion A is V_A (i.e., 1, 2, 3, etc.), then $Q = 9650 V_A$ absolute units. Hence E in e.m.u. is $E = \frac{4.18 \times 10^7 \Delta H_A}{9650 V_A}$, where

ΔH_A is the heat of reaction calculated in terms of the moles of substance A consumed and V_A is the valence of substance A . If we had chosen, we could have written $E = \frac{4.18 \times 10^7 \Delta H_B}{9650 V_B}$, where ΔH_B is the heat of

the replacement reaction when one mole of ion B is liberated and V_B is the valence of ion B . E can be expressed in volts by division by 10^8 .

Hence $E_V = \frac{4.18 \Delta H_A}{96,500 V_A}$.

If we had chosen to describe the reaction in terms of the heat of reaction of 1 gram of ions of A instead of 1 mole, the heat of reaction ΔH_A becomes that of 1 gram Δh_A , and E_V may be written $E_V = \frac{4.18 \Delta h_A M_A}{96,500 V_A}$, where M_A is the atomic weight of A for $\frac{\Delta H_A}{M_A} = \Delta h_A$.

Frequently $\frac{M_A}{96,500 V_A}$ is expressed by a single symbol S_A , the mass of substance A consumed per coulomb, and termed the electrochemical equivalent. Hence $E_V = \frac{4.18 \Delta h_A M_A}{96,500 V_A} = 4.18 \Delta h_A S_A = \frac{J \Delta h_A S_A}{10^7} \cdot E_V$ could equally well be calculated from the similar quantities expressed in terms of the mass, valence, and heat of reaction for the ion B liberated in the form

$$E_V = \frac{4.18 \Delta h_B M_B}{96,500 V_B} = \frac{J \Delta h_B S_B}{10^7}.$$

Actually this equation as given is not accurate, for it turns out that there is in most thermal processes a certain amount of the energy ΔH_A involved in the change in entropy concerned in the reaction. Thermodynamical reasoning, for which there is no place in this book, yields as the accurate basic equation

$$E = \frac{J \Delta H_A}{Q_A} - T \frac{dE}{dT} \quad \text{or} \quad E_V = \frac{4.18 \Delta H_A}{96,500 V_A} - T \frac{dE_V}{dT}.$$

Here T is the absolute temperature and $\frac{dE_V}{dT}$ is the rate of change of electromotive force of the cell with temperature. If $\frac{dE_V}{dT}$ is positive, then E_V is less than $\frac{4.18 \Delta H_A}{96,500 V_A}$, the expression from simple theory. $\frac{dE_V}{dT}$ varies with temperature, character of ions present, and concentration, and must be determined experimentally.

Example. The Daniell cell may be considered an example. In this cell the metallic zinc dissolves, giving ZnSO_4 , and replaces copper ions in solution as CuSO_4 , giving rise to metallic copper. The heat of formation per mole of ZnSO_4 , together with its heat of solution, as used in the cell, gives roughly $H_A = 248.1 \times 10^3$ calories, while for CuSO_4 , the heat of formation and hydration is roughly $H_B = 194.7 \times 10^3$ calories. Hence $\Delta H_A = 53,400$ calories per mole of $\text{Zn} = \text{ion } A$. The valence of Zn^{++} is 2. Hence $E_V = \frac{4.18 \times 53,400}{96,500 \times 2} = 1.16$ volts. The observed e.m.f. of the

Daniell cell varies with the concentration of the solutions and with amalgamated electrodes (this reduces the effect of impurities in the Zn) is observed to be about 1.08 volts. The agreement, it is seen, is very good. The differences are in part due to the fact that H_A and H_B did not correspond accurately to the concentrations used in the amalgamated cell and in part to a neglect of the term $T \frac{dE}{dT}$. In most cells the latter term *cannot* be neglected.

54. CAUTIONS IN ELECTROPLATING

There is one point of great importance in the electroplating of metals which the question of solution tension brings out. As stated in the last chapter, the passage of the electric current causes the deposition of the ion to take place. It must be borne in mind, however, that in order to cause zinc to deposit from a zinc sulfate solution the potential of the electrode upon which the zinc is being deposited must be more negative than the potential which a zinc electrode immersed in the solution would take on as the result of the tendency of the zinc to go into solution. That is, in any electroplating bath care must be taken to have the potential sufficiently great to ensure that the substance will be deposited. Thus, if a solution contains an iron electrode which is to be plated with zinc, the zinc having a higher solution tension than iron, it must be remembered that Fe and Zn form a cell which has an e.m.f. tending to cause Zn to dissolve and to deposit Fe. Thus to electroplate the Fe with Zn the P.D. must be greater than the e.m.f. of the Fe-Zn cell. It is also useful to know, for instance, that if the problem is to plate an electrode with a given metal and metals are at hand which are more readily deposited than that metal the other metals will also tend to precipitate out when the potential is applied to the bath. One method of separation of copper in a pure form is by means of electrolysis with carefully controlled potentials so that the copper deposits whereas other more electropositive impurities do not.

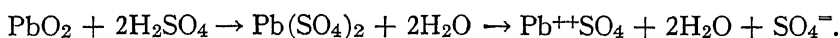
It should be added that for good uniform deposits in electroplating low current densities are desirable, as well as low potentials. Rotating the electrodes also assists in obtaining more uniform deposits. Unless care is taken deposits are likely to be spongy or of loose crystalline character and they may not adhere uniformly to the surface.

55. STORAGE CELLS

The peculiar reversibility of a cell, namely, the fact that copper and zinc in the proper solutions will, if connected, deposit copper and dissolve zinc, giving us an electrical current, and if a sufficiently high potential be applied to the cell to reverse the action, the cell will consume electricity and deposit zinc putting copper into solution, leads at once to the principle of the storage battery. The storage battery is merely a convenient reversible form of cell. In such a cell, when the electricity is permitted to flow into the cell, a deposition of the element having the greatest solution tension results. When all this element has been deposited on the proper electrode, then, by connecting this electrode to the other electrode, the reaction may be reversed and current obtained at the expense of the chemical energy stored up. In order to have the greatest capacity and potential with as few cells as possible, and in as convenient a form as possible for

practical purposes, a great many investigations have been made. There seem to be two types of cells on the market which are fairly successful. The first of these is the lead storage battery and the second is the Edison cell.

In the lead storage battery, the electrodes consist of grids in a framework of lead filled with a paste of lead and lead sulfate as one electrode and lead peroxide made into a paste with lead sulfate as the other electrode. The elements of the battery consist then of electrodes of lead and lead peroxide, PbO_2 . The mechanism of the reaction is summed up in the following equation:



That is, PbO_2 reacts to form $\text{Pb}(\text{SO}_4)_2$ which ionizes, sending SO_4^{--} ions into solution, leaving the PbO_2 plate positively charged and filled with PbSO_4 . As PbSO_4 is insoluble there is little Pb in ionized form in the solution and the ions depositing on the Pb plate are other ions; i.e., the Pb electrode is the inert electrode. As the solution contains an excess of SO_4^{--} ions these tend to deposit on the lead plate, charging it negatively. On closing the circuit between the PbO_2 plate and the Pb plate, equilibrium is disturbed, causing more $\text{Pb}(\text{SO}_4)_2$ to ionize and more SO_4^{--} ions to deposit on the Pb plate. These ions on losing their charge will react with water to give O, which will in part react to oxidize the Pb plate to PbO, and in part will come off as O_2 gas, depending on the rate of discharge of the battery. The action will continue until all the PbO_2 is changed to PbO or all the Pb is changed to PbO. Under these conditions the cell is said to be discharged.

In the process of discharge the SO_4 is taken by the Pb or PbO on both plates, reducing the specific gravity of the electrolyte in the cell. Hence a discharged modern type of cell has a specific gravity of electrolyte of about 1.175, as against 1.275 in a fully charged cell.

If, now, an external electromotive force is applied to the cell, the positive terminal being on the PbO_2 electrode and the negative terminal on the PbO electrode, and *the electromotive force exceeds that of the storage cell*, the following reaction will take place: the acidulated water will furnish hydrogen ions which will at once migrate to what was initially the lead plate if the negative potential is great enough to cause hydrogen ions to deposit. If the hydrogen ions give up their charge, atomic hydrogen is liberated. This reacts with the oxygen on

* Actually cells are initially made up of pastes of mixtures of PbO, yellow oxide of lead, and Pb_3O_4 , red lead of very high purity. The future negative plate has mostly PbO while the future positive plate has some PbO but is 60 to 80 per cent Pb_3O_4 . Expanders of lampblack or BaSO_4 are added to make the paste stay in the grids. These are cast of pure Pb-Sb alloys with 5 to 12 per cent Sb. The plates are "formed" by appropriate periods of charging and discharging. Users of storage batteries are urged to read a book by G. W. Vinal (of the National Bureau of Standards), *Storage Batteries*, New York, John Wiley & Sons, Inc., 1940.

the lead electrode, giving water and leaving pure lead. At the positive and what was initially the PbO_2 electrode SO_4^- ions will be attracted. Arriving at the electrode they will give up their charge and react with the water, liberating atomic oxygen. This oxygen will at once react with the PbO giving PbO_2 . After all the PbO of the positive plate has been converted to PbO_2 the cell will be in its initial state, and is said to be charged, the acid in this case being largely restored to the solution.

The cells can act in this fashion indefinitely until mechanical wear and tear destroy them. To reduce the internal resistance and increase the capacity of the cell it is necessary to have the plates as large as possible in area and as close together as possible. Contact between the plates is prevented by thin porous separators. If one portion of a plate is closer to its opposite plate than the rest the current flow will be concentrated at this point. The result will be that the plate will wear or erode unequally. Thus, buckling of the plates leads to very rapid destruction of the cell. If the cell is charged or discharged very rapidly, the mechanical action of the gases involved as well as the heat produced by the resistance of the cell leads to buckling of the cell plates and to rapid destruction. The evolution of gases also serves to loosen mechanically the paste in the grids.

Because in the process of charging and discharging the water is electrolyzed and because gradual evaporation takes place, it is necessary that the batteries have their solution replenished by the addition of fresh *distilled* water from time to time; tap water would introduce Ca and Ba salts, which would precipitate H_2SO_4 .

If a battery remains unused for long periods of time there is a gradual solution of the lead and precipitation of some lead sulfate at the bottom of the cell. This process destroys the cell and removes sulfuric acid. The plates, on standing unused, gradually become coated with large insoluble crystals of PbSO_4 , which reduce their effective area and increase the internal resistance of the cell. This process is known as "sulfating." To keep cells in good condition regular charging and discharging must be carried on to prevent sulfating. Cells should never be left standing for more than a day or two without charging and discharging. All cells have instructions giving the percentage of acid present when charged and when discharged in terms of the specific gravity of the solution, and the cell should be tested both when charged and discharged by measuring its specific gravity. The efficiency of the lead storage cell, if properly handled, is from 75 to 85 per cent. With good care a cell may last many years.

The Edison cell, which consists of a hydrated nickel oxide electrode on the one hand and an iron electrode on the other, is far more mechanically robust. On discharging, the iron oxidizes and the nickel oxide is reduced. The cell uses alkali as an electrolyte instead of acid. Owing to the lower specific gravity of iron and nickel it weighs half as much as the lead storage cell for the same volume. On the

other hand, its e.m.f. is only 1.25 volts, whereas that of the lead cell is about 2.2 volts. This means that while the Edison cell weighs half as much as the lead storage cell nearly twice as many cells are required to give the same potential. The efficiency of the Edison cell is only 50 per cent. Deterioration of the Edison cell, however, is far less, and it can stand much mishandling.

56. DRY CELLS

Besides storage cells, the only other cells which are of particular interest today are the zinc-carbon cells or their modifications known as "dry batteries." These batteries consist of a zinc electrode surrounding the cell with a central core of graphite surrounded by a paste of *moist* MnO_2 . These cells are polarizing cells, and their action is very much like the action of the acid cell. Zinc goes into solution at the negative pole and hydrogen is liberated on the carbon; the aqueous solution is replaced by a *moist* paste of gypsum with ammonium chloride solution. The manganese dioxide reduces the polarization, but heavy currents cannot be drawn from the batteries for any length of time without a fall in potential. The internal resistance of the 1-volt cells runs about 0.1 ohm and they yield about 1.5 volts per cell.

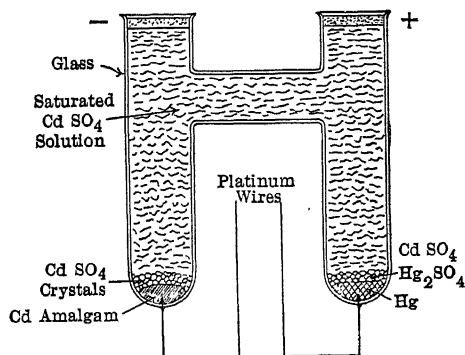


FIG. 62. Weston standard cell.

57. STANDARD CELL

Standard cells are cells which are used as standards of comparison of electromotive forces. Several types are on the market, and they are so designed as to give an accurately reproducible and constant electromotive force when made up according to formula. The Weston standard cell consists of an H-shaped tube, as shown in Fig. 62. At one lower end of the H a platinum wire is sealed into the tube. This

makes contact with the solution by being covered with an amalgam of cadmium and mercury. Next to this amalgam is a saturated solution of cadmium sulfate, a few crystals of the latter lying on the amalgam. On the other side of the H, another platinum wire is sealed in, and this is covered by a mercury globule. Over the mercury is a paste of mercurous and cadmium sulfates. The cross part of the H is filled with a saturated solution of cadmium sulfate. The two upper arms of the H are sealed off. The e.m.f. given by this cell should be 1.0183 volts at 20° C. The e.m.f. of all cells varies with the temperature, and standard Weston cells should always be used at their rated temperature or corrected by the expression $E_t = 1.0183 [1 - 0.0000406 (t - 20) - 0.00000095 (t - 20)^2]$ volts. This cell is the international standard of potential.

58. MOST EFFICIENT ARRANGEMENT OF CELLS

In discussing cells the question of the most efficient use of cells when they are used in quantity might be discussed at this point. There are two different ways of using cells in a circuit, adapted to two extreme types of usage. In the first place, cells may be connected in parallel. In this connection all the negative poles are connected to one wire and all the positive poles to the other wire; the e.m.f. is that of 1 cell, and the internal resistances are in parallel. The use of cells connected in parallel is of importance where a heavy current is required and the external resistance R_e is less than the internal resistance R_i . Assume all the cells to have the same e.m.f. equal to E . Since all the cells are in parallel, they pass the current i through the resistance. The equation then is

$$E = i(R_{\text{cells}} + R_e).$$

In this equation, R_{cells} is the total resistance of the n cells in parallel. Since

$$\frac{1}{R_{\text{cells}}} = \frac{1}{R_i} + \frac{1}{R_i} + \frac{1}{R_i} \dots = \frac{n}{R_i},$$

therefore,

$$E = i\left(\frac{R_i}{n} + R_e\right).$$

Thus the current

$$i = \frac{E}{\frac{R_i}{n} + R_e}.$$

If R_e is small compared to R_i , the current i will then be proportional to n , the number of cells in parallel.

Again suppose the cells to be placed in series. That is, cells are connected so that the positive terminal of the one goes to the negative terminal of the next, and so on. In this condition the e.m.f.'s of the cells add, as do the resistances. The arrangement of cells in this form is of special use where the external resistance is great compared to the internal resistance R_i of the cells. If there are n cells

$$E_{\text{total}} = nE = i(nR_i + R_e),$$

hence

$$i = \frac{nE}{nR_i + R_e}.$$

If R_e is very great compared to R_i , i is proportional to n .

In general, where many cells are to be used and a specific circuit is to be run in the most efficient manner, that is, to get the greatest

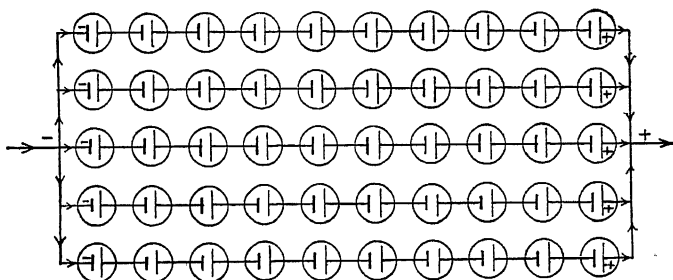


FIG. 63. A bank of 50 cells arranged with 10 cells in series and 5 rows of cells in parallel.

current, what the best arrangement of the cells will be cannot be decided offhand. Assume that there is a total of n cells. Suppose in the most efficient arrangement, which we intend to calculate, that x groups of cells should be in parallel. $\frac{n}{x}$ cells will then be in series for $x \left(\frac{n}{x} \right) = n$, by the geometry of series and parallel connections. This relation is illustrated in Fig. 63, where $x = 5$, $n = 50$, whence $\frac{n}{x} = 10$ cells are in series. The current in this case will be expressed by an equation

$$i = \frac{\frac{n}{x} E}{\frac{n}{x} \frac{R_i}{x} + R_e} = \frac{E}{\frac{R_i}{x} + \frac{x}{n} R_e}.$$

This equation may at once be set up from what has gone before concerning the current when cells are in series and in parallel separately.

Now i is to be a maximum. For what value of one of the quantities the curve representing an equation is a maximum can be determined by calculus, by differentiating the quantity with respect to the variable in which one is interested. In setting the derivative equal to 0 the solution of the resulting equation at once gives the condition which must be fulfilled to give the maximum or minimum current. Thus the maximum effectiveness, or perhaps a minimum effectiveness, is found by setting

$$\frac{di}{dx} = 0$$

$$\frac{di}{dx} = 0 = \frac{-E\left(\frac{-R_i}{x^2} + \frac{1}{n} R_e\right)}{\left(\frac{R_i}{x} + \frac{x}{n} R_e\right)^2}.$$

Solution gives us the result that

$$\frac{R_i}{x^2} = \frac{1}{n} R_e,$$

or that

$$\frac{R_e}{R_i} = \frac{n}{x^2} = \frac{1}{x} \left(\frac{n}{x}\right).$$

To test whether this is a maximum or a minimum the second derivative of the expression must be taken and the sign of the result must be observed. It is hardly necessary to carry this out here, and it suffices to state that the condition laid down *is* the condition for a *maximum current*. The relation

$$\frac{R_e}{R_i} = \frac{1}{x} \left(\frac{n}{x}\right)$$

in words reads as follows:

Number of cells in series $\left(\frac{n}{x}\right)$
 Number of cells in parallel (x) = Ratio of external resistance to internal resistance.

If the two resistances are equal (i.e., if $R_e = R_i$), the number of cells in series equal the number in parallel.

Example. To see the general application of the problem assume that there are 100 dry cells to connect so as to give the maximum current. Assume that the external resistance is 10 ohms and that the internal resistance of a cell is 2 ohms. The rule says that:

$$\frac{\text{Number of cells in series}}{\text{Number of cells in parallel}} = \frac{10}{2} = 5 = \frac{1}{x} \left(\frac{n}{x}\right).$$

That is, consequently, $5x = \frac{n}{x}$. Therefore $x^2 = \frac{1}{5}n$. Since $x = \sqrt{20} = 4.48$, the result says that there should be 4.48 cells in parallel and consequently $\frac{100}{4.48}$ cells in series. Actually cells cannot be split up into fractions. *Consequently the closest whole number to 4.48 cells must be taken, and this will be the most effective way in which the cells can be combined.* Thus, if 4 cells are placed in parallel and 25 in series the maximum current would be obtained. Since 4.48 is so nearly equal to 4.5 this indicates that almost as much current would be obtained if 5 cells are placed in parallel and 20 in series. To prove this the current is calculated in the two cases. Substituting the values for x and $\frac{n}{x}$ as well as the resistances in the equations, for the 4 cells in parallel

$$i = \frac{25 E}{25(\frac{1}{4}) + 10} = 1.11 E.$$

And for 5 cells in parallel

$$i = \frac{20 E}{20(\frac{1}{5}) + 10} = 1.11 E.$$

Thus in this example the two are just equal. However, with 2 cells in parallel current is distinctly less:

$$i = \frac{50 E}{50(\frac{1}{2}) + 10} = 0.833 E$$

Had 10 cells in series and 10 in parallel been chosen

$$i = \frac{10 E}{10(\frac{1}{10}) + 10} = 0.833 E.$$

Such calculations are often needed in electrical circuits to get the best combinations of cells, since the more cells there are in parallel the less will be the current and hence the less the drain per cell.

CHAPTER XII

THERMOELECTRICITY

59. THE PELTIER AND THE SEEBECK EFFECTS

In 1826, Seebeck observed a curious effect. He connected the ends of a strip of antimony wire to two strips of bismuth, the ends of which were connected to the terminals of a sensitive galvanometer. When one of the bismuth-antimony junctions was heated to a higher temperature than the other, he found, the galvanometer showed a deflection indicating a current. This became known as the Seebeck effect, and is illustrated in Fig. 64a.

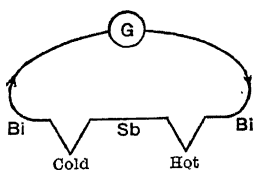


FIG. 64a. Seebeck effect: produces current.

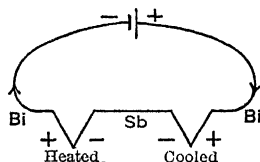


FIG. 64b. Peltier effect: cooling and heating at junction when a current flows.

In 1835, Peltier observed that if a pair of junctions similar to that used by Seebeck — i.e., a strip of antimony connected at its two ends to two strips of bismuth — were attached to the terminals of a battery so that a current flowed through the junctions, it was possible to observe a cooling of the junction where the current went from bismuth to antimony, and a heating of the junction where the current went from antimony to bismuth. This was called the Peltier effect: it is illustrated in Fig. 64b.

The two effects were soon discovered to have a common origin, and its explanation followed what was originally supposed to be the correct interpretation of metallic conduction. It was assumed that the metals bismuth and antimony had free electrons present, but that bismuth had more free electrons than antimony. At a junction between the two, owing to the higher concentration of free electrons in bismuth, a diffusion of these from bismuth to antimony took place. Thus, the bismuth metal was left positive and the antimony became negative with a potential π across the junction. The electrification was,

however, confined to the actual junction of the two metals. This is illustrated schematically in Fig. 65.

Now it was further believed that, if such a junction were heated, owing to the increased temperature agitation the bismuth free electrons were able in greater number to traverse the boundary into the antimony. Thus, with two junctions, the hotter one would have the bismuth-antimony potential π_H greater than the potential π_C for the cold junction; see Fig. 65. The net result would be that the current would flow from the more positive hot bismuth to the less positive cold-bismuth through the galvanometer. This explanation, however, disagreed with the experimental facts as depicted in Fig. 64a in that the current from the Bi at the heated junction flowed in a sense indicating that the Bi became *less* positive and not more so on heating.

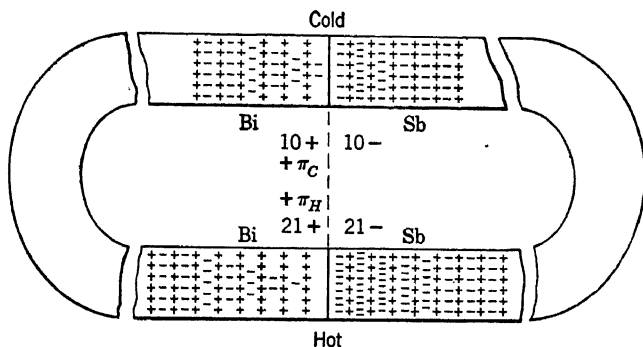


FIG. 65. Section of wires near junction of bismuth and antimony, enlarged. The diagram indicates how potentials at boundaries could be built up by the classic mechanism of the diffusion of electrons under the influence of heat. The change in concentration of electrons, diffusing with heat in this diagram from Bi to Sb, is faulty as it is not corrected for the effect of heat on nondegenerate electrons.

The effects observed also were orders of magnitude less than those calculated from reasonable assumptions as to electron concentrations. This failure of the simple picture of the phenomenon is today explained by changes in concentrations of the *nondegenerate free electrons* with temperatures which produce changes in the potentials at the junctions that may be greater and even opposed in sign to the changes produced by the increase in thermal agitation of the electrons. It has been shown likely that while there are as many free electrons as there are valence electrons in the atoms of the metal lattice, the great majority are in a peculiar state termed degenerate, in which they do not share in the usual thermal agitation. It is therefore only the very few nondegenerate electrons, of a number that is temperature-dependent, that play a rôle in these phenomena. In fact, the electron theory of the metallic state proposed by Arnold Sommerfeld and later elaborated

by many others (see page 511, Chapter XXVIII) makes it possible correctly to predict the Seebeck effect in magnitude and direction when proper data are at hand.

The explanation of the Peltier effect is exceedingly simple on the assumption that the electron diffusion potentials π_H and π_C exist at the boundaries of the two metals. Assuming, owing to the agents just discussed, that the Bi is positive to the Sb at a junction as shown in Fig. 64b, if a current is passed from the Bi to the Sb tending to annihilate the field at the junction, this field can be maintained only at the expense of the heat energy causing the electron diffusion. Hence that junction will be cooled. At the other junction the current is doing work against the field and this work manifests itself as heat at that junction. It is of interest to note in passing that the thermal effects at the junctions in the Peltier effect are such as to build up Seebeck potentials tending to oppose the current. Otherwise perpetual motion would result.

The heating and cooling at junctions because of the Peltier effect requires that the Joule law of heating be modified for those sections of any circuits where junctions of dissimilar metals appear. The work in ergs done in a time t if a current i_a flows across a junction potential difference π is $\pi i_a t$ if π and i_a are in absolute units, or $\pi i t$ in joules, if π is in volts, i is in amperes, and t in seconds. Thus, Joule's law at any junction reads $H = 0.24(i^2 R t \pm \pi i t)$. Here the $-$ sign indicates the flow with the sense of the potential π and a cooling, and the $+$ sign the heating effect at the opposite junction. π is the e.m.f. at the junction and is a characteristic constant for each pair of metals and temperature. It is called the *Peltier coefficient*. As π is usually fairly small -10^{-3} to 10^{-4} volt— $i^2 R t$ is generally greater than $\pi i t$, so that even at the cooled junction there will be a *net* heating, when $\pi i t$ is negative. However, the cooling is sufficient to manifest itself clearly if temperatures at both junctions are taken.

Early studies of the Seebeck effect showed that:

1. The thermal e.m.f. for any temperature difference between two junctions is always the same provided the metals are of the same purity and in the same physical state with regard to the previous heat treatment. In other words, the e.m.f.'s are an intrinsic property of the metals and the crystalline state of the metals involved.

2. For small temperature differences, the thermal e.m.f.'s *appear* to be proportional to temperature differences between the junctions. For extended temperature ranges this does not apply.

3. The algebraic sum of the several thermal e.m.f.'s in a circuit will give the total e.m.f., if the separate e.m.f.'s are added with due regard to sign. This merely indicates that the effects at one junction are independent of those at another, which follows from the very local character of the effects at the junctions.

60. THERMOELECTRIC POWER AND THE CALCULATION OF THERMAL E.M.F.'S

A study of the e.m.f. given by two metals A and B , if one junction was kept at a constant temperature, e.g., 0°C , while the other junction was heated to progressively higher temperatures, showed that the e.m.f.'s observed did not vary linearly with the temperature. For some pairs of metals the e.m.f. increased rapidly at first and then more slowly, reaching a maximum and then decreasing to negative values after passing through zero. Such curves are general, if the sense of the e.m.f. is disregarded, but the vertices of the parabolas may occur for different metal pairs at quite different temperature ranges. Hence in the region of room temperatures widely different aspects of curves are observed. Thus, for example, for one pair of metals the curve may be like the Fe-Al couple curve with one junction at 0°C , as shown in Fig. 68. At other times the peak may occur at such low temperatures that with the cold junction at 0°C , only the declining aspect of the curve may be observed. It should also be noted that the perfect parabolic shape of the curves may be interrupted at any temperature by sudden changes caused by changes in crystalline state of either metal. Thus when the temperature is reached where iron goes from one crystalline form, the β type to the γ type, abrupt changes in thermal e.m.f. curves can be expected. However, as long as such changes do not occur the shape of the e.m.f. temperature relation is along curves of the parabolic type indicated in Fig. 68.

The significance of this observed effect was first noted by Lord Kelvin. If it is assumed that the Seebeck effect is merely the result of the two Peltier potentials, π_H and π_C , at the hot and the cold junctions, we can reason as follows. If a current i flows for a time t the quantity q of electricity has passed. Then the heat *given out* at the one junction is $\pi_C q$, whereas that *absorbed* at the other junction is $\pi_H q$. The temperatures at the hot and the cold junctions are T_H and T_C . These thermal junctions which give current with a temperature difference, and absorb and emit heat with current flow, constitute an ideal heat engine. Now the second law of thermodynamics says that for such an engine the efficiency

$$E = \frac{\text{Work out}}{\text{Work in}} = \frac{J(Q_H - Q_C)}{JQ_H} = \frac{T_H - T_C}{T_H}.$$

Here Q_H is the heat absorbed at the hot junction, which is $\pi_H q$, and Q_C is the heat liberated at the cold junction, which is $\pi_C q$, and J is the mechanical equivalent of heat. Thus $\frac{\pi_H q - \pi_C q}{\pi_H q} = \frac{T_H - T_C}{T_H}$. Since the Seebeck electromotive force $E = \pi_H - \pi_C$, as a consequence of the

second law it can be written that

$$E = \pi_H - \pi_C = \pi_H \frac{T_H - T_C}{T_H} = \pi_H \frac{\Delta T}{T}.$$

This says that the thermal e.m.f. is proportional to the temperature difference ΔT , since π_H and T are constant or nearly so.

Experiment, however, shows that E is a *parabolic function of t* and is not proportional to ΔT . Thus, Thomson argued that there must be *other sources of e.m.f. in the circuit* that cause the variation. The only source he could think of was that by heating a wire at one end and cooling it at the other *an e.m.f. is set up in the wire*. This is not unexpected, for if by heating a section of wire the concentration of nondegenerate electrons is increased, or changed, and also their energy of heat motion is increased, there will be a diffusion of electrons down, or up, the wire, leading to a potential difference at its ends. That is, by having a temperature gradient, or change, along a wire of the same material an e.m.f. will be generated. Thus in addition to π_H and π_C for the hot and the cold junctions of the two metals, A and B , the e.m.f.'s caused by the temperature difference down the wires A and B must be added. The Thomson coefficients, named for Sir William Thomson (Lord Kelvin), are the e.m.f.'s, σ_A per degree difference in temperature for metal A and σ_B per degree difference in temperature for metal B . Thus the true thermal e.m.f. will be given by

$$E = \pi_H - \pi_C - \int_{T_C}^{T_H} \sigma_A dT + \int_{T_C}^{T_H} \sigma_B dT.$$

For the increments of e.m.f. $\sigma_A dT$ and $\sigma_B dT$ in the circuit from T_C to T_H must be summed up, i.e., integrated, and added in. They cannot be just added or subtracted $(\sigma_B - \sigma_A)(T_H - T_C)$, since σ_B and σ_A may change with temperature in different ways and in a nonlinear fashion.

To test for the presence of the e.m.f. produced by a temperature difference, Lord Kelvin performed the following ingenious experiment. He heated a piece of uniform metal sharply at one region. He then passed a current down the metal. On each side of the heated strip there were cooled portions. At one side of the heated portion the current passed from hot to cold, i.e., across an e.m.f. due to the temperature gradient. At the other side it passed from cold to hot, across an oppositely directed e.m.f. As in the Peltier effect, there would be expected to be a cooling at one spot and a heating at the other. In fact, at one side of the heated portion a cooling by the current was observed, and on the other side heating was observed. This is shown experimentally by means of an iron rod bent in the form of a U, as shown in Fig. 66. The two ends, A and C , pass into mercury pools connected to a d-c current main giving 10 amperes. These are main-

tained cold by dipping them into an ice bath. The loop L of the U is heat-insulated from the straight portions by an asbestos baffle B and is heated strongly with a flame F . Between the mercury cups and the baffle B on each limb AL and CL , i.e., at the points where the gradient of temperature is greatest, two fine platinum resistance wires, P and Q ,

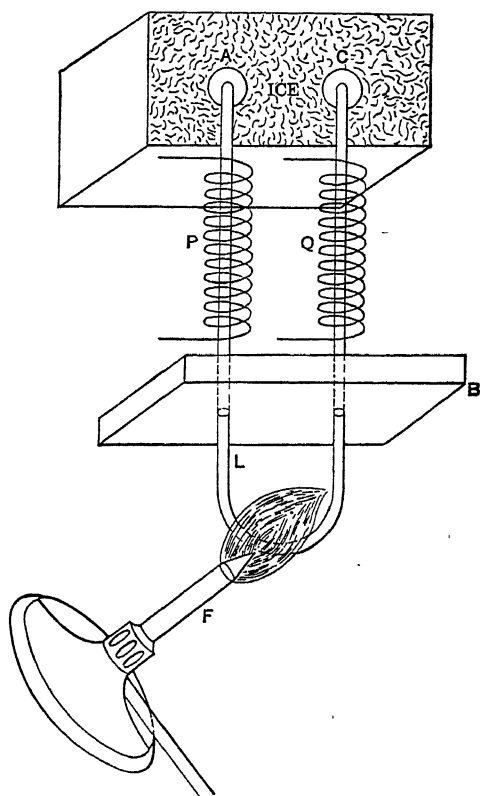


FIG. 66. Experimental arrangement to illustrate the Thomson effect.

are insulated electrically from the rods by thin mica and wrapped about the rods CL and AL . They are surrounded on the outside by asbestos insulation and connected into two arms of a Wheatstone bridge. The bridge is balanced when the current in the rod ALC is off. When the current of about 10 amperes flows in ALC the bridge goes out of balance owing to a difference of temperature and hence a change of resistance in the two resistance thermometers P and Q . The phenomenon is called the Thomson effect in honor of William Thomson (Lord Kelvin), the discoverer.

Thus, in addition to the thermal e.m.f.'s at the two junctions of dissimilar metals A and B , whenever there is a difference of tempera-

ture between two junctions the thermal e.m.f.'s along metals A and B must be added because there is a difference of temperature $t_H - t_C$ between their two ends. A study of the Thomson effect for various metals revealed the fact that for one metal, namely, pure lead, the Thomson effect was zero. That is, there is no appreciable thermal e.m.f. down a lead wire when two parts are at different temperatures. Lead was accordingly chosen as a reference substance in thermocouple work, for in dealing with thermocouples made of a metal A with lead it is necessary only to deal with the differences of the e.m.f.'s at the hot and cold junctions of lead and A and with the e.m.f. due to a temperature gradient in the metal A . This then throws the lack of proportionality of the e.m.f. with $(t_H - t_C)$ entirely onto the Thomson effect in A . Under these simplified conditions it was found that by measuring the value of the thermal e.m.f. for one degree difference in temperature $t_H - t_C = 1^\circ \text{C}$ between the hot and cold junctions at different values of t_C , it was possible to describe the behavior of each lead-metal couple as a function of t in terms of a relatively simple equation in the form $P = a + bt$.

Since the thermoelectric power P has been defined as the e.m.f. for 1° difference in temperature, for a $Pb-A$ thermocouple: $P_A = \frac{E_{AH} - E_{AC}}{t_H - t_C} = \frac{dE_A}{dt}$. Experimentally it is found to be given by

$$P_A = \frac{dE_A}{dt} = a_A + b_A t. \quad \text{For some other metal } B, P_B = \frac{dE_B}{dt} = a_B +$$

$b_B t$ can be written, and so on. Extensive measurements have yielded the table of sample values given here. It is seen that some equations hold only over limited temperature ranges. The values of a range from about one to tens of microvolts for most substances and are positive and negative in sign. The values of b range in the tenths to hundredths of microvolts and are either positive or negative in sign.

It is seen that, with any two metals A and B , $P_A - P_B = a_A - a_B + (b_A - b_B)t$, since the Pb aspect of the equations for P_A and P_B cancel out. Thus from the values of the thermoelectric powers P_A and P_B taken from published tables, the thermal e.m.f.'s at any temperatures can be calculated for any couple for which values of a and b are given.

The calculation can be done in one of two ways, either by the calculus, or by graphical means.

Since

$$P_A - P_B = \frac{dE_A}{dt} - \frac{dE_B}{dt} = a_A - a_B + (b_A - b_B)t,$$

the e.m.f.,

$$E = \int_{t_C}^{t_H} (P_A - P_B) dt = \int_{t_C}^{t_H} \left(\frac{dE_A}{dt} - \frac{dE_B}{dt} \right) dt = (E_A - E_B)_{t_C}^{t_H}$$

THERMOELECTROMOTIVE POWERS FOR THE ELEMENTS AT 20° C AND AS A FUNCTION
OF TEMPERATURE t IN DEGREES CENTIGRADE

(The standard metal in these couples is lead and the power is in volts per degree.)

Element	Power at 20° C	Power at t° C
Al.....	$- 0.68 \times 10^{-6}$	$(- 0.76 + 0.0039 t) \times 10^{-6}$
Au.....	$+ 3.0 \times 10^{-6}$	$(+ 2.8 + 0.0101 t) \times 10^{-6}$
Sb.....	$+ 6.0 \times 10^{-6}$	
Bi (commercial).....	$- 97.0 \times 10^{-6}$	
Bi (pure).....	$- 89.0 \times 10^{-6}$	
Cd.....	$+ 3.48 \times 10^{-6}$	$(+ 2.63 + 0.0424 t) \times 10^{-6}$
Cu.....	$+ 1.52 \times 10^{-6}$	$(+ 1.34 + 0.0094 t) \times 10^{-6}$
Fe.....	$+ 16.2 \times 10^{-6}$	$(+ 17.15 - 0.0482 t) \times 10^{-6}$
Ni (-18° C to 175° C.....	$- 22.8 \times 10^{-6}$	$(- 21.8 - 0.0506 t) \times 10^{-6}$
Pt (hard).....	$+ 2.42 \times 10^{-6}$	$(+ 2.57 - 0.007 t) \times 10^{-6}$
Pt (malleable).....	$- 0.818 \times 10^{-6}$	$(- 0.60 - 0.0109 t) \times 10^{-6}$
Pt-Ir alloy..... (85% Pt - 15% Ir)	$+ 8.03 \times 10^{-6}$	$(+ 7.90 + 0.0062 t) \times 10^{-6}$
Pb.....	0	0
Se.....	$+ 807.0 \times 10^{-6}$	
Ag.....	$+ 2.41 \times 10^{-6}$	$(+ 2.12 + 0.147 t) \times 10^{-6}$
Steel.....	$+ 10.62 \times 10^{-6}$	$(+ 11.27 - 0.0325 t) \times 10^{-6}$
Sn.....	$- 0.33 \times 10^{-6}$	$(- 0.43 + 0.0055 t) \times 10^{-6}$
Zn.....	$+ 2.79 \times 10^{-6}$	$(+ 2.32 + 0.0238 t) \times 10^{-6}$

Here t_H and t_C are the temperatures of the hot and the cold junctions. Thus,

$$E = \int_{t_C}^{t_H} (P_A - P_B) dt = \int_{t_C}^{t_H} [a_A - a_B + (b_A - b_B)t] dt$$

$$= (a_A - a_B)(t_H - t_C) + \frac{1}{2}(b_A - b_B)(t_H^2 - t_C^2).$$

It is seen that this value of E for metals A and B with junctions at t_H and t_C is a parabola as shown in Fig. 67.

The evaluation of E can be done graphically as follows:

Suppose as in Fig. 68 metal A is Fe, with $P_{Fe} = (17.15 - 0.048 t) \times 10^{-6}$ volt, and metal B is Al, with $P_{Al} = (-0.76 + 0.0039 t) \times 10^{-6}$ volt. Plot P as ordinates and t as abscissas. Then Pb will give the axis of abscissas, as its power P is set as 0 and is constant with t . The plot for Fe on this diagram will give a straight line with a steep downward slope as t increases, starting at +17.15 microvolts at 0° C. Al, on the other hand, starts with -0.76 microvolt at 0° C and rises linearly and very slowly as t increases. It is to be noted that at about 330° the Fe and the Al lines cross, i.e., the *thermoelectric powers are equal* in magnitude but opposite in sign. Above this temperature Al becomes positive to Fe.

Now if P_{Fe} were constant with t at 17.15 microvolts, then if one junction were at 0° and the other were at x degrees above zero, the thermal e.m.f. between Pb and Fe would be merely $P_{Fe}x$. $P_{Fe}x$ is the

area of the rectangle and it would give the e.m.f. between 0 and x if P_{Fe} were constant. Actually, P_{Fe} falls from P_{Fe0} at $t = 0$ to P_{Fez} at $t = x$. Since the fall is linear we can get the average value of P_{Fe} by taking $\frac{P_{Fe0} + P_{Fez}}{2}$. Then the e.m.f. would be just the area $x \left(\frac{P_{Fe0} + P_{Fez}}{2} \right)$.

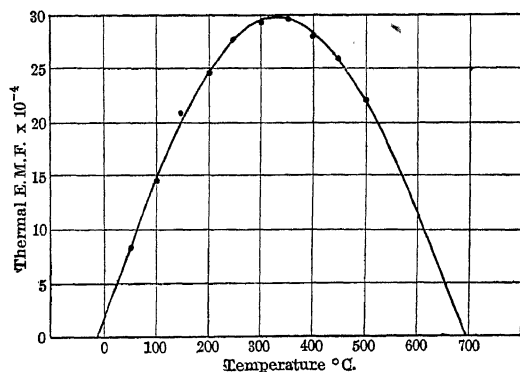


FIG. 67. Thermoelectromotive force of an Fe-Al thermocouple as a function of temperature with cold junction at -15°C .

Geometrically, this can readily be seen to be the area of the *trapezoid* described by the 0 line, P_{Fe0} , P_{Fez} , and the abscissa x .

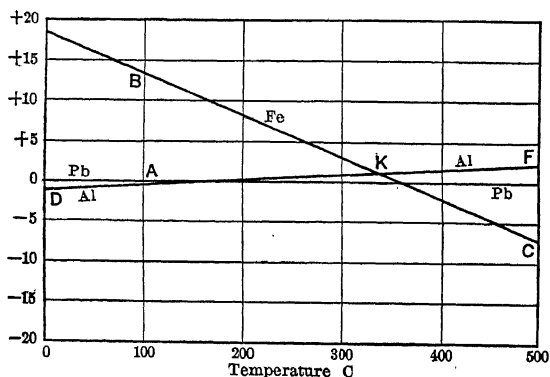


FIG. 68. Thermoelectromotive power against temperature in degrees C. (Thermoelectromotive power to be multiplied by 10^{-6}).

Thus it can be seen that the thermal e.m.f. is just the area of the trapezoid cut out by the powers at the two temperatures and by the temperatures on the lead base line. This can at once be generalized by saying that for any two metals at any two temperatures *the thermal e.m.f. is given by the areas of the trapezoids cut by the intersection of the*

two temperature verticals and the two lines for the powers. The areas can be determined graphically. Precisely the same thing was done when the equation for $P_A - P_B$ was integrated, for this merely yields the equation for calculating this area.

Considering the curves for P_{Fe} and P_{Al} in Fig. 68, it is then possible, as an example, to proceed to calculate the e.m.f.'s for an Fe-Al couple, with one junction t_C at zero and the other one t_H at any temperature, $t_H = 50^\circ, 100^\circ, 150^\circ, 200^\circ$, etc., as follows. Draw the verticals at the various values t_H chosen. Then calculate the areas of the trapezoids between $t_C = 0$ and the several values of t_H . These can be plotted against t_H as shown in the plot of Fig. 67.

It is there seen that calculated the e.m.f. rises sharply from 0, where $t_H = 0$, since the areas of the trapezoids are relatively large near zero and the proportionate decrease in height is small. However, as $t_H = 330^\circ$ is approached, it is noted that the areas added by successive 50° temperature intervals are progressively decreasing to 0. Hence the e.m.f. increases more slowly. At the temperature where the power curves cross the highest point of the e.m.f. curve, i.e., the vertex of the parabola, is reached. Thereafter, since the powers P_A and P_B have changed sign relatively, with the Al positive to the Fe, the areas to the right of the crossing point subtract. Thus the area decreases and the descending curve of the couple results. At 660° , as can be seen, the areas to the right and the left of the crossing point are equal and the thermal e.m.f. is zero. Beyond 660° it becomes reversed in sign, or negative. The point at which the thermoelectric power lines cross, or the vertex of the parabolic thermal e.m.f. curve, is called the *neutral point*, and the temperature is designated as t_n . At this point $\frac{dE_{AB}}{dt} =$

0. It might be noted that the graphical solution can be applied between any two temperatures t_C and t_H by using appropriate areas.

The value of t_n can at once be calculated from its definition for any pair of metals since at t_n it can be written that $\frac{dE_{AB}}{dt} = P_A - P_B = 0$, or that $P_A = P_B$. Accordingly, at t_n , $a_A + b_A t_n = a_B + b_B t_n$. Thus $t_n = \frac{a_A - a_B}{b_B - b_A}$. This relation leads to several commonly used equations for calculating thermal e.m.f.'s from the tabulated powers.

The integration to give the value of E_{AB} led to the expression,

$$E = (a_A - a_B)(t_H - t_C) + \frac{1}{2} (b_A - b_B)(t_H^2 - t_C^2).$$

Collection of terms and algebraic manipulation at once lead to the expression,

$$E = (a_A - a_B)(t_H - t_C) \left[1 + \frac{1}{2} \frac{b_A - b_B}{a_A - a_B} (t_H + t_C) \right].$$

With the use of the expression for the temperature of the neutral

point, and calling the value of the power at $t = 0$, $a_A - a_B = P_0$ it is possible, from the equation above, to write that,

$$E = P_0(t_H - t_C) \left[1 - \frac{1}{2t_n} (t_H + t_C) \right].$$

This equation can also be inferred from the geometry of the plots of Fig. 68. Another form which follows from the relations above is,

$$E = (a_A - a_B)(t_H - t_C) \left\{ \frac{t_H + t_C}{2} - t_n \right\}.$$

If the equation is written in terms of the power, P_C at the temperature of the cold junction t_C when t_C differs from 0, the expression takes on a still simpler form found in the literature. This reads

$$E = P_C(t_H - t_C) \left[1 - \frac{\frac{1}{2}(t_H - t_C)}{t_n - t_C} \right].$$

It is now necessary to digress for a moment to show the relation between the Thomson and the Peltier coefficients and the experimental constants a and b expressing the thermoelectric power, which are easily experimentally determined constants. It was seen that from Thomson's theory,

$$\begin{aligned} E &= \int_{T_C}^{T_H} \left\{ \frac{d\pi_{AB}}{dT} - (\sigma_A - \sigma_B) \right\} dT \\ &= \pi_{ABH} - \pi_{ABC} - \int_{T_C}^{T_H} (\sigma_A - \sigma_B) dT, \end{aligned}$$

with
$$P = \frac{dE}{dT} = \frac{d\pi_{AB}}{dT} - (\sigma_A - \sigma_B).$$

Now the second law of thermodynamics says that the integral of the heat exchanges divided by the absolute temperature, around a closed reversible cycle of operations, is equal to 0, i.e., that $\oint \frac{dQ}{T} = 0$.

Evaluating dQ from the equation for thermoelectric potentials and q , the quantity of electricity passing, it can be shown that this yields the relation that,

$$\frac{d\left(\frac{\pi_{AB}}{T}\right)}{dT} - \frac{\sigma_A - \sigma_B}{T} = 0.$$

From $P_{AB} = \frac{d\pi_{AB}}{dT} - (\sigma_A - \sigma_B)$ and the expression above, it

follows that $P_{AB} = \frac{\pi_{AB}}{T}$ and

$$\frac{dP_{AB}}{dT} = d\left(\frac{\pi_{AB}}{T}\right) dT = \frac{\sigma_A - \sigma_B}{T}.$$

Experiment, however, yielded the relation,

$$P_{AB} = a_A - a_B + (b_A - b_B)t$$

Since $\frac{dP_{AB}}{dT} = \frac{dP_{AB}}{dt},$

then $\frac{dP_{AB}}{dt} = (b_A - b_B) = \frac{dP_{AB}}{dT} = \frac{\sigma_A - \sigma_B}{T},$

whence the following identifications follow.

$$\sigma_A = b_A T, \quad \sigma_B = b_B T$$

$$P_{AB} = \frac{\pi_{AB}}{T} = (a_A - a_B) + (b_A - b_B)T.$$

Again, since $P_{AB} = \frac{\pi_{AB}}{T}$ and since $P_{AB} = \frac{dE_{AB}}{dT}$ then $\frac{\pi_{AB}}{T} = \frac{dE_{AB}}{dT}$, or $\pi_{AB} = T \frac{dE_{AB}}{dT}$. Thus, the Peltier coefficient is the rate of change of thermoelectromotive force with temperature multiplied by the *absolute* temperature. Hence, we can finally write,

$$\pi_{AB} = (P_A - P_B)T = [(a_A - a_B) + (b_A - b_B)t]T.$$

This is the quantity required in the modified Joule's law equation for heating at junctions. It can be calculated from the observed values for the power.

61. USES OF THERMOCOUPLES

Outside of its theoretical interest, the thermoelectric effect is of great practical use in temperature-measurement applications. Practically all the temperature measurements in the regions between 300° and 1200° C are now made by means of the thermocouple. The thermocouple is nothing but a thermal junction of two dissimilar metals, one junction of which is kept at a constant temperature, generally that of melting ice, the other junction being at the place where temperature is required. The advantages of thermocouples are that they possess a small heat capacity, that they make an almost instantaneous response to the temperature because of their high conductivity and low heat capacity, that they can be inserted into positions inaccessible to ordinary thermometers, that if they are carefully calibrated they will give temperatures as accurately as the e.m.f. can be measured, and, finally, that they can be made of metals which are sufficiently temperature-resistant so that temperatures which could not be studied with ordinary substances such as glass can easily be studied.

The measurement of temperatures with thermocouples has resulted in a great diversity of combinations being developed for different uses.

Where small temperature differences are involved, at about room temperatures, Cu-Bi, Sb-Bi, and Fe-Ni couples can be used. Ag-Cu couples give powers that are nearly parallel. Thus though the e.m.f. is low it is nearly proportional to temperature. For more extended ranges conventional use is made of copper constantan couples, constantan being an alloy of very reproducible properties. For higher temperatures iron alloys of chromium and aluminum manufactured under the trade names of Chromal and Alupal couples are used, especially where the oxidation conditions can be considered rough. For very high temperatures couples are made of platinum and platinum-iridium, or platinum-rhodium, alloys (1 to 15 per cent). These give low powers but are heat-resistant and have long ranges over which they can be used.

The couples are best used with potentiometers or with special potentiometer boxes that read microvolts directly. High-resistance galvanometers can be used if necessary. The standard couple materials have their e.m.f.'s given in tables of physical constants as a function of temperature. Thus it is possible to read the e.m.f. and to get the temperature from a curve plotted from such sources. Usually it is safer to calibrate the couples by placing them in known temperature baths. Both texts on heat and tables of constants give a list of the accurately measured melting, boiling, and transition temperatures which make it possible to calibrate couples for any temperature range directly in terms of the observed e.m.f. at those fixed points. This should be done where the purity of the couple material is in doubt and great accuracy is needed.

It must be realized that while thermal e.m.f.'s are small, i.e., run in the order of 10^{-6} up to 10^{-2} volt or so, they can give rise to heavy currents. If a couple gives an e.m.f. of 10^{-2} volt and the resistance of the metal circuit is 5×10^{-5} ohm the current will be 200 amperes. Demonstration experiments designed to develop such potentials in circuits of such resistance show the presence of heavy currents with one junction at 500°C and the other at 80°C by the powerful magnetic effects yielded by such currents.

The thermocouple has been developed for use in measuring very small temperature differences in the form of the *thermopile*. Such temperature differences arise in the measurement of the radiation from stars, in measurements of the intensity of spectral lines, and in the measurement of high-frequency alternating currents of small magnitude.

The thermopile consists of a number of couples in series such that the effects add. For radiation measurements the couples are arranged so that the Bi to Cu junction of each couple is brought next to, but not in contact with, the others, on a thin insulating flake of mica or similar substance. These junctions are blackened for radiation measurements. The Cu to Bi junctions are fastened to a thin insulator attached to a large cool mass of copper or other good conductor. This

arrangement can be achieved by use of a ring arrangement, as indicated in Fig. 69. By placing such a pile in vacuum, its efficiency is increased by reducing air cooling. For some work the couples are made by evaporating thin films of Cu and Bi on the appropriate surfaces and

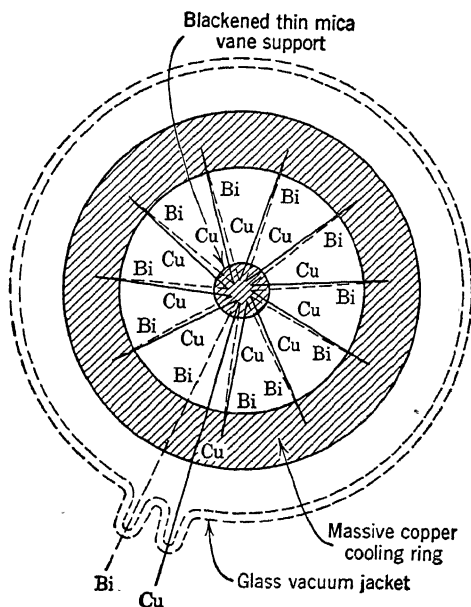


FIG. 69. Vacuum thermopile.

using as many as twenty to sixty contacts. These are easier to achieve than making the same joints in fine wires.

The Boys' radio micrometer is a simple adaptation of the principle. It has a single hot junction blackened at the lower end, as shown in Fig. 70. The upper cold junction is fastened to the mirror and suspension assembly. Using strong permanent magnets and a sufficiently low resistance in the wire loops a good deflection can be obtained for very feeble light intensity. Boys claims to have detected a candle flame at a distance of 2 miles with this micrometer.

There are no instruments for the measurement of weak electrical alternating currents above 10^4 cycles per second, since iron does not respond to alternating frequencies above this value. To this end the principle of the thermal cross, as shown in Fig. 71, may be employed. It consists of a pair of dissimilar thermocouple wires such as Fe and Ni made into a cross by spot-welding or soldering two *fine* wires together to ensure contact. The flow of the alternating current through one side of the cross heats the junction and the increase in temperature at that point registers on the sensitive galvanometer as a thermal e.m.f. In the thermogalvanometer or thermomicroammeter, the cross is

replaced by a series of sensitive junctions as with the thermopile, though the ring structure is not needed here. The one set of junctions are free and the other set are insulated and attached to a cold block, as shown in Fig. 72. The high-frequency alternating current heats the

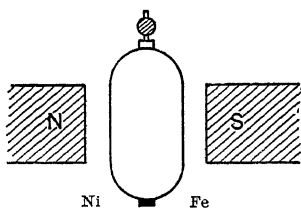


FIG. 70. Boys' radio micrometer.

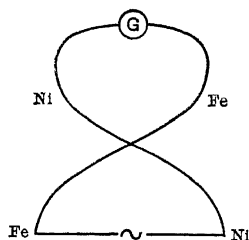


FIG. 71. The thermal cross.

free junctions while the other junctions are kept cool by conduction. The couple then is short-circuited across a galvanometer. It may be

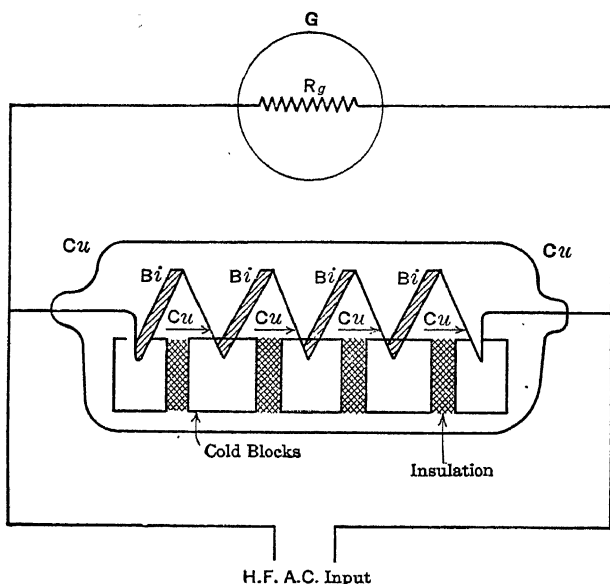


FIG. 72. Principle of the vacuum-jacketed thermopile, which can also be used as a thermogalvanometer.

placed in a vacuum jacket to increase its sensitivity. The couple and the a-c source can be in series with the galvanometer or the couple and the galvanometer can be in parallel across the alternating current as in the thermal cross. The alternating current does not affect the galvanometer. Such devices are widely used in radio circuits.

CHAPTER XIII

STATIC ELECTRICITY I: CHARGES AND FIELDS

62. ELECTRIFICATION AND ITS DETECTION

As was stated in Chapter I, electrification was known to the Greeks. It was later studied by William Gilbert, physician to Queen Elizabeth. Gilbert found that, besides amber and jet, many other nonconductors when rubbed with other nonconductors were able to pick up light objects. Electrification therefore manifests itself by attractions, and it is only by a manifestation of the attractive and repulsive forces that we know of the existence of electricity. It was later found that the forces between electrified particles are not only those of attraction but also those of repulsion. Shortly after Gilbert's time two kinds of electricity were discovered and named. They were typified by the electrification produced when a resinous substance like sealing wax or hard rubber was rubbed with wool or cat's fur, and when substances like glass were rubbed with silk. The electrification obtained from resinous materials was called *resinous* electricity. The electrification obtained from glass-like substances was called *vitreous* electricity. Investigation showed that two bodies charged with the same kind of electrification repelled each other, while two bodies charged with the two different sorts of electrification attracted each other.

The repulsive force between two charges was utilized in the so-called electroscope which was early used to measure and study charges. In Fig. 73, *B* is a metal box with glass windows on two sides. *W* is a wire passing through an insulated plug *A* consisting either of amber or sulfur. *L* is a small strip of gold leaf fastened to *W* at one point. When vitreous or resinous electrification is put on *W* the like charge which reaches *L* causes *L* to be repelled and to give a deflection which will later be seen to measure the electric potential.

It was found that, if an uncharged insulated body was connected by a metal wire to a charged body, the charge would travel along the wire to the uncharged body. This was called *electrical conduction*. The ability of electricity to move along conductors differentiates it from magnetism in which the two magnetic entities appear in equal amounts and are fixed in the material in which they occur. The con-

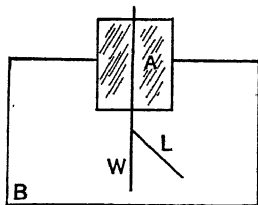


FIG. 73. Gold-leaf electroscope.

ductivity of electricity makes it possible to measure experimentally the units of electricity and compare them much more accurately and easily than is the case for magnetism. The fact that some bodies do not carry electricity is utilized in isolating conductors for experimental purposes. Such isolating, nonconducting bodies are called *insulators*.

63. ELECTROSTATIC INDUCTION

If a charged body be brought near a neutral uncharged one, the uncharged body exhibits charges at its ends, as shown in Fig. 74A.

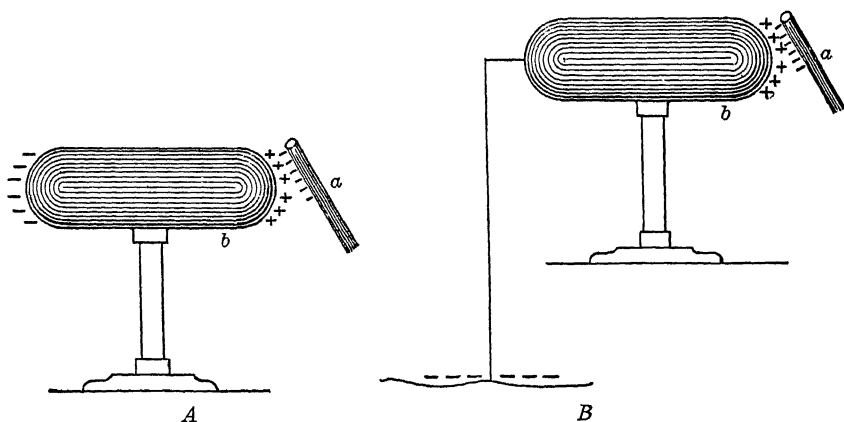


FIG. 74. Charging by induction.

On the end nearest the charged body it has electrification of the opposite type; on the farther end it has electrification of the same type as indicated in the diagram of Fig. 74A. *This phenomenon is known as electrostatic induction.*

If the charged body is brought up to the insulated, uncharged body, and if this is touched by a conductor connected to earth, its contact next being broken, and the charged body removed from the neighborhood of the uncharged body, the uncharged body will now be observed to have a charge. The mechanism of charging by induction is simple and is easily understood by regarding the two diagrams of Fig. 74. In diagram A, the resinous electricity of the charged body *a* attracted a charge of the opposite sign on the near side of the insulated conductor *b*. The charge of like sign attempted to get as far away from the charged body as possible. Making contact between the uncharged body and the ground gave the charge of the same sign as the charged body the opportunity of getting farther away, as shown in diagram B of Fig. 74. On breaking the contact while the charged body *a* was near the insulated body the vitreous electrification could

not return to *b*. Therefore, on removing the charged body *a*, the electrification of opposite sign to the charged body on the insulated body *b* remained and spread over the surface of the insulated body. Thus, one body can be charged with the electrification of opposite sign by bringing a charged body near it, touching it, breaking contact, and removing the charged body. *This is known as charging by induction.* It has very important consequences. One of them is that if the quantities of electrification produced be measured, the electrification of both kinds produced by induction in a body appears in equal amounts. That this must be so follows from the fact that, before bringing the charged body near the uncharged body, the body *b* was as a whole neutral.

64. ONE- AND TWO-FLUID THEORIES OF ELECTRICITY

The phenomenon just described together with the conduction of electricity led Franklin to postulate a theory of electricity. He assumed that there was one electrical fluid* which he called positive electricity. It was identified with the vitreous electrification. Then there was another type of electrification which was the result of the removal of the positive electrical fluid from neutral bodies. Electricity was looked on as a fluid because it could move through a conductor. In contradistinction to the positive electrification the residual electricity was called negative.

At about the same period another theory of electricity had been put forward, which was that both positive and negative electricities were separate fluids, and so a neutral body contained equal amounts of the positive and negative fluids.

Today it is known that in the sense of a single fluid the Franklin theory was correct with regard to metals. One of the types of electricity, the negative electricity, is today known to consist of minute particles (not a continuous fluid), which are called electrons. Because of their small size and mass they are free to move. *They then constitute the "fluid"*

* It is interesting to note that in the period during which these researches developed, beginning with 1750 and extending well through the succeeding 150 years, there was a peculiar type of viewpoint in the study of matter. This viewpoint came from the attempt to explain the universe in terms of a mechanical system based on Newton's laws. With this attempted "explanation" there developed, in order to solve the problems of motion involved, the mathematical methods of the calculus. Because of the ease with which problems could be handled by the calculus, which deals with continuously varying functions, the attempts to describe nature were all made in terms of concepts based on the continuous. Thus heat was supposed to be a weightless fluid, as was the case with electricity. In fact, it has been only since 1890 that the great importance of the discontinuous in physics, has begun to be realized, and the era which has given us the atom, the electron, the proton, and the quantum of energy is making us search hard for an adequate mathematical and physical system of representation of a world which has been found to be made up of particles instead of continuous fluids.

postulated by Franklin but are negative in sign, and consist of particles. The atoms of the matter, when the negative electrons are removed, are positively charged. They constitute the residual "negative" charge of Franklin. *Where the charged atoms and molecules (ions) are free to move as in liquids, electricity moves accordingly to the two-fluid theory.* Hence, the bitter controversy which raged over this question was entirely futile and unnecessary, for both views have been shown to be partly right and partly wrong. However, physics still retains the terminology of the Franklin theory in that it is assumed that the electricity flows from positive (vitreous) to negative despite the fact that the flow in metals is contrary.*

65. CONSEQUENCES OF INDUCTION: THE ICE-PAIL EXPERIMENT

The phenomenon of induction leads to several important applications. The first one is the phenomenon of electrical screening. Suppose

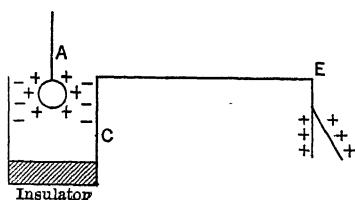


FIG. 75. Faraday's ice-pail experiment.

that a charged metal body is placed inside of a box; on introducing the charged metal body the electricity of opposite sign will flow to the inside of the box tending to neutralize the charge placed inside. The electrification of the same sign as the charged body will flow to the outside of the box, and if the box is attached to earth will flow away. Thus, outside of the box, if

earthed, there will be no manifestation of electrification, the electrification inside of the box neutralizing the field of the charge.

This *screening effect*, so called, is proved by Faraday's famous ice-pail experiment, shown in Fig. 75. In this experiment, the charged body *A* with positive electricity is brought inside of a metal cup *C* which is connected to the electroscope *E*. As a result of the positive charge, negative electrification will be drawn to the part of the cup nearest *A*. The positive electricity will then remain on *E* causing the leaves of the electroscope to diverge. If now the *positively charged body A be touched to the inside wall of the cup C* and then removed, *no change will be observed in the leaves of the electroscope.* They will stay separately exactly as far as before. This means that *the induced positive charge on the electroscope is exactly the same as the charge that was communicated to the electroscope by A.* Thus it is seen that the charge on *A* was completely neutralized by the negative charge which resided in *C*, the equivalent of this in positive charge being given to *E*.

* The reason such a convention is retained is that all the literature, tables, machinery, and equipment of the past is couched in this terminology. It would be too costly to change even if there were much more cogent reasons than to be consistent with electrical flow in metals and radio tubes. As long as all follow the convention and know its weakness there is no trouble in its retention.

The consequences of the ice-pail experiment are greater than they would seem at first sight, for besides leading to the phenomenon of screening, *they lead to a proof, of very great precision, of the law of force between electrical charges.* Of this, more will be said later.

66. USES OF INDUCTION

Induction can be utilized to produce electrification. In its simplest form induction is achieved by the instrument known as the *electrophorus*. This consists of an *insulating* plate, of hard rubber, for instance, which can be charged by friction. An insulated metal disk is brought as near as possible to the charged plate without making contact except at one or two isolated high points. The metal disk is then touched for an instant to ground giving the electricity of the same sign as the charged ebonite plate a chance to flow off. The ground is then removed, and when the metal disk is removed it will be found to be highly electrified. This electricity can be communicated to any system, the plate thus discharged, and being again brought near the charged ebonite can be charged again. Thus the process of charging and discharging can be carried on indefinitely with the same quantity of electricity on the ebonite. *The energy for producing this electrification is furnished by the work of bringing the metal disk into the neighborhood of the ebonite plate and lifting it away against the attractive forces.* Thus electric energy is not obtained for nothing. In fact, the energy obtained this way under ideal conditions would be equal to the work done. Mechanical devices to perform this operation continuously, which store the electricity obtained in proper containers, have been invented.* They are called electrostatic machines, and served in the early days as the only source of electrification. They were also used in the early days to get high voltages for x-ray work. Today, we have far more constant and efficient sources for this purpose.

It is perhaps of interest to illustrate the principle of the electrostatic induction generators by describing the latest and largest device. This device was designed and developed by R. J. Van de Graaff for the generation of high-potential d-c sources to be used in nuclear disintegration experiments. It has been built under the auspices of the Massachusetts Institute of Technology at Round Hill, Massachusetts, with the hope of achieving potential differences up to 10^7 volts. It is schematically represented in Fig. 76. Two large 15-foot-diameter hollow aluminum spheres S_+ and S_- are mounted on 22-foot-high hollow insulating columns 6 feet in diameter. They in turn are mounted on steel trucks. The motive mechanism is mounted on the

* The earlier types were the Toepler and the Wimshurst machines. The principle on which they operate is essentially that of the electrophorus. The detailed mechanism is, however, quite complicated and is not worth while presenting here. Details can be found in any of the textbooks on electricity antedating 1910.

trucks which are enclosed so that the whole system inside the spheres, columns, and generator mechanism can be air-conditioned. Motors on the trucks and in the spheres rotate two *insulated* steel rollers R_1 and R_2 at desired speeds (about 3600 r.p.m.). These rollers carry endless belts BB of suitable insulating fabric, paper, silk, etc., at a linear speed of about 5650 feet per minute in the direction indicated by the arrows. Opposite the lower rollers R_2 are corona discharge points D_1 connected to the ground G .

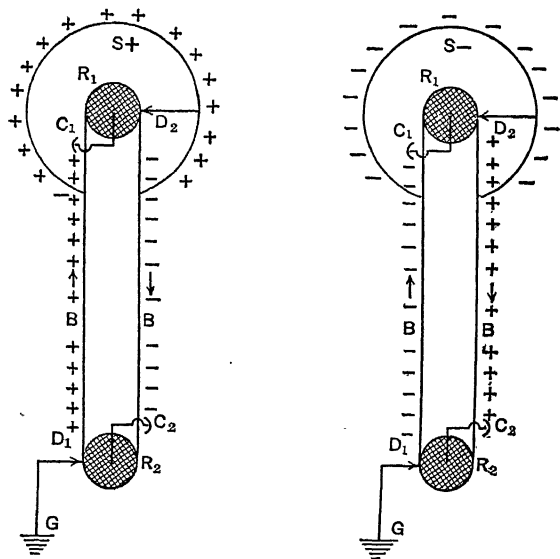


FIG. 76. Schematic diagram of the operation of the Van de Graaff high-voltage static generator.

If initially the system belonging to S_+ has R_2 charged to 20,000 volts negative relative to ground, the corona point D_1 sprays positive ions onto the outside of the belt B through induction. This charge is rapidly conveyed upward by the belt B and reaches a small shielded corona collector C_1 connected to the roller R_1 . This collector charges R_1 positively. When R_1 reaches a sufficiently high potential the corona point D_2 attached to the sphere S_+ , by induction sprays negative charges onto the belt. This leaves a positive charge on S_+ which, as the sphere is a Faraday cage, then appears on the outside. The belt B conveys the negative charge downward to a corona collector C_2 connected to R_2 . The collectors C_1 and C_2 could be brush contacts. As these damage the belt, shielded corona collectors are used. In the system connected to S_- the mechanism is the same except that R_2 is initially positively charged, thus leading to a negative charge on S_- , relative to ground. It will be seen that the charges on R_1 and R_2 in

both cases are self maintaining at the corona point, the essence of the mechanism being to draw charge from S_+ or S_- by induction through D_2 and to run this charge by the mechanical action of the belt to R_2 whence it escapes to the ground G by induction through D_1 . All that is needed beyond the moving belt which does the work of separating charges is an atmosphere dry enough so that the belt is insulating and the placement of the initial charges on R_2 by means of generators, transformer, and rectifier. After generation begins, the chargers of R_2 can be disconnected. Obviously a large undertaking of this sort is far more complicated than indicated by the schematic diagram given, the function of which is solely to illustrate the physical principles of charging by induction. With two sets of belt in each tower, the maximum charging current obtained to date at Round Hill is 2.1 milliamperes. The highest voltage obtained consistently without sparking is 2.4×10^6 volts on S_+ and 2.7×10^6 volts on S_- , giving 5.1×10^6 volts between terminals. At this voltage 1.1 milliamperes of current are available for operating the atomic disintegrator, once a vacuum tube standing this potential can be built between the spheres. The magnitude of voltage fluctuation is about 0.1 per cent under operating conditions. Several successful devices of this kind operating between 1×10^6 and 2×10^6 volts are in use for atomic disintegration studies. The power available once the disintegration tube is perfected will be $5.1 \times 10^6 \times 1.1 \times 10^{-3} = 5.62$ kw, or perhaps more. The attempt to use the two spheres in open air with a tube between them failed owing to spark over. Using a single generator inside a pressure chamber with a spark-inhibiting gas (Freon) Westinghouse Research Laboratories under E. U. Condon achieved 4×10^6 volts on a Van de Graaff. These generators were used during the World War II to give 2.5×10^6 - volt x-rays for radiography. They will have some use as initial accelerators for some of the new high-energy accelerators of the future. They are the most successful of the high-voltage generators to date.

67. QUANTITATIVE TREATMENT: COULOMB'S LAW

The qualitative facts of static electricity having been discussed, it now becomes necessary to take up the quantitative facts. As the phenomenon of electrification is characterized by attractions and repulsions electrification is regarded as a manifestation of the action of forces. *These forces differ from forces such as gravitation in that they are produced on bodies by different means. Forces of the type above described, that are produced by frictional processes, by processes going on in chemical solutions, or by the action of magnetic fields on moving conductors, are defined as electrical forces.* The only way in which electrification can be measured is in terms of such forces. It is therefore essential that the law of force acting between electrical charges be known and that this law be used to define the unit of electrification.

Coulomb (1785), using the torsion balance, was the one who formulated the laws of electrostatic attraction and repulsion. Cavendish had also deduced the law at an earlier date (1762), but as he did not publish his results, the credit has been given to Coulomb. Coulomb found that the force was proportional to the product of the charges on the two bodies and that it was inversely proportional to the square of the distance. It was later observed that the force was inversely proportional to a constant of the medium in which the charges were immersed. This constant, designated by the letter D , is called the *dielectric constant*. More will be said of this in a later chapter. Symbolically this law is written

$$f = \frac{qq'}{Dr^2}.$$

In this equation q and q' are the two charges and r is the distance between them, while f is the force.

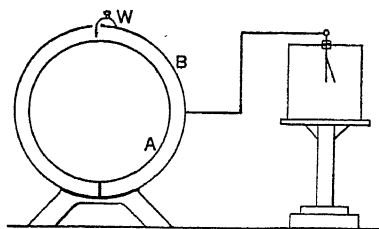


FIG. 77. Modified ice-pail experiment.

If now B be charged and touched to A no decrease in the charge on B will be noted and no charge will appear on A . This result is closely similar to the one obtained in the ice-pail experiment (when the sphere produced no change in charge on being touched to the walls) but is more likely to be accurately verifiable than the cruder demonstration experiment.

The experiment must be interpreted as meaning that there is *no electrical force on electrical charges between the outer and inner spheres*, that is, a charge on the outer sphere exerts no resultant force on the inner sphere or on charges between the inner and outer sphere, as no electricity flows between A and B in either sense when connected together. This result is perfectly general for all spheres A and B so that it may be generalized to include any point within B .

The situation can now be idealized by taking a point P at any place between A and B , as in Fig. 78, and considering it alone inside the sphere B . It is known that there was no resultant force on any charge at P due to the charge on B , as there was no change in the state of B on connecting B and any point P electrically. Draw through P a plane indicated in cross section by the line COD , which passes through O , the center of B . Then pass a plane EPF perpendicular to COD

through P . Two cones may then be passed through P in opposite senses and of equal solid angle $d\omega$. These cones are represented in section by the two elongated triangles Px_1ds_1 and $Pxds$ in intersecting the spherical surface in the small surface elements represented in section by the lines ds_1 and ds . It is seen by joining O , the center of B , to the two ends of one of the sides of the cone that the intercepted

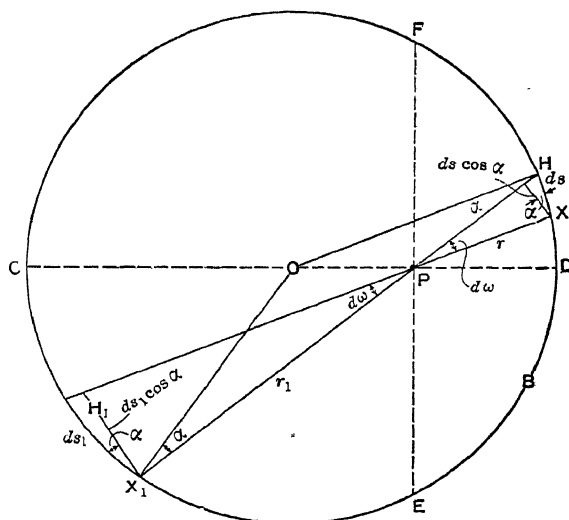


FIG. 78. Proof of the inverse square law from the ice-pail experiment.

surface elements ds and ds_1 make angles $\alpha = \alpha_1$ with the perpendicular lines H and H_1 , representing the right sections of the two cones. The areas of the right sections represented by H_1 and H are from geometry $r_1^2 d\omega$ and $r^2 d\omega$, where $d\omega$ is the solid angle of the cone, and r_1 and r are the distances of P from the right sections H_1 and H at one point of each. It then follows at once that $ds = \frac{r^2 d\omega}{\cos \alpha}$ and $ds_1 = \frac{r_1^2 d\omega}{\cos \alpha}$.

Now if the charge had been uniformly distributed over the sphere B as has been assumed then let the surface density (number of charges per square centimeter) be σ charges per square centimeter, and the charges on ds_1 and ds will be σds_1 and σds , or $q_1 = \frac{r_1^2 \sigma d\omega}{\cos \alpha}$ and $q = \frac{r^2 \sigma d\omega}{\cos \alpha}$. Now if a charge q_2 is placed at P it would be attracted, if

Coulomb's law is correct, by a force $f_1 = \frac{q_1 q_2}{r_1^2}$ toward ds_1 and by a

force $f = \frac{qq_2}{r^2}$ by ds , or in one sense by a force $f_1 = \frac{q_2 r_1^2 \sigma d\omega}{r_1^2 \cos \alpha}$ and in the other sense by a force $f = \frac{q_2 r^2 \sigma d\omega}{r_2 \cos \alpha}$. It is obvious that these two forces are on this assumption independent of the values of r_1 and r , and are equal. Therefore, the resultant force, on the Coulomb law which says that f is proportional to $\frac{1}{r^2}$, is 0 on a charge at P , a fact which is borne out by observation. Since the cones were chosen completely at random, as was the point P , this reasoning holds equally well for any other elements ds and ds_1 chosen lying on two sides of the line EPF and for any point within the sphere B . The conclusion which is drawn, therefore, is that if Coulomb's law holds there will be no force on a quantity q placed anywhere within sphere B acting to move it toward or away from O the center of B . Thus a charge on a wire between the spheres A and B would move neither toward A nor B as was observed to be the case.

Had, however, the law of force been $f = \frac{q_2 q_1}{r^n}$, where n could have a value different from 2, the reasoning would have led to the conclusion that

$$f_1 = \frac{q_2 r_1^2 \sigma d\omega}{r_1^n \cos \alpha} \text{ and } f = \frac{q_2 r^2 \sigma d\omega}{r^n \cos \alpha}, \text{ or } f_1 = \frac{q_2 \sigma d\omega}{\cos \alpha} \left(\frac{1}{r_1^{n-2}} \right) \text{ and } f = \frac{q_2 \sigma d\omega}{\cos \alpha} \left(\frac{1}{r^{n-2}} \right).$$

If then n were greater than 2, f_1 would have been less than f since r_1^{n-2} would have been greater than r^{n-2} , as r_1 is greater than r for all cones through P for all points P . Hence if n is greater than 2 the positive charge on A of Fig. 77 would move toward B , or there would have been a resultant force away from the center O . If n is less than 2, r_1^{n-2} and r^{n-2} have negative exponents and f_1 will be greater than f . Thus in this case the positive charge would have moved from B to A , i.e., toward the center O . As a result of measurements which are very sensitive to small changes in potential for A while the value of σ on B is very great it can be shown that n has the value 2 to better than one part in many millions.

The essential notion in this proof is simple for if conditions are chosen for test which are simple, i.e., concentric spheres for which calculation is very simple, then as the areas of the surfaces intercepted on the sphere by the cones drawn through any point P and hence the charges on these surfaces are in direct proportion to the squares of the distances from the point P , there must be a resultant force on P toward or away from the center of the sphere unless the law of force between the surface charges and the point varies inversely as the square of the distance. *This reasoning shows that the only law of force*

compatible with the fact that a charge inside of a conducting body exerts no force is the inverse square law of force.

The original experimental proof of the extent to which this law holds was the work of Bertrand, Maxwell, and Faraday. Faraday experimentally went inside a conductor charged to potentials of hundreds of thousands of volts with the most sensitive electrometer available but no field inside the conductor was discovered. The above-mentioned experiment establishes the inverse square law very accurately for fields of considerable magnitude. The range of distance between bodies over which the inverse square law has been observed to hold is enormous. It holds as far as we can tell directly for distances of several meters. The experiments of Rutherford on the deflection of rapidly moving positively charged atoms of helium by the positive nuclei of the elements has proved that this law holds true within 1 per cent down to distances of 10^{-12} cm. Beyond this, the law ceases to hold. These dimensions are therefore supposed to indicate the order of magnitude of the dimensions of the ultimate particles of matter, namely, protons, neutrons, and electrons.

Definition of Quantity in the Electrostatic System. Returning now to the expression for the force given above, viz., $f = qq'/Dr^2$. If D is set equal to unity* and is made q equal to q' , a result easily achieved, for instance, by touching two charged conducting spheres of equal size together, then one may write,

$$q = \sqrt{fr^2}.$$

Actually D is unity for empty space only. In air, however, it is so near to 1 (1.0006) that it can be called unity. It is then possible to write that q equals 1 electrostatic unit of electricity when r equals 1 cm and f equals 1 dyne. *This gives an arbitrary definition of electrostatic unit of quantity in terms of the force produced at the unit distance. This is recognized by regarding the value of the charge on the true unit of electricity of which the value is 4.8×10^{-10} of the electrostatic unit.*

In words, the unit quantity of electricity on the electrostatic system can be defined as *the quantity of electricity which repels an exactly equal quantity of electricity at a distance of 1 cm with the force of 1 dyne (in vacuo).*

It is seen that this unit is *small*, for a dyne is a weak force. The electrostatic unit of quantity might then be expected to be less than the electromagnetic unit of quantity, and, in fact, 1 electromagnetic unit of quantity = 3×10^{10} electrostatic units of quantity, and 1 coulomb equals 3×10^9 electrostatic units of quantity.

It should be pointed out that here again for convenience only a purely arbitrary unit of charge in the c.g.s. system has been established. When the electron, the natural unit of charge, was discovered

* The question of the dimensions of D is treated on pages 34 and 417.

it was found to be a very much smaller quantity; in fact, the charge on the electron is 4.803×10^{-10} of the absolute e.s.u.

68. DEFINITION OF CURRENT IN THE ELECTROSTATIC SYSTEM

The quantity of electricity in terms of the electrostatic system of units has now been defined. This at once makes it possible to use this concept to define current in terms of the electrostatic system, that is, to define current in terms of a new set of phenomena. For since current is the quantity of electricity passing a point per unit time, it is merely necessary to write that $i = \frac{q}{t}$, where i is the current, q is the

quantity, and t is the time.* Then $i = 1$ electrostatic unit of current if $q = 1$ electrostatic unit of quantity and $t = 1$ second. Thus current on the electrostatic system is defined as *the quantity of electricity transported past a given point divided by the time taken to transport it*. Since 3×10^{10} e.s.u. of quantity are equal to 1 e.m.u. of quantity, therefore 1 e.m.u. of current equals 3×10^{10} e.s.u. of current, and 1 ampere is 3×10^9 e.s.u. of current.†

It may be noted in passing that the *e.m.u. of current* was *directly defined* and measured by its magnetic effect. The *e.m.u. of quantity* was thus defined as $q_{e.m.u.} = i_a t$. In the *electrostatic system* quantity was *defined directly* by its electrostatic repulsive effect. Current then is given by the relation $i_{e.s.u.} = q_{e.s.u.}/t$.

The small magnitude of the statically generated currents in earlier days made the direct detection of their magnetic effects impossible. Arago, however, early discharged a statically charged Leyden jar through a small solenoid containing a steel needle and magnetized the needle. In this case, while q was small, t was still smaller for the discharge and the currents active were of the order of magnitude of amperes. With the development of the moving-coil galvanometer, the currents produced by laboratory static machines can be easily measured today.

Since the e.s.u. of current is so small the currents usually dealt with represent large numbers of e.s.u. and the unit is not ordinarily used. When, however, currents due to gaseous ionization or occurring through insulators are measured currents of the order of 10^{-15} ampere or 3×10^{-6} e.s.u. per second and less are encountered. Such currents are of the order of 6000 electrons per second. Since they are measured by static means, i.e., by the rate of charge or discharge of small capacities, e.s.u. are frequently used.

* It is to be noted that $i = q/t$ for a constant current. If i changes with t , $i = \frac{dq}{dt}$ must be written.

† A true unit current would be one of a single electron per second. This amounts to 4.8×10^{-10} e.s.u. per second or to 1.6×10^{-19} ampere.

69. DEFINITION OF ELECTRIC FIELD STRENGTH

It is obvious that, with a charged body, there is at any point in space surrounding it a force manifested on another charged body placed at that point. Thus it can be said that the charged body has a field of force about it. Again, in electricity as in magnetism, the intensity of this field of force at a point is represented by *the force in dynes which would be exerted on a unit positive charge * placed at that point in the field*. Thus electrical field strength X at a point is defined as the force per unit positive charge placed at that point, i.e., electrical field strength X is $X = \frac{f}{q}$, where f is the force in dynes and q is

the charge * in e.s.u. The unit of field strength is then dynes per unit quantity. Such a force has magnitude and direction. It is therefore *a vector quantity*. Consequently the force at any point in the field caused by a number of charges can be determined by adding *vectorially* the separate forces produced at that point due to the separate charges.

For the field due to an isolated positive charge q , at a point P , r , centimeters distant, Coulomb's law at once gives the magnitude of the field. The field will be directed away from q along the line joining q and the point P . The force will be given by $f = \frac{qq'}{r^2}$ on a charge

q' at P . The field (i.e., the force per unit charge) $X = \frac{f}{q'} = \frac{q}{r^2}$.

If a number of charges are used their fields add vectorially and the procedure is similar to that given in Chapter III on magnetism. For the continuous distribution of charges resident on metal surfaces the calculations can become quite involved.

Again as in magnetism an electrical field can be conventionally represented by hypothetical lines of force joining the positive and the negative charges, the direction of the lines being that along which an isolated positively charged point would move. These lines are pictured as acting like stretched elastic rubber bands that repel each other.

By convention unit electrical field is represented as having 1 line of force crossing an area of 1 cm^2 taken perpendicular to the lines. Thus a field of 4 units would have 4 lines per square centimeter. If it were represented by a drawing in a plane this drawing would have $\sqrt{4} = 2$ lines per linear centimeter, which means a distance of $\frac{1}{\sqrt{4}} = \frac{1}{2}$ cm between lines.

Since the lines of force are assumed *perpendicular* to the plane representing the area, it is to be noticed that there is *no component of electrical force along this plane*, the force everywhere being perpendicular.

* A quantity of static electricity is often referred to as a charge.

Since at $r = 1$ cm from an isolated charge q the field is $X = \frac{f}{q'} = \frac{q}{r^2} = q$ units, the field X will be represented by q lines of force per square centimeter emerging normal to the surface of this *unit* sphere. Thus there will be a total of $4\pi q$ lines of force emerging from the sphere for the q charges at its center.* Hence *each charge emits* 4π lines of force. This result will be of immense value to the student later on, for it is possible in many cases to know the number of charges σ on unit area of surfaces. If all the lines of force emerge normal to the surface the field at the surface at once is given by $X = 4\pi\sigma$, where σ represents the number of charges per square centimeter, i.e., the surface density of charge. This relieves one of the tedious necessity of computing X by the vectorial addition of the fields produced by the different charges scattered over a surface.

* This is in exact analogy to the case of magnetic poles if q replaces m ; see Fig. 20, page 49.

CHAPTER XIV

STATIC ELECTRICITY II: POTENTIAL

70. THE CONCEPT OF POTENTIAL DIFFERENCE IN THE ELECTROSTATIC SYSTEM

In the last chapter it was shown that at any point in the neighborhood of an isolated charge q there is a force on another charge q' . Accordingly, about a positively charged body q at Q there is a field of electrical force surrounding the charge q . Suppose a quantity of electricity q' is at a point A in the field of q and assume that q' is positive, as shown in Fig. 79. Let us move $+q'$ from A to another point B nearer Q . Since the force is one of repulsion, work must be done on the charge $+q'$ to move it. Now, since the strength of the field at a point distant r from a charge q is

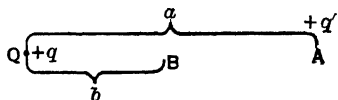


FIG. 79.

$$X_r = \frac{q}{r^2},$$

the work done in moving a quantity q' a distance dr from A toward B is given by

$$dw = \text{force} \times \text{distance} = -Xq' dr = -\frac{qq'}{r^2} dr.$$

Thus the total work for all elements of path dr in going from A to B is given by

$$W_{AB}q' = \int_a^b dw = -q' \int_a^b X dr = -qq' \int_a^b \frac{dr}{r^2} = qq' \left(\frac{1}{b} - \frac{1}{a} \right).$$

If q' were equal to unity, this would be the work done to bring a unit positive charge from A to B , or the work done to move unit positive charge from one point to another in the field. But the *work to move unit charge from one point to another in a wire was called the potential difference between the two points when the electromagnetic system was being discussed*. Therefore, the potential difference in the electrostatic system between the points A and B , distant from a charge q by the distances a and b , respectively, is

$$\frac{W_{AB}q'}{q'} = \text{P.D.}_{AB} = q \left(\frac{1}{b} - \frac{1}{a} \right).$$

Accordingly it is seen that, owing to the existence of an electrostatic field of force about a charge q , there is at every point a force which will act to produce or require the production of work whenever a charge $+q'$ is moved in the field. By dividing the work to move q' from one point to another by q' the work required to move *unit* quantity is obtained. This work, it is seen, is due to the electrostatic field and is quite characteristic of it, being determined by q and the points in question. Thus the conditions at two points about q due to its static charge can be described in terms of the work to move unit quantity between them as well as by the force per unit quantity, or the field at the point. At times it will be found very convenient to use the properties of this new method of description.*

71. DEFINITION OF THE ELECTROSTATIC UNIT OF POTENTIAL DIFFERENCE

Thus, in analogy to the definition for potential difference causing current flow in the electromagnetic system, for convenience a new concept for describing *electrostatic conditions*, i.e., the concept of electrostatic potential difference, will be introduced. This concept is defined as *the work to move unit positive static quantity from one point to the other in the field*; i.e., electrostatic potential difference is defined as $P.D. = \frac{W}{q}$, where W is the work to move a static quantity of charge q .

The electrostatic *unit* of potential difference may at once be defined by saying $P.D. = 1$ absolute e.s.u., when $W = 1$ erg and $q = 1$ absolute e.s.u. of quantity. The potential is regarded as *positive when it takes work to bring up positive charge against the field*. Now the electrostatic unit of quantity is a small unit while the absolute electromagnetic unit is a large unit. Hence since $P.D._{es} = \frac{1 \text{ erg}}{q_{es}}$ and $P.D._{em} = \frac{1 \text{ erg}}{q_{em}}$, it is clear that the absolute e.s.u. of potential difference will be large relative to the e.m.u. of potential. Now 3×10^{10} e.s.u. of quantity = 1 e.m.u., so that 3×10^{10} e.m.u. of potential difference = 1 e.s.u. of potential difference. Since the volt = 10^8 e.m.u., the volt = $\frac{10^8}{3 \times 10^{10}}$ e.s.u. = $\frac{1}{300}$ e.s.u., i.e., 300 volts = 1 e.s.u.

72. FURTHER DISCUSSION OF THE CONCEPT OF POTENTIAL DIFFERENCE

Returning for the moment to the equation derived for the potential difference between two points A and B distant a and b centimeters

* The reason for this convenience lies in the fact that fields are vector quantities and require account being taken of the angles when they are added. Potential is a scalar quantity and potentials add algebraically. Since in many instances the potential gives all the information needed it is found convenient to use this scalar quantity.

from an isolated positive charge q ,

$$\text{P.D.}_{AB} = q \left(\frac{1}{b} - \frac{1}{a} \right),$$

it is seen at once that if $a = \infty$, $\text{P.D.}_B = \frac{q}{b}$. That is, the work to

move a positive unit charge from infinity up to B is merely $\frac{q}{b}$. This is called the *absolute potential* of the point B , or more simply the *potential* of the point B . Thus, to every point in space about q at any value of the distance b from it, there is a value of the potential $\frac{q}{b}$ characteristic of that point. Hence just as one speaks of a *field of force* about a charge, so one can speak of a *potential field* which is also characteristic of it.

It is further to be observed that the potential at each point has a value, but that potential is directionless. Hence unlike electrical force and field strength, which are *vector* quantities, electrical potential is a *scalar* quantity, i.e., it has magnitude but not direction. This has many advantages. For instance, if the potential at a point B due to a charge $+q$ is $+\frac{q}{b}$ and if at a distance c there is a charge

$+q_c$ giving a potential at B of $\frac{q_c}{c}$, the resultant potential at B due to

q and q_c will be $P = \frac{q}{b} + \frac{q_c}{c}$, irrespective of the directions of q and q_c relative to each other from B . Thus more generally the absolute potential P of a point in space is given by $P = \sum_i P_i$, where \sum_i is the sum of all i potential sources P_i giving rise to P . Since for many purposes potentials can be used for the solution of problems its scalar properties make it a most useful concept.

More generally, at any point in space the sum of the potentials due to surrounding charges gives a value to the absolute potential at that point which can be measured by the work done in bringing a unit $+$ charge up to the point from infinity. While this idealized concept of absolute potential is useful in considering problems and guiding our thinking, we cannot on the earth deal practically with *absolute* potentials. The reason is that we are bound to the earth, a point in the universe, of which the absolute potential is unknown as its net charge is unknown. Practical considerations then lead us to deal largely with potential differences. For convenience a reference potential of zero has been arbitrarily assigned to the earth as a whole (which means to the electrically conducting elements of the earth's surface). All measurements of potential are then made relative to the

earth, and one speaks mostly in terms of potential difference referred to the earth as 0.

Some further advantages of potential must now be taken up. One of these lies in the fact that between any two points A and B the potential difference is always the same irrespective of the path taken in going from one point to the other. To prove this, consider two points A and B for simplicity placed in a uniform field between two

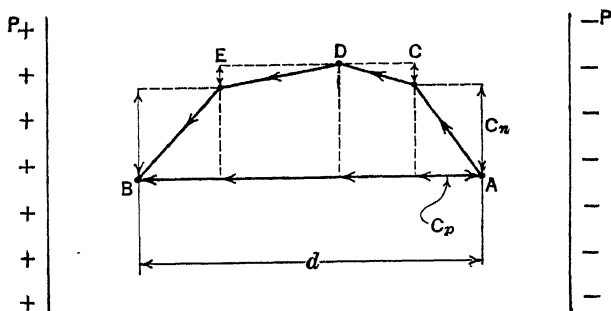


FIG. 80. Work to move a charge in an electrical field.

plates P_+ and P_- of Fig. 80. The field X goes from P_+ to P_- along AB and has no component of force perpendicular to AB . If d is the distance between A and B , and X is uniform and constant, the work to move a charge $+q$ from A to B is $W = fd = qXd$ and $P.D._{AB} = \frac{W}{q} = Xd$. Now suppose one goes from A to B via $ACDEB$. The path AC can be resolved into a path AC_N perpendicular to AB and AC_P parallel to AB . The work to move q from A to C_N perpendicular to the field is 0, for the force is zero along AC_N . The work to move q from A to C_P along AB is $XqAC_P$. In a similar fashion CD , DE , and EB can be broken up into components CD_N , DE_N , and EB_N normal to AB , and CD_P , DE_P , and EB_P parallel to AB . The normal components all require no work to be done, while the work along the components $AC_P + CD_P + DE_P + EB_P$ is $qX(AC_P + CD_P + DE_P + EB_P) = qXd$. This is the same as that obtained by going directly from A to B along the field. This proof may be generalized for a field oriented in other directions and except for adding mathematical complications will lead to the same result in the end, so that the statement may be considered as proved.

Another proof that this must be so follows from the law of conservation of energy. For if the work to move q along the path $ACDEB$ were greater than to move it along AB , q could be moved by some device from A to B along AB and then could be repelled back to A

along $BEDCA$. In doing this more work would be gained than was put in along AB . This would give work or energy out of nothing, a result which might seem desirable to achieve but one in which Nature does not appear to favor the human race.

Thus the important conclusion can be drawn that whenever a charge q is moved from a region where the potential is P_1 to one where it is P_2 , the work $(P_1 - P_2)q$ has been done on it or by it. This work will be $W = q(P_1 - P_2)$ and, if done on it, may appear as kinetic energy $\frac{1}{2}mv^2$ if it moved freely through space, or it may be stored up as chemical energy or lost as heat in overcoming resistance, etc.

Consider next the isolated charge $+q$ and regard the expression that the potential at a point B at a distance b from q is $P = \frac{+q}{b}$.

Now there are, besides the point B , an infinity of other points in the space about q of which the distance from q is b centimeters. These points define the surface of a sphere of radius b about q as center. At each of these the potential is also $\frac{q}{b} = P$. Hence this surface about q as

center is a surface having the *same potential* as that of B . Thus a surface in space of *constant potential* which is called an *equipotential surface* can be conceived of. Along such a surface the work to move a charge $+q'$ is 0. For since the work done to move a body between two points is dependent on the potential difference of the points, and since this is zero, the work is zero. This means that the *electrical field* creating the potential *has no component of force along an equipotential surface*. That is, the *electrical field must everywhere be normal to an equipotential surface*, and lines of force can always be *drawn normal to such a surface*. Had another radius a about q been chosen, a new equipotential surface would have been found of which the potential was $+\frac{q}{a}$. This would also be a sphere about q as a center, and the

potential difference between the surfaces is $\frac{q}{a} - \frac{q}{b}$. We can thus draw about a charge an infinitude of equipotential surfaces, each one characterized by its radius, and choose such as suit our purpose.

In the more general example of any field, it must be clear that it will usually be possible to find a series of points defining a surface over which the potential is constant and of a particular value such that there is no force parallel to the surface, while the lines of force are everywhere perpendicular to it. In particular, consider the uniform field between the plates A and B of Fig. 81. At the point B the work to move a $+$ charge q from the plate B to A is given by $W = q\overline{XAB}$, and the potential of A , if B is grounded, is by convention $\frac{W_A}{q} = P = \overline{ABX}$. Now since the field is strictly normal to the plate B plane

through C parallel to B will everywhere be \overline{BC} centimeters from B , and hence $P = X\overline{BC}$ will be constant. Thus the surface equipotential to C is the plane through C normal to the field X and parallel to B . Such planes can be drawn through any points of given potential such as $+2$, $+4$, $+6$, $+8$, etc. The equipotential surfaces for two isolated charges $+q$ are shown in Fig. 82, and those for an isolated charge $-q$ and one $+q$ is shown in Fig. 83. Note the broken lines of force normal to the equipotentials in the upper left-hand quadrants.

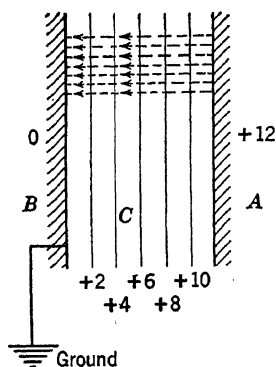


FIG. 81. Equipotential surfaces for parallel plates separated by 12 cm with $+12$ potential on A and plate B grounded. Broken lines of force perpendicular to equipotentials. Note parallelism of equipotentials and uniformity of field.

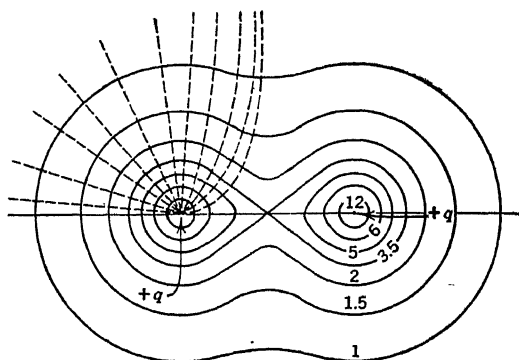


FIG. 82. Equipotential surfaces for two positive quantities ($q = 12$), separated by 14 cm. Solid curves are the equipotentials. Numbers give the values. Broken curves in the upper left quadrant are lines of force perpendicular to the equipotentials.

Now it happens that the concept of the equipotential surface is a most useful one for the following reason. All conductors have free charges of one form or another on them. If we place a charge on one point of a conductor there is an accumulation of electricity at that point and the potential differences due to the accumulations cause the charges to flow all over the surface. In due time (which may be very short) it will be observed that the currents due to the potential differences have ceased and no more electrification moves. This means that all the charges have so arranged themselves over the conductor surface that *there is no longer any potential difference between points on the surface to cause a flow and there are therefore no electrical forces parallel to the surface*. Thus the conducting surface has become in effect an equipotential surface. It does so the more rapidly the lower the resistance. Thus any metallic surface which has no continuous current flowing along it is to all intents and purposes an equipotential surface very shortly after the charges are placed on it. Thus it is

known that the electrical field of any statically charged conductor is always normal to the surface (i.e., the lines of force emerge normal to the surface) and that its potential is uniform. This fact is of great practical importance as will be seen from certain applications.

For many experiments it is awkward to deal with point charges q , although they are easy to obtain. With such charges the fields and potentials always vary with the distance r . It is desirable to create uniform fields in which the field X is constant over the length of the field, in which the lines of force are straight and of a known direction, and in which the equipotential surfaces are plane and parallel. Such uniform fields can be achieved by the *parallel-plate condenser*.

Suppose that there are two conducting metal plates of linear dimensions very large compared to the distance d between them. Let the area be S . Let a $+$ charge q be placed on one of the plates B and connect A to ground. The metal being a conductor, the charges will spread all over B . When they come to rest the following situation will exist. The $+$ charge on B will have called forth an equal $-$ charge from the ground to A opposite it in an attempt to neutralize itself. Again at any distance from the edges the uniformity of conditions over the surface will be such that σ , the number of charges per unit area of surface, will everywhere be the same. It is then possible, except near the edges, to set the *charge density* $\sigma = \frac{q}{S}$ and assume that it is uniform.

Now each unit charge has by convention 4π lines of force emerging from it. Again Faraday's ice-pail experiment says that no force may exist inside a conducting surface of the metal, and since the surface is equipotential all the $4\pi\sigma$ lines of force in a square centimeter of plate surface emerge perpendicular to the plate B . Hence there are $4\pi\sigma$ lines of force leaving normal to B per square centimeter. Opposite the $+\sigma$ charges per square centimeter there are the negative charges, $-\sigma$ per square centimeter, drawn by induction from the ground. These have each $4\pi\sigma$ lines of force per square centimeter entering them from the $+$ charges on plate B . Since they try to neutralize the $+$ charges

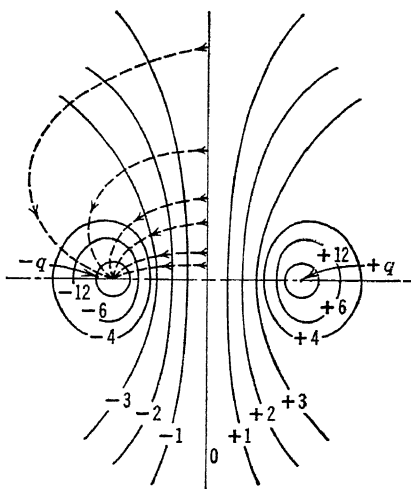


FIG. 83. Equipotential surfaces for two quantities ($q = +12$, $q = -12$), separated by 12 cm. Solid curves are the equipotentials. Numbers give the values. Broken curves in the upper left quadrant are lines of force perpendicular to the equipotentials.

on B , they are also symmetrically disposed over plate A . Thus in the space between A and B the lines of force are uniform in distribution and $4\pi\sigma$ in number. As there is no reason for departing from their direction of emerging normal to the plates A and B , they run across the intervening space d in that direction. Thus the field X away from the edges is everywhere constant, uniform, and parallel and is $X = 4\pi\sigma$ units in strength.

At the edges the lines of force bulge outward in the middle owing to their self-repulsion with a configuration about that shown in the diagram of Fig. 84. To avoid edge effects a condenser such as the one pictured is protected by a guard ring. That is, if measurements are to be made involving plate B while A remains continuous and larger than

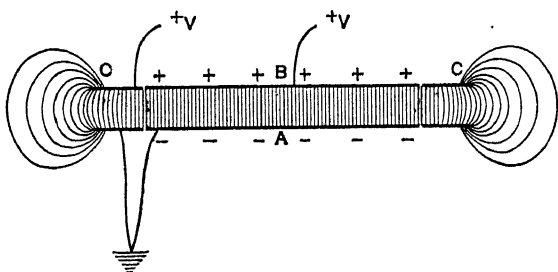


FIG. 84. Production of a uniform field in a parallel-plate condenser with a guard ring.

B , an annular plate C is placed in the same plane and concentric with B and separated from it by the smallest possible clearance. C is then maintained at the same potential as B . Under these conditions the disturbing effect of the small gap on the field between B and C is negligible. It can be seen that such an arrangement giving a uniform field will have many important practical uses.

Again one may regard the equation for the potential of an isolated charge q . It was seen that in its deduction

$$\text{P.D.}_{AB} = \frac{W}{q} = \int_a^b \frac{q}{r^2} dr = \int_a^b X dr,$$

where X was the field strength at some point r due to q . There is no reason, however, why this procedure and the relations which it yields need be limited to the field X produced by a point charge q . Hence in general it may be written that $\text{P.D.} = \int X dr$, and per contra that

$\frac{d(\text{P.D.})}{dr} = X$. Thus the potential difference P.D. is the integral of the field strength X along the field direction over the distance r between the points; and vice versa, the field strength along a given direction

is the *rate of change of potential along* that direction. This is an exceedingly useful relation, for it happens that it is often simpler to calculate the potential at a point and from this to derive the field strength than it is to calculate the field strength directly.

This relation is particularly valuable in dealing with a uniform field and has already been utilized in discussion without stating that it was being used. For if X is independent of r , i.e., if X is constant from one point to another, as in the condenser discussed above, then

$$\text{P.D.} = \int_A^B X dr = X \int_A^B dr = X d;$$

and vice versa, if P.D. and d are given, $X = \frac{\text{P.D.}}{d}$. This has led to the

practical unit for describing the field strength X . For X will be a practical unit if P.D. is 1 volt and d is 1 cm. Thus a field strength of so many volts per centimeter is spoken of. The absolute e.s.u. of field strength is then e.s.u. per centimeter. It is seen that this also makes it possible to relate charge density to the potential in this

condenser. For $X = 4 \pi \sigma = \frac{\text{P.D.}}{d}$. If P.D. is in e.s.u., $\sigma = \frac{\text{P.D.}}{4 \pi d}$

e.s.u. per square centimeter.

73. CHARGE DENSITY AND ITS APPLICATIONS

The concept of density of charge on a conducting surface which has been introduced has some interesting applications. Surface density of electrical charge has been defined as the number of charges per square centimeter of surface. Assume an isolated sphere of metal of radius r with q charges put on it. The surface of the sphere is $4 \pi r^2 \text{ cm}^2$, and

the charge density is $\sigma = \frac{q}{4 \pi r^2}$. Now this sphere will have $4 \pi \sigma$ lines

emerging per square centimeter normal to the surface, and since it is isolated these lines of force continue outward radially indefinitely. There are thus $4 \pi r^2 \times 4 \pi \sigma$ total lines of force emerging radially from the surface of this conducting sphere *as if they came from its center*.

Since now $\sigma = \frac{q}{4 \pi r^2}$, then $4 \pi r^2 \times 4 \pi \sigma = \frac{4 \pi r^2 \times 4 \pi q}{4 \pi r^2} = 4 \pi q$ lines of

force emerge from the sphere. But $4 \pi q$ is just the number of lines of force that would have emerged from the charge q had it been a *point charge at the center of the sphere*. Now to another charge q' outside the *metal* sphere the only important factor is the number of lines of force per square centimeter encountered and their direction. There is no

difference in direction or number between the lines of force emerging from a conducting sphere of radius r with a charge q and those emerging from a point charge q placed at the center of the sphere. Thus in practical application a charged conducting sphere can be substituted for the point charge, provided it is not so near other charges that it is polarized. If this occurs the effect can in any case be calculated and corrected for. Rough measurements of charges are thus greatly facilitated in comparison with magnetic measurements.

These considerations may next be carried over to any conducting body whatever its shape. Any curved surface can at a point be approximated to by a spherical surface circumscribing and tangent to it at the point. Now the surface of a charged conductor is an equipotential surface of potential V . If such a circumscribing conducting spherical surface of radius r be imagined at the same potential, it is

at once known that its potential must be $V = \frac{q}{r}$. Hence it is possible

to derive the value of the quantity of charge q which would have to be placed on the charged sphere to give it the potential of the surface V as $q = Vr$. But the charge density on such a spherical surface is

$\frac{q}{4\pi r^2} = \sigma$ whence $q = 4\pi r^2\sigma$. Accordingly, $4\pi r^2\sigma = Vr$ and $\sigma = \frac{V}{4\pi r}$.

Now since the surface of the circumscribing sphere has a charge density

$\sigma = \frac{V}{4\pi r}$, it is clear that this must also be the charge density of the surface at that point.

Thus, if on an irregularly shaped conducting surface a charge q is placed giving a potential V over the surface, the electrical charge density σ will vary over the surface

in direct proportion to $\frac{1}{r}$, the reciprocal of the radius of curvature.

It will then be expected that the charge density at sharp points on conductors is exceedingly high, for r is small. This can be illustrated experimentally by touching a "proof plane" to two points on the egg-shaped charged metal body shown in Fig. 85. The proof plane is an insulated metal plane that must be small, and its area of contact with the surface of large radius of curvature should be the same as with the surface having the smaller radius of curvature. On charging the body and touching the proof plane to the two portions of different curvature in succession and carrying the charge acquired to an electroscope, the more sharply curved surface will give the greater charge to the electroscope. The charge given is

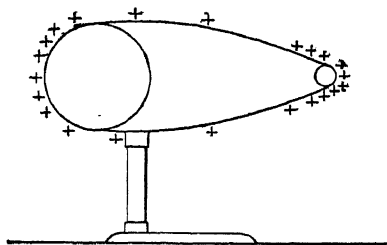


FIG. 85. Surface distribution of charge on an irregular conducting surface.

not proportional to σ in the two cases, as the areas of contact cannot be made equal.

In extreme cases such as needle points, the values of $\sigma = \frac{V}{4\pi r}$ and the field strength $X = 4\pi\sigma = \frac{V}{r}$ become so large that the few electrons produced by radioactivity and cosmic rays in the air about the point begin to ionize the air by impact. That is, the electrons of charge e

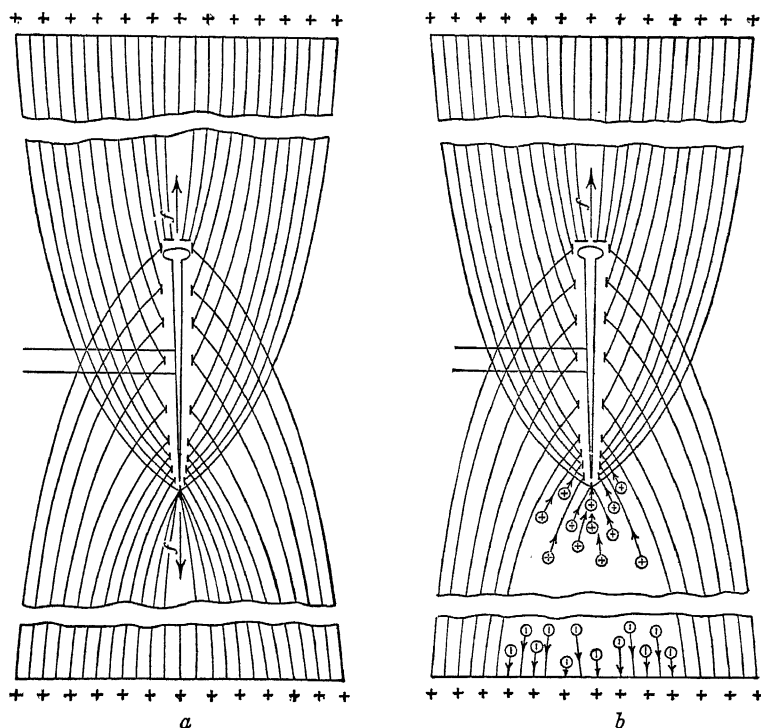


FIG. 86. Field and force distribution about a body with ends of different curvatures, a , in the absence of ionization by collision, b , with ionization by collision at the sharp end.

gain so much energy in the electrical field X because of the high value of Xe that they tear more electrons out of the neutral gas molecules and make the air conducting. If then a needle point in this condition be mounted with its other end in a blunt conductor the needle will *move away* from the sharp end in the direction of its axis. This is the basis of the action of the electrical pinwheel.

The character of the mechanism is quite clearly described as follows. In Fig. 86a the needle is at rest with a high charge but no ionization and the integrated pull of the walls of the room in opposite direction

at the two ends of the needle is balanced. When, as in Fig. 86*b*, the air becomes conducting at the point, the negative ions are repelled from the point. The $+$ ions are attracted to the point and form a cloud about the point. These $+$ charges in the air effectively neutralize the charge on the point. The $-$ ions driven to the wall neutralize the $+$ charge on the wall. Thus the pull of the wall on the blunt end of the needle remaining the same, the sharp point now being pulled only by the yielding $+$ air ions, the needle suffers a resultant force toward the blunt end. Hence the pin-wheel shown in Fig. 87 will rotate in the sense away from the sharp points as indicated, when V becomes sufficiently high and r sufficiently small. Since the $-$ ions moving toward the wall are striking gas molecules, they give their momentum acquired from the field to the neutral gas molecules. There is thus produced

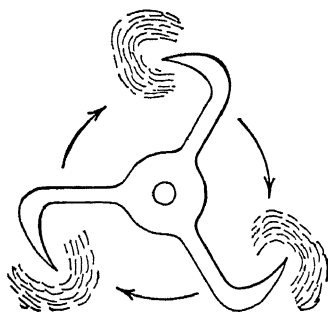


FIG. 87. The electrical pin-wheel or electrical whirl.

an air current (the reaction to the backward pull on the needle) which may attain considerable velocity. This is known as the *electrical wind*, which can be illustrated by its action on a small vane or a candle flame.

74. THE ABSOLUTE ELECTROMETER

The definition of difference of potential given on page 68 makes it possible to measure the potential in absolute value in terms of electrostatic units of potential. If the ratio of the electrostatic unit and the electromagnetic unit of potential is known, it is possible by its means to evaluate potential in e.m.u., directly in absolute measure. The instrument used for the purpose is called the *absolute* or *attracted disk electrometer*. In Fig. 88, two parallel circular plates are seen. The upper one has been cut so that it is formed of two parts, an inner disk and an outer annular disk (guard ring) with a very minute air gap G separating the two. The inner circular disk of the upper system is fastened to the arm of a very sensitive beam balance B . If, now, the balance, the upper plate and annular ring is grounded and attached to one side of the source of potential while the other insulated plate K is attached to the other side of the source, a force of attraction will exist between the two plates. It will be necessary, therefore, to change the weights on the balance pan p to counteract the attractive force and maintain A in its initial position.

Call A the surface of the central upper plate; call d the distance between this plate and the lower one. If, now, σ is the charge density, that is, the number of units of charge per square centimeter of the plate, the total charge q on the plate of area A is $q = A\sigma$. The total

number of lines of force emerging is equal to $4 \pi q = 4 \pi A \sigma$. Between such plates with an annular disk, which is called a *guard ring*, the field is uniform. The value of the field strength of such a field is the number of lines per square centimeter, or $X = 4 \pi \sigma$. Now the force per square centimeter on a surface with σ charges per square centimeter (or a charge density σ), in a field X is given by the expression

$$f_1 = \frac{X\sigma}{2}.$$

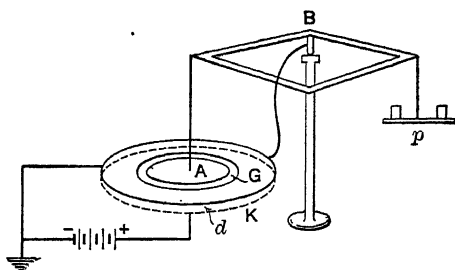


FIG. 88. The absolute electrometer.

That this must be so follows from the fact that inside the charged conductor the field is 0, i.e., a charge σ would experience no force inside the surface. On the other hand, just *outside* of the surface the field X equals $4 \pi \sigma$, and a charge σ experiences a force $X\sigma$ or $4 \pi \sigma^2$. A charge σ *exactly on the surface* must have a force intermediate between 0 and $(4 \pi \sigma)\sigma$ acting on it as a result of a field $X = 4 \pi \sigma$, which is $(2 \pi \sigma)\sigma$ or $\frac{X\sigma}{2}$.

Therefore

$$f_1 = 2 \pi \sigma^2 = \frac{X^2}{8 \pi}.$$

Again the potential difference between the plates, P.D._{AK}, is

$$\text{P.D.}_{AK} = Xd,$$

where d is the distance between the plates.

Therefore

$$X = \frac{\text{P.D.}_{AK}}{d}.$$

This at once gives us the total force f_A on plate A as

$$f_A = f_1 A = \frac{X^2}{8 \pi} A.$$

Accordingly

$$f_A = \frac{(\text{P.D.}_{AK})^2}{d^2} \frac{A}{8 \pi}$$

or

$$\text{P.D.}_{AK} = d \sqrt{\frac{f_A 8 \pi}{A}}.$$

Since f_A , the electrical force, can be measured by the weights m added to balance (for $f_A = mg$, where g is the gravitational acceleration), and since A and d are known accurately, P.D._{AK}, the potential difference, is given accurately in absolute *electrostatic units*. This method is absolute in the electrostatic system. It can be made accurate to 0.1 per cent or better. It is limited to a relatively small range of potentials, viz., from 150 volts to 10,000 volts. It is not sensitive enough below 150 volts and is subject to electrostatic breakdown at fields much above 10^4 volts per centimeter between plates. Its use requires many precautions.

75. GOLD-LEAF ELECTROSCOPE

Frequently, for experimental work, a more convenient instrument known as the gold-leaf electroscope is used for measuring the potential difference. Instruments fitted with a pivoted aluminum needle and a vertical support, similar in form to the gold-leaf electroscope but of sturdier construction, are also often used for rough potential measurements. They are known as the Braun electrostatic voltmeters. Such instruments must be *calibrated* in terms of the absolute electrometer or other potential-measuring devices.

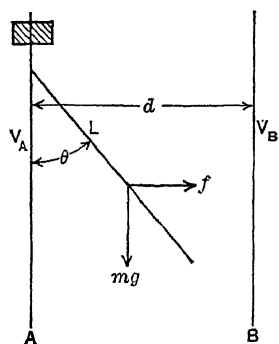


FIG. 89. Principle of the gold-leaf electroscope.

In the gold-leaf electroscope, a gold leaf L , Fig. 89, is fastened to one plate A of a parallel-plate condenser system. The plate A to which it is attached is insulated and connected to the source of potential to be measured. The other plate B is grounded or attached to the other side of the circuit. There is then a field X between A and B which, if it were not distorted by the gold leaf, would be given by $\frac{\text{P.D.}_{AB}}{d}$, where d is the distance between A and B . The charges q' on the leaf act in X to produce a force $f = Xq'$, driving the leaf from A to B . This is counteracted by the force of gravity mg acting downward to cause it to move towards A . The leaf will then take on an equilibrium position, making an angle θ with A roughly given by $\frac{f}{mg} = \frac{Xq'}{mg} = \frac{\text{P.D.}_{AB}q'}{dmg} = \tan \theta$. Thus $\text{P.D.}_{AB} = \frac{mgd}{q'} \tan \theta$. Again, roughly, if the system has a capacity C that is nearly constant with deflection, it will be seen in Chapter XV that $q' = C \text{P.D.}_{AB}$. Hence $\text{P.D.}_{AB} = \frac{mgd}{C \text{P.D.}_{AB}} \tan \theta$ so that $\text{P.D.}_{AB} = \sqrt{\frac{mgd}{C}} \tan \theta$. For a gold leaf of mass 0.005 gram with $C = 10$ cm and $d = 10$ cm, $\text{P.D.}_{AB} =$

$2.2\sqrt{\tan \theta}$ in e.s.u. or $P.D._{AB} = 660\sqrt{\tan \theta}$ volts. Thus a deflection 45° would signify roughly 660 volts. Actually the distortion of the field by the needle and the variation of C with θ make calibration essential.

76. THE QUADRANT ELECTROMETER

A third important instrument for measuring very small differences of potential is the quadrant electrometer. In Fig. 90, the object $ABCD$ is a cylindrical brass box with top and bottom, which has been sawed into four equal quadrants, A , B , C , and D , so that each quadrant is separated from its neighbor by a small gap. A , B , C and D are each mounted on amber insulators EE and the opposite quadrants A and C ,

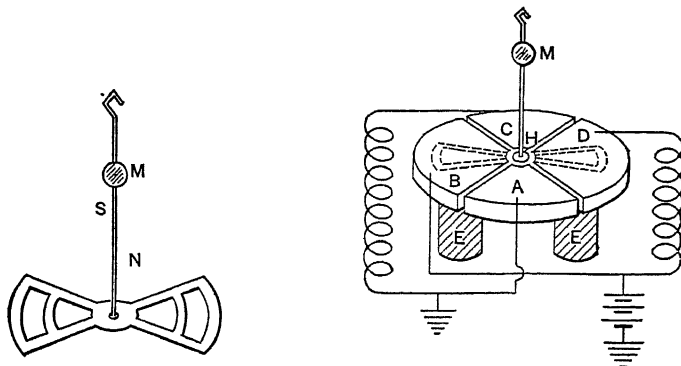


FIG. 90. The quadrant electrometer.

and B and D are connected together. A very light conducting needle, the shape of which is shown in the diagram as N , is suspended inside the cylindrical box with the upright support at its center passing through the hole H in the center of the quadrants. The needle is charged to 100 volts or so by the conducting suspension, generally a silvered quartz fiber or Wollaston wire, and insulated from the quadrants. The stem S carries a light mirror M by which the deflection of the needle can be read by telescope and scale. One pair of quadrants is grounded (i.e., at zero with reference to the needle); the other pair is connected to the source of potential to be measured. When a P.D. is applied between AC and BD , the needle is attracted or repelled by the quadrant pairs in a sense depending on the sense of the P.D. and the potential on the needle. Thus the needle suffers a torque and the suspension is twisted through an angle roughly proportional to the potential difference.

The whole instrument must be covered with a grounded screening case, and all leads to the sensitive quadrant must be screened with

grounded shields. Some electrometers will deflect a beam of light so as to displace the spot of light reflected from the mirror on a scale at 1 meter's distance by 10,000 mm for 1 volt P.D. and have a period of only 30 seconds. Capacities as low as 10 cm can be achieved in the newer instruments. The screening is necessary to avoid disturbances due to the induced static charges caused by charged objects in the room. The deflection is proportional to the potential under some conditions of operation, but the instrument needs calibration in terms of a known potential difference, which can be accomplished by the use of the potentiometer and a standard cell.

For alternating potentials the needle and one pair of quadrants can be joined to one terminal, the other terminal and the second pair of quadrants being grounded. This connection is called *idiostatic* and leads to deflections proportional to the square of the potential difference. It is a useful method for measuring low radio-frequency a-c potentials.

CHAPTER XV

STATIC ELECTRICITY III: CAPACITY

77. DEFINITIONS OF CAPACITY AND THE UNITS OF CAPACITY

It now becomes necessary in pursuit of the study of static electricity to consider the ability of electrical systems to hold or store electricity. Although the practical device for storing electricity, the Leyden jar, was discovered and used very early in the history of electricity (1746), the concept and definition of electrical capacity did not come until near the middle of the electrotechnical period, 1838-1840. The reason for the delay must be ascribed to the fact that capacity is one of the *derived* concepts dependent in its definition on the electrostatic concept of potential, a concept which arose through the work of Gauss in 1838. Again, though capacity is today one of the most important concepts in view of its many applications and uses in all fields of electrical endeavor, its importance was not manifest in the early days, so that its properties were not intensively investigated until Faraday's classic studies in 1837.

As has been stated, the faculty of the Leyden jar for storing electricity was early known, and some of the characteristics that increased its capacity for holding electricity must have been empirically known for some time. This knowledge, however, did not satisfactorily answer the question as to how much electricity a given object or condenser could hold or store, a question which would at once occur to any physicist or engineer of today.

In considering the capacity of a system for storing electricity, or more briefly *capacity*, as it will now be called, the same sort of difficulty presents itself which is encountered in attempting to define the gas-storing capacity of a steel gas cylinder or bomb. For apparently more and more charge can be piled on a body, by raising the electrical pressure, until the air or insulation breaks down. In close analogy, the amount of gas stored in a pressure bomb depends on the pressure under which it is forced into the bomb, which is limited only by the bursting strength of the bomb. Since the amount of gas stored in the bomb is directly proportional to the pressure, the capacity of the tank is defined as the quantity of gas Q (expressed in cubic feet or pounds) per atmosphere or per unit of pressure p . Hence the capacity of a gas container is $C = \frac{Q}{p}$. As stated, it is possible to crowd charges of electricity onto a condenser up to its breakdown strength as long as

the electrical pressure, i.e., the potential, is continuously increased. Accordingly, the quantity q of electricity stored depends on the electrical pressure or potential to which the condenser is charged and on the electrical capacity of the condenser.

Electrical capacity can then be defined just as capacity for the gas tank is defined, that is, as the quantity of electricity required to raise the potential by unity. Hence electrical capacity C is defined by the equation $C = \frac{q}{\text{P.D.}}$, where q is the quantity, and P.D. the potential

difference between the two sides of the condenser. It is at once clear that in the electrostatic system $C = 1$ e.s.u. if $q = 1$ e.s.u. of quantity and P.D. = 1 e.s.u. of potential difference.

Thus in the electrostatic system the unit of electrostatic capacity of a conductor is the capacity which requires 1 electrostatic unit of quantity to change the potential by 1 electrostatic unit of potential. The value of this unit must be found from dimensional reasoning.

It was found that the quantity in the electrostatic system is defined by

$$q = \sqrt{fr^2},$$

where f is the force between the two equal charges q , and r is the distance between them. Thus dimensionally

$$q = \sqrt{ML^3T^{-2}}, \text{ in the electrostatic system.}$$

Again potential

$$\text{P.D.} = \frac{\text{Work}}{q} = \frac{ML^2T^{-2}}{\sqrt{ML^3T^{-2}}}, \text{ in the electrostatic system.}$$

Therefore

$$C = \frac{q}{\text{P.D.}} = \frac{q^2}{W} = \frac{ML^3T^{-2}}{ML^2T^{-2}} = L, \text{ in the electrostatic system.}$$

Therefore, the electrostatic unit of capacity has the dimensions of a length. Since q and P.D. in absolute electrostatic units are in terms of the absolute c.g.s. system, the *c.g.s. electrostatic unit of capacity is the centimeter.*

That capacity has the dimensions of a length can be shown by considering the capacity of a sphere. The potential of a conducting sphere infinitely far away from any other charges was given as q/r (page 187), where r is the radius of the sphere. C therefore is

$$\frac{q}{q/r} = r, \text{ which is a length.}$$

Thus a sphere of 1 cm radius or 2 cm diameter in empty space has a capacity C of 1 cm.

In electromagnetic units, the unit of capacity is the capacity which requires 1 electromagnetic unit of quantity to change the potential by 1 electromagnetic unit of potential, i.e., $C = 1$ e.m.u. if $q = 1$ e.m.u. of quantity and P.D. = 1 e.m.u. of potential difference.

It is seen that since the electromagnetic unit of quantity is a very large quantity and the unit of P.D. is a very small quantity, this unit of capacity must be a tremendous unit.

In terms of the practical system, the unit of capacity is amperes times seconds divided by volts, or

$$\frac{\text{coulombs}}{\text{volts}} = \frac{Q}{V}.$$

This unit is called the farad. Therefore C (practical) equals

$$\frac{\text{coulombs}}{\text{volts}} = \text{farads.}$$

Now 10 coulombs equals an absolute e.m.u., and 1 volt = 10^8 absolute e.m.u. Therefore, 1 unit of capacity in absolute e.m.u. is equal to

$$\frac{10}{10^{-8}} = 10^9$$

farads or practical units of capacity. Again the absolute electrostatic unit of potential is 3×10^{10} e.m.u. and the unit of quantity in the electrostatic system is

$$\frac{1}{3 \times 10^{10}}$$

absolute e.m.u. of quantity, therefore

$$C \text{ in e.m.u.} = \frac{3 \times 10^{10} q}{\text{P.D.}} = 9 \times 10^{20} C \text{ in e.s.u.}$$

$$\frac{3 \times 10^{10}}{3 \times 10^{10}}$$

Thus, unit capacity in electromagnetic units is 9×10^{20} cm.

The practical unit of capacity, the farad, is so large a unit that it is impractical to use it. Most capacities used in experiments and industrial work are of the order of $1/1,000,000$ of a farad. This leads to another unit. It is called the *microfarad*, that is, the microfarad is 10^{-6} farad = $10^{-6} \times 10^{-9} = 10^{-15}$ absolute electromagnetic unit. The microfarad is therefore 9×10^5 absolute electrostatic units. Or, the microfarad = 900,000 cm. In static electricity and for high-frequency radio oscillations capacities are sometimes given in centimeters, otherwise capacities are given in microfarads, or micro-microfarads, i.e., 10^{-6} microfarad or 10^{-12} farad or 0.9 cm.

78. CAPACITY OF THE SPHERICAL CONDENSER

Having defined the units and given their relative magnitudes, a study of the capacities of various electrical systems of conductors is now in order. It will be seen at once from the definition of capacity

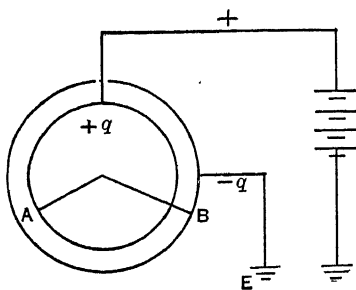


FIG. 91. Spherical condenser.

that capacity is a derived unit analogous in this respect to resistance in the electromagnetic system. Resistance was derived as the ratio between current and potential, the two fundamental quantities in current electricity. Resistance was shown to be a *characteristic of the particular electrical system of conductors used*. Likewise, capacity is the ratio between quantity and potential which are the fundamental quantities of importance in the electrostatic system. As will be seen,

this property is again a property of the conductors and their positions in space.

Let us calculate the capacity of the concentric spherical condenser.

In the system shown in Fig. 91, there are two concentric spheres of radii A and B separated from each other and insulated, with all substances removed from between the plates, including air. The outer sphere of radius B is connected to the earth E . The inner sphere is connected to the battery or source of potential the other terminal of which is also connected to the earth. In making the connection assume that a quantity $+q$ has been placed on the inside sphere. A quantity $-q$ runs from the earth to the inside of the outer sphere. The potential of the sphere of radius A is by Chapter XIII equal to q/A ; that inside of B , owing to the charge on B , is $-q/B$. Thus the resultant potential difference between A and B is

$$\text{P.D.}_{AB} = \frac{q}{A} - \frac{q}{B}.$$

Since by definition

$$C = \frac{q}{\text{P.D.}}$$

Therefore

$$C = \frac{q}{\frac{q}{A} - \frac{q}{B}} = \frac{BA}{B - A}.$$

Thus it is seen that the capacity of a condenser made of concentric spheres is a function of its dimensions only. It is seen that the greater A and B , the greater is C , and the smaller $B - A$, the greater is C . It

is thus seen that C depends on (1) the absolute size of the conductor, and (2) the distance to the nearest body or bodies of opposite sign depending on the nature of the system. In the assumption made in the deduction of this equation all material was removed from between the spheres. Had different material substances been placed between the spheres the capacity would have been found to depend on the nature of the substance placed between the two spheres. Thus, it is seen that capacity finally depends on (3) the material between the two surfaces. This last item is an *experimental* observation originally due to Faraday (1837). He found that the capacity of a spherical condenser changed with the material between the conductors. The constant which determines the value of this change in capacity is today called the *dielectric constant* and is designated by the symbol D .

It was by means of the spherical condenser that Faraday first measured dielectric constants of substances. If the dielectric constant of the material in the equation for the spherical condenser is included, the equation for the capacity of such a condenser must be written as

$$C = \frac{D(BA)}{B - A}.$$

79. THE DIELECTRIC CONSTANT, DIELECTRIC POLARIZATION, AND HYSTERESIS

Originally when Faraday observed the change in capacity C with the change in the material placed in his condenser he measured the ratio D of the capacity with the substance between his condenser plates to that when the space was filled with air. This ratio he termed the specific inductive capacity. Later more refined measurements showed that even gases had a value of D relative to *empty space* (or vacuum), and the term *specific inductive capacity* today represents the *ratio* of the capacity of the condenser with a substance *relative* to that when the condenser is evacuated. It is thus purely a numerical ratio and has no dimensions.

A quantity of the same relative value for different substances, however, also appears in another connection, that is, in the evaluation of the Coulomb static force of attraction. In this case it indicates that the quantity D must be placed in the denominator of the Coulomb force equation $f = \frac{qq'}{Dr^2}$. This quantity is needed to give the reduction of the Coulomb electrostatic forces under the influence of the surrounding medium. Here again D can be used as a number which gives the ratio of the force reduction in different media relative to empty space. Since, however, it now appears in the fundamental defining equation for electrostatic quantity, it is not possible to consider D as merely representing a ratio. Thus, depending on the choice of unit systems and the assignment of dimensions to other electrical quantities,

D may be endowed with an absolute value and dimensions. These must then depend on the character of the units used and the dimensional assignment chosen in any system. This character can be differentiated from the mere ratio given by the condenser comparison by saying that the *dielectric constant* D is the divisor in the Coulomb law of force. It is also the multiplier in the formulas for the capacity of condensers which accounts for the influence of the medium surrounding the charges or between the condenser plates.

As will be seen later, when the systems of units are studied, it is unity for empty space in the electrostatic system and in the Gaussian system of units. In the Gaussian system (*which is the system used in this book*) and in the electrostatic system it is also dimensionless. In the electromagnetic system of units D has the dimensions of the reciprocal of a velocity squared the numerical value of which in empty space is the reciprocal of the velocity of light squared. Hence both in the Gaussian and electrostatic systems of units, D , the dielectric constant, has the same numerical value as the specific inductive capacity.

The relation between the two terms, specific inductive capacity and dielectric constant, is in essence quite similar to that between specific gravity and density. Specific gravity is a ratio independent of any system of units. Density has dimensions and varies in value in each system. The c.g.s. system of units was designed to make density have the same numerical values as specific gravity.

The specific inductive capacity, and the dielectric constant in the Gaussian system, can be evaluated by measuring the change in capacity of a condenser in vacuum and when it is filled with the dielectric. The frequency of oscillation of a circuit with self-induction and capacity is critically determined by the value of the capacity. With a standard crystal-controlled oscillator deviations between the frequencies of two circuits can be detected to a few parts in a million by the method of beats. It therefore is possible to measure the slightest changes in capacity produced by the introduction of a dielectric. Hence not only can D be measured today for gases like He, but the changes in D with temperature in gases like HCl and NH_3 can also be measured.

To get some idea of the character of D a few values for common substances can be listed.

GASES		LIQUIDS AT ABOUT 20° C†	
Empty space	1.000000	Octane	1.93
Helium	1.000074	Ammonia, liquid at 14° C . . .	16.0
Hydrogen	1.000273	Acetone	26.6
Air	1.000590	Nitrobenzol	35.0
Ammonia	1.00718*	Pure water (highest value known)	81.0

* D for NH_3 changes with t according to $D_t = D_0 - 5.45(t - t_0) \times 10^{-6}$, while that for the other gases is nearly constant with temperature.

† All these liquids except octane vary rapidly with temperature.

SOLIDS*

Sulfur.....	4	Quartz (fused).....	3.8
Diamond.....	16	Ebonite (hard rubber).....	2.55
Ice at -18°C	3.1	Paraffin.....	2.00
Mica.....	7.1-7.7	Paper.....	1.72
Glass.....	6-8	Titanite.....	85.0

* These depend very much on purity and crystalline state.

The nature of the property of atoms causing a decrease of force between charges and an increase in the capacity of the condenser is easily understood. Since the attraction between charges is depicted as being the result of lines of force between the charges, it is clear that a decreased force means that fewer lines run across from one charge to another

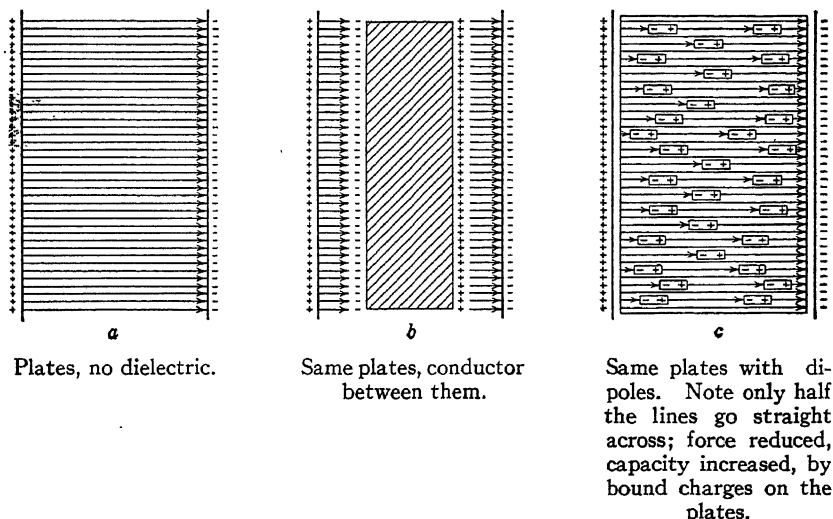


FIG. 92. Indicating the nature of dielectric action.

the other in a dielectric than in empty space. This can occur if in the medium the lines of force previously going from one charge to the other now end respectively in positive and negative charges of the medium as seen in Fig. 92c.

If a good conductor in which charges are mobile lies between the charges but does not touch them, the force between the charges is obviously zero and the phenomenon is called electrical screening; see Figs. 92a and 92b. As in general a conductor of any marked conductivity will act as a screen, it is obvious that *dielectrics in general must be insulators*, with very few mobile charges. The *true dielectric action then is not the result of free charges in the medium*.

A *fictitious* dielectric constant may appear if the charges in a body are few and slightly mobile but cannot leave the dielectric. In this case the fictitious value of the dielectric constant and the charging

current varies with time. On reversing the field on such a body there is an initial heavy current that slowly falls to zero as the neutralizing charges accumulate at the two sides of the dielectric. Heated Pyrex glass between lead electrodes shows this behavior clearly. Care must be used at times in being sure that an apparent dielectric constant is not due to this limited form of conduction.

It is clear then that the charges in the dielectric which alienate the lines of force cannot be mobile charges and that they must be bound in the material in some fashion. It was early suspected that the charges in a dielectric were the result of electrical doublets or dipoles, that is, to positive and negative charges $+q$ and $-q$ bound together at some distance l apart. Such electrical dipoles then act like magnetic dipoles, i.e., small magnets. For they have an electrical moment $\mu = ql$ and orient in the field direction. If a dielectric is unpolarized and is not in an electrical field all the doublets, much as the domains in a bar of soft iron, neutralize each other in groups. (See page 23.) In a field these dipoles line up with the positive charges toward the negative side of the field and the negative charges toward the positive side of the field. In this position they take up some of the lines of force of the charges causing the field and reduce the force of attraction; see Fig. 92c. They consequently partially neutralize those charges the lines of which they take up, thus requiring that more charges be added to the conductors producing the field to raise its potential to the initial value. Hence they increase the capacity of the system, for $q/V = C$ is increased.

Concerning the nature of the dipoles, not much was known for a long time. When, however, the electrons were discovered in 1896, it at once appeared that the dipoles might be made of the atoms themselves. For on such a viewpoint the electrons in the atoms are the mobile displaceable units and by being crowded to the positive side of an imposed electrical field would create temporary atomic dipoles which disappear as soon as the field is removed. Even before this time Maxwell had imagined some such action and had deduced that even in the rapidly alternating electrical field of an electromagnetic light wave the atoms would polarize. On the basis of his electromagnetic theory, such polarization should reduce the velocity of light in the medium. Since the index of refraction of light depends on the velocity of the light in a medium compared to that in free space, Maxwell inferred from his theory (see page 408) that the index of refraction n should for transparent colorless bodies be related to the dielectric constant by the simple relation, $n = \sqrt{D}$. This proved to be nearly correct for a number of substances, but ran into serious contradictions when it was applied to substances of particularly high values of D .

What this displacement polarization means in the modern nuclear atom with its central positive nucleus and its concentric shells of

electron orbits is merely the following. In a normal neutral atom the center of electrification of the negative electron swarm is on the average completely symmetrical about the nucleus so that at any appreciable distance from an atom the electrical fields of electrons and nucleus are quite superposed and no displacement of charges is noted. In a field, however, the electron shells are ever so slightly dis-

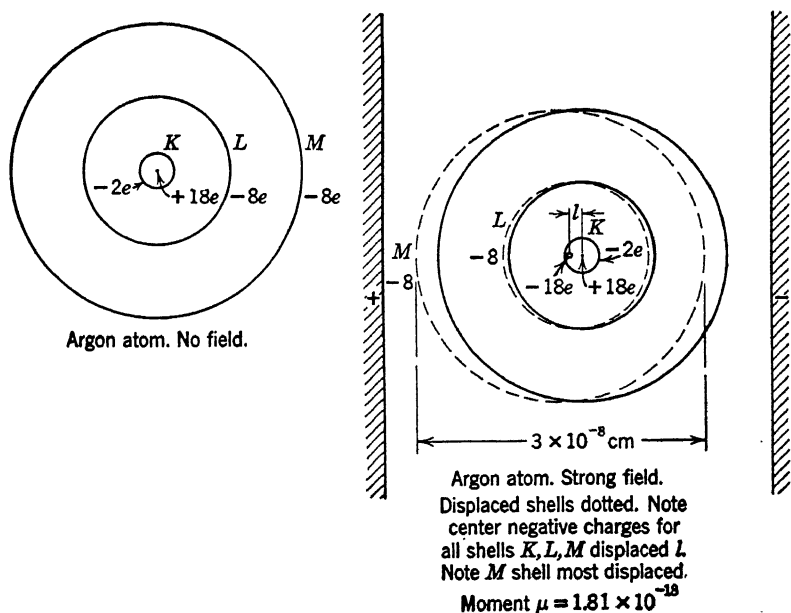


FIG. 93. Displacement polarizability.

placed toward the positive side of the field relative to the nucleus, thus producing centers of positive and negative charge effectively separated by a slight distance, giving a moment $\mu = ql$; see Fig. 93. The more electrons there are in the outer shells and the more loosely they are bound, the greater the effective displacement l and the greater D . Helium, with the least electrons and these very tightly bound, has a low D , while xenon has a relatively high D . It is clear that this displacement polarizability is an atomic property shared by *all atoms* but in, of course, varying amounts. Molecular binding forces are small compared to the inner forces on the electron shells. It might thus be supposed that the displacement polarizability characterizing atoms when they are free would be retained by them in molecular compounds. Hence the molecular polarizability of a substance would be an additive function of the constituent atoms. This, in fact, is found to hold for the atoms C, H, N, and O in compounds that give $n = \sqrt{D}$. Finally, since electrons are mobile, the polarization of an

atom in a field proceeds in time intervals comparable to the *electronic* rotation in atoms, i.e., of some 10^{-16} second. Hence, except near resonant frequencies of the atoms, the displacement polarization should be independent of the frequency with which the applied field varies and independent of such slow phenomena as *molecular* rotations of 10^{-11} second. Thus temperature, which acts to cause such disorienting rotations, should not influence the value of D . This was again observed for those substances which obeyed the $n = \sqrt{D}$ relation and the additive relationship. However, substances like NH_3 gas, H_2O vapor, etc., obeyed none of these rules, and for them D was a marked function of T , D decreasing as T increased.

It was Peter Debye who, in 1912, solved the problem. He recognized that all atoms must have displacement polarizability and that this polarizability must contribute to D in the measure of $n = \sqrt{D}$, must be additive, and must be independent of temperature. He pointed out, however, that many *molecules* occurred in which, from the existence of electrolytic dissociation, the molecule must be thought of as consisting of two ions, positive and negative, bound together by electrostatic forces. In such molecules as NaCl , HCl , NaOH , NH_3 , H_2O , etc., which show electrolytic dissociation, it is possible that, besides the displacement polarizability which adjusts to the field, there are *permanent dipoles resulting from spatial segregation of charges in the molecule*. Thus HCl consists of an H^+ ion which has lost an electron bound at a distance d from a Cl^- ion. This distorts the shells so that the effective positive and negative centers of charge shown as $+e$ and $-e$ of Fig. 94a are displaced a distance l . Hence if there is such segregation these molecules have on hand at all times a moment μ (permanent dipole moment) given by $\mu = ql$. Since in such molecules μ can be large, the contribution to D may be large, in fact, in some cases many times larger than the displacement effect. Now in a field these dipoles will tend to line up with the H^+ toward the negative side and Cl^- toward the positive side. But in this endeavor they are constantly thwarted by the heat motions and impacts of their neighbors; see Fig. 94b. However, the lower the temperature, the less the disorientation and the more readily will they line up. Hence D , which is determined by the number of these dipoles per cubic centimeter and the average value of the moment in the field (i.e., the average value of $\mu \cos \theta$, where θ is the angle of the dipole axis with the field), will vary with T , which continually disorients dipoles and reduces $\mu \cos \theta$. As the polar molecules are composed of atoms having electrons in shells which are not too different from atomic shells such *molecules* also exhibit displacement polarizability moments. These are independent of temperature.

As a result of the analysis of the effect of temperature on the rotation of magnetic dipoles in solution developed by Paul Langevin, Debye was able to derive the equations given later relating μ with T ,

the absolute temperature, and hence from the variation of D with temperature to enable physicists and chemists to evaluate μ .

In this way he found for HCl that $\mu = 1.034 \times 10^{-18}$ e.s.u. of charge \times centimeters. If in HCl an electron had actually been separated in the Cl^- from the positive charge, since $\mu/q = l$, l could be evaluated as given by $l = \frac{1.034 \times 10^{-18}}{4.803 \times 10^{-10}} = 0.215 \times 10^{-8}$ cm. The

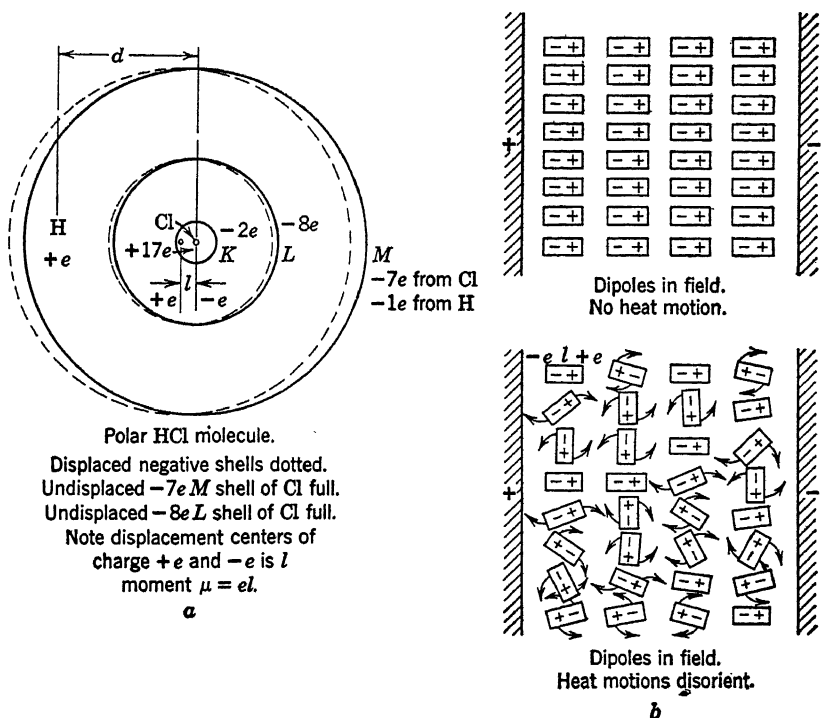


FIG. 94. Polar molecules and the action of heat in disorienting them.

distance between the H atom and the Cl atom in HCl is known, and varies with the state of rotational energy. It lies between 1.276 and 1.34×10^{-8} cm. The outer electron shell of the Cl in HCl probably extends completely around the H^+ ion, since the H^+ charge quite heavily polarizes and displaces toward itself the octet of the Cl^- ion which contains its own original electron partner. Hence it is probably not proper to think of the HCl dipole as a $+$ charge and a $-$ charge separated by 0.215×10^{-8} cm. The HCl molecule, however, acts as a dipole of $\mu = 1.034 \times 10^{-18}$. Analogous conditions hold for all polar molecules.

From a study of the dipole moment coupled with nuclear distances and force constants from band spectra, the chemist is able to learn a

great deal about the structural shape of molecules. Thus for years the molecule of water H_2O was assumed to be a chain compound $\text{H}-\text{O}-\text{H}$. This can have *no* dipole moment. From the value of μ it was possible to show that the form of the H_2O molecule is trian-

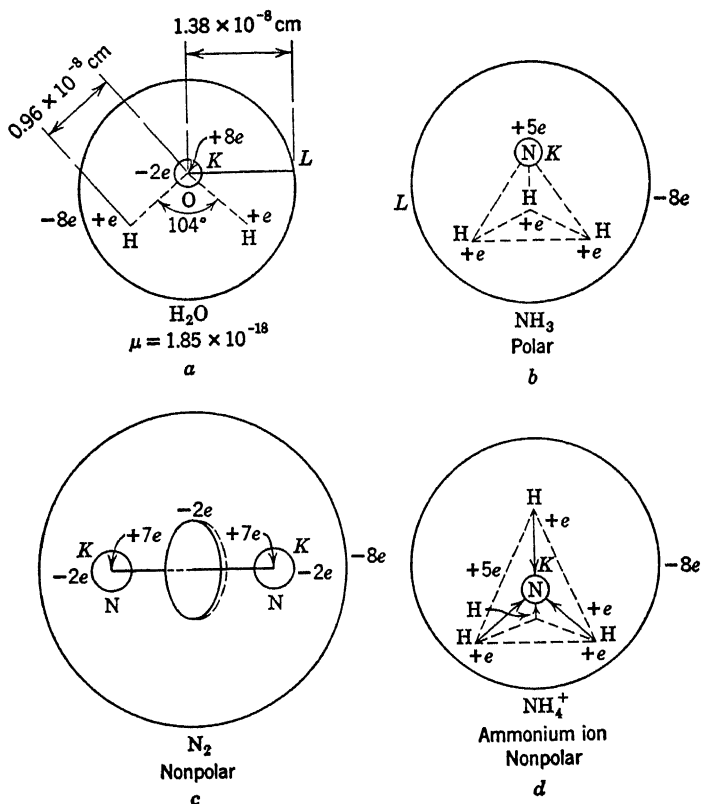


FIG. 95. Diagrams of polar and nonpolar molecules.

gular, the protons being enclosed in the outer octet electron shell of the O molecule, making an angle of $104^\circ 31'$, the distance from H to O being obtained from band spectra as seen in Fig. 95a. CO_2 , on the other hand, has no permanent dipole moment, and for this reason the structure is known to be linear and symmetrical, i.e., $\text{O}-\text{C}-\text{O}$. Correspondingly, NH_3 has a pyramidal form, the N being at the apex, the three protons being at the base as seen in Fig. 95b. N_2 on the contrary is symmetrical and nonpolar as seen in Fig. 95c. If a fourth proton is added to NH_3 the dipole moment vanishes in the NH_4^+ ion, the nitrogen being in the center of a very symmetrical tetrahedron of protons as seen in Fig. 95d.

From an analysis of the temperature variation of D in gases and in solutions of polar substances in nonpolar solvents, it has been possible to determine what fraction of D is due to displacement polarization and what is due to permanent dipole moments. Thus, for example, He, Ar, H_2 , N_2 , Cl_2 , CH_4 , C_3H_8 , and CCl_4 show only displacement polarization. HCl , SO_2 , NH_3 show mostly a strong dipole moment but have some displacement polarizability. In NH_3 , 16 per cent of D is at room temperature due to displacement polarization and 84 per cent dipole moment, whereas in ethyl ether the dipole moment makes up only some 14 per cent of D , displacement polarization being responsible for 86 per cent.

Determination of Dipole Moments. For those interested in the study of the character of the dielectric action, the following information is of interest. Given a block of dielectric material of area A square centimeters and length l in an electrical field X , it must be realized that, owing to the polarization of the material by the field, σ new electrical positive charges will appear at the end of the block toward the negative pole of the field and an equal number of negative charges on the end toward the positive pole. The total number of lines of force entering or leaving this material per square centimeter is then $B = DX = X + \frac{4\pi\sigma}{A}$. Multiply the top and bottom of the term $\frac{4\pi\sigma}{A}$ by l and the result is $\frac{4\pi\sigma l}{Al}$, where $\sigma l = M$ is the electrical moment of the

block and $Al = V$ is its volume. Thus $\frac{M}{V}$ is the moment per unit volume of the sample. Now this moment results from the polarization of the individual molecules in the volume V of dielectric. It is equal to the moment m of a molecule multiplied by the number of molecules per unit volume N . Hence $\frac{M}{V} = mN$. Again, the moment m of the molecule is induced by the field, and m should be proportional to the field. However, the field *inside* the dielectric is not X , the *external* field. It has a value E (the inner field) composed of X and the contributions to the field of the polarized molecules surrounding the molecule in the volume. Thus the inner field $E = X + \nu mN$. Here ν is a numerical factor which depends on the way in which the molecules are distributed in space. For weak fields and random distribution of molecules $\nu = \frac{4\pi}{3}$. Thus $E = X + \frac{4\pi}{3} mN$. But, as was stated, the molecular moment m is proportional to the field E polarizing the molecule, so that $m = E\gamma$. Here γ is the polarizability of a single molecule. Hence $E = X + \frac{4\pi}{3} \gamma NE$, so that, solving for E ,

the result is $E \left(1 - \frac{4}{3} \pi \gamma N \right) = X$ or

$$E = \frac{X}{1 - \frac{4}{3} \pi \gamma N}.$$

Accordingly

$$B = X + 4 \pi \frac{M}{V} = X + 4 \pi \gamma N E = X \left(1 + \frac{4 \pi \gamma N}{1 - \frac{4}{3} \pi \gamma N} \right).$$

But since the dielectric constant D represents the factor by which the charges on a conductor are neutralized by the dielectric when X is acting before the dielectric is introduced, it is clear that $B = DX$. For the lines of force normally in the space have been effectively multiplied

by D . Thus $B = DX = X \left(1 + \frac{4 \pi \gamma N}{1 - \frac{4}{3} \pi \gamma N} \right)$. Hence, from the

value of D , the dielectric constant, γ , the molecular polarizability, can be calculated through the relation $D = 1 + \frac{4 \pi \gamma N}{1 - \frac{4}{3} \pi \gamma N}$.

Now study has shown that all molecules have some polarizability as a result of the displacement of their electron shells by the field, which contributes a part γ' to the total polarizability of the molecule. If in addition there is in the molecule a permanent electrical dipole of moment μ , kinetic theory shows that, owing to its partial orientation by X resisted by heat motion, it adds to γ' an amount $\frac{\mu^2}{3 k T}$. Thus

the total polarizability γ is $\gamma = \gamma' + \frac{\mu^2}{3 k T}$. Here T is the absolute temperature and k is the average energy of a molecule per degree centigrade. Thus by studying D as a function of T , it is possible to evaluate γ , γ' , and μ . The theory leading to this expression for γ is due to the genius of P. Debye. The importance of the determination and study of μ in the investigation of molecular structure is at present inestimable, and today a knowledge of this quantity for a given molecule is almost as important to the structural chemist as its heat of formation.

Dielectric Polarization and Hysteresis. Having indicated the character of dielectric action, it is now essential to discuss other aspects of the phenomenon. The displacement polarizability responds to rapidly varying fields and is an atomic phenomenon. It suffers no frictional difficulties, and as rapidly as the field causing polarization is

removed, the dielectric depolarizes and releases the bound charge on the plates. It becomes of importance only in the optical frequencies, and there causes the well-known phenomena of anomalous dispersion. In the case of gaseous dielectrics containing permanent dipoles, the violent and rapid character of molecular heat motions causes depolarization and release of bound charges within 10^{-9} second or less of the removal of the polarizing fields. It is only at near radar frequencies that anomalies from this cause are observed.

The same behavior does not occur with liquid and solid dielectrics. In many solids the molecular forces of friction bind the dipoles oriented by a field more or less permanently in position. Thus, there is quite a delay in the release of the bound charges caused by polarization. To a far smaller degree the situation is the same with liquids.

The behavior can be well illustrated by a cylindrical or cup condenser of which the inner and the outer layers of conductor can be separated from the solid dielectric between. The condenser is charged, and the coatings are removed by insulating tongs. When discharged, they show the quantity of electricity that the condenser with empty space between plates plus the *displacement* polarization of the dielectric would bind. After discharge, the coatings are replaced. The subsequent discharge of the reassembled condenser will, for a dielectric of high D , release a considerable charge. This charge was that bound up in the semifrozen dielectric either by dipoles or ionic polarization layers if these are present. The retention of the charge in some dielectrics can be shown in other ways. Choose a poor dielectric, such as fiberboard soaked in oil. Then charge the condenser and discharge it at once. If, next, a galvanometer is placed in the circuit, it will be seen that instantaneous currents continue to flow on repeated subsequent short-circuiting of the plates over quite a period of time. While such action can also be caused by the migration of ions, a considerable amount of it results from polarization. There are certain mixtures of waxes which, if charged in the molten state, retain their polarization indefinitely if they solidify in the electrical field. These are called electrets. When working with condensers of large capacity and high voltage, such as the Pyranol condensers, the residual charge accumulated after initial discharge and standing may be sufficient to give painful, if not dangerous, shocks. As a common safety measure, when the system is not operating these condensers should at all times be short-circuited by very high resistance bleeders, i.e., resistances drawing on the order of a milliampere of current.

In the same way that it takes time to release the bound charges, it also takes time to build up, i.e., to charge such condensers to their full capacity. As a result of such actions a phenomenon is produced that is known as *dielectric hysteresis* where alternating currents are used. Thus the state of charge or discharge of the condenser, i.e., its charging per area A of condenser surface, always lags behind the

charging field X , in time. If a complete cycle of charge, discharge, reverse charge and discharge operations as with ac is gone through, the $\frac{q}{A} - X$ curve will cut out an area in the $\frac{q}{A} - X$ plane. Since $X = \frac{V}{d}$ and $\frac{q}{A} = \frac{CV}{A}$, this area has the dimensions of $\frac{CV^2}{v}$, where $v = Ad$, the volume of dielectric. $\frac{CV^2}{v}$, as will be seen represents an energy per

unit volume. The curve is called the *dielectric hysteresis curve* and represents the energy expended in charging and discharging the dielectric. Very similar curves are obtained when a body is magnetized and demagnetized and the phenomenon is termed magnetic hysteresis (see page 239). Such energy consumption is manifested in the liberation of heat. Thus, a poor dielectric will consume energy and heat under alternating electrical fields. Since condensers are of vital importance in all a-c circuits, and especially in high-frequency and radio circuits, the problem is of great importance. Certain plastic dielectrics will boil and disintegrate at frequencies of the order of 10^7 cycles, even with fairly low field strengths. The study of the polarization of liquid dielectrics has revealed *characteristic* frequencies of the dipoles for rotation, ranging from 10^5 to 10^8 cycles. The periods of oscillation are called *relaxation times*. When such characteristic frequencies are passed through, as the frequency of oscillation is increased, an increase in the angle of phase lag, a decrease in the value of D and marked energy absorption are observed.

The advent, during World War II, of ultra-high frequencies, in the techniques known as radar, made the search for nonpolarizing solid dielectrics critical. Most glasses are very poor at high frequency. One of the best substances at all frequencies is pure quartz. It has, however, a low value of D . Various titanium oxide combinations, "titanites," have a very high D sometimes reaching values of 200 and a very low hysteresis loss at ultra-high frequencies. These high values of D are strongly temperature-dependent. Certain organic condensation products, polystyrene and polyethylene, also have remarkably low hysteresis loss at all frequencies combined by high resistance.

In discussing dielectric polarization and loss caused by dipole action, it must be remarked that other actions occur which can cause dielectric loss, which do not depend on dipole *rotation*. In ionic crystals the positive and the negative ions can be acted on by the field to displace the ions relative to each other, thus producing an ionic polarization. At electrical oscillation frequencies, which correspond to natural vibrations of ions in the crystal lattice, absorption of energy to this type of activity may occur. Some of the action observed by titanites is assumed to be of this character. Such action is notably temperature-dependent.

Again, when dielectric polarization was at first observed, J. C. Maxwell proposed another mechanism to account for the observed action. This was later put to test by Wagner and proven to occur. It occurs if we have a nonhomogeneous dielectric composed of two substances of dielectric constants D_1 and D_2 , differing appreciably in value. It then happens that when the field is applied, the polarization of the dielectric of high value being greater, causes a current to flow, producing maximum fields across the particles of low dielectric constant and weak fields across those of high. Since this charging takes a finite time, the polarization and discharge take a finite time. With an alternating potential this produces phase changes and at appropriate frequencies leads to energy loss. The character of the effects, as well as the magnitude, have been shown by Sillars to be strongly influenced by the shape, size, and distribution of the dissimilar dielectric particles. Strangely enough, the equations derived for loss under the polarization mechanism of Debye and those on the Maxwell-Wagner mechanism are, except for identification of the constants, so similar in form that it is impossible without additional studies to decide which mechanism is active. This situation led in the past to rather intense controversy. Today we know that both mechanisms have been found to be active. The Maxwell-Wagner mechanism can be expected in liquid and solid dielectrics of a gross to a microscopically heterogeneous nature, e.g., oil paper, paraffin paper, and resin-loaded oils. Pure dipole action alone can be looked for in pure homogeneous bodies or in solutions of insulating dielectrics. The natural frequencies involved in this mechanism are usually much lower than those resulting from dipole action.

It must further be remembered that if ions are present in an insulating medium so that they can move within it but cannot be liberated at the electrodes to carry the current around the circuit, a temporary *conduction polarizing* current arises. This current ceases when the space charges neutralize the fields in the dielectric. The effect on a condenser can simulate superficially a polarization produced by dipoles or by the Maxwell-Wagner mechanism. Such action can be differentiated from the other mechanisms by looking for the accumulation of electrical space charges near the electrodes in one of several ways. Conduction currents of such character also cause a phase lag and dielectric loss with alternating fields. The frequencies involved in conduction mechanisms can be very low. The equations again formally resemble the polarization equations and further study is required to identify the loss mechanism.

80. THE CAPACITIES OF PARALLEL-PLATE AND CYLINDRICAL CONDENSERS

1. The Parallel-Plate Condenser. With two parallel plates separated by a distance d , small compared to the linear dimensions of the

plates, the capacity can be calculated very easily. Thus in Fig. 96, *the field is uniform*, that is, the lines of force run from one plate directly to the other normal to the surface of the plates except at the edges. (See page 190.) Now the number of lines of force per square centimeter in this case gives the field intensity, in other words, the force on unit charge. Thus it can be

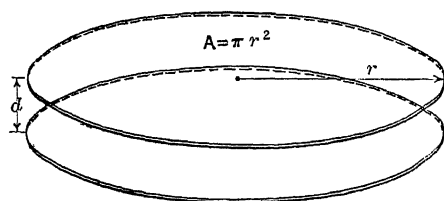


FIG. 96. Parallel-plate condenser.

written that if there are σ charges per square centimeter of surface, the field X equals $4\pi\sigma$. If there is a dielectric of value D between the plates,

$$X = \frac{4\pi\sigma}{D}.$$

Now in the last chapter, it was shown that the potential difference P.D. is the work to move unit charge the length d between the plates. Therefore, since the field is uniform,

$$\text{P.D.} = Xd = \frac{4\pi\sigma d}{D}.$$

If the area of the plates be A ,

$$q = A \times \sigma.$$

Therefore

$$C = \frac{q}{\text{P.D.}} = \frac{DA\sigma}{4\pi\sigma d} = \frac{DA}{4\pi d}.$$

An interesting variant of this arises if the material between the plates is made of two dielectrics of constants D_1 and D_2 , say, for instance, air and glass. Let x be the thickness of D_1 . Then $d - x$ is the thickness of D_2 . The fall in potential P.D. will be broken into two parts: a potential drop P.D.₁ across the distance x of dielectric constant D_1 , and a drop P.D.₂ across the distance $d - x$ of dielectric constant D_2 . The field at the surface of the plate next D_1 is $X_1 = \frac{4\pi\sigma}{D_1}$. At the plate near D_2 the field is $X_2 = \frac{4\pi\sigma}{D_2}$. If the fields are

uniform, P.D.₁ = $X_1x = \frac{4\pi\sigma x}{D_1}$ and P.D.₂ = $X_2(d - x) = \frac{4\pi\sigma(d - x)}{D_2}$

Thus

$$\begin{aligned} \text{Total P.D.} = \text{P.D.}_1 + \text{P.D.}_2 &= 4\pi\sigma \left(\frac{x}{D_1} + \frac{d - x}{D_2} \right) = \\ &= 4\pi\sigma \frac{D_2x + D_1(d - x)}{D_1D_2}. \end{aligned}$$

As again $q = A\sigma$,

$$C = \frac{q}{\text{P.D.}} = \frac{AD_1D_2}{4\pi[(D_2 - D_1)x + D_1d]} = \frac{AD_2}{4\pi\left[d + x\left(\frac{D_2}{D_1} - 1\right)\right]}.$$

2. The Cylindrical Condenser. Two concentric cylinders of radii a and b are seen in Fig. 97, the inner cylinder a being connected to one terminal of the battery, the outer cylinder b and the other terminal of the battery being grounded. Since the field is uniform except at its

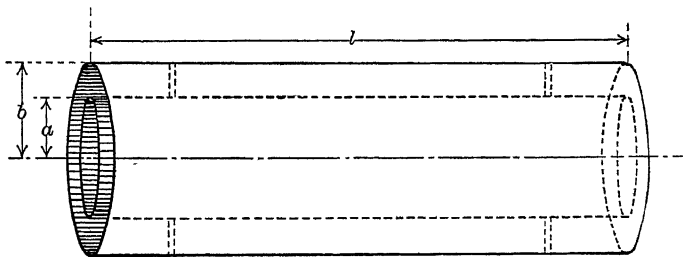


FIG. 97. Cylindrical condenser.

ends, for a cylinder in which the distance $b - a$ is small compared to the length l of the cylinder, the lines of force run radially from a to b and are symmetrically spaced about the axis of the cylinder. The field strength X at any distance r from the axis of a cylinder carrying a charge q per centimeter length is given by $2q/Dr$. (See S. G. Starling, *Electricity and Magnetism*.) The potential difference

$$P_a - P_b = \int_b^a X dr = \int_b^a -\frac{2q}{Dr} dr = \frac{2q}{D} \log \frac{b}{a}.$$

Since per unit length of the cylinder the charge on a is the charge of the system, and since b is earthed, and consequently $P.D._b = 0$, therefore

$$\text{P.D.}_a = \frac{2q}{D} \log \frac{b}{a}.$$

The quantity q of electricity per unit length of cylinder on a is equal to q . Therefore since $C = q/\text{P.D.}$ we have

$$C = \frac{q}{\frac{2q}{D} \log \frac{b}{a}}$$

as the capacity per unit length of a cylindrical condenser. Thus for a cylindrical condenser

$$C = \frac{D}{2 \log \frac{b}{a}}$$

per unit length. For a condenser of length l filled with a substance of dielectric constant D the capacity C is

$$C = \frac{Dl}{2 \log \frac{b}{a}}.$$

Condensers of this type are very easily made. For a small distance between a and b compared to the length, the capacity may be computed from this formula with a satisfactory degree of precision. Such condensers can thus furnish easily reproducible standards for all electrostatic capacity measurements. While the concentric spheres have less correction owing to end effects than the concentric cylinders, the difficulty of turning out accurately constructed spherical condenser is too great to warrant their use. By placing guard cylinders separated by minute air gaps at the ends of the condenser, the guard cylinders being of the radii a and b and concentric with the condenser, and by raising them to the same potentials, the end corrections are made negligible. In making standard condensers, separators or spacers must be used. One must be sure to include the dielectric constant correction for the spacers.

81. USES OF CONDENSERS, ENERGY OF CHARGE, ENERGY IN THE ELECTROSTATIC FIELD, CONDENSERS IN SERIES AND PARALLEL

Uses of Condensers. The ability of a condenser to hold electricity in large quantities at a low potential makes it useful for many purposes. The uses of condensers are, among others:

(1) As an accumulator of electricity. (The condenser is used in static machines and rectifiers for storing the charge until it is utilized.)

(2) As a potential multiplier. (The property of the condenser which leads to this use will be discussed later.)

(3) As a coupler of circuits. (For oscillating currents when it is unsatisfactory to make direct electrical connection with a circuit in which oscillation is taking place, use may be made of the induced oscillations set up in an insulated system through the inductive action of the condenser.)

(4) As a producer of oscillations. (The condenser is frequently used in connection with an inductance for producing periodic variations of electrical intensity.)

(5) As a potential divider in the measurement of high potentials with low-range static instruments.

(6) For measuring small currents from the relation $i = C \frac{dV}{dt}$, in

which C is the capacity and $\frac{dV}{dt}$ is the time rate of change of potential which can be measured.

Energy of Charge of Condensers. Before taking up the various combinations of capacities, it may be worthwhile to calculate the energy of the charge on a condenser. By definition, dw , the work done in bringing up a quantity of electricity dq to a condenser against a potential P.D., is given by $dw = \text{P.D.} \times dq$, for by definition, $\text{P.D.} = \frac{dw}{dq}$.

Accordingly as a condenser is charged up by transferring to it in succession a quantity dq it will be found that the work required to add each succeeding dq is greater than the preceding one, because as each successive quantity dq is added the potential has been increased by a certain amount. Thus to determine the energy necessary to charge a condenser the little increment of work $dw = \text{P.D.} \times dq$ must be summed up for all values of q beginning at zero and going to the total charge on the condenser. This is done by the process of integration. Hence it follows that

$$w = \int_0^q \text{P.D.} \, dq.$$

Since by definition

$$C = \frac{q}{\text{P.D.}}, \text{ P.D.} = \frac{q}{C},$$

and with $w = E$ (the energy put into the charged condenser), one may write

$$w = \int_0^q \frac{q \, dq}{C} = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} \text{P.D.} \cdot q = \frac{1}{2} C \text{P.D.}^2$$

Accordingly, when energy is stored on an ideal condenser the energy of charge is $\frac{1}{2} q \text{P.D.}$ or $\frac{1}{2} C \text{P.D.}^2$. This energy can be liberated in discharging the condenser either in the form of light, noise, and heat, as in the spark, as $i^2 r$, heat in a conductor, or in part as radiated electromagnetic waves if the discharge is oscillatory.

Since the internal resistance of a charged condenser is very low, the discharge of a large capacity at a high voltage will give instantaneous currents of high value. In this way, Dr. John A. Anderson at Mount Wilson Solar Observatory "explodes" fine wires of substances and gets instantaneous temperatures of $20,000^\circ \text{C}$, i.e., about four times as high as that of the sun.

Energy of Charge of a Condenser and Energy in the Electrostatic Field. The storing of energy in a condenser needs more comment. It will be noted that in a charged condenser the quantity of electricity is bound to the surface of the plates by the dielectric. When the condenser is discharged the quantity q is restored to the system, but energy is expended. To study this, consider a condenser of capacity C_1 charged to V_1 and connect it to a condenser of capacity C_2 with zero charge. After connecting the two the potential is V_2 . The quantity

q on C_1 is now divided between C_1 and C_2 . One can then write, $q_1 = C_1 V_1 = (C_1 + C_2) V_2$ whence $V_2 = V_1 \frac{C_1}{C_2 + C_1}$. One may at once calculate the energy in the systems E_1 , before and E_2 after charge. Thus, $E_1 = \frac{1}{2} C_1 V_1^2$, while $E_2 = \frac{1}{2} (C_1 + C_2) V_2^2 = \frac{1}{2} \frac{C_1^2}{C_1 + C_2} V_1^2$.

Thus it can be noted that in connecting C_1 and C_2 there was lost an amount of energy given by $E_1 - E_2 = \frac{1}{2} C_1 V_1^2 \left(1 - \frac{C_1}{C_1 + C_2} \right) = \frac{1}{2} q(V_1 - V_2)$. The energy lost was dissipated in the spark, if a spark passed, as $i^2 R$ heat in the resistance of the connecting wires or as radiated electromagnetic waves, if the charge oscillated on connection.

One may now inquire where this energy comes from. It is clear that the *electricity*, q , was conserved. However, it was at a higher potential V_1 before connecting and de-graded to a lower potential V_2 after connecting. In fact, manipulating the equations just given shows that the loss is just $\frac{1}{2} q (V_1 - V_2)$.

As to where the energy that is liberated resided in the condenser a further word might be said. The *charge*, q , is bound on the plates of the condenser by the polarized dielectric. Thus, electrically, polarized dielectric, including empty space for which $D = 1$, has energy stored in it through the potential of the bound charge. The empty space, or the dielectric in a condenser with a charge on it, is under electrical stress in an electrical field. The field in a plane parallel-plate condenser with empty space is X . The field in a charged condenser with dielectric, D , is $B = DX$. Thus the stressed condition of an electrical field can be associated with the energy stored in the dielectric.

In many problems where the field can be calculated the energy stored by the dielectric in virtue of the field can also be calculated. This knowledge is of vital importance when it is necessary to calculate the electromagnetic radiation of energy. From the energy of charge of a condenser the energy stored in an electrical field X in empty space may at once be deduced in a simple fashion. Consider a plane parallel

condenser of capacity $C = \frac{A}{4\pi d}$, charged to a potential V . The energy

of charge is $E = \frac{1}{2} CV^2 = \frac{1}{2} \frac{A}{4\pi d} V^2$. Multiply top and bottom of the last expression by d , the distance between the condenser plates. Then

$E = \frac{Ad}{8\pi} \frac{V^2}{d^2} = v \frac{X^2}{8\pi}$. Here, $v = Ad$, the volume of the dielectric in-

involved. The field strength is $X = \frac{V}{d}$. Thus at once it appears that the *energy per unit volume*, E/v , stored in a uniform electrical field X in empty space, is merely $\frac{X^2}{8\pi}$. Where the field is not uniform, energy is

still stored but the calculation requires integration of the variable field over the space using $\frac{X^2}{8\pi}$ as the density at the point where the field is X .

While this calculation was done for a simple example, there is no reason why this value should not apply anywhere where a uniform field X exists in empty space. Hence quite generally in creating a uniform electrical field X in empty space, an amount of electrical energy $E = \frac{X^2}{8\pi}$ per cubic centimeter is stored up. E will be in ergs if X is in absolute c.g.s. units. When the field disappears the energy is released. Where a region undergoes periodic building up and annihilation of fields as would be produced by an alternating potential this constitutes a radiation of electromagnetic energy.

As will be seen later on, a magnetic field H in empty space also represents stored magnetic energy and the density of energy stored will be $\frac{H^2}{8\pi}$.

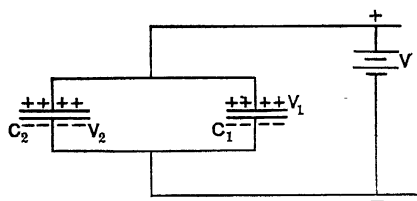


FIG. 98. Condensers in parallel.

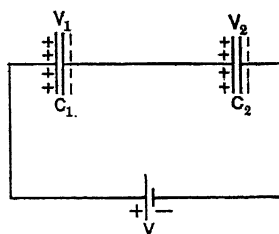


FIG. 99. Condensers in series.

Capacities in Parallel. In Fig. 98 two condensers C_1 and C_2 are connected in parallel. The potential V_1 across C_1 and that of V_2 across C_2 are both equal to the potential V across the line. By definition

$$V_1 = q_1/C_1,$$

$$V_2 = q_2/C_2,$$

and

$$q = q_1 + q_2.$$

Therefore since $q = CV$ (where C is the total capacity)

$$q = CV = C_1V_1 + C_2V_2;$$

and hence

$$C = C_1 + C_2.$$

Capacities in Series. If two capacities are in series across a battery, as shown in Fig. 99, the capacities having the values C_1 and C_2 , it can be seen from the laws of electrostatics that the quantity of elec-

tricity q_1 on C_1 must equal the quantity of electricity q_2 on C_2 . Each of these are equal to the quantity q of electricity on the combined capacity C , of C_1 and C_2 . That this must be so is seen from the following considerations.

If a plus charge be placed on C_1 , negative electricity will be drawn from the insulated section containing one plate of C_1 and one plate of C_2 . The amount bound by the positive charge on the left-hand side of C_1 must be equal to the positive charge on C_1 . On the other hand, since initially the insulated section was neutral, the equal quantity of positive electricity will be left on the left-hand side of C_2 . The latter will in turn bind an equal quantity of negative electricity on the right-hand plate of C_2 . Thus the charges on the condenser C_1 , on the condenser C_2 , and on the condenser made of the left plate of C_1 and the right plate of C_2 must all be equal. Furthermore, since there is no current flowing, the potential V of the battery must be distributed in two parts—one across the condenser C_1 , and the other across the condenser C_2 . In equation form this is expressed by

$$V = V_1 + V_2.$$

Therefore

$$\frac{q}{C} = \frac{q_1}{C_1} + \frac{q_2}{C_2}$$

and $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ or, in this case, $C = \frac{C_1 C_2}{C_1 + C_2}$.

For more than two condensers the law can be extended to read

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}, \text{ etc.}$$

Furthermore,

$$q_1 = C_1 V_1 = q_2 = C_2 V_2.$$

Therefore

$$\frac{C_1}{C_2} = \frac{V_2}{V_1}.$$

The last equation says that the *fall of potential across the two condensers C_1 and C_2 is in the inverse ratio of their capacities*. Accordingly this principle can be utilized in the measurement of potentials with electrostatic voltmeters. If a total fall of potential exists across two condensers C_1 and C_2 , by placing the voltmeter across C_2 and making its capacity, C_2 , large compared to C_1 , then the reading of the potential across C_2 will be a fraction of the potential across C_1 and C_2 . This follows from the fact that

$$\frac{C_2}{C_1} = \frac{V_1}{V_2},$$

whence

$$\frac{C_2 + C_1}{C_1} = \frac{V_1 + V_2}{V_2} = \frac{V}{V_2}.$$

If now it is required that the potential across C_2 which is V_2 will be $1/x$ times as great as the total potential V , all that need be done is that the capacities C_1 and C_2 be adjusted in the following fashion. From the above equation

$$\frac{V}{V_2} = \frac{C_1 + C_2}{C_1} = x,$$

and thus

$$C_2 = C_1(x - 1).$$

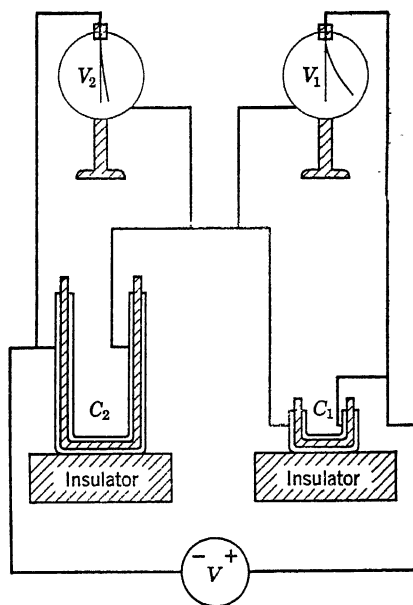


FIG. 100. Demonstration of a fall of potential across condensers. Crosshatching indicates insulation material. Note the potential across small C_1 , greater than across large C_2 .

This means that, if the capacity C_2 be made $x - 1$ times as great as C_1 , the fall of potential across C_2 multiplied by x will give the potential across the circuit. This device is frequently used for distributing potential drops and for measuring high potentials with instruments of a low range. An experimental demonstration of the phenomenon is shown in Fig. 100.

Another use of capacity is in multiplying the effects of small quantities of electricity. Assume a gold-leaf electroscope, G , Fig. 101,

and, fastened on the insulated electrode, a large disk A , which forms the base of a parallel-plate condenser. Separated from this plate by thin, accurately equal, insulators, is placed a second plate B of the parallel-plate condenser which is earthed and which is provided with means for removing it easily. If now a charge be placed on the lower plate A , the upper plate B and A again form a condenser. After charging the system, assume that the contact to the outside be broken,

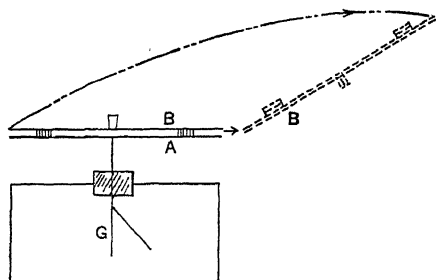


FIG. 101. Electrostatic potential multiplier.

and that the upper plate be removed to a considerable distance. Then the charge which had been placed on A and had charged the capacity of AB , which will be designated as C_1 , to a potential V_1 will reside on a system consisting of the plate A alone, of capacity C_2 which is much smaller. If the ratio between C_1 and C_2 is known, then the ratio between the quantities $q = C_1 V_1 = C_2 V_2$ will also be known. Thus:

$$\frac{V_2}{V_1} = \frac{C_1}{C_2},$$

which says that the deflection given by the electrostatic voltmeter when the upper plate has been removed will be to the deflection when the upper plate is present as the capacity when the upper plate is present is to the capacity when the upper plate has been removed. In this way, the potential of a dry battery, which is very much too small to cause a gold-leaf electroscope to deflect, can be made to give a measurable deflection on a gold-leaf electroscope. By accurately determining the capacities, this multiplication can be made as accurate as is desired.

The recent need for high potentials and high potential surges in the study of nuclear physics and of transient effects on power lines has led to the utilization of this principle in the lightning surge generator. In essence this consists of a group of n capacities which are charged in parallel and by a switching device are suddenly placed in series. The quantity placed on n of them in parallel at a potential V is $q = nCV$.

In connecting them together in series, q is reduced to $\frac{q}{n}$ by connecting

n alternate plates together. In series the capacity is $\frac{C}{n}$. The quantity $\frac{q}{n} = \frac{C}{n} V'$. Thus $\frac{nCV}{n} = \frac{C}{n} V'$, so that $V' = nV$. The action can be understood by observing Fig. 102. S_1 and S_2 are switches connecting the two condensers C in parallel to a source of potential $V = q/2 C$.

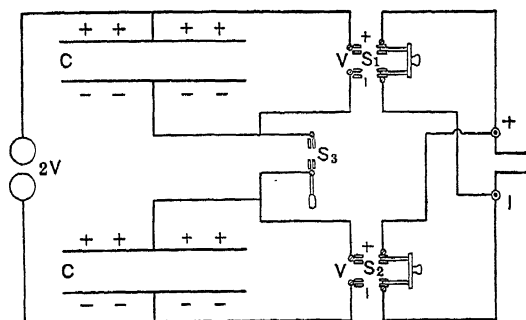


FIG. 102. Principle of the potential multiplier and the lightning surge generator.

After closing S_1 and S_2 and charging the condensers C in parallel they are opened. When switch S_3 is closed, the charges on the adjacent plates of the condensers C are mutually bound. The condensers C are in series with $\frac{q}{2}$ of charge bound, but with a capacity $\frac{C}{2}$ giving a potential $V' = 2V$ across the line. Thus on discharge the bound charge $\frac{q}{2}$ is lost and the quantity $\frac{q}{2}$ on the outer plates at a potential $V' = 2V$ is discharged.

CHAPTER XVI

ELECTROMAGNETICS

82. DEFINITION OF CURRENT AND WORK DONE IN MOVING A CONDUCTOR IN A FIELD

Before proceeding further in the study of the magnetic properties of materials, the magnetic circuit, and the applications of these studies to the analysis of the fundamental actions of motors and dynamos, it becomes necessary further to develop our knowledge of the magnetic fields of currents and their interactions. To do this, the alternative definition of an electrical current which was given in Chapter V and applied to the analysis of the moving-coil galvanometer in Chapter VI must be recalled. This definition reads that a straight conductor of length l perpendicular to a uniform magnetic field of strength H carrying a current i_a experiences a force f at right angles to itself and to H given by the relation $f = i_a l H$. f will be expressed in dynes if i_a is in absolute electromagnetic units of current, l is in centimeters, and H is in oersteds. As will be seen in the ensuing sections, this law can at once be applied to the study of forces between conductors carrying currents and between such conductors and magnetic fields.

It will first, however, be applied to the solution of a problem which would otherwise be exceedingly difficult, namely, that of calculating the magnetic field in an infinitely long solenoid. As has already been stated, the study of problems of magnetic measurement and many problems involving the interactions of currents and fields could be very much simplified if *uniform magnetic fields* of any specified magnitude could be obtained. While uniform fields can be obtained by the use of Helmholtz coils or similar systems these are primarily used for weaker fields and for precision work. For practical purposes such as use in electrical machinery, the creation of electromagnets, etc., more compact coils are needed. This need in all its phases can be met by the use of the helical coil or solenoid. Uniformity of field will be satisfactory if a coil long compared to its diameter be used. It is therefore necessary to carry out an analysis of such fields. As was seen in Chapter V, it is possible to calculate the field at any point on the axis of a plane circular coil carrying a current i . The field on the axis inside a long solenoid could then be calculated by summing up the fields contributed at the point by the separate elements of equivalent plane circular coils consisting of the separate turns of the solenoid on both sides of a point chosen. This calculation, as can be seen from the equations, is somewhat difficult. It will be even more complicated

mathematically when this calculation is attempted for points off the axis of the solenoid.

It was stated in Chapter XIV that many electrical calculations can often be simplified by using the concept of potential instead of field strength, i.e., by utilizing the energy or work expended in a circuit. This statement applies equally well to problems dealing with magnetic fields, and the concept of magnetic potential can be developed in analogy to static electrical potential. In electrostatics the potential

difference was defined by the expression $P.D. = \frac{W}{q}$, where $\frac{W}{q}$ is the work to move unit quantity from one point to another in the field. Analogously magnetic potential difference can be defined as the work W to move unit magnetic quantity from one point to another in a magnetic field. Thus $P.D._m = \frac{W}{m}$. It is thus clear that, by studying

the work done in moving magnetic poles in fields due to magnets or currents, possible simple solutions of otherwise complex problems might be arrived at.

Toward this end, let us consider a straight conductor of length l lying perpendicular to a uniform magnetic field H and carrying in it a current i_a . This experiences a force f . Now suppose that the conductor is moved perpendicular to the field H a distance x against the acting force of the field. The work done will be $W = i_a H l x = f x$. Now $l x = A$, the area of magnetic field cut* out by the conductor, and AH is the total number of lines of magnetic force cut by the conductor. Thus a flux $\phi = AH$ of lines is cut* by the conductor carrying the current, and the work done is $W = i_a A H = i_a \phi$. This then says that, when a conductor carrying a current i_a cuts a flux of ϕ lines of force, work is done in the measure $W = i_a \phi$. It furthermore puts no restriction in any way on how magnetic lines of force are cut, i.e., whether a magnetic pole is moved about the conductor or the conductor is moved about a magnetic pole or in a magnetic field. With this knowledge we can proceed to the problem of evaluating the field in an infinitely long solenoid.

83. FIELD IN AN INFINITELY LONG SOLENOID

Consider a coil of n_1 turns of wire carrying a current i_a . Then take a magnetic pole of strength m and move it once around the wires of the coil. Each wire in this coil has cut the $4 \pi m$ lines going out of the magnetic pole once. Thus when a pole of strength m is carried around a coil of n_1 turns, carrying a current i_a , each wire is cut by

* Since moving a conductor normal to a magnetic field so that the conductor sweeps across the hypothetical lines of force would cause the conductor to cut the lines were they stretched elastic bands, such action will be designated, in the following paragraphs, by the expression, "cutting lines of force."

$4 \pi m$ lines of force, and, from the above, the work done, $W = 4 \pi i_a m n_1$.

If the coil is long (by this is meant a solenoid the diameter of which is small compared to its length) an interesting situation arises. If the direction of the lines of force encircling each wire for two adjoining turns is studied, it will be observed that these lines of force annihilate each other except parallel to the surface of the coil on its inside and on its outside. The field, therefore, represented in such a section consists of lines of force parallel to the axis of the coil running in one sense inside the coil and in the other sense outside the coil.

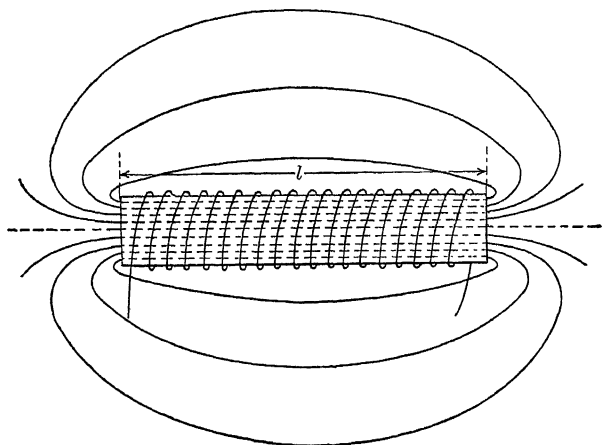


FIG. 103. Magnetic field of a solenoid.

This is quite similar to the flux of lines of force inside and outside an ideal cylindrical bar magnet. There is thus no loss of lines of force through the sides of the coil, and all lines of force originating in the coil run through the length of the coil emerging at its ends. As a result of this, the lines of force distribute themselves uniformly inside the coil and run parallel to its axis as seen in Fig. 103. Therefore, there is a uniform magnetic field down the center of the coil.

If now the coil with n_1 turns is bent into a circle of radius r so that the two ends are together, the circuit of the lines of force are continuous inside the coil, as none would emerge; see Fig. 104. For in this case each wire is encircled by all the lines of force inside the toroidal solenoid without the need for any outside. Actual calculation based on a superposition of the fields for all the wires in this form of coil will show that the field *outside* the ideal toroid must be 0. Thus the lines of force give a uniform field, H , inside the coil of length $2 \pi r$, which is the length of the coil. The work done in moving a pole of strength m once around inside the circuit is then $W = 2 \pi r H m$. If on the other hand the pole m had been carried around each wire once separately, the work would have been $W = 4 \pi n_1 i_a m$. The work in both cases would have been

the same, as in each case m was carried around each wire just once. Therefore, an equation can be written

$$W = 4 \pi n_1 i_a m = 2 \pi r H m$$

or

$$H = \frac{4 \pi n_1 i_a}{2 \pi r}.$$

Now assume that the radius r of the circle into which the large coil was bent is *very large*. A limited section of the coil is nearly a straight

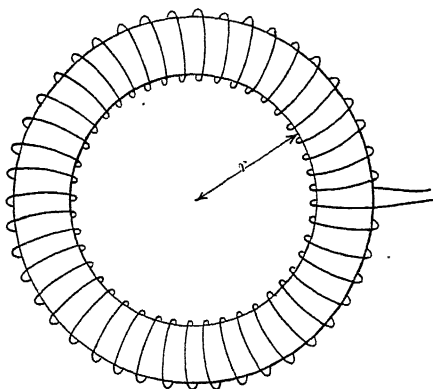


FIG. 104. Coil bent into a circle.

line, and the field except near the ends is H , which is the same as that above. Therefore the field can be set in such a section as given by $H = \frac{4 \pi n_1 i_a}{2 \pi r}$. In this section, however, we have not n_1 but n turns and a length for these n turns of l centimeters. Since for a uniform coil $\frac{n_1}{2 \pi r} = \frac{n}{l}$, we may write

$$H = \frac{4 \pi n i_a}{l},$$

where l is the length of the section. Thus the field in a straight section of a solenoid the length of which is great compared to its diameter is approximately $H = \frac{4 \pi n i_a}{l}$. If i is in amperes the field in the solenoid can be written as, $H = \frac{4 \pi n i}{10 l}$.

This is, however, only an approximation which is the more correct the more nearly the ideal conditions above are approached. For ratios of length to diameter of the order of 10 to 1 the equation is good to 2 per cent in the center of the coil. The deviations are greatest at the

ends of the coil, as shown in Fig. 104, where the lines of force begin to diverge. They are also serious at distances from the wires of the order of the diameters of or distances between the wires. Corrections for these effects can be found in the *Bulletin* of the National Bureau of Standards and certain advanced textbooks. Coils of this design are of great value, as they enable uniform and calculable magnetic fields, which are especially important in the study of electromagnetism, to be produced. Uniform fields may also be obtained by the use of Helmholtz coils or similar arrangements, as indicated on page 61.

There is perhaps one more property of a current flowing around a single turn of wire which might be of interest. If such a turn of wire with a current in it is suspended in a magnetic field, it suffers a torque, as was shown in the deduction of the galvanometer formula. Now a bar magnet suspended in a field also suffers a torque under the same conditions. In fact, if the magnet or coil were fixed in a box so that the lead wires of the coil were not visible, the reaction of the box to the field would leave the observer in ignorance of the nature of the origin of the magnetic action. Indeed, from the magnetic study alone, there would be no way of knowing which one was the causative agent of the box's behavior.* This was indicated on page 14 when magnetism was defined. Now for the torque G on the bar magnet of moment M making an angle θ with a field H , it can be written that

$$G = MH \sin \theta.$$

Again a coil of a single turn enclosing an area A with a current i_a in it, according to the galvanometer formula, suffers a torque $G = i_a AH \cos \phi$ when placed with its plane at an angle ϕ with the field H . If its axis makes the angle θ with the field, this becomes $G = i_a AH \sin \theta$. Accordingly it can be written that *magnetically the coil is equivalent to a bar magnet with a moment $M = i_a A$* , for $G = MH \sin \theta = i_a AH \sin \theta$. More complicated analysis leads to the proof that in its action a single-turn coil of wire carrying a current i_a enclosing an area A acts like a very thin magnetic shell of area A , of moment $M = i_a A$, with N polarity on one side and S polarity on the other, as given by the right-hand rule. This analogy simplifies the calculation of fields due to currents, for these fields are the same as would be given by the equivalent magnetic shell. If there are n turns of wire $M = ni_a A$.

84. INTERACTIONS BETWEEN FIELDS AND CONDUCTORS CARRYING CURRENTS

The study of the interaction between current and magnetic field is essential in order to predict the direction of motion resulting from

* Actually, as was indicated in sections 5 and 87, all magnetism is the result of the motion of electrical charges, which accounts for its circuital character. Thus, strictly speaking, even magnets are simply accumulations of oriented electrical currents.

such an interaction. The study of these laws is often facilitated by the use of certain rules known as the dynamo-and-motor rules. As these are hard to remember and easily confused, a simple device for determining the direction of motion is much to be preferred. The method to be outlined in what follows is very simple. It requires memory only of the *right-hand rule*, which gives the direction of the magnetic field circling a wire carrying a current.

Consider the wire represented by i in Fig. 105a. This is in a uniform magnetic field H at right angles to the wire. The field about the wire is indicated by the arrows. In cross section the fields would be seen as indicated in Fig. 105b. It is seen there that, on the upper side of the wire, the lines of the field about the wire are in the same direction as the field of the magnet. On the lower side, the lines of force are contrary in sense. These result in a field of the form illustrated in Fig. 105c. This occurs since two superposed fields in opposite senses when added give a weaker resultant field. The field H below is therefore weakened and is represented by fewer lines per unit area. It is seen

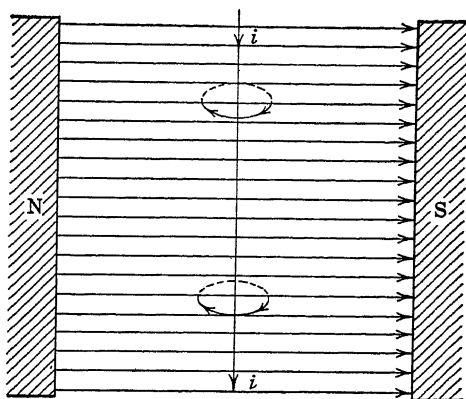


FIG. 105a.

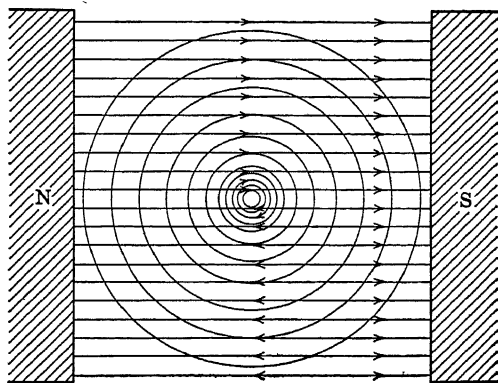


FIG. 105b.

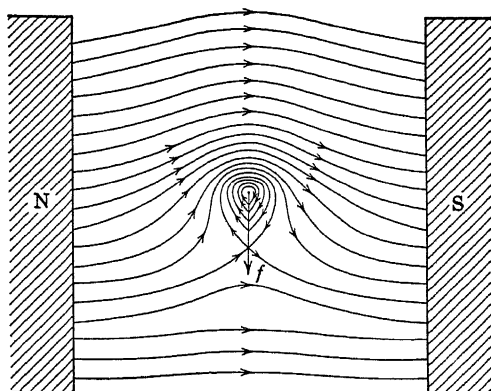


FIG. 105c.

FIG. 105. Determination of direction of motion of a conductor carrying current in a magnetic field.

that the lines of force on the upper side of the wire are crowded together. On the lower side, they are somewhat apart. Lines of force as imagined by Faraday act like stretched elastic rubber bands which exert stresses as a result of distortion. These stresses have components perpendicular to the general trend of the lines. Since, on the upper side of the figure, the number of lines compressed together is greater than on the lower side, the resultant force on the wire would be

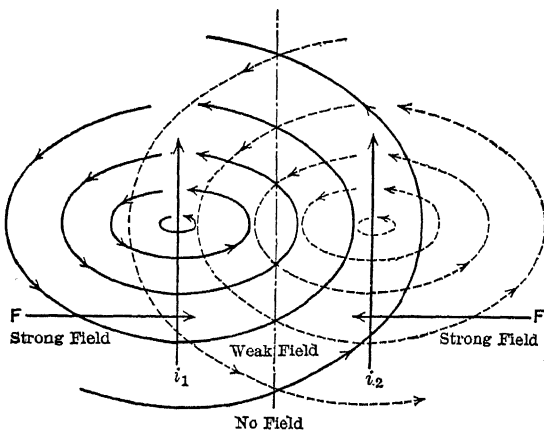


FIG. 106a. Fields about parallel conductors: currents in the same direction.

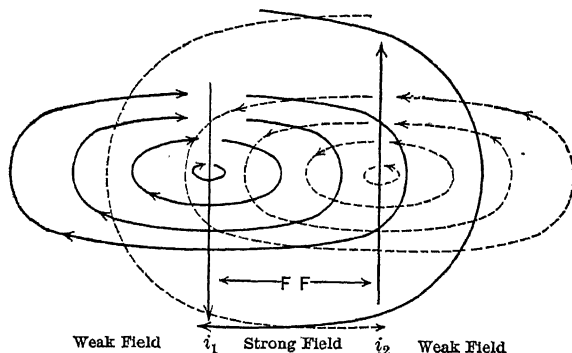


FIG. 106b. Field about parallel conductors: currents in opposite directions.

to move the wire downward relative to the magnet. In general, on plotting the lines of force about the conductor where the lines of force go in the same direction as the field, the force is greater than on the side on which they go in the opposite direction. Motion will always be from the side of the greater number of lines to the lesser.

This rule may now be applied to several simple examples.

1. Consider two wires with the currents flowing in the same direction. In what way will these wires react in reference to each other?

Figure 106a illustrates the direction of the fields about the two wires. It is seen that outside of the two wires the forces produced by the one wire and the other are in the same direction whereas between the wires the forces annihilate each other. If the currents in the two wires were equal, there would be no field in the neighborhood of the center as one wire is approached from the other. Since the field increases as the wire is approached, there will be a few lines of force and those will increase in number as the wire is approached. The result will be that the wires will act as if they were being drawn together, that is, they will attract each other.

2. If the current flows upward in one wire and downward in the other, as shown in Fig. 106b, the lines of force will add up in the space between the wires while in the region surrounding both wires they will annihilate each other. Thus there will be a crowding of the lines of force between the wires and the wires will appear to repel each other.

The forces can be easily calculated as follows: It is assumed that the wires act as if they were infinitely long, compared to their distance of separation. Then Biot and Savart's law may be applied. Such an assumption is realized to a fairly high degree of approximation if the distance between the wires is about one-tenth the length of the wires.

At r centimeters from a wire a , with current i_a , its field is $H = \frac{2 i_a}{r}$ from Biot and Savart's law. The force of H on wire b of length l with a current i_b is $H i_b l$. Therefore, the force between wires a and b is one of attraction or repulsion which is given by

$$f = \frac{2 i_a i_b l}{r}.$$

This force is in dynes if i_a and i_b are in absolute e.m.u.

3. Another interesting combination is what is known as the Faraday disk. It is also called Barlow's wheel. This consists of a copper disk* mounted on an axle, as shown in Fig. 107. This is placed between the poles of a powerful magnet N - S . The lower end of the disk dips into a trough of mercury Hg while the axis and the trough of mercury are connected to the terminals of a source of potential. The current then flows into the axle down through the mercury and back to the battery. This current is acted on by the magnetic field causing the disk to rotate in a sense determined by the direction of the field and the direction of the current. The force then acts tangentially on the wheel causing it to turn. This constitutes one of the first electric motors; it is inefficient, however, because of the effect of eddy currents which will be discussed in a later chapter. The inefficiency of this

* The Barlow wheel will operate in the absence of the magnetic field introduced from outside. This seems paradoxical at first, until it is recognized that the field produced by the circuit $ABCD$ carrying the current to the wheel itself can produce a field which will act in the same way as the field of the magnet.

motor led Faraday to conclude that the practical application of his discoveries would never be of any value. Not very long after these statements, a more practical form of motor was devised. This simple motor, which may now be discussed, is shown in Fig. 108.

4. A coil of wire is placed between the poles of a magnet N - S . One end of the coil of wire is connected to the half of the split ring a ; the other end, to the half b . The segments of this split ring are insulated from each other. The terminals of the battery are connected to the

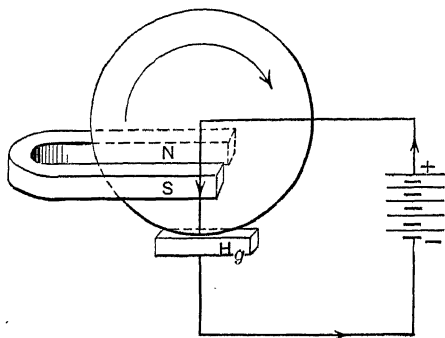


FIG. 107. Barlow's wheel.

two terminals which are at the two ends of the diameter of the ring. When the current flows through the coil, the wires which are normal to the lines of force of the field will have forces exerted on them urging the one wire up and the other wire down, provided the current and field are properly arranged. The forces on the two wires produce rotation in the same sense and there is a couple acting to cause the motor to turn. The wires turn in such a fashion that as they reach a position of equilibrium (when the plane of the coil is perpendicular to the field), in which the field acting on the wires reverses in direction with regard to the flow of current in the wire, the current is also reversed in direction by the contacts striking the next segment of the split ring. Thus the two wires are caused to rotate continuously about their axis. It is seen that when the coil is perpendicular to the field the force is zero. Accordingly, unless the inertia of the rotating system carries the coil past this neutral position the coil will cease to rotate. The motor coil is then said to be on a dead center, and it must be displaced from this before the field is turned on to start the motor. This action is very well illustrated in the cheap commercial toy motors in which friction is high and the inertia of the armature is so small that the coil readily finds itself in a position of dead center. The energy consumed and the considerations of the torque produced by the motor will furnish material for another chapter.

Still another example of the force between wires is illustrated by

a loose coil of spring wire suspended vertically which carries a current and dips into a cup of mercury. Two succeeding turns of the wire carry the current in the same sense. This results in the field produced in the two turns being such that the turns attract each other. If the spring be suspended so that one end dips into a trough of mercury and the trough of mercury be connected to a battery the other end of which

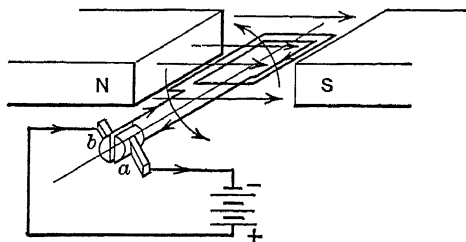


FIG. 108. Simple motor.

goes to the farther end of the coil, then as soon as contact is made a current flows through the coil. The coil contracts against gravity, owing to the current and to its elastic forces (for it is stretched) breaking the contact. On breaking contact the force is removed and the coil again makes contact. The result will be that the coil oscillates back and forth as long as the current flows through it. This oscillating coil is depicted in Fig. P45 in the Problems at the end of the book.

A large number of other examples might be cited, the mechanics of each of which could be worked out satisfactorily by means of the rules laid down.

CHAPTER XVII

MAGNETIC PROPERTIES OF MATERIALS

85. THE MANNER OF REGARDING A PIECE OF IRON IN A MAGNETIC FIELD

As was stated in the second chapter, substances which show ferromagnetic properties are either permanently or temporarily magnetized in a magnetic field. They show various degrees of retentivity. Those with a high degree of retentivity are the permanent magnets; those with a low degree are temporary magnets. The magnetization of a specimen depends on the magnetic field and at first increases slowly, then rapidly as the field increases, finally increasing more and more slowly, eventually reaching saturation. The explanation of the latter effect was made on the basis of the theory of domains.

In the last chapter it was seen that in a solenoid a uniform magnetic field with lines of force parallel to the axis of the coil can be obtained. Consider more closely what happens when iron or steel is placed in such a field. The apparent result is an increase in the number of lines of force over what there was when there was no field acting. This increase in the number of lines of force may be treated mathematically in *two ways*. The two ways of looking at the phenomenon are both useful for different applications to magnetic problems. The one, using the concepts of intensity of magnetization and susceptibility, lends itself to a study of the fundamental atomic nature of magnetism. The other, using the concept of permeability, lends itself especially well to the engineering problems of magnetic circuit analysis. The first point of view regards the increase in the lines of force from the point of view of the *creation of new magnetic poles in the iron*. The second point of view regards the increase of lines of force as an *increase in the number of lines of force threading the iron*. It really considers the iron as a magnetic conductor. An analogous analysis was made in Chapter XV concerning the induced charges in dielectrics.

Taking the first point of view. Assume a piece of iron of length l . It acts like a magnet of pole strength m and moment M . The magnet of pole strength m creates new lines of force. This magnet is, however, produced by the field H already existing in the coil. If the magnet exists only as long as the field exists (as with soft iron) the magnetization is not permanent and may be proportional to the field. This is the simplest example to consider. The new poles contribute $4\pi m$ lines of force. The density of flux where iron or magnetic materials are concerned is called the *induction* and is represented by the letter B .

It is the total number of lines of magnetic force entering unit area of the surface normal to the lines.* Thus for a piece of soft iron in a field H :

$$B = H + \frac{4 \pi m}{A}.$$

Here A is the area of cross section of the iron normal to the field, and the $4 \pi m$ represents the lines due to the new pole. If the top and the bottom of the expression $\frac{4 \pi m}{A}$ are multiplied by l , the length of the magnet, then

$$\frac{4 \pi ml}{Al} = \frac{4 \pi M}{V},$$

where Al is the volume V of the magnet and ml is the magnetic moment M . Thus

$$B = H + \frac{4 \pi M}{V} = H + 4 \pi I.$$

I is a new quantity defined as the intensity of magnetization. It is the ratio of the magnetic moment to the volume of the magnet. Now I is produced by the field. Thus

$$I = \frac{M}{V} = \kappa H.$$

Here κ is a constant, over a small range of values of H , called the susceptibility. It measures the susceptibility to magnetization by the field H . Therefore

$$B = H + 4 \pi \kappa H$$

and

$$B = H (1 + 4 \pi \kappa).$$

From the other point of view, iron is regarded as a magnetic conductor. Thus more lines of force flow per unit area taken perpendicular to the iron than would flow through the same space were the iron absent. These so-called magnetic conductors† have a greater permeability than empty space. This is stated in equation form by writing that $B = \mu H$. For magnetic conductors (para- and ferro-magnetic substances) μ is a number greater than 1 which gives the

* Note lines of magnetic force always leave the surface of a magnetic conductor normal to the surface.

† That μ is actually equivalent to a magnetic conductivity is seen in the expression on page 268 which draws a parallel between $R_0 = \frac{1}{\sigma_0}$, with R_0 specific resistance and σ_0 specific conductivity, and $\frac{1}{\mu}$.

amount by which H inside the conductor must be multiplied to give the induction B .* For empty space μ equals 1; for iron it attains values of 1000 or more. For air μ is very nearly equal to 1, having the value of 1.0000004. Therefore μ the *permeability* can be related to the quantities defined on the basis of the other viewpoint. Since

$$B = \mu H,$$

and

$$B = H + 4 \pi I = H (1 + 4 \pi \kappa).$$

Therefore

$$\mu = 1 + 4 \pi \kappa, \quad \text{or} \quad (\mu - 1) = \frac{4 \pi I}{H}.$$

It should be noted that this quantity μ is also the quantity that appears in the defining equation for magnetic pole strength, namely,

$$f = \frac{mm'}{\mu r^2}.$$

86. THE CURVE OF MAGNETIZATION AND HYSTERESIS

It is noted that it was convenient for some purposes to treat magnetic materials as conductors of magnetic lines of force and to describe their behavior in terms of an equation $B = \mu H$. Now, for weakly magnetic substances, such as diamagnetic and paramagnetic materials, this treatment is quite justified by observation. For it is observed that not only is B dependent on H , but B is directly proportional to H and μ is closely a constant for any given substance.

While B is determined by H in strongly magnetic materials, such as the ferromagnetic substances, it is nowhere, except over very limited ranges of H , proportional to H . That is, μ *varies* with H . Worse than that, it depends for its value at a given H on the past history of the fields to which the material was subjected. Because the use of μ has certain conveniences the quantity is used in applications even though its use cannot be rigorous and thus leads to peculiar inconsistencies. Where these are encountered, they must be discounted because of the character of the function. In practice, where μ is used the curves are sufficiently single-valued not to give serious trouble.

The variation of B with H is in general as follows. At first as H increases B increases slowly, then more rapidly, and finally as the magnetization approaches saturation B becomes constant. A series of typical B - H curves for different types of metals are shown in Fig. 109.

The quantity B may be measured in three ways: First, by measuring the field in the coil with a *magnetometer* when the iron is present. (See Chapter III.) Second, by knowing the value of the field H exist-

* Note that B is expressed in gauss and H in oersteds. In this textbook written in the Gaussian system with μ dimensionless and equal to unity for empty space there is no basic difference and the units could be used interchangeably.

ing in the solenoid from the number of ampere turns in the coil producing H , $H = \frac{4\pi ni}{10l}$, and measuring the deflection produced in a secondary coil when the field producing H is cut off in two cases: (1) when the iron is absent; (2) when the iron is present. The current in the secondary coil depends on the change in flux through the coil, and the ratio in the change in flux on breaking the current in the absence and presence of iron gives the ratio of B to H . Knowing H , B can be obtained. (See Chapters XIX, section 99, and XXII, section 116.) Third, B can also be measured by what is known as traction methods. In such methods the force required to separate a bar of the substance

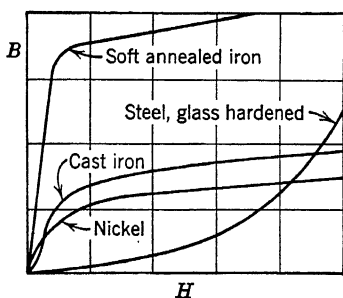


FIG. 109. Examples of B - H curves.

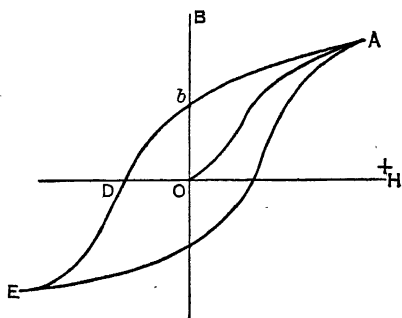


FIG. 110. Hysteresis loop, or a complete B - H curve.

joining two poles of an electromagnet of known characteristics from the two poles is measured. This force F is proportional to B^2 .

A peculiar phenomenon is observed when a piece of ferromagnetic material is slowly magnetized and then demagnetized. If a piece of steel is magnetized a B - H curve as pictured in Fig. 110 will be obtained.

As the field is increased from O , the curve as in the figure goes from O to A , where it attains saturation. If now the magnetic field H is *decreased* it will be found that B lags behind the value of H and the curve is traced from A until it cuts the B axis at b . Here H is 0, but the specimen is still magnetized. From there on it goes down until it cuts the H axis at D . At this point the magnetism is 0 but it has taken a negative field H to produce it. As negative values (which means with a reverse field) are continued negative saturation is reached at E . The curve retraced follows the line EA . Thereafter the magnetization and the demagnetization will follow the curve $AbDE$ indefinitely. The *residual magnetism* when H is 0 is represented by Ob , and is called the *remanence*. OD is called the *coercive force*. The retentivity is measured by the ratio Ob to the ordinate at the saturation value of the curve.

The magnetization loop cutting out an area in the B - H plane just

described is called a hysteresis loop. The failure of the magnetism to retrace its path as the field H varies is characteristic of a series of processes in nature all termed *hysteresis*. Thus elastic hysteresis, dielectric hysteresis, and magnetic hysteresis can be observed.

If the reduction of the field in Fig. 110 had been interrupted at $H = 0$, b , and H again increased, the curve traced out would have been that shown as loop bA (dashes) in Fig. 111. If the magnetization had not been carried to approximate saturation as in A , but had gone only

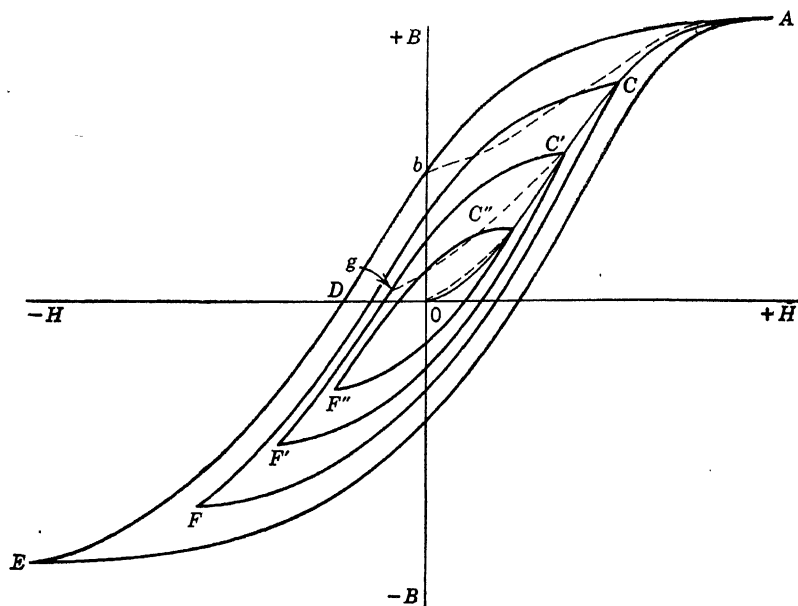


FIG. 111. Hysteresis loops.

as far as C , the loop traced would have been that shown as CF (all in Fig. 111). If the magnetization had only gone as far as C' or C'' in Fig. 111 the loops would have been those indicated as $C'F'$ and $C''F''$. It is seen at once that in each case there is a hysteresis loop which is smaller the lower the stopping point in H . The end points of all these loops define a curve $OC''C'CA$, which is called the *normal magnetization curve*. It is fictitious, as the specimen in magnetization never strictly follows it. It is useful, as it gives the locus of the *end points* of the loops at successive values of H . The curve of magnetization of the specimen beginning at $B = 0$, $H = 0$, i.e., on a demagnetized specimen, is called the *initial magnetization curve*. Actually, except near the origin, where as in Fig. 111 it is a line of dashes, it follows the normal curve very closely. No points in the magnetization of a given specimen can be outside of the area enclosed in the hysteresis loop which is run to saturation. Within the loop any value can be reached

by proper cycling. If the loop beginning at C' has diminishing slowly at g by increasing the field, the loop covered would be of the coil and indicated by $C'g$ of Fig. 111. This peculiar character of the magnetization curves where there can be as many as two or three values of B against hence μ for one value of H makes the significance of μ doubtful.

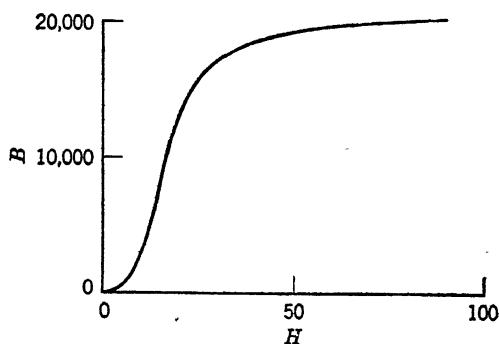


FIG. 112a.

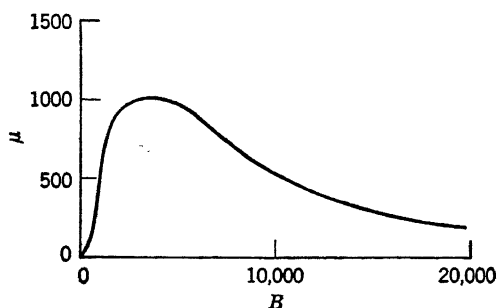


FIG. 112b.

B - H and μ - B curves on the same specimen.

particularly troublesome situation arises in the value of μ when the curve $AbdD$ of Fig. 111 passes through $H = 0$. Here $\mu = \frac{B}{H}$ would be infinite. It is clear that the use of μ in such an example is *meaningless because of the effect of retentivity*. For most purposes the values of μ are taken from the initial magnetization curve. Actually where μ is used we deal with soft materials where the hysteresis is small and the initial magnetization curve suffices. For the solution of certain problems it is desirable to have μ - B curves for materials used. These are obtained from the initial magnetization curves by dividing B by H . Such a curve is shown in Fig. 112a, while the corresponding μ - B curve is shown in Fig. 112b.

described is called *reversal*. Once we start to magnetize a ferromagnetic material it retraces its path, follow an S-shaped curve in the B - H plane. This process, in reverse, does not retrace itself, as H is reduced, and thereafter dielectric varied back and forth over the range of values a continuous

If this is retraced that never restores the magnetic material back to its zero value. This behavior can be ascribed to a sort of "magnetic memory" of the substance. Its significance lies in a reluctance of the substance to change its state of magnetism as indicated in the discussion of the Barkhausen jumps and related phenomena presented on page 23.

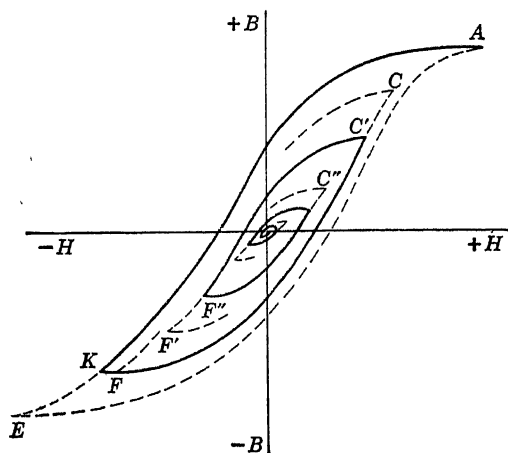


FIG. 113. Demagnetization by reversals. Compare to Fig. 111.

To demagnetize the specimen, i.e., to restore it back to the condition $B = 0$ at $H = 0$, the procedure is as shown in Fig. 113. Starting at a point, A , corresponding to A of Fig. 111, H is reduced and the negative field $-H$ is carried back to a point K . This is shown by the point K Fig. 113 and is short of the negative saturation value indicated by the dotted section KE Fig. 113 corresponding to E of Fig. 111. At this point the values of $-H$ are reduced and a loop corresponding to FC of Fig. 111 is followed. This loop is cut off short of the value of $+H$ corresponding to C at C' and $+H$ is reduced along the curve corresponding to $C'F'$ but stops short of F' at F'' . By successively reducing H and $-H$ at each reversal, the loops described close down around the origin $B = 0$, $H = 0$, as seen in Fig. 113. This process is known as *demagnetization by the method of reversals*. It is the method used by the jeweler in demagnetizing a watch. The watch is placed in a solenoid with an alternating current. In the center of the coil H is large and alternates from $-H$ to $+H$ as the current i alternates. If the watch is drawn slowly out of the coil it is undergoing hysteresis

cycles following H . The maximum swing of H is diminishing slowly as the watch enters the low-field regions at the end of the coil and beyond.

During World War II the necessity of making ships safe against magnetic mines required the removal of the longitudinal component of the ships' permanent magnetism. This was an enormous task for ships 500 feet long and over, with their complex steel structures. The demagnetization was accomplished by the process of "deperming." In this process hairpin-like loops of heavy flexible copper cable were slung under the keel of the ship and brought up along its rails. Such loops were placed at 10- to 20-foot intervals and cross-connected over the decks so as to make a continuous solenoid. The earth's field was neutralized by a separate set of loops of smaller wire or by a biasing current. The deperming by reversals was then carried out by passing currents in the many thousands of amperes from large banks of storage batteries, first in one sense and then in the reverse with reducing amplitudes. This required a proper schedule of reductions of the current at each reversal down to zero field. Once the difficulties were mastered the results were remarkably successful.

By placing the object to be magnetized in a field of known strength and starting at very high currents and reversing them to low currents, as in deperming, any desired degree of magnetism can be induced into the object. The process was used in the war and is called magnetic *shaking*. Magnetism "shaken in" in this fashion is quite stable. It is important to remember that the act of magnetization takes a finite time. This has two causes. If a field H is created about a ferromagnetic body, which is always a conductor of electricity, *temporary eddy currents are induced* (see section 112). These currents in small objects do not last very long as the resistance is low and the currents are feeble. Hence 60 cycles per second of alternating current can be used to demagnetize a watch as the eddy currents are of very short duration. In a steel ship with its complex structure, or even in a solid steel propeller shaft on a large ship, the situation is very different. Eddy currents continue to flow for seconds and even minutes after the field is turned on. These currents flow so as to give a field that opposes H . If, then, it is required to have H act fully, these currents must be allowed to die out to zero before cutting off H . In deperming the current shots thus lasted in the order of 30 seconds. Even where eddy currents decay rapidly, time must be allowed for domains to flop over, as indicated by the nature of the Barkhausen jumps discussed on page 25. Finally, in any case, the orientation of the domains into the field direction can only occur with the velocity of sound, or of mechanical sound vibrations in the metal. It is thus not strange that for most ferromagnetic materials the time required to magnetize and demagnetize even small specimens cannot go much below 10^{-4} to 10^{-5} second. Thus, not very much gain can be expected

from the use of ferromagnetic substances to increase magnetic effects when alternating currents of frequencies beyond 10^4 or 10^5 cycles per second are used.

The theoretical significance of the hysteresis loop becomes clear by dimensional analysis. Since $B = \mu H$ and since an area in the B - H

plane is $A = \oint B dH$, which has the dimensions of BH , it is possible

to write that $A \equiv BH = \mu H^2$. Now m , the pole strength, by definition

is $m = \sqrt{\mu f r^2} = \sqrt{\mu M L^3 T^{-2}}$. Thus field strength H is given by $H =$

$$\frac{f}{m} = \frac{M L T^{-2}}{\sqrt{\mu M L^3 T^{-2}}}. \text{ Whence } BH = \mu H^2 = \frac{\mu M^2 L^2 T^{-4}}{\mu M L^3 T^{-2}} = M L^{-1} T^{-2} =$$

$$\frac{M L^2 T^{-2}}{L^3} = \frac{\text{Energy}}{\text{Volume}}. \text{ Accordingly, the area of the hysteresis loop}$$

represents the energy per unit volume of specimen expended in magnetizing and demagnetizing the specimen in the process of carrying it around its cycle. This energy manifests itself in heat and thus represents a *loss of energy*. It arises from the work done mechanically, in stressing the crystals by aligning the domains from the direction of easy magnetization into the field direction. This results in mechanical working of the crystal lattice which appears as heat.

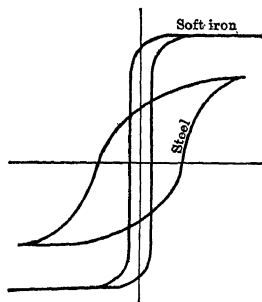


FIG. 114. Hysteresis loops for soft iron and steel.

As seen in Fig. 8d, a pure single crystal oriented with the direction of easy magnetization parallel to the field has very high values of μ and practically no area in the B - H plane. It would represent an extreme case of the soft iron curve shown in Fig. 114. Where the usual polycrystalline ferromagnetic materials are being dealt with larger areas to the curves can be expected as a result of random orientation of the crystals. In general, the mechanically harder and more rigid the microcrystalline substance the larger the area of the

hysteresis loop. This is indicated schematically in contrasting steel with soft iron in Fig. 114.* Other than for most ferromagnetic materials there is very little area to the B - H curve. Empty space obviously shows no hysteresis loop and paramagnetic substances show practically none. It is possible that certain long chain magnetic molecules in solution and perhaps in solids could exhibit hysteresis phenomena.

The phenomenon of magnetic hysteresis is of great industrial

* Note that whereas the *remanence* is higher in soft iron than in steel the *coercive force* is very small. Thus soft iron demagnetizes spontaneously whereas steel retains its magnetism. The saturation value for the soft iron need not exceed that for the steel. Figure 114 is merely schematic and all ranges of saturation values exist for

importance since it represents a power loss. Thus all electrical machinery such as dynamos, motors, and transformers, in which the iron undergoes periodic changes in magnetic field, suffers an energy loss because of hysteresis loops. For such purposes it is desirable to have materials with low hysteresis loss. On the other hand, where permanent magnets are required, a material is desired that is as magnetically "hard," i.e., as retentive as possible. For many sensitive detectors and for use in electronic circuit transformers, substances with a very high permeability and which saturate at very low magnetic fields are desired. These have led to the development of numerous alloys in recent years. These go by trade names of Permalloy, Mu metal, and Hypernic (for transformers). Transformer iron for transformers and electrical machinery has reached a high degree of perfection. In fact, power loss attributable to hysteresis and eddy currents has now been reduced to less than $\frac{1}{2}$ a per cent in certain better-class transformers. Magnet materials have been vastly improved in the form of alloys such as the Alnico series ranging from numbers one to five.

87. THE MAGNETIC BEHAVIOR OF ATOMS: PARAMAGNETISM, DIAMAGNETISM, AND FERROMAGNETISM

As has been stated earlier (page 238), there are besides the powerfully acting ferromagnetic materials other bodies which show somewhat different magnetic behavior. Actually, three essential types of magnetic behavior are recognized today. These are the diamagnetic, paramagnetic, and ferromagnetic types of action. They are distinguished by characteristic manifestations and are in essence ascribable to different basic agencies or mechanisms. These are determined, in part, by the electronic organization of the atoms composing the materials as related to the periodic table of the elements. In part they are determined by the state of the material rather than by the particular atomic arrangement alone. Thus, paramagnetism in the metallic state has a different origin from that in the gaseous or combined state in nonmetallic compounds. The properties of the three types of magnetic behavior are best summarized by the table on page 246.

Actual exhibition of these properties is not always easy to achieve. As noted in the table ferromagnetism is observed in the solid metallic state in three elements only, Fe, Co, Ni, in an oxide of one of them, Fe_3O_4 , or in alloys containing bordering elements Cr and Ni and Cu of that triad group. Ferromagnetism is, because of its magnitude, the first magnetic manifestation noted.

Diamagnetism can be observed only in substances which have no paramagnetism, since this phenomenon masks it. It is exhibited by the inert gases (N_2) and other molecular gases, and by ionic substances

DIAMAGNETIC, PARAMAGNETIC, AND FERROMAGNETIC BEHAVIOR

Type	In Uniform Field, Sets its Long Axis	In Divergent Field, Is	Value of μ	Magnitude of Effects	Where Found	Active Agent
Diamagnetic	Normal	Repelled	$\mu < 1$	Very feeble: $\sim 10^{-6}$	All atoms. Masked by paramagnetism in some. Inert gases. Ions, atoms with completed shells.	Precession of electronic gyrostats in magnetic field.
Paramagnetic	Parallel	Attracted	$\mu > 1$	Weak: $\sim 10^{-5}$ to 10^{-4}	A. Unbalanced electron orbits in some atoms. Gases, liquids, ionic solids. B. Free electrons in most metals: Na, K, etc.	Unbalanced <i>net</i> moments of orbital electrons. Spin moments of free electrons aligned by change of energy level by field.
Ferromagnetic	Parallel	Attracted	$\mu \gg 1$	Strong to very strong: 10 to 10^4	Free electrons in triad of transition elements (Fe, Co, Ni) and alloys of Cu, Mn, or Cr.	Spontaneously magnetized domains due to partially filled 3d shells with exchange forces.

with complete shells and complete octets as mentioned in section (45) such as Na^+ , K^+ , Cl^- , O^{2-} etc. Impurities can mask the effect in substances otherwise only diamagnetic. Of the metals, Bi is the only one to show it strongly. It is said to be shown by pure Cu and Ag, but most specimens are too impure and the effect is too feeble to demonstrate conveniently.

Paramagnetism is exhibited by atoms of H, Na, K, etc., and molecules like O_2 and NO in the gaseous form. It is also shown by ions of various substances in the crystalline form. For example, the ions Sc^{++} , Ti^{+++} , Cr^{+++} , Sm^{+++} , Eu^{+++} , Dy^{+++} , Mn^{++++} , Fe^{+++} , Fe^{++} , Co^{++} , Ni^{++} , and Cu^{++} are paramagnetic in solutions and in salts. Moments of this nature are studied by placing the materials in properly shaped capsules of known magnetic properties and determining the change in the magnetic moment M when empty and when filled with the ions. An ion like Fe^{+++} would have to be combined with a nonparamagnetic ion like Cl^- in FeCl_3 to be studied. Paramagnet-

sm is also exhibited by many metals having free electrons in the metallic state, e.g., Na, K, etc.

The nature of the causes for the behavior characteristic of diamagnetism on the one hand and paramagnetism and ferromagnetism on the other is easily seen. Paramagnetic and ferromagnetic bodies have μ greater than unity. This means that in a field they have a north pole induced in them at the end toward the south magnetic pole of the field and a south pole at the other end. If they have one axis larger than the others the torques from the moments induced are greater about the normal to the long axis and will swing the long axis

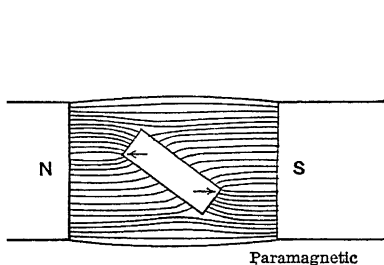


FIG. 115. An oblong paramagnetic body in a uniform field.

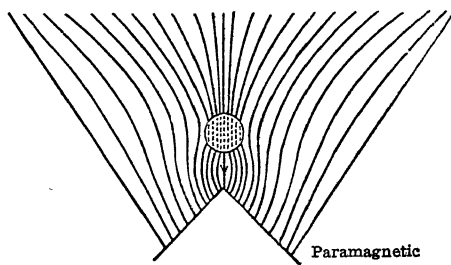


FIG. 116. A paramagnetic body in a divergent field.

into line with the field. In consequence of the induced poles the increased lines of force will also try to thread through as great a length of the conductor as possible. This is another way of indicating the tendency to orient with the long axis parallel to the field and is shown in Fig. 115.

Again, the effect of a divergent field is to concentrate the lines of force through paramagnetic and ferromagnetic bodies in such a position that the component of force *toward* the nearest pole is greater than that away from it, as seen in Fig. 116. It is also clear that a south magnetic pole will induce a north magnetic pole in the nearest portion of the material. Since the attractive force of this near pole will be greater than the force of repulsion on the more distant induced pole on the other side of the specimen, the attraction will predominate.

Diamagnetism at first sight is more difficult to account for. Since the long axis sets itself normal to a uniform field and since diamagnetic bodies are repelled in a divergent field it is clear that magnetic lines of force try to avoid the body. If this were so it would cause the effects as illustrated in Figs. 117 and 118 for a uniform and for a divergent field.

The significance of the avoidance of the material by lines of force is that μ is less than unity. This can only mean that there are induced on the ends of the body in the field H , poles of the same sign as those opposite them and producing H . That is, a diamagnetic body has

north magnetic polarity induced by the north pole of the field near it and a south pole at the south pole of the field. In other words, a field H induces in a diamagnetic body a magnetic polarity *opposing the field H* . Such behavior follows at once from the modern understanding of atomic structure.

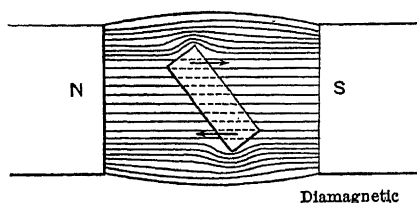


FIG. 117. An oblong diamagnetic body in a uniform field.

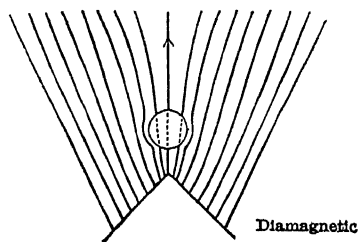


FIG. 118. A diamagnetic body in a divergent field.

88. DIAMAGNETIC ACTION

Atoms consist of a central positive sun, N (Fig. 119, the nucleus), with negative electrons, such as E , describing virtual orbits about it, see page 119. An electron moving about a nucleus constitutes a small gyroscope. When a magnetic field H is placed in the neighborhood

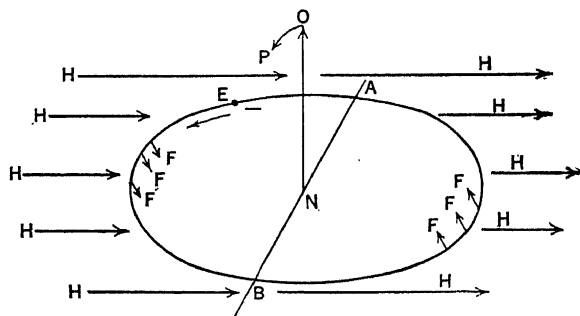


FIG. 119. Diagram illustrating the nature of diamagnetism.

of the atomic gyroscope, the electron in its orbit which constitutes a current suffers a torque or force F about the axis ANB . The torque acting on a gyroscope causes the gyroscope to execute what is known as a precessional motion. This precessional motion causes the plane of the orbit to describe an orbit (indicated by the motion of the point O out of the plane of the paper towards P). It does so in such a fashion that the electron moves partly so as to create a current producing a field parallel to the magnetic field H which caused the disturbance but opposite in sense to H . There is, as will be seen in a later chapter, a law which says that when a circuit is so disturbed by a magnetic

field as to have a current set up in it, the current set up will flow in such a sense as to counteract the magnetic field producing the disturbances. If this were not so perpetual motion could be attained. The little electronic orbital gyrostat in the magnetic field is thus caused to precess so that the electrical gyrostats of the atoms as a whole set up a field opposing the inducing field. This causes the diamagnetic

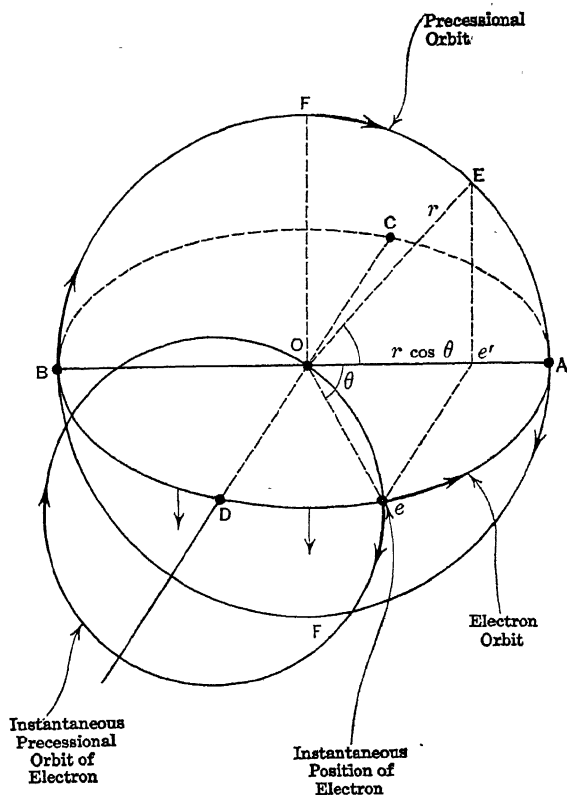


FIG. 120. Electronic precession in a magnetic field. Field axis is DOC , electron e , electron orbit BDe which rotates about DOC in precession.

behavior. All substances should show diamagnetism, and they *apparently* do. In the ferromagnetic and paramagnetic substances, however, the ferromagnetic and paramagnetic effects so far outweigh the diamagnetic effects in magnitude that the latter are not noticeable.

Quantitative Analysis of Diamagnetic Action in Elementary Form.

In a field H imagine an electronic circular orbit of radius r , electronic charge e , angular velocity ω , and mass m , as seen in Fig. 120. The plane of this orbit is parallel to an imposed external magnetic field H . The electron in the orbit is equivalent to an electrical current

$i = e \frac{\omega}{2\pi}$, for the electron passes a given point $\frac{\omega}{2\pi}$ times a second and i is then the equivalent number of electrons passing a point per second. In this calculation e is to be taken in electromagnetic units. This current i in the orbit or loop of area $A = \pi r^2$ produces a mechanical torque acting on the orbit to cause a rotation about a diameter AB perpendicular to the field H . This torque is $G = i_a A H$, according to the galvanometer deduction on page 104.

Now a mass m moving in an orbit with a linear velocity $v = r\omega$ has a moment of inertia $I_1 = mr^2$, a moment of momentum $I_1\omega = mvr$, and constitutes a miniature gyroscope. In a gyroscope a torque such as the orbit above discussed experiences in the field H will produce a precessional motion of the electron which is equivalent to a rotation of the plane of the orbit about an axis (CD in Fig. 120) at right angles to the axis about which the torque is acting, i.e., a rotation of this orbit about a diameter parallel to the field H .^{*} The angular velocity of this rotation ω' of the gyroscope is related to the torque exerted by the equation $G = I_1\omega\omega'$, as can be seen in any elementary textbook on mechanics. From what has gone before it is possible to write

$$i_a A H = e \frac{\omega}{2\pi} \pi r^2 H = I_1 \omega \omega'.$$

As $I_1 = mr^2$, one gets at once $\omega' = \frac{eH}{2m}$.

It is now of interest to calculate the magnetic moment \bar{m} per unit applied field H produced by the orbital precession with an angular velocity ω' . This moment, because of Lenz's law of electromagnetic induction, will always be so directed as to give a field opposing H . Now a current i' caused by ω' flowing around an area A gives a magnetic moment $\bar{m}_H = i' A$, as shown on page 230. Thus the current $i' = \frac{\omega'}{2\pi} e$, flowing because of the electronic precession, produces a moment $\bar{m}_H = i' A'$, where A' is the area of the orbit of the precessing electron. Now this area is not $A = \pi r^2$, for as the electron moves about its plane circular orbit of radius r with the velocity ω it is, as regards this orbit, rotating about the field axis sometimes at the distance r from that axis, and sometimes on the axis. Hence the area A' is an *average area* corresponding to the time spent by the electron at

^{*} That an electron orbit or a gyroscope has its maximum velocity of precession displaced 90° from the position of *maximum* torque follows from the fact that the electron possesses inertia and while the torque is a maximum at D it must move to A before it has its maximum velocity. At A the torque reverses in sense and the velocity decreases. The whole matter is discussed in the *American Physics Teacher*, Vol. 6, p. 66, 1938.

each distance from O to r from the axis CD of precessional motion. This average area can be calculated to be $A' = \frac{2}{3} A$.

The calculation of \bar{r} and $\pi\bar{r}^2$ is basically simple but requires spherical trigonometrical analysis and averaging which finds no place in this textbook. The averaging must not be carried out for the special case of the orbit oriented with its plane parallel to the field. It must be carried out for all orientations of the plane of the orbit from parallel to normal to the field, since for the average atom electron orbits of various inclinations will occur. The result of this averaging process is to make the average radius $\bar{r} = \sqrt{\frac{2}{3}} r$ and the area $A' = \pi\bar{r}^2 = \frac{2}{3} \pi r^2 = \frac{2}{3} A$. The fact that the orbit has all possible orientations does *not* alter $\omega' = \frac{eH}{2m}$, the angular velocity for precession about the field calculated for the special parallel orbit shown. Sir Joseph Larmor has shown that, in general, the frequency of precession ω' , the *Larmor precession*, is invariant.

Accordingly, it is possible to write,

$$\bar{m}_H = i' A' = \frac{\omega' e}{2\pi} \frac{2}{3} \pi r^2 = \frac{2}{3} \frac{e^2 H}{4m} r^2 = \frac{e^2 H}{6m} r^2.$$

$$\bar{m} = \frac{\bar{m}_H}{H} = \frac{e^2}{6m} r^2.$$

The magnetic induction was shown to be $B = (H + 4\pi I)$, where $I = \frac{M}{V}$, with M the magnetic moment of all the molecules in V . Now the moment M results from the polarization of all the n molecules in the volume V . Thus we should, as in the dielectric case, write that $\frac{M}{V} = I = \bar{m}_H n$, or that $\bar{m}_H = \frac{I}{n}$, where \bar{m}_H is the average moment per atom in the field H and n is the number of atoms per cubic centimeter.

Again we wrote that $I = \kappa H$ and thus that $\frac{I}{n} = \bar{m}_H = \frac{\kappa}{n} H = \chi H$,

where $\chi = \frac{\kappa}{n}$ is the *molecular susceptibility*. This can hold true only if

the H *inside* the material is the same as the H *outside* the material; that is, if the *inner field* (due to the outer field H plus the contributions by the polarized molecules) is sensibly equal to the outside field. It will be seen that for the dielectric case, where the dielectric constant is *large*, the *inner field* is materially larger than the outer field and a correction had to be made. For paramagnetic and even more for diamagnetic action, the effects produced by the feeble polarization of molecules are entirely negligible compared to H , and it is proper to set $\frac{I}{n} = \chi H$. For *ferromagnetic* substances this is not true. Thus we write

$\chi = \frac{\overline{m}_H}{H} = \frac{I}{nH} = \frac{e^2 r^2}{6m}$, where e is in electromagnetic units.

Since for a substance like bismuth or argon μ can be measured and since $\kappa = \frac{\mu - 1}{4\pi}$, κ can be determined. Again, as e and m are known, the value of r for the orbit can be calculated. In a complex atom like bismuth or argon there are a number of orbits, and what would be evaluated is the sum of the quantities r^2 for the orbits present. Hence for a molecule of j electrons in orbits $\left(\frac{\kappa}{n}\right)_j = \chi_j = \frac{e^2}{6m} \sum_i \overline{r}_i^2$. Here

Σ stands for the sum of the average value of r_i for these orbits. This calculation holds good for all atoms, and estimates of \overline{r}_i^2 from theory agree well with the values of κ observed.

Unfortunately, however, the values of κ that result from diamagnetic behavior are small, owing to the low values of ω' . Hence the *diamagnetic effect can be observed only in the absence of the strong paramagnetic effects*. For let us consider the simple single electron orbit used in this calculation. The electron in this orbit has, aside from its precession and any intrinsic magnetic properties of its own, a magnetic moment of the orbit as a whole along its axis of motion OF . This electron e moving with velocity ω in its orbit of radius r has a magnetic moment of value $\overline{m}' = \frac{\omega}{2\pi} e\pi r^2$. The diamagnetic moment owing to

precession is $\overline{m}_H = \frac{2}{3} \frac{\omega' e}{2\pi} \pi r^2$, and $\frac{\overline{m}_H}{\overline{m}'} = \frac{2\omega'}{3\omega}$. Now $\omega' = \frac{eH}{2m}$, and ω for a unit Bohr orbit for hydrogen is $\omega = 4.14 \times 10^{16}$ radians per second. For precession with $H = 10,000$ gauss (a high field), $\omega' = 7 \times 10^{10}$ radians per second. Thus the precessional moment is about 10^{-6} of the natural moment of the Amperian molecular magnet caused by the electron motion. In actual practice in gases and liquids where molecules or atoms are free to move the temperature agitation acts to disorient the orbital moments. Thus the *average* value of \overline{m}' in any field is much less than \overline{m}' . As observed in practice, therefore, paramagnetic effects are only from 10 to 100 times as large as diamagnetic effects. It is clear that only in atoms in which for some reason the net vector sum of the moments of the differently oriented orbits in the atom is zero can the feeble diamagnetic properties be observed. This is the case for bismuth and especially for the inert gases, the positive ions Na^+ , K^+ , Li^+ , etc., and the negative ions F^- , Cl^- , Br^- , and I^- . For atomic structure shows that in these the atomic orbital moments are 0.

89. THE EXISTENCE OF AMPÈRE'S MOLECULAR MAGNETS, THE MAGNETON, THE QUANTIZATION OF ATOMIC MAGNETS, PARAMAGNETIC AND FERROMAGNETIC BEHAVIOR IN SOLIDS, GASES, AND METALS

Atomic and Molecular Magnetism in Gases, Liquids, and Solids. The Bohr Magneton. The character of paramagnetism is well understood and quantitative agreement with wave mechanical theory is quite satisfactory. The treatment of the subject is well beyond the scope of this book. It is, however, essential that the elementary character of the agencies responsible for these phenomena be briefly presented. In thus proceeding, the approximate picture of virtual electron orbits in the Bohr type of atom, as mentioned on page 120, will again be resorted to. In so doing it will be recognized that in *reality* atomic electrons are not mass points in orbits, but are described by the abstract relations of wave mechanics.

In the preceding section it was indicated that if electrons are in orbits in the atoms, then, aside from the gyroscopic behavior discussed under diamagnetism, the electron *orbits* have magnetic moments of their own. The value of the moment was in section 88 given for a circular orbit as $\bar{m}' = \frac{\omega}{2\pi} e\pi r^2 = \frac{er^2\omega}{2}$. Hereafter the orbital moment will be designated by the symbol μ .* Now, on the Bohr theory, the radii and the angular velocities of electrons in the allowed simplest circular type of orbits are determined by the principle quantum number n (see page 121). There n takes on successive values 1, 2, 3 for different sized orbits, with the moment of momentum set by the relation

$p = mvr = \frac{nh}{2\pi}$. In this relation the quantity m is the electron mass, $v = r\omega$ is the linear velocity, and h is the Planck unit of quantum action. These relations taken together with the one above for $\bar{m}' =$

μ , at once fix the value of μ . This is $\mu = \frac{nh}{4\pi m} \frac{e}{r}$, for any given circular orbit of quantum number n . If n is unity, which is the value for the smallest Bohr type orbit, the result is the *Bohr unit of magnetic moment*,

$\mu_1 = \frac{h}{4\pi m} \frac{e}{r}$. This unit is called the *Bohr magneton*. It is thus seen that in terms of modern atomic theory Ampère's hypothesis of atomic magnets due to circulating currents of electricity in the atom has been realized. What is better, the value of the moment for a single circular orbit can easily be calculated.

Some few more things must now be stated about these orbital moments. In the orbit the magnetic moment is normal to the plane

* This is not to be confused with the use of μ for permeability. There are just not enough symbols to go around.

of the orbit. In the simpler circular orbit the magnetic moment vector is located at the center. The direction of this vector is downward for an electron circulating in a counterclockwise or right-hand sense in the orbit. This comes from the negative sign of electron charge.

The *mechanical moment* of momentum of the orbit is $p = mvr$ and is *conventionally chosen* as an upward vector for counterclockwise motion. It is normal to the plane of the circular orbit at its center. Its value is given by $p = mvr = \frac{nh}{2\pi}$. It is to be noted that the ratio

$\mu/p = \frac{e}{2m}$ for such an orbital electron. This ratio μ/p is called the *gyromagnetic ratio*. The value is characteristic of all types of simple orbital electrons when no spin complications occur.

We thus see that the values of the magnetic moments in the various circular Bohr orbits are fixed by n . It was, however, shown on page 121 that for a given value of n there were possible n differently shaped orbits designated by the azimuthal quantum number l . l can take on values 0, 1, 2, 3 with the maximum value of $l = n - 1$. If $l = 0$ the orbit is quite elliptical and has a moment of momentum and thus also a magnetic moment of zero. If l is $n - 1$ in any orbit, the orbit is circular with the moment of momentum given by l and not by n . Hence, for a given value of n , n different types of orbits having the moment of momentum determined by l are possible. Since it is the moment of momentum that determines the value of μ in an orbit, for $\frac{\mu}{p} = \frac{e}{2m}$, then it is l that fixes μ and not directly n . As n sets the values l can have, it limits μ but does not fix it. Thus, wave mechanics alters the values of μ as would be calculated by Bohr's simple circular orbit theory.

There is now another complication to be considered. It was discovered in 1925-1926 that the orbital moment of momentum of the electron is not the only moment of momentum to be considered in an orbit. The *electron itself appears to have the properties of a mass of negative electricity spinning about an axis. Thus the electron, irrespective of whether it is in an orbit or not, has a mechanical spin moment of momentum of its own.* This was discovered to be $p_s = \frac{nh}{4\pi} = \frac{1}{2} \frac{nh}{2\pi}$

$= \frac{nh}{2\pi} s$. Here s is the spin quantum number, which is always $\pm \frac{1}{2}$.

It must therefore also have a magnetic moment because of the spin.

Its value was found to be just one Bohr magneton, e.g., $\mu_s = \frac{nh}{4\pi} \frac{e}{m}$.

The combination of mechanical moment and magnetic moment for the electron spin gives the gyromagnetic ratio characteristic of a free electron as $\frac{\mu_s}{p_s} = \frac{e}{m}$. This has some significance in magnetic studies,

as will be seen. In any case, when the mechanical and the magnetic properties of an orbit are considered, it is clear that the combined effect of orbital motion and spin must be discussed.

As a result of the electron spin a new difficulty thus presents itself when an orbit is considered, for the spin and the orbital moments may be compounded. This is complicated by the fact that the electron may be spinning in the same sense as its orbital motion. Then its mechanical moment adds to the moment from its orbital motion. If the spin opposes the orbital motion it subtracts from the mechanical moment of the orbit. Thus, the magnetic moments of electron spin and the orbit can add or subtract. This would not be very troublesome if the spin and the orbital moments were collinear. However, in consequence of wave mechanical quantum relations, and the differences of the gyromagnetic ratios for orbit and spin, the *orbital-spin moment and electron-spin moment are inclined to each other at certain calculable angles*. In consequence, they precess about *their resultant* mechanical moment of momentum vector, which is now no longer parallel to the resultant magnetic moment. The resultant magnetic moment of the combined electron spin and orbital motion can, however, be calculated in terms of what is known as the Landé splitting factor, g . Thus the combined net orbital moment is now no longer μ , but $m_l = \mu_l j g$, with j the resultant mechanical moment of momentum derived from the sum of l and s , i.e., $j = l + s$.

So far it has been seen that the net resultant magnetic moment m_l for an electron in any orbit can be determined by spin number, s , plus the values of n and l . However, single orbits in most atoms are not usually dealt with. As was seen, atoms are composed of a nucleus of charge $+Z$ and Z electrons in Z different orbit types defined by the four quantum numbers n , l , s , and m_l . As indicated on page 120, m_l , the magnetic quantum number, determines the angle of the orbit axis relative to some fixed direction, i.e., a field or an atomic axis. There are $2l + 1$ possible values of m_l ranging from $-l$ through 0 to $+l$. The values of m_l represent the *projections of the total orbital mechanical moment j^** (vector sum of orbital and spin moments) as *whole multiples of $\frac{h}{2\pi}$* on this axis. This condition at once fixes the

angles of the axes. The condition therefore also fixes the angles for the resultant orbital magnetic moments. The vector sum of all these separate orbital magnetic moments for all electron orbits in the atom gives the value of the *atomic magnetic moment \bar{m}* for an atom.

It is at once seen that such a resultant moment \bar{m} for a given atom of Z electrons could in theory have almost any value from 0, where all the moments cancel, to quite large values where all electron and orbital magnetic moments nearly add in the same sense. Accordingly, one would not, on measuring paramagnetic moments, expect to find them in general equal to μ_l , or 1 Bohr magneton, nor to whole multiples thereof. They could well be smaller or larger.

This is precisely what has been observed. Pierre Weiss, following the experimental methods of Pierre Curie and the mathematical analyses of Paul Langevin before the time of the Bohr theory, evaluated \overline{m} for a large number of atoms and ions. He observed that all his thirty-odd values of \overline{m} were approximate multiples of a given value. This numerical value Weiss called unit magnetic quantity and named it the *magneton*. When later Bohr denominated *his magneton* the Weiss quantity was found to be nearly, but *not* accurately, one-fifth of a Bohr magneton. Today the experimentally determined value is called the Weiss magneton. The reason for its value being one-fifth of a Bohr magneton is at present obscure, if it is anything beyond a fortuitous agreement. On modern theory there is at present no justification for such agreement.

While to the reader the calculation of the resultant atomic magnetic moment \overline{m} from the vector sum of the Z separate orbital m_l values for any atom may present a formidable task, Nature has been kind to us. It happens that the limitations imposed by the Pauli exclusion principle and the symmetrical and systematic characters of the quantum numbers n , l , m_l and s , cause the resultant mechanical moments of momenta j and magnetic moments m_l in the closed electron shells or levels to combine so that both j and m_l become zero. That is, the *oppositely directed spins and mechanical moments cancel each other in closed shells*. Thus, as indicated in section 45, the completed K , L , M , etc., electron shells, completed electron pairs, and completed octets have $J = 0$ and $\mu = 0$.

Hence the *inert gases* have *no resultant magnetic moments*. This applies to atomic ions which have achieved their outer octets by acquiring or losing electrons. It also applies to certain molecular gases like N_2 . The atoms having resultant paramagnetic moments are those having unbalanced, uncompleted shells. Hence atoms and some ions of the elements in the center of the long periods and atoms having odd numbers of electrons in their outer valence shells are primarily paramagnetic. Thus except for the atoms in the long periods, including the rare earths and atoms with uncompleted octets, or pairs, in their outer valence shells, the majority of isolated atoms are not paramagnetic. All such nonparamagnetic atoms should show only diamagnetism. This simplifies magnetic studies considerably, since it is only necessary to concentrate on the uncompleted, or unbalanced, shells in the atoms. Thus if one is going to look for paramagnetic properties it would be sensible to choose certain gases, solutions of atoms, ions, or compounds, and certain salts and crystalline ionic solids.

If, however, the attempt is made to measure the paramagnetic moments of these, it would be discovered that the atomic susceptibilities were very small and markedly temperature-dependent. This result need not surprise anyone, however. Just as in polar dielectrics when trying to orient the dipoles by an electrical field X , the heat

motions disorient them, so with magnetic dipoles. As soon as the field H tries to align these magnetic moments the rotations and dislocations caused by heat impacts disorient them. It was Pierre Curie who found experimentally that the magnetic susceptibility of paramagnetic substances varied either according to the law $X_m = \frac{C}{T}$, or the

law $X_m = \frac{C_m}{T - \theta}$. Here T is the absolute temperature and C , C_m

and θ are constants. This work started the development of the theory. Paul Langevin carried out the analysis which showed that the *average observed value* of the paramagnetic moment, \bar{m}_a , in a field H was related to the atomic magnetic moment \bar{m} , by the relation $\bar{m}_a = \frac{\bar{m}^2 H}{3 k T}$. Here

T is the absolute temperature and k is the gas constant per atom. Wave mechanics has changed this rule in detail to read, $\bar{m}_a = \frac{J(J+1)g^2\mu_1^2}{3 k T} H$. Here J is the resultant *atomic mechanical moment of momentum* quantum number, μ_1 is the Bohr magneton, and g is the Landé splitting factor. It is thus seen that the temperature reduces the effective value of the orbital moment and thus gives low values for the observed paramagnetic \bar{m}_a .

A very satisfactory direct confirmation of all this theory came from the experiments of Stern and Gerlach between 1921 and 1924, wherein moments were measured on single atoms. The results were much more accurately verified in later years as techniques improved. The experiments made it possible directly to measure the atomic paramagnetism of single atoms, to test the rules for quantization and to evaluate the Bohr magneton.

If atoms of H in the normal state, or atoms of the monovalent elements of the first column of the periodic table, e.g., Na, K, Ag, etc., are considered, it will be noted that they have saturated inner shells with $J = 0$ and $\mu = 0$, and consequently they possess only one valence electron. These electrons in normal, unexcited atoms are in an $l = 0$ type of orbit. The *orbital* magnetic moment as well as mechanical moment of momentum is thus 0. The only mechanical and magnetic moment associated with this type of orbit is thus the mechanical electron spin moment, and the magnetic spin moment,

$\mu_1 = \frac{h}{4\pi} \frac{e}{m}$, of 1 Bohr magneton.

Quantum theory therefore predicted that if a beam of such atoms were passed through a magnetic field H , one half the atoms would have magnetic spin moments oriented in the field direction, and the other half would have the spins oriented in the opposite sense. With this situation in mind, the experimental arrangements used by Stern and Gerlach for observing the consequences of these effects can be turned to.

These two investigators passed a ribbon-like beam of silver atoms with its plane perpendicular to the magnetic field axis, and the beam-length parallel to the magnet pole faces through the very inhomogeneous magnetic field produced by the magnet in high vacuum, as shown in Fig. 121. In such a beam the Ag atoms traverse the field

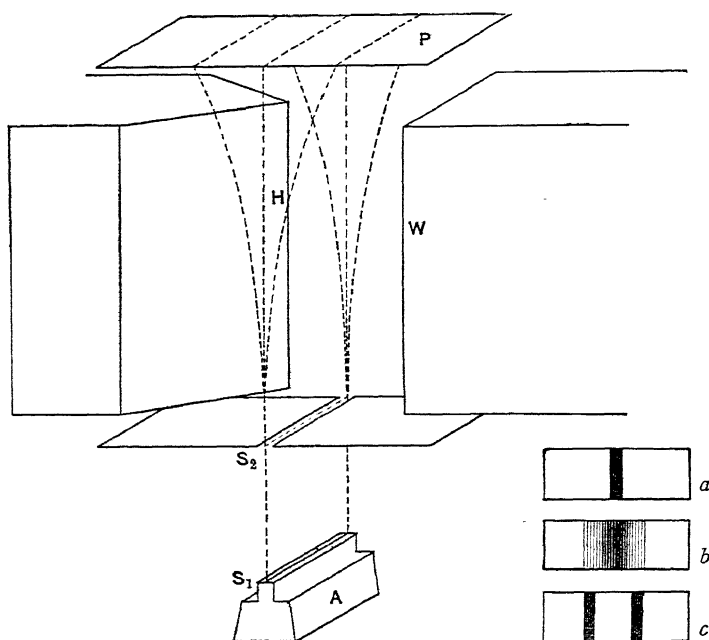


FIG. 121. The Stern-Gerlach experiment.

in vacuum in some 10^{-4} second and suffer no disorienting collisions with other atoms. The atoms emerge from the oven source *A* with the magnetic spin moments of their orbital electrons oriented in all directions. On the wave mechanical assignment of quantum numbers the value of *l* for the single Ag valence electron orbit is zero, and thus it is expected that merely its spin is magnetically active. Thus the spin moments emerging from the oven are randomly oriented. If there is no field acting on the beam showing in the diagram of Fig. 121 the slits *S*₁ and *S*₂ will confine the atoms from the oven *A* to a ribbon-like beam which will proceed in a straight line and impinge on the glass plate *P*. There the Ag atoms condense and leave a deposit. The deposit can be studied by a microphotometer and its density and character determined with some precision. In the absence of the field it will be an image of the rectangular slits shown in Fig. 121a.

When the field *Z* between the magnetic knife edge, *H*, and the solid block, *W*, has been turned on the beam will be deflected. If classic

theory holds, all the moments m_1 of the atoms that are parallel to the field at S_2 would be attracted to H and all those with moments anti-parallel — namely, at 180° — would be repelled. Those in between, with axes making an angle θ with the field, would be attracted for $0^\circ < \theta < 90^\circ$ and repelled for $90^\circ < \theta < 180^\circ$, but to a less extent. In fact, the force acting would be proportional to $m_1 \cos \theta$.

If the magnetic field varies from H to W at the rate $\frac{dZ}{dx}$ the force on the atomic moment m_1 will be $f = m_1 \frac{dZ}{dx} \cos \theta$. With such a force the ribbon-like beam, in going the length L of the magnetic field in t seconds, will be deflected by an amount S according to the well-known formula, $S = \frac{1}{2}at^2$. Here $a = \frac{f}{m}$ is the acceleration, and m is the mass of the atom. Now the time t is related to the path-length L from the slits to the plate by the relation $L = vt$, where v is the velocity of heat motion of the atom leaving the oven at a temperature T . On the average $v = \sqrt{\frac{4kT}{m}}$, where k is the gas constant per atom. Thus it can be expected that the atom in the center of the beam will be deflected from its $Z = 0$ point of impact on the plate P by an amount,

$$S = \frac{1}{8} \frac{m_1 L^2}{R T} \frac{dZ}{dx} \cos \theta,$$

when the field Z acts. Since θ varies from 0 to 180° , deflections can be expected to the right and the left of the undisplaced deposit with $Z = 0$.

Now the beam emerging from the oven A has a uniform distribution of angles θ as stated. If the field Z in the 10^{-4} second of passage along L does *not* disorient this random distribution the trace of the beam a of Fig. 121 would appear as b .

Quantum theory, however, is very definite as to what will happen. It says that unlike classic theory the field will at once at S_2 orient all the atoms to angles $\theta = 0$ or $\theta = 180^\circ$, there being equal numbers in each group. Thus the single deposit a observed with $Z = 0$ would become a split deposit with two images as in c if quantum theory applies. If classic theory applied and the atoms could reorient at once near S_2 , in the field Z , it would be expected that all the moments m_1 would be attracted to the pole face H . In this case, only one of the displaced lines in c , the one to the right, would be observed.

The experiments of Stern and Gerlach were unequivocal. The beam was split sharply into two components. Evaluation of S , L , T , and $\frac{dZ}{dx}$ gave m_1 as $m_1 = \mu_1$, or exactly 1 Bohr magneton. The experiment was later repeated for H by T. H. Johnson and for Na, Li, K,

in 1933, by Scheffers and Meissner. By this means the latter two evaluated μ_1 , to 0.5 per cent accuracy.

Metallic Paramagnetism. The para magnetism which has to do with electronic *orbits* and which has been explained before is not, however, the only manifestation of paramagnetism. If a group of atoms of metallic substances with their valence electrons are condensed to the solid state, the resulting solid is a metal and an electronic conductor, as indicated on page 78. In the free state metal atoms like Na or Ca have one or more outer valence electrons in $l = 0$ type, very elliptical orbits. These orbits are exactly defined by the various quantum numbers. As a result of these and the Pauli exclusion principle the separate electrons are in clearly defined, well-separated, and nearly independent orbits, or states. Characterizing these states, besides the quantum numbers and Z , is a clearly defined negative potential energy; see page 444. It is the energy required to remove the electron in that state from the atom. These energies are closely related to the frequencies of spectral lines emitted and can be evaluated accurately from spectroscopic data.

The minute, however, that we attempt to crowd the single atoms together in the solid state, the situation changes. The forces are such that the individual atomic ions, with their inner shells, are closer together than the virtual radii of the elliptical valence electron orbits in the free state. Thus the valence orbits no longer can exist. The various positive atom ions of the solid exert forces on electrons that belong to their neighbors. Thus the electrons cease to be associated with any particular ion. They wander freely through the ion lattice of the metal. They are also crowded very close together so that they interact on each other. Thus electrons which were once in clearly defined isolated energy levels in single atoms now can have a multiplicity of possible energy values; see page 515 and Fig. 122.

Of course, the electrons in the inner shells of the ions, such as the K and L electrons of Na^+ in the lattice, are so close to the nuclei that their energies are little affected and they are, unlike the valence electrons, in their original discrete states.

In consequence, the valence electrons, which in the atoms were in clearly defined single levels as seen in Fig. 122*a*, when in the solid metallic state have their energies widened to bands, as shown in Fig. 122*b*. The assignment of the energies in the band to various electrons is determined by the Pauli exclusion principle, the crowding together of the electrons and atoms in the lattice, and various other lattice factors. They may be computed in some cases. Figure 122*b* will therefore be modified by plotting for a given ionic separation (lattice configuration) the number of the electrons in a given state of energy along the axis of abscissas, and the value of the energy in the state along the axis of ordinates. To assist in later discussion it will be necessary to have two diagrams for each of these energy level con-

tours. One will apply to electrons with left-hand spins and the other to electrons with right-hand spins. For in a metal it would be expected that both types of electron spin are present in equal numbers under ordinary circumstances.

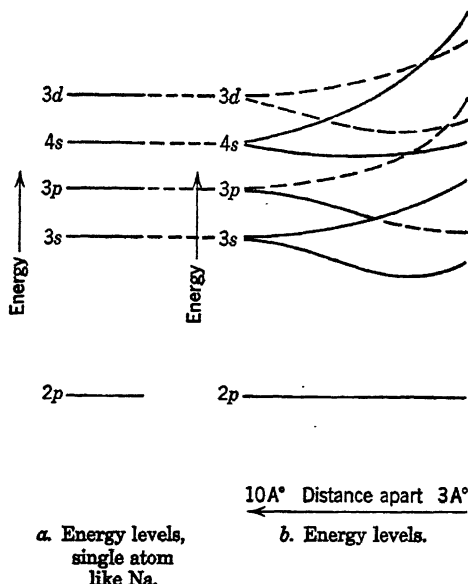


FIG. 122. Spreading of atomic energy levels as atoms approach each other.

Such levels are shown for valence orbits of three types in Fig. 123*a*, *b*, *c*, *d*. The first is for a metal like Na with single valence electrons and the level not filled by all the electrons of the same type. For example, Na has its valence electron in a $3s$, $n = 3$, $l = 0$, type of orbit. It could have another electron in the same type of orbit, but with opposite spin. Hence its s , $l = 0$ level is only half-filled. This is indicated by the shading in the diagram of Fig. 123*a*. The second type is for a metal like Ca with all $4s$, $n = 4$, $l = 0$ states filled with electrons, as in Fig. 123*b*. Another would be copper, where the $4s$ shell is only half-filled, but the $3d$, $n = 3$, $l = 2$, shell of ten electrons is complete, as shown in Fig. 123*c*. The last type is the element Co, shortly preceding Cu in the periodic table. Co has its $3d$ shell only six-tenths filled, as shown in Fig. 123*d*.* For metals like Cu and Mn, Fe, Co, and Ni, there is a rather free exchange between the $3d$ and $4s$ levels, and in the metallic state the $3d$ electrons act as free valence electrons.

* Note that the $4s$ shell for Co is not shown in Fig. 123*d* as it is for Cu in 123*c* merely for convenience. It should, in general, be included.

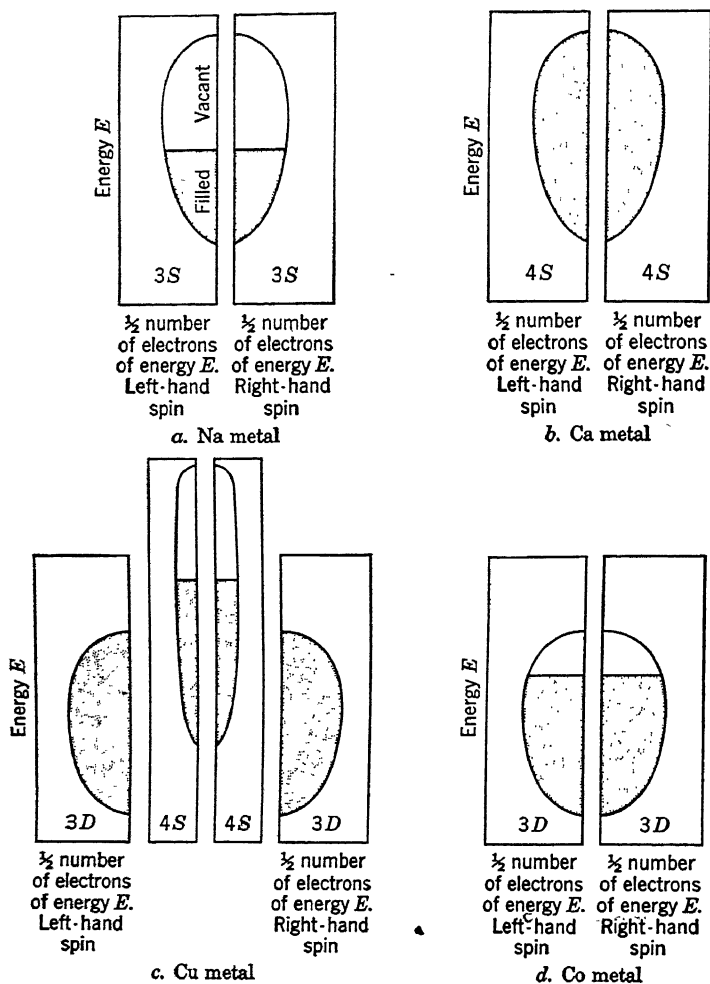


FIG. 123. Energy band contours in different metals.

Returning now to Na it can be seen how the paramagnetism of this element develops. For consider the diagram of Fig. 123a reproduced in Fig. 124a and that this represents the energies when no magnetic field is present. The levels for right- and left-hand spin electrons are identical and the top of the level of right- and left-hand spin electrons in both groups is the same. Now let a magnetic field be applied. The energy level diagram with the electrons in the left-hand spin group happens to have its electrons naturally so aligned that they are parallel with this applied field. That is, they are in a position of *minimum potential energy*. Thus the left-hand spin energy

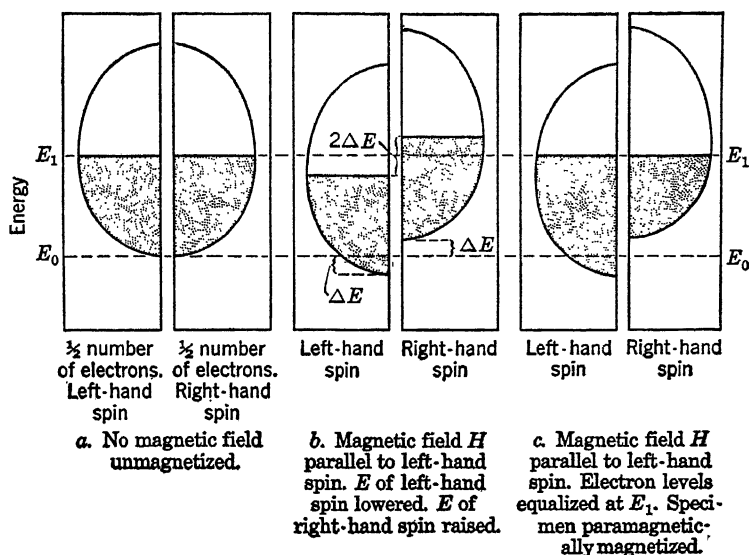


FIG. 124. Paramagnetism of Na metal.

level is, by the imposed field, *lowered* by that field, relative to the right-hand spin level. The relative amount of lowering will be the greater the magnetizing field. This lowering is indicated in Fig. 124b. But the electron levels of right- and left-hand spins cannot exist side by side with the tops of their filled levels at different energies because of the interaction of the electrons. This means that as indicated in Fig. 124c the right-hand electrons flop over to the left-hand group until the top of the level is the same in both groups. When this occurs we see that the magnetic field, H , by lowering the level corresponding to its direction, has resulted in more left-hand spin than right-hand spin electrons. That is, *the metal is magnetized*.

The magnetization is determined by the contour and energy of the levels, by the number and the density of the electrons, and by the field. Calculations for metals like Na, etc., give satisfactory values for the paramagnetic susceptibility of the metal. It is to be noted that elevation in temperature will counteract such alignment of spins by increasing exchange of electrons between the two groups. Thus, above certain critical temperatures this paramagnetism disappears. The paramagnetism is one caused entirely by electron spins. Orbits are not involved. Thus, if the gyromagnetic ratio for the paramagnetism of such metals can be measured it will have a value $\frac{e}{m}$, and not

$g \frac{e}{2m}$, as is the case with the other orbital type of paramagnetic substances.

Ferromagnetism. In 1914, S. J. Barnett measured the gyro-magnetic ratio of iron. In 1915 this ratio was again measured with another technique by Albert Einstein and Arthur Erich Haas. As techniques in these difficult measurements improved over the years, the value of these ratios was found to be $\frac{e}{m}$. This at once indicates that *ferromagnetism is caused by electron-spin moments of free electrons.*

In discussing the various types of electron levels in the metallic

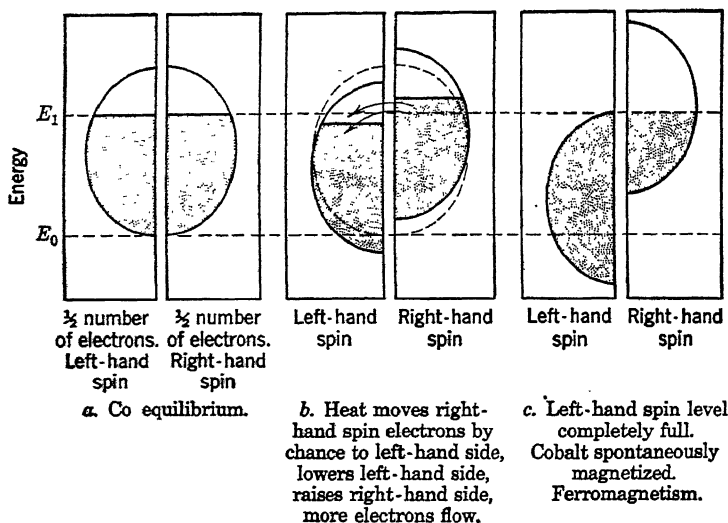


FIG. 125. Spontaneous magnetization of Co.

state the case of Co was indicated in Fig. 123d. It was also pointed out that the electrons in the uncompleted shells were among the free "valence" electrons in these elements. Now, it is to be noted that for Fe, Co, and Ni these 3 *d* shells, which were less than half full in Cr and Mn, go from half full for Fe to filled with Cu. In such a group of electrons, which interact *very strongly* because of the crowding, there are certain powerful exchange forces active. Hence, if the electron groups with right- and left-hand spins are present as shown in Fig. 123d, and as again depicted in Fig. 125a, a strange thing happens. The levels in the 3 *d* shell are equal for left- and right-hand spins in Fig. 125a. But if as a result of heat motions a few of the electrons in excess migrate from the right-hand to the left-hand spin, the exchange forces *lower the energy level* of the left-hand group. Thus, more electrons pour from the right-hand to the left-hand level, which sinks as this happens until it is full. At this point the top of the left-hand spin level coincides with the top of the partially filled right-hand spin group. The metal has *spontaneously magnetized itself with left-hand*

spins, in the amount of the difference in numbers of electrons in the two levels. Analogously, had the migration caused by heat been from left-hand spin to right-hand spin the right-hand level would have filled and sunk as for the left-hand level. Thus, in the latter case, *spontaneous magnetization in the sense of the right-hand spins results*. The spontaneous left-hand magnetization is shown in Fig. 125*b* and Fig. 125*c*.

This means that Fe, Ni, and Co, or alloys of Mn, Cr, and Cu, which in the metallic state furnish (1) enough densely packed electrons to give powerful exchange forces, and (2) enough electrons between the two levels completely to fill one level with electrons of one spin and still have some electrons left over, will *spontaneously magnetize themselves*. They do this *in the absence of any magnetic field*. They will do it *only if the temperatures are below some critical value*, above which exchange forces cannot take control against heat migrations of electrons. This spontaneous magnetization can occur only for the elements listed. Although the triad elements Ru, Rh, Pd and Os, Ir, Pt are filling in higher uncompleted shells (and should thus show similar action), the exchange forces appear not to be adequate to cause the phenomenon with these elements.

Thus it can be concluded that ferromagnetic behavior is caused by spontaneous orientation of the electron spins in a certain group of elements with proper electron configurations. It differs from metallic paramagnetism in that it is *spontaneous* and needs no imposed field. It is noted that the direction of the spontaneous magnetism, i.e., right-hand or left-hand spins, is governed by pure chance in any given specimen. It may be this factor among others that leads to the spontaneous magnetization of any extensive specimen in the oppositely oriented domains discussed on page 23. Since the further explanation of ferromagnetic behavior is treated at that place, all that is needed here is to indicate why and how the spontaneous magnetization causing domains arises.

It is seen that now, after some 350 years of observation, mankind has at last arrived at a plausible picture of the various types of magnetic behavior.

CHAPTER XVIII

THE MAGNETIC CIRCUIT

90. CONCEPT OF MAGNETOMOTIVE FORCE AND DEFINITION OF RELUCTANCE

The rapid evolution of electromotive and electrogenerative machines about 1870 made a development of methods of calculation of the various fields produced by electromagnetic devices imperative. A very convenient method of treatment was devised largely by H. A. Rowland, an American physicist, in 1873. He made use of the idea that a current flowing in a conductor produces a magnetic field, that is, a flux of magnetic lines of force through the circuit. To drive a unit magnet pole around a circuit against the field representing this flux requires work. In analogy to the work done by an electromotive force in driving current through a circuit, Rowland called the work to carry the magnet pole once around the magnetic circuit, or system, the *magnetomotive force*. By dividing magnetomotive force by flux in analogy to dividing electromotive force by current a quantity Z , called the reluctance of the magnetic circuit, is obtained. This is equivalent in its mathematical behavior to the quantity called resistance in the flow of electrical currents. Since means are devised to calculate the reluctance of the elements of a circuit, as was the case for resistance, the reluctance of a circuit can be determined. This allows the flux to be calculated if the magnetomotive force is known. If the flux is given then the magnetomotive force needed to give that flux can be calculated. It also happens that the reluctances in series are additive and when placed in parallel they follow the same laws as resistances in parallel. It is thus possible to compute the flux in branched circuits of all sorts. It must be borne in mind, however, that *the analogy is purely a formal one, for in currents there is actually a transfer of electricity whereas in magnetism there is no actual flow.*

With this outline of the situation we can proceed to a quantitative formulation of the problem with illustrative applications. It was found in Chapter XVI that the work to carry a unit pole about a coil of n turns carrying a current i_a was given by $\frac{W}{m} = W_1 = 4 \pi n i_a$, where i_a is in absolute units. If the current is spoken of in amperes then $i_a = \frac{i}{10}$ and the work W_1 per unit pole becomes $W_1 = \frac{4 \pi n i}{10}$. Thus the work is proportional to ni , that is, to the ampere-turns.

This work (per unit pole in analogy to work per unit quantity in the case of current (electromotive force) is called the *magnetomotive force*.

Thus magnetomotive force or m.m.f. = $\frac{4 \pi ni}{10} = \frac{4 \pi}{10}$ ampere-turns.

As a result of m.m.f. in a circuit a current i flows. In analogy, a magnetomotive force produces a flux of ϕ lines in the magnetic circuit.

For the current i and R , the resistance, a constant of the materials, their distribution and dimensions in the circuit. In the case of the

magnetomotive force we now write $\frac{\text{m.m.f.}}{\phi} = Z$, where Z is called the

reluctance of the circuit. Z represents the equivalent of resistance (magnetic resistance) to the flux ϕ in a circuit. This is also a characteristic constant of the magnetic materials, their distribution and dimensions in the circuit.

It has become the custom in recent years to redefine magnetomotive force as the actual ampere-turns, m.m.f. = ni and to write

$$\frac{\frac{4 \pi}{10} m}{\phi}$$

The $\frac{4 \pi}{10}$ then comes into the equation as a constant divisor and the

equation is written

$$\frac{\text{m.m.f.}}{\phi} = \frac{Z}{\frac{4 \pi}{10}} = \frac{1}{\frac{4 \pi}{10}}$$

91. THE VALUE OF THE RELUCTANCE

It is possible at once to deduce the value of the reluctance for a circuit. Consider a complete magnetic circuit as that shown in Fig. 126. The lines of force can be traced all around the circuit. The work done in threading once around the circuit for unit magnet pole is $\frac{4 \pi ni}{10}$. This work, however, is the sum of the products Hdl all

around the circuit, i.e., force H on unit pole times distance. Here H is the value of the field in a small length of the circuit dl within which H is sensibly constant. Thus

$$\frac{W}{m} = W_1 = \int_0 Hdl = \frac{4 \pi ni}{10}$$

where the sign \int_0 means the integral, or sum, of the quantities Hdl

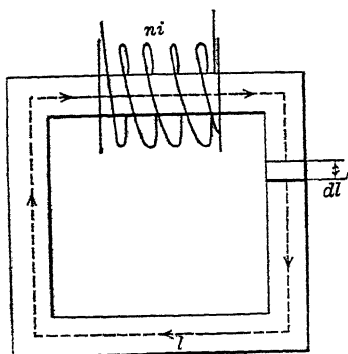


FIG. 126.

around the circuit. In a general H varies along the circuit. This integral is called the *line integral of the field*. It is of great importance in all problems dealing with magnetic circuits of any sort, electrical or mechanical. For a circuit of uniform cross-section, a tube of force for which the total number of lines is constant, we can write $\phi = BA = \mu HA$, where B is the induction, A the area of cross-section (which may vary), and μ the permeability. Thus $H = \frac{\phi}{\mu A}$. In the equation above be-

comes

$$W_1 = \frac{4\pi}{10} \text{ m.m.f.} = \int_0 H dl = \frac{4\pi}{10} \text{ m.m.f.} = \phi \int_0 \frac{dl}{\mu A}.$$

Thus

$$\frac{\text{m.m.f.}}{\phi} = Z = \frac{10}{4\pi} \int_0 \frac{dl}{\mu A}.$$

In general, μA is not constant, and integration must be carried out over the whole circuit. However, the circuit may be broken up into sections where μA is constant over a length l , so that $Z = \sum \frac{l}{\mu A}$ may be written. This says that Z is the sum \sum of the terms $\frac{l}{\mu A}$ in the

circuit, in which over a length l , A is constant so that $\int \frac{dl}{\mu A} = \frac{l}{\mu A}$.

Thus it is seen that in practice in a magnetic circuit it is frequently possible to express Z as the sum of a group of component reluctances

$$Z_1 = \frac{l_1}{\mu_1 A_1}, Z_2 = \frac{l_2}{\mu_2 A_2}, Z_3 = \frac{l_3}{\mu_3 A_3}, \text{ and so forth.}$$

Since $Z = \frac{l}{\mu A}$ a close analogy to resistance is again apparent, for with resistance $R = R_0 \frac{l}{A}$, where R_0 is similar to $\frac{1}{\mu}$, and l and A are

strictly analogous. More accurately μ is equivalent to $\frac{1}{R_0} = \sigma_0$, so that permeability can be considered a sort of specific magnetic conductivity. It may also be noted that the reluctances Z in series are additive as are resistances R . For two reluctances in parallel it may again be noted

that as for resistances $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ so with reluctances $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$. Thus the values of the reluctance for a magnetic shunt can easily be computed. In general, then,

$$\phi = \frac{4 \pi \text{ m.m.f.}}{10 \sum \frac{l}{\mu A}} = \frac{4 \pi \text{ ampere-turns}}{10 \sum \frac{l}{\mu A}}.$$

Applications. Three types of problems may be considered. (1) The calculation of ϕ for a given circuit such as those depicted below in Figs. 127a and 127b may be attempted. (2) Again it may be desired to find a given flux ϕ in a region, and having fixed on the form of circuit necessary to know how many ampere-turns will be needed to give the flux. (3) Finally it may be found necessary to calculate the effect of a magnetic shunt.

Let us consider the circuit of a horseshoe-shaped magnet as shown in Fig. 127a, with the coil having an ampere-turn value ni , with an area of cross-section A_1 , a permeability μ_1 , and a length l_1 in the horseshoe part. The iron keeper separated from the magnet by a small air gap at each pole of the horseshoe has a length l_2 , an area A_2 , and a permeability μ_2 . The length l_3 of each of the two air gaps is so short that the area may be considered equal to A_1 . For these air gaps $\mu = 1$, and it is seen that in this case most of the reluctance comes in the gap as μ_1 and μ_2 are of the thousands. The solution* of the problem is then merely

$$\frac{4 \pi ni}{10 \left(\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{2 l_3}{A_1} \right)}.$$

2. It is desired to calculate the ampere-turns ni necessary in a coil about the horseshoe-shaped magnet of Fig. 127a so that the magnet will just support the iron keeper carrying a weight W_1 at its center. The problem is further complicated by the additional fact that owing to the air gaps at the two ends of the keeper there is a leakage of lines

* This solution for simplicity assumed that μ_1 and μ_2 were known. Actually, there is at hand for a metal only a $B - \mu$ curve derived from the initial magnetization $B - H$ curve for the metal. If B , the induction, is known, the μ corresponding to it can at once be read from the curve. To solve this problem as it is usually met, a method of successive approximations must be gone through. If ϕ is assumed constant, i.e., with no leakage at the air gaps, then one has $\phi = A_1 B_1 = A_2 B_2$. If l_3 is large, so that it predominates, it is necessary to solve for a value of ϕ' , using ni and l_3 . From the relation above one can solve for B_1' and B_2' . Then one can get values of μ_1' and μ_2' from the curves. With these a new value of ϕ , ϕ'' , which is more accurate, can be obtained. Then inserting the new values of μ_1'' and μ_2'' a better value of ϕ is obtained. If l_3 is not very important some likely value of μ_1' and μ_2' is chosen from the $B - \mu$ curve and successive approximations are then worked.

of force in fractional amount f (say of 10 per cent, or $f = 0.1$) which do not enter the keeper through the area A_1 of contact with the horseshoe.

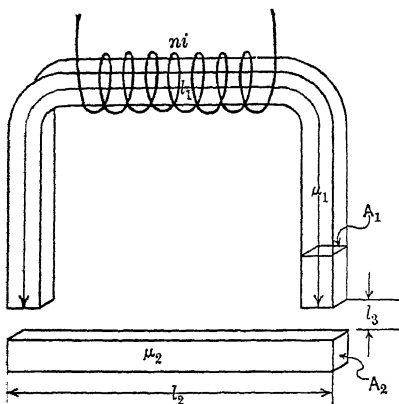


FIG. 127a. Calculation of flux through magnet and keeper.

In this problem as in the design of practical electromagnets the areas and dimensions of the parts $l_1 A_1$ and $l_2 A_2$ of the horseshoe and keeper are dictated by expediency or circumstances. In addition to obtain the values of μ_1 and μ_2 a B - H curve has been run on a piece of the iron. From the B - H curve a B - μ curve can be plotted for the material giving the values of μ appropriate to the values of B obtained from computations to come. The total weight of the keeper and its load W_1 be called W in dynes, where g is the acceleration of gravity. One may then proceed as follows.

The force Mg is borne by the magnetic traction at the ends of the horseshoe. Thus each end supports $\frac{Mg}{2}$ dynes.

it will be seen that $f_A = \frac{Mg}{2} = \frac{AB^2}{8\pi}$. Here A is the area A_1 of contact and B is the induction B_3 entering the keeper over an area A_1 . Thus, from $\frac{Mg}{2} = \frac{A_1 B_3^2}{8\pi}$, one can evaluate B_3 .

With a value of B_3 it can be realized that the flux φ_1 in the armature is $A_1 B_3$ plus the fractional loss of flux f in the air gap. Thus $\varphi_1 = A_1 B_3 \frac{1}{1-f}$ can be evaluated; and since $\frac{\varphi_1}{A_1} = B_1$ it is possible from the

μ - B curve to evaluate μ_1 . Now since by definition $ni = \frac{10}{4\pi} Z\varphi$, it can at once be written that the value of ni required to give the flux φ in A_1 is $n_1 i_1 = \frac{10}{4\pi} \varphi_1 \frac{l_1}{\mu_1 A_1} = \frac{10}{4\pi} A_1 B_3 \frac{1}{1-f} \frac{l_1}{\mu_1 A_1}$. Now to drive a flux $\varphi_3 = A_1 B_3$ across the two gaps requires $n_3 i_3$ ampere-turns, given by

$$n_3 i_3 = \frac{10}{4\pi} \varphi_3 \frac{2 l_3}{A_1} = \frac{10}{4\pi} A_1 B_3 \frac{2 l_3}{A_1},$$

since $\mu_3 = 1$ for air.

Finally a flux $A_1 B_3 = \varphi_2$ enters the keeper. Since this has an area A_2 the value of B in the keeper is $B_2 = \frac{\varphi_2}{A_2} = \frac{A_1 B_3}{A_2}$. From B_2

μ_2 is evaluated for the keeper using the μ - B curve. Then it at once follows that the ampere-turns $n_2 i_2$ needed to maintain a flux φ_2 in the keeper is

$$n_2 i_2 = \frac{10}{4\pi} \varphi_2 \frac{l_2}{\mu_2 A_2} = \frac{10}{4\pi} A_1 B_3 \frac{l_2}{\mu_2 A_2} = \frac{10}{4\pi} A_2 B_2 \frac{l_2}{\mu_2 A_2}.$$

Whence to maintain the various fluxes throughout the circuit the total value of ni required is $ni = n_1 i_1 + n_2 i_2 + n_3 i_3$ or

$$ni = \frac{10}{4\pi} A_1 B_3 \left[\frac{1}{1-f} \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{2 l_3}{A_1} \right].$$

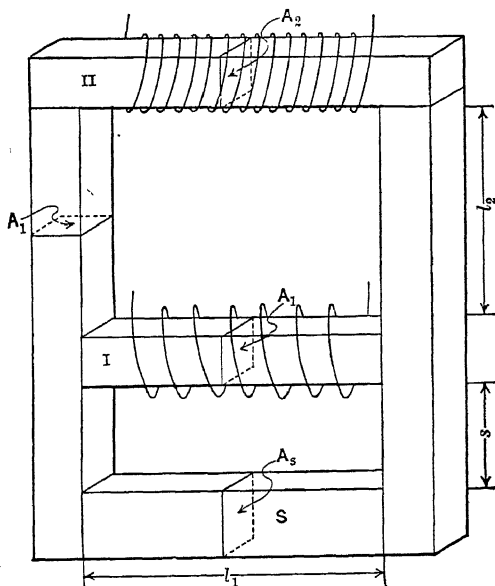


FIG. 127b. Calculation of magnetic shunt for a transformer.

Then from the character of the available power supply and considerations of cooling, etc., the value of i and hence of n can be determined.

3. There is a third type of problem concerning the design of a magnetic shunt. Consider the transformer frame shown in Fig. 127b. It is required that neglecting magnetic leakage a shunt be designed so that a given flux φ_2 goes through the secondary branch of area A_2 while x times the flux goes through the shunt A_3 . In addition, it is required to find the ampere-turns needed in the primary to accomplish this end. Such branched circuits are occasionally required in magnetic control mechanisms. In this magnetic circuit the length l_1 and area A_1 of the primary are known, the length l_2 and area A_2 of the secondary branch are known, and the area A_3 of the shunt is known, but its length l

remains to be determined. It is assumed that the μ - B curve for the iron is known. Let us for analysis set $x = 3$. It could as well be 2 or 0.5 or $\frac{1}{3}$. One may then at once write down the following relations:

$$\varphi_s = x\varphi_2, x = 3;$$

therefore $\varphi_s = 3\varphi_2$. Whence, as $\varphi_s + \varphi_2 = \varphi_1$,

$$\varphi_1 = 4\varphi_2 = \frac{4}{3}\varphi_s.$$

This then enables the values of B and μ to be determined as follows:

$$B_s = \frac{\varphi_s}{A_s} = \frac{3\varphi_2}{A_s}; \text{ whence from the table } \mu_s \text{ is evaluated.}$$

$$B_1 = \frac{\varphi_1}{A_1} = \frac{4\varphi_2}{A_1}; \text{ from the table evaluate } \mu_1.$$

$$B_2 = \frac{\varphi_2}{A_2}; \text{ from the table evaluate } \mu_2.$$

Then the reluctances Z_1 and Z_2 may be evaluated as follows:

$$Z_1 = \frac{l_1}{\mu_1 A_1}; \quad Z_2 = \frac{l_2}{\mu_2 A_2}.$$

To evaluate the reluctance Z_s , the length l_s must be known. Now the primary circuit gives a certain magnetomotive force which drives the flux through Z_2 and Z_s . Hence we can write

$$Z_2 \varphi_2 = \frac{4\pi}{10} \text{ m.m.f.} = \varphi_s Z_s.$$

$$\text{Thus } \frac{\varphi_2}{\varphi_s} = \frac{Z_s}{Z_2} = \frac{1}{3}. \quad \text{Hence } Z_s = \frac{1}{3} Z_2 = \frac{l_2}{3 \mu_2 A_2} = \frac{l_s}{\mu_s A_s}.$$

$$\text{This evaluates } l_s \text{ as } l_s = \frac{\mu_s A_s Z_2}{3}.$$

It is next required to evaluate the m.m.f. to cause this flux. The total reluctance Z of the circuit is the sum of the reluctance Z_1 and the reluctance Z_p , the parallel reluctance of Z_2 and Z_s . Thus $Z = Z_1 + Z_p$, where Z_p is the sum of Z_2 and Z_s in parallel, which is $\frac{1}{Z_p} = \frac{1}{Z_s} + \frac{1}{Z_2}$
or

$$Z_p = \frac{Z_s Z_2}{Z_s + Z_2} = \frac{\frac{1}{3} Z_2^2}{\frac{4}{3} Z_2} = \frac{1}{4} Z_2.$$

$$\text{Thus } Z = Z_1 + \frac{1}{4} Z_2, \text{ so that } ni = \frac{10^7}{4\pi} \varphi_1 Z = \frac{10}{4\pi} 4\varphi_2 (Z_1 + \frac{1}{4} Z_2)$$

Thus we have determined the length of the shunt l_s needed to give the required reduction in flux and in addition have calculated the ampere-turns to give the required flux ϕ_2 .

92. DEMAGNETIZATION, LEAKAGE, STRAY FLUX, EFFECT OF AIR GAPS, ELECTROMAGNETS, LIFTING POWER OF MAGNETS

We now come to the questions of demagnetization, leakage, stray flux, the effect of air gaps, the structure of electromagnets, and the lifting power of magnets.

1. Demagnetization. By looking at a magnet it is seen that while outside the magnet the lines of force run from north to south, inside the lines of force run from north to south also. The direction of the external field of the magnet is therefore such as to tend to cause the domains to reorient themselves (see Fig. 128). The result is that,

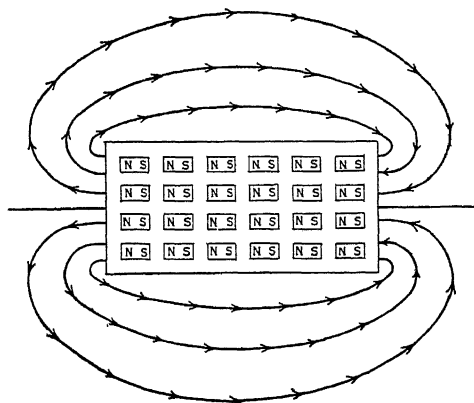


FIG. 128. Demagnetizing field of a short magnet.

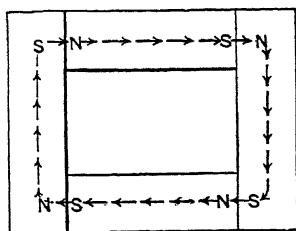


FIG. 129. Action in keepers on magnets.

owing to the very high permeability inside the iron and the very low permeability of the air outside, the tendency of the flux outside a bar magnet is to demagnetize the bar. If the bar magnet is provided with a complete circuit of iron of high permeability, the demagnetizing action will not exist, and the magnet will retain its magnetism indefinitely because the induced poles produced tend to oppose the demagnetizing field, as shown in Fig. 129. Thus a short isolated bar magnet will lose its magnetism, whereas a horseshoe magnet with a keeper will not lose its magnetism. The effect is the greater the shorter and broader the bar magnet since a greater portion of the magnet is exposed to the field. The effect is small for long thin magnets and still less for horseshoe magnets.

A striking example of the demagnetization can be shown by the

experiment depicted in Fig. 130. The horseshoe-shaped magnet M is made of very soft iron. It has two coil windings C about its vertical legs. A very soft iron yoke Y provided with a scale pan and weights W can be lifted up to contact and the switch S closed. The current produces a lifting force sufficient to support some 5 pounds on W .

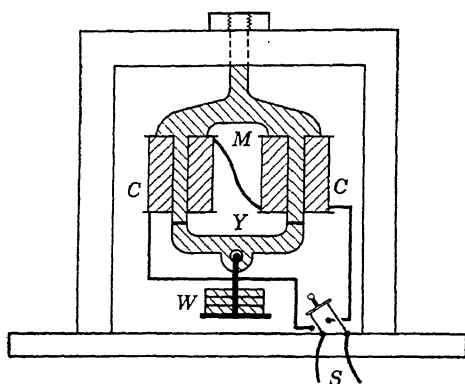


FIG. 130. Illustrating demagnetization in the absence of a keeper.

If S is opened Y remains in place for quite a long time. If the yoke Y is separated from C by hand for an instant after the current is off, it will be found that M will no longer support W , even without weights. Separation has completely demagnetized the systems.

2. Stray Effects and Leakage. If the horseshoe magnet with a keeper separated by an air gap from the ends of the magnet is carefully regarded, it is obvious that, if the keeper is some distance away from the end of the magnet, the flux of lines of force through the air from one pole to the other may be as great as the flux from one pole to the keeper and back to the other pole. Thus the lines of force will go straight across the air gap rather than go through the keeper. (See Fig. 131*a*.) When the keeper is in contact with the ends of the magnet, if the contact were perfect, there would be no stray flux. Between these two states, as the air gap is increased, all degrees of loss of flux or leakage can occur, as shown in Fig. 131*b*. These must be taken account of when, as above, circuits are considered.

3. The Effect of Air Gaps. Besides increasing the leakage, the effect of an air gap on the hysteresis loop, or the B - H curve, is to make higher fields essential to produce the same degree of magnetism. The B - H curve will thus take on an elongated shape instead of the more compact shape. This is because μ for air is small and to get the same flux the m.m.f. must be increased. The area of the hysteresis loop is, however, not increased for the hysteresis, for empty space is 0.

4. The Structure of Electromagnets. Electromagnets are of two types. Where a high resistance is permissible and a small current

flow is required, the magnet is wound with a large number of turns of as low a resistance as possible. Then for a small current, the magnetic effect is a maximum. Under some conditions, where it is possible to get very high currents, as from some of the modern high-current generators, the number of turns wound on a magnet is comparatively small but the currents are prodigious. To prevent heating with such currents, the armature windings are made of tubes through which water is circulating to carry off the excess heat. In general, these are wound about magnetic circuits with poles close together. Fields of the order of 50,000 gauss can be obtained with such magnets in short air

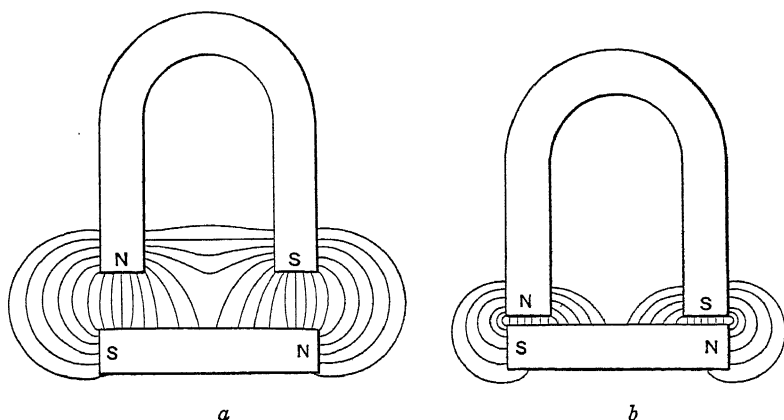


FIG. 131. Leakage caused by air gaps.

gaps. Higher magnetic fields have been obtained by Kapitza for short intervals of time by the discharge of a high-capacity storage cell of low internal resistance through a solenoid. The limitation on magnetic fields depends on the saturation value of the magnet material, and on the ampere-turns. The latter are limited by the problem of removing the i^2R heat in the windings, which presents formidable difficulties in industrial use.

5. Lifting Power of Magnets. When two magnets are brought close together, the plane surface of the magnet, or magnet and keeper with induced magnetism, *act like magnetic shells** with a density of mag-

* A magnetic shell is merely a thin piece of magnetized material the area of which has linear dimensions of much greater magnitude than has its thickness. If this shell is magnetized with all its north poles on one surface and all the south poles on the other, it constitutes a magnetic shell. Unlike a conducting metal surface with σ charges on it (the *lines of force of which all emerge from the surface normal to it* as there is no field inside the conductor), the σ north (or south) poles on the surface of the shell send but one-half their lines of force outward into the space, the other half ($2\pi\sigma$) lines running from the north (south) to the south (north) pole through the shell as the shell is a magnetic conductor.

netization per square centimeter equal to σ . The number of lines going out from one surface to the other is $2\pi\sigma$ per square centimeter. The force on the density σ of one pole exerted by the $2\pi\sigma$ lines from the other pole is the product of those two and is expressed by $f = 2\pi\sigma^2$. If there is no divergence of lines of force, which is the case when the two surfaces are parallel and close to each other, H (the field strength) equals $4\pi\sigma$ in the gap. Thus

$$\sigma = \frac{H}{4\pi} \quad \text{or} \quad f = \frac{H^2}{8\pi}.$$

Thus the force between the two surfaces of area 1 sq cm is

$$f = \frac{H^2}{8\pi}.$$

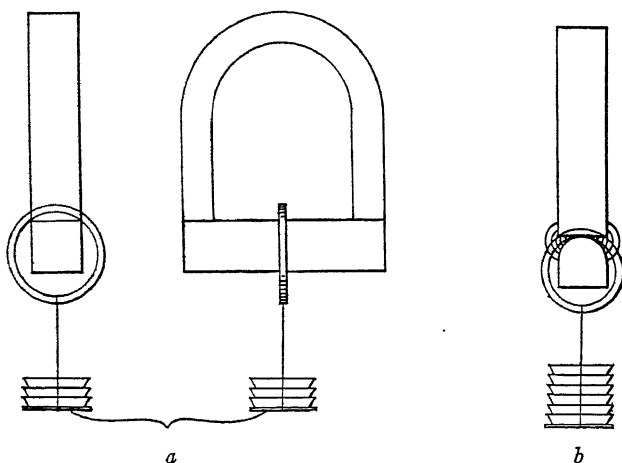


FIG. 132. Experiment showing the lifting power of a magnet to be proportional to AB^2 .

If there is no divergence of the lines, that is, if there is no stray field, which is true for a short gap, $H = B$, where B is the induction through the iron, it follows that,

$$f = \frac{B^2}{8\pi}, \text{ as the force per square centimeter.}$$

Thus if the induction through the iron is known the force of traction between magnet poles can be calculated; or, vice versa, if the lifting power of a magnet is measured, the induction through it provided the air gap is small can be determined. If the area of the poles is A the force f per square centimeter must be multiplied by A to give the total force f_A on area A . That is, the total force is $f_A = \frac{AB^2}{8\pi}$.

The fact that the attractive force between a magnet and a piece of iron is proportional to the area times the square of the induction is nicely illustrated by the experiment shown in Figs. 132*a* and 132*b*. In this experiment, which depicts the side view of an iron horseshoe magnet supporting a keeper carrying a scale pan with weights by means of string, it will be observed that if the keeper is perfectly flat (Fig. 132*a*), it will support less load than the keeper with the curved surface in contact with the iron, illustrated in Fig. 132*b*. In Fig. 132*a* while the area A is large the induction is the total flux of the magnet through the keeper divided by the area of the keeper in contact with the magnet. In Fig. 132*b*, since the area A of the keeper in contact with the magnet is a small fraction of the area in contact in the first case, the induction, except for the small loss of lines of force due to leakage, is increased in inverse proportion to the area of contact; for here, except for the flux lost to leakage, most of the flux is found to cover a much smaller area and thus B is increased accordingly. Therefore, B^2A is distinctly greater in the second case than in the first, as shown by the greater traction.

CHAPTER XIX

INDUCED ELECTRIC CURRENTS

93. DESCRIPTION OF INDUCED CURRENTS, LENZ'S LAW, DIRECTION OF INDUCED E.M.F.

In 1831 Faraday made a discovery the consequences of which have led to the development of modern industrial electrical engineering. As it had been observed that a current produced a magnetic field, he decided that, vice versa, a magnetic field should produce a current. In looking for this effect he found that in fact under certain conditions a magnetic field will produce a current in a conductor. From the first discovery of the effect, in ten days of experimentation conducted over a three-month period, he had completed an investigation of the phenomenon, and the present-day knowledge of the phenomenon hardly exceeds what he discovered in those ten days.

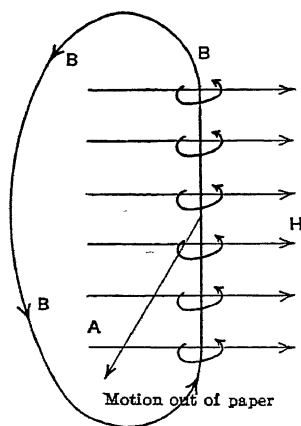


FIG. 133. Direction of an induced current caused by a conductor cutting lines of force.

This is called *electromagnetic induction*. The direction in which this current flows can be deduced at once from the law of conservation of energy.

The law for the direction of flow of current is known as *Lenz's law*. The current set up in the conductor must flow in such a direction as to build up a magnetic field opposing the change in the circuit. Otherwise, the magnetic field set up by the current flowing would increase the motion and generate more current. This process could not go on indefinitely increasing the motion and building up more and more current at the expense of nothing. In a more general form, Lenz's law could be stated by saying that the current induced by any magnetic changes in the regions around a conductor flows in such a way as to produce a magnetic field which opposes the change or results in mo-

tions such as to oppose the change. This is illustrated for one case in Fig. 133. The magnetic field H is given by the horizontal lines. The vertical wire is being moved outward from the paper as indicated by the arrow A . The electrical current generated flows around the circuit in the direction of the arrows B and goes upward in the conductor. The magnetic field produces currents in the wires so that the side of the wire outward from the paper has the lines of force flowing in the same direction as the applied external field H . From Chapter XVI it will be seen that this results in a force on the wire urging it into the paper.

94. ELECTROMAGNETIC INDUCTION AS A CONSEQUENCE OF ELECTRON THEORY

That a conductor, especially a metallic conductor, should have an e.m.f. induced in it when it moves in a magnetic field so as to cut lines of force is to be expected on the electron theory of metals. Analysis of the problem from this viewpoint leads to an evaluation of the e.m.f. of self-induction.

Consider the block of metal shown in the diagram of Fig. 134. It is in a uniform magnetic field H lying in the plane of the paper. Its length is l . It is moved out of the paper perpendicular to H with a velocity v . If the metal is regarded as a rigid lattice of positive ions (+) with free electrons (−) moving among the ions it can be reasoned as follows. The positive charges constitute a current out of the paper of strength i_a . This current suffers a force of magnitude $f = Hi_al$ upward. This force acts on the ions of the lattice. The electrons also suffer a force downward as they have a negative charge. These, however, are *free to move in the lattice*, and as a result a negative charge will accumulate at the lower end until the potential becomes so great that the force of repulsion of the field produced by the electron migration balances the force due to the motion. These forces can be calculated.

Since $f = Hi_al$ and since $i_a = e/t$, where e is the number of units of charge carried past a point in a time t , $f = H \frac{e}{t} l = He \frac{l}{t}$. But $\frac{l}{t} = v$, the velocity of motion. Hence the force of a field H on a charge e moving with a velocity v is Hev . $f = Hev$. Again the electrons pile up on the lower end until the electrical field X reaches such a value that the force $f = Xe$ on each electron equals Hev , i.e., equilibrium is reached for the condition that

$$Xe = Hev \quad \text{or} \quad X = Hv.$$

But the field X is the potential difference, or better the electromotive force E , divided by the distance d over which it exists, i.e., the top and bottom of the block. Hence $X = E/d$, whence $E = Hvd$. Now

vd is the area swept out in space by the block in unit time, i.e., it is $\frac{dA}{dt}$. $E = H \frac{dA}{dt}$. But $\frac{HdA}{dt} = \frac{d\phi}{dt}$, i.e., it is the time rate at which lines of force are cut by the conductor. Thus it follows that $E = \frac{d\phi}{dt}$.

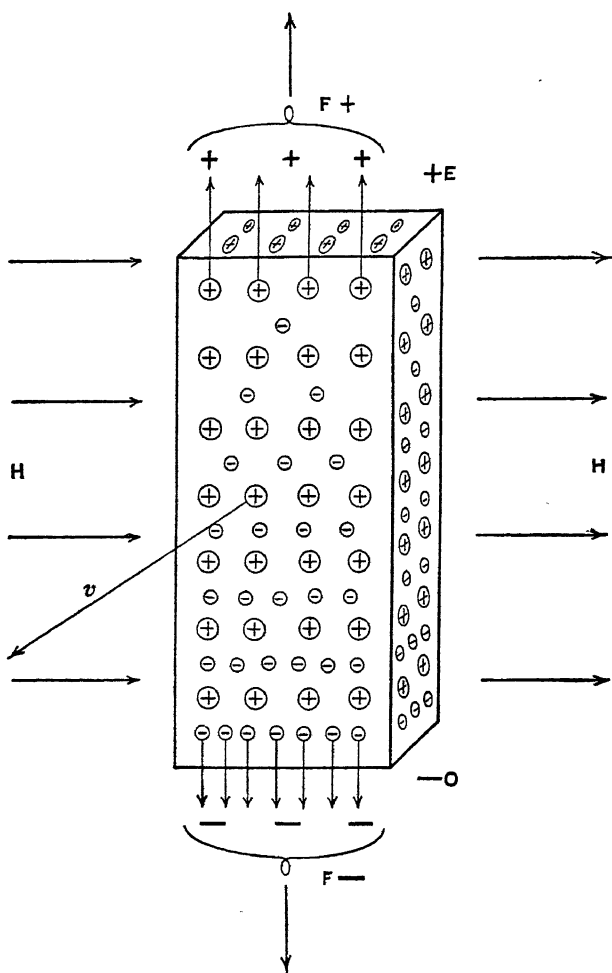


FIG. 134. Production of an e.m.f. when a conductor cuts magnetic lines of force H . The block moves out of the paper along v . The small circles represent electrons and these are forced downward.

This result can also be deduced by classic methods from energy considerations as will be seen below. Thus it is clear that the electromagnetic induction is the direct result of the fact that a magnetic field exerts a force on a current and that one of the electrical units in a metal

is free to move. The action is no longer mysterious and is to be expected. Hence the totality of electromagnetic interactions lead back to the fact that one of the manifestations of electrical charges in motion is the presence of a magnetic field. This is the cause of the electronic magnetic fields, of the Ampèrian atomic magnetic fields, and, in fact, of all magnetic action.

95. MAGNITUDE OF INDUCED E.M.F.

The magnetic field H of Fig. 135 is represented by dots, the lines of force running out of the paper. Two conducting wires connected to a resistance coil form a sort of track. The distance between them is l . A conductor C is laid perpendicular to these and is moved a distance dx parallel to the rails. Suppose that moving the conductor generates an e.m.f. of value E . Suppose also that this e.m.f. causes a current i_a to flow through the wire during the time dt while the conductor is moved dx . The work done $W = Edq = Ei_a dt$. On the other hand, if the current i_a is flowing through the conductor as a result of the induced e.m.f., it is known that the force f , acting on i_a resulting from the field H which opposed the motion will be given by $f = i_a l H$, therefore the work done $W = f dx = i_a l H dx$. Thus, since this work is the same work as that done against the e.m.f.,

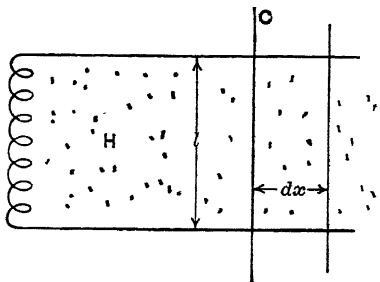


FIG. 135. Calculation of the magnitude of an induced e.m.f.

$$Ei_a dt = i_a l H dx,$$

or

$$E = Hl \frac{dx}{dt}.$$

Now $l dx$ is A , the area of the field swept out, but the area of the field swept out is nothing else than the change in area inside the circuit, and multiplying this change in area by the number of lines of force per unit area the *change in flux* through the circuit is obtained. If $Hl dx = d\phi$, is the change in flux, the equation becomes

$$E = \frac{d\phi}{dt},$$

that is, *the e.m.f. is rate of change of flux in the circuit*. It is, however, to be noted that this change of flux *always* implies a component of relative motion between conductor and lines of force *normal* to the lines of

force; thus generation of an e.m.f. is associated with the concept of cutting across lines of force.

96. DEFINITION OF E.M.U. OF POTENTIAL

The *electromagnetic unit of potential* can then be defined by saying that *it is the potential difference produced by a given rate of cutting of lines of force. By this equation $E = 1$ e.m.u. of potential when one line of force is cut per second. The volt is the e.m.f. produced when 10^8 lines of force are cut per second.* This is a definition of potential on the electromagnetic system which is a perfectly legitimate one and could replace the fundamental work definition. Since, however, this definition is directly related to the work done by means of the work definition which has already been given, and inasmuch as it connects this unit to the absolute c.g.s. system, the original definition of the unit of potential difference on the electromagnetic system is the one which will be adhered to.

The definition given in terms of cutting is frequently found in engineering textbooks. Although correct, it represents a particular and narrow professional point of view. Potential differences are generated by many other means and potential, both static and electromagnetic, is of much larger scope. A modern engineer should be aware of, and be able to use, these broader implications.

97. E.M.F. PRODUCED BY CUTTING LINES OF FORCE

It makes no difference whether the conductor moves and lines of force are cut or whether the lines of force move and cut the conductor. This fact has led to a loose definition of the process by which the e.m.f. is produced, for it is responsible for the statement that the e.m.f. is produced and is proportional to the rate of change of magnetic flux through the circuit. As seen from the discussion of electron theory, there must *always be a relative motion between the field and the conductor having a component normal to the field*, i.e., "cutting the lines of force." Hence it is *not the rate of change of flux, but the cutting of the lines of force associated with the rate of change of flux, which causes the e.m.f.*

Thus $\frac{d\phi}{dt}$ is merely an expression for the rate of cutting of the lines of force. The loose definition that the e.m.f. is equal to the rate of change of flux again results from too close an adherence to engineering practice. Actually as was seen from the nature of the electronic interpretation of electromagnetic induction it requires a normal component of relative motion of conductor and field to produce an e.m.f. It is, in fact, possible to obtain an e.m.f. by cutting lines of force while the flux is not changed through the circuit. In Fig. 136, N is the north pole of a soft iron bar magnetized by means of the coil C at its lower

end. From this pole lines of force emerge and swing around downward to the south pole. W is an inverted L-shaped wire loop the lower end of which dips into a trough of mercury marked Hg , pivoted so that the loop can turn freely about N . Between N and the mercury trough there is connected a galvanometer G through a wire. It will be observed that as the wire W rotates around the magnet as an axis a current is generated in the circuit and is recorded by the galvanometer. It is obvious that, since the field around the magnetic pole N is uniform, the flux through the wire W and through the circuit as a whole is constant. That is, the lines of force through the circuit do not change. The upper part of the wire W , however, is continually cutting the downward component of the lines of force emerging from N , with the result that an e.m.f. is caused while there is no change of flux in the circuit.

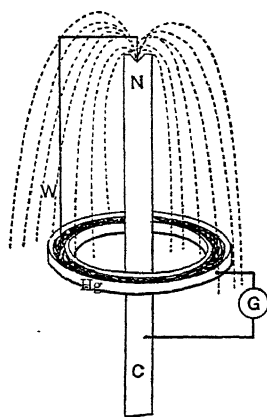


FIG. 136. Experiment showing the e.m.f. to be generated by a cutting of the lines of force.

98. PRINCIPLE OF THE EARTH INDUCTOR

In an era of cyclotrons and betatrons, where large-scale magnetic fields require measurement as well as where there is need of measuring uniform weak magnetic fields to a high degree of precision, the application of the principle of electromagnetic induction in the form of the earth inductor is invaluable. This instrument or its variant the flip coil adapted to various forms of recording is the basis of most magnetic field measurement today. This excepts special methods such as the fields of ships, geophysical magnetic studies, or the measurement of fields varying rapidly with distance. Its principle must be studied. Suppose that a rectangular coil of n turns be mounted about a horizontal axis, and assume it placed so that its axis is perpendicular to the lines of force of the earth's field. Then, as it rotates about its axis, the components of the n wires of the rectangular coil parallel to the axis of the coil cut the earth's lines of force at right angles. An e.m.f. is then induced in the coil and may be measured as detailed below. This instrument is called the *earth inductor*. Assume an earth inductor having a coil of n turns, and that the earth's field is H_T . Assume that the area of the coils is A . As the earth inductor is rotated the areas perpendicular to the earth's field swept out in equal intervals of time dt by the component of the wires parallel to the axis of the inductor are not equal. Thus the e.m.f. generated is not constant, and

the instantaneous value of the e.m.f. takes on the form

$$E_{\text{inst.}} = \frac{d\phi}{dt} = H_T n \frac{dA}{dt}.$$

In the first quarter-turn the two ends of the rectangular coil together cut an area of magnetic flux A equal to the area of the coil, that is, they cut $H_T A$ lines of force. In the second quarter-turn, the lines of force cut are again $H_T A$ for each wire of the conductor, thus $2 H_T A$ lines of force are cut in a half-turn. In the second half-turn, lines will be cut with the coils-moving in the opposite sense, and the current would flow in the opposite direction. The current can, however, be rectified by a commutator. We may consequently confine ourselves to the motion for one-half a turn only. The average e.m.f. for n wires for one-half turn is then

$$\bar{E} = \frac{2 n H_T A}{\tau},$$

where τ is the time of one-half revolution. Since $\bar{i}_a = \frac{\bar{E}}{R_a}$,* then

$$\bar{i}_a = \frac{2 n H_T A}{R_a \tau}.$$

But

$$\bar{i}_a \tau = q = \frac{2 n H_T A}{R_a} \text{ in absolute e.m.u.}$$

Now q can be measured by what is known as the ballistic galvanometer (see Chapter XXIII). This is an instrument in which the time of deflection is large compared to the time during which q passes. It is also an instrument in which there is little damping. In such an instrument

$$q = \frac{K' T \theta \rho^{1/2}}{2 \pi}.$$

Here K' is the galvanometer constant, and θ is the deflection of the galvanometer when the field H_T is cut, ρ is the damping factor, and T is the period of the galvanometer. Thus, q being known, H_T can at once be obtained. This quantity H_T which is measured is the *total intensity* of the earth's magnetic field. The horizontal component H may then be obtained by knowing the angle of dip.

This principle can be applied by using a small coil of accurately known area and number of turns rotated continuously by air turbine or remote control by a motor. Motor fields must not interfere. In

* Note that the current is in absolute units i_a only if \bar{E} and R_a are in absolute e.m.u.

this case the sinusoidally varying e.m.f. can be picked up by slip rings and read on an a-c instrument. Otherwise it can be rectified and the peak value of the e.m.f. can be read directly. The precision can easily be 0.1 per cent or better. The results give field strength in absolute measure.

99. APPLICATION OF ELECTROMAGNETIC INDUCTION TO A SECONDARY COIL ABOUT A PRIMARY IN WHICH THE CURRENT CHANGES

Another application of great practical importance is one in which an e.m.f. is induced in one coil of wire by a change of current in a neighboring coil. This is used in the measurement of magnetic induction and in estimating the values of magnetic field strengths. Consider the long coil represented in Fig. 137 by n_1 , with n_1 turns of wire in it. Assume that its area is A and that it has a current of i amperes flowing through it. About it is a second small coil of n_2 turns. The ends of this small coil are connected to a galvanometer G of the ballistic type. While the current i is flowing through the large coil n_1 there is a flux φ equal to

$$\frac{4 \pi n_1 i A}{10 l}$$

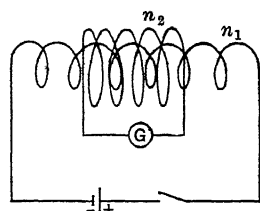


FIG. 137. E.m.f. induced in a secondary coil.

through it, where i is in amperes. If the current in n_1 be suddenly broken the magnetic flux goes from φ to 0 in a time τ which is unknown. The time depends on the way in which a spark is drawn out at break. Thus on breaking the circuit the average change in flux per unit time is

$$\frac{\varphi}{\tau} = \frac{4 \pi n_1 i A}{10 l \tau}.$$

In one turn of the coil n_2 the average e.m.f. in volts will then be

$$E = \frac{4 \pi n_1 i A}{10^8 \times 10 l \tau}.$$

For n_2 turns the average e.m.f. in volts will be

$$E_2 = \frac{4 \pi n_1 i A n_2}{10^9 \tau l}.$$

If τ is known, only the average value of E_2 , the e.m.f. in the coil n_2 , may be computed, for it is obvious that as the current i dies out nonuniformly, the rate of cutting of lines of force will not be uniform, and thus E_2 will not be uniform. The study of i as a function of time which would be needed in an accurate investigation of this circuit is

only possible by means of the cathode-ray oscillograph. For practical purposes this may not be possible, and the same device mentioned in connection with the earth inductor, i.e., the ballistic galvanometer, must be used. Thus, if E_2 is short-circuited through a ballistic galvanometer of resistance R the average current flowing is then given by

$$\bar{i}_2 = \frac{\bar{E}_2}{R} = \frac{4 \pi n_1 n_2 i A}{10^9 \tau l R}.$$

The quantity Q of electricity in coulombs which flows is therefore

$$Q = \bar{i}_2 \tau = \frac{4 \pi n_1 n_2 i A}{10^9 R l}.$$

Since the ballistic galvanometer measures Q by means of the expression $Q = \frac{k \theta_m T \rho^{1/2}}{2 \pi}$, where k is the figure of merit and θ_m is the deflection in millimeters at 1 meter, one can write

$$\theta_m \left(\frac{k T \rho^{1/2}}{2 \pi} \right) = \frac{4 \pi n_1 n_2 i A}{10^9 R l} = \frac{n_2}{10^8 R} \varphi,$$

where $\varphi = \frac{4 \pi n_1 A i}{10 l}$ and represents the flux through the circuit in the absence of iron. Therefore $\theta_m = C \varphi$, where C is a constant involving k , T , $\rho^{1/2}$, n_2 , and R , which can be evaluated from the constants of the circuit but is more conveniently determined otherwise. To do this it is necessary to know only A , n_1 , and l and by breaking or making the circuit with different values of i evaluate φ and observe θ_m . From the average ratio of $\frac{\theta_m}{\varphi}$ one at once evaluates $C = \frac{\theta_m}{\varphi}$. Then it is clear that an observed value of θ_m can be associated with φ through the relation $\varphi = \frac{\theta_m}{C}$.

Now the field in the coil is $H = \frac{\varphi}{A} = \frac{4 \pi n_1 i}{10 l} = H_1 i$, where H_1 is the field per unit current $H_1 = \frac{4 \pi n_1}{10 l}$, which is a constant of the coil n_1 and is readily calculated. Thus H and φ can be determined from the current i and θ_m , the deflection. If now a piece of iron of area of cross section A' is placed inside n_1 , the deflection θ_m for a given change in current from 0 to i will differ from that in the absence of iron resulting from the induction B in the iron which adds BA' to the flux, in place of HA' . Thus the new flux $\varphi' = H(A - A') + BA' = \frac{\theta'_m}{C}$. Since H is known from the change in current i , the equation can be solved for B .

Actually, owing to hysteresis, the iron has a sort of magnetic memory, and it is first necessary to wipe this out by placing the iron within an alternating electrical field and removing it slowly. This leaves the iron unmagnetized; see page 242. Next, however, the field H must be continuously increased in a regular fashion by adding increments to i and observing the number of new lines of force $\Delta\phi'$ added each time. The procedure then is the following. The demagnetized iron is placed in the coil and its ends are magnetically "shorted" by a large yoke of soft iron to prevent the *demagnetizing* field of the magnetized iron from changing the flux in A .* Then the current i is made and increased in small increments Δi . That is, the battery has its circuit through n_1 closed by closing a key k_1 . This leaves in the circuit with the battery a high resistance R which allows a small current Δi_1 to flow in n_1 . Then by closing a series of other keys k_2, k_3, k_4 , etc., in succession, the resistances R_1, R_2, R_3 composing R are short-circuited out increasing the current by $\Delta i_2, \Delta i_3, \Delta i_4$. The currents are read by ammeter. As the keys k_1, k_2, k_3 , etc., are closed the deflections $\theta_{m1}, \theta_{m2}, \theta_{m3}$ are observed. On this basis the following record is obtained:

Close Switch Deflection	k_1 θ_{m1}	k_2 θ_{m2}	k_3 θ_{m3}	k_4 θ_{m4}
$\Delta\phi$	$\Delta\phi_1 = \frac{\theta_{m1}}{C}$	$\Delta\phi_2 = \frac{\theta_{m2}}{C}$	$\Delta\phi_3 = \frac{\theta_{m3}}{C}$	$\Delta\phi_4 = \frac{\theta_{m4}}{C}$
ϕ	$\Delta\phi_1$	$\Delta\phi_1 + \Delta\phi_2$	$\Delta\phi_1 + \Delta\phi_2 + \Delta\phi_3$	$\sum_1^4 \Delta\phi$
i (amperes)	Δi_1	$\Delta i_1 + \Delta i_2$	$\Delta i_1 + \Delta i_2 + \Delta i_3$	$\sum_1^4 \Delta i$
$H = iH_1$	$H_1\Delta i_1$	$H_1(\Delta i_1 + \Delta i_2)$	$H_1\sum_1^3 \Delta i$	$H_1\sum_1^4 \Delta i$
$H(A - A')$	—	—	—	—
$\phi - H(A - A')$	—	—	—	—
$B = \frac{\phi - H(A - A')}{A'}$	—	—	—	—

In this table the symbol $\sum_1^4 \Delta\phi$ stands for the sum of $\Delta\phi$ from $\Delta\phi_1$ to $\Delta\phi_4$.

To complete the cycle and obtain the complete hysteresis loop, it is necessary to close switches k until B reaches a saturation value, then to begin opening the switches in reverse order, beginning with k_n , the last switch closed, until all switches are open. Next the current from the battery must be reversed and the switches k_1, k_2 , etc., closed again in succession. As H becomes increasingly negative, θ_m will become negative and the switches must be closed until the negative saturation

* If the specimen is more than twenty times its diameter in length the demagnetizing field ceases to be very important if the field coil giving H is not too large in diameter relative to the sample.

value of B is reached. Finally the switches must be opened in reverse order, beginning at k_n and going to k_1 . Reversal of the battery again and a closing of k_1, k_2, k_3 , etc., in succession should, on reaching saturation, complete the cycle and close the B - H curve.*

100. APPLICATION OF ELECTROMAGNETIC INDUCTION TO FARADAY'S DISK

The Faraday Disk. Faraday was the first to show the continuous production of an e.m.f. by cutting a magnetic field. The square H in Fig. 138 represents a magnetic field perpendicular to the plane of

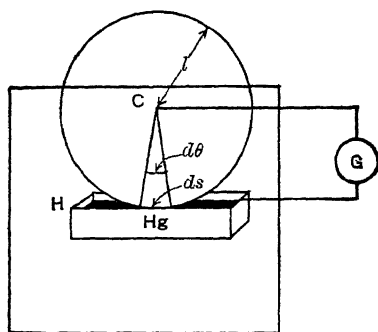


FIG. 138. Faraday disc.

a copper disk of radius l mounted on a horizontal axis C . This disk is rotated so that the angle $d\theta$ is swept out in the time dt . Its lower end dips in the mercury in the trough Hg . The axle and the mercury trough are connected to a galvanometer G . The e.m.f. E_{volts} generated will be then given by

$$E_{\text{volts}} = \frac{H}{10^8} \frac{dA}{dt},$$

where dA is the area swept out in dt . Now

$$\frac{dA}{dt} = \frac{1}{2} l \frac{ds}{dt},$$

where ds is the element of arc swept out in the time dt . But

$$\frac{ds}{dt} = \frac{ld\theta}{dt}.$$

* Since errors in such a procedure are cumulative, the hysteresis loop may not close unless the work is very carefully done. Heating of the wires by the current, thus changing R and i after readings were made, can also spoil the curve.

Therefore

$$\frac{dA}{dt} = \frac{\frac{1}{2}l^2 d\theta}{dt}.$$

Now θ equals 2π radians for one revolution. For N revolutions, it equals $2\pi N$ radians. Thus

$$\frac{d\theta}{dt}$$

for N revolutions per second is equal to $2\pi N$. Whence

$$E_{\text{volts}} = \frac{H}{10^8} \frac{1}{2} l^2 2\pi N.$$

As

$$\pi l^2 = A \text{ (the area of the disk).}$$

$$E_{\text{volts}} = \frac{HAN}{10^8}.$$

This expression is the prototype equation for generation of an e.m.f. by cutting lines of force by rotational motion.

CHAPTER XX

SIMPLE DYNAMOS AND MOTORS

101. DERIVATION OF EQUATION FOR SIMPLE DYNAMO

In the simple dynamo there is a rectangular coil of wire rotating about the axis A normal to the plane of Fig. 139; B and B' are the cross sections of the ends of the rectangle viewed parallel to the axis at two instants dt apart. The sides of the rectangle parallel to the paper are of no concern as they move parallel to the magnetic lines of force. The magnetic field in which the system moves is indicated by the horizontal lines H . The axis is perpendicular to these lines as is the vertical AC from which the angle θ of the armature with its neutral position C is measured.

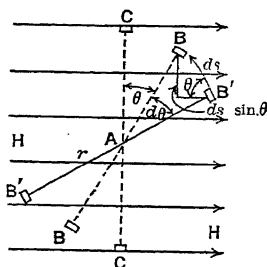


FIG. 139. Theory of the simple dynamo.

Assume that there are n_1 turns in the coil, that the length of the armature coil normal to the paper is l , and that the distance from the axis to the coil is r . Assume a field strength H . E_{dt} , the instantaneous electromotive force, is then $n_1 \frac{d\phi}{dt}$, where $d\phi$ is the change of flux through the coil during dt , and n_1 is the number of turns in the coil. Assume that the dynamo armature revolves through an arc ds in the time dt , when it makes an angle of θ with the neutral position. If it moves a distance ds , the distance moved perpendicular to the field is $ds \sin \theta$. Since the length of the conductor perpendicular to the field H is l , the area swept out in dt is $lds \sin \theta$.

The field has a strength H and there are n_1 turns in the armature, whence

$$n_1 \frac{d\phi}{dt} = \frac{n_1 H l ds \sin \theta}{dt}.$$

Now $ds = r d\theta$, where r is the radius of the coil, and

$$E_{dt} = n_1 \frac{d\phi}{dt} = 2 r l \frac{n_1 H \sin \theta d\theta}{dt},$$

for the whole coil since the wires B' at the other end of the coil are also producing an equal e.m.f. in the same direction. Again,

$$\frac{d\theta}{dt} = 2 \pi N,$$

where N is the number of revolutions per second, so that E_{dt} in volts,

$$E_{vdt} = \frac{2 r l N H 2 \pi n_1 \sin \theta}{10^8}.$$

Now $2 r l n_1 = A$ where A is the total area of the n_1 turns in the armature. Then

$$E_{vdt} = \frac{2 \pi A N H \sin \theta}{10^8}.$$

It is thus seen that the e.m.f. generated by the dynamo depends on the angle θ , on the number of revolutions per second N , on the field strength H , and on the total area A of the armature.

In practical discussions variants of this equation are used. One of these replaces A by $n_1 A_1$. The quantity $2 \pi H A_1$ which is 2π times the flux through the area of one turn of coil, is designated by the symbol $U = 2 \pi H A_1$, and $2 \pi H A = n_1 U$, whence it follows that

$$E_{vdt} = \frac{n_1 N U}{10^8} \sin \theta. \text{ This is convenient as it makes } E_{vdt} \text{ vary with}$$

$n_1 U$, and N , the principle variables. The expression is 0 when $\sin \theta = 0$, i.e., at the point C . It is a maximum when $\sin \theta = 1$ and $\theta = 90^\circ$.

Thus we have $E_{\max.} = \frac{n_1 N U}{10^8}$.

102. DYNAMOS, ALTERNATING CURRENT, AND DIRECT CURRENT

We now turn to the question of dynamos and generators. It was found above that the e.m.f. of a simple dynamo was given by the expression

$$E_{vdt} = \frac{n_1 N U}{10^8} \sin \theta.$$

Where N is the number of revolutions of the dynamo per second, n_1 the number of windings in the armature, and U a constant of the dynamo. It is seen that the e.m.f. varies as the sine of the angle. If the e.m.f. generated by the coil were picked up by two brushes on the armature mounting making sliding contact with two rings connected to the two ends of the armature, one would actually get an electromotive force which varies sinusoidally with the angle of the armature and consequently with the time, as seen in Fig. 140A. Dynamos of this sort are used to a very great extent in generating power for transmission.

Power of this sort can be generated at high potential and transmitted to stations where the potential is lowered by transformers to enable it to be used for lighting circuits. The maximum value of the

e.m.f. is given by the peaks of the curve when $\theta = 90^\circ$. It is $E_{\max.} = \frac{n_1 N U}{10^8}$ volts, as seen above.

In order to generate a *direct* current from a dynamo a *commutator* must be used. With the simple dynamo just referred to, the commutator consists of a ring cut in two equal segments by saw cuts parallel to the axis at the ends of a diameter of the ring. The one segment is insulated by a small gap from the other segment. One end of the armature winding is attached to one segment; the other is attached to the other segment. Two fixed brushes at opposite ends of a diameter attached to the armature mounting make contacts with the ring. Thus as the armature turns through the position where it starts

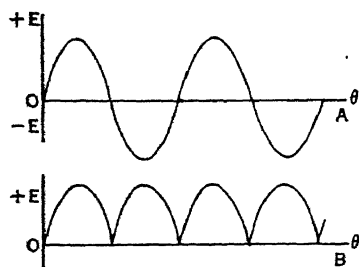


FIG. 140. E.m.f. of a simple dynamo. A, From collector rings. B, Rectified by a split ring commutator.

to cut the lines of force in such a way so as to reverse the flow of current, and thus reverse the potential, the commutator brushes make contact with the opposing segment. Hence one brush continually picks off only the positive side of the sine curve and the other brush picks off only the negative side. Since, however, there are positions where $\sin \theta$

is 0 it is seen that there are points in the revolution at which the potential difference between these two brushes is 0. If the potential difference is plotted against the time with such a commutator a variation will be found as indicated in Fig. 140B instead of the sinusoidal variation shown in Fig. 140A.

Such an electromotive force would give a *fluctuating* current through the circuit. To overcome this the number of coils in the armature are multiplied. By multiplying the number of coils, which also means multiplying the number of segments in the commutator, the commutator segments are continually picking off the electromotive force of the coil that is passing through its maximum e.m.f.

This multiplication of armature coils results in smoothing out the electromotive force given during one revolution; the result is that, instead of the sinusoidal e.m.f. given by an alternating-current dynamo, an e.m.f. of the form shown in Fig. 141 is given.

The magnetic field of a dynamo may be excited by the dynamo itself or it may be excited separately as occasion demands. Self-exciting dynamos depend on their residual magnetism for starting the excitation. There is also a class of dynamos so wound that the armature wires do not cut the field in the same fashion as for the simple dynamo of Fig. 139. The wires are wound around an iron core as shown in Fig. 142. When the axis of the core is parallel to the field H , the flux is

a maximum. When it is at right angles to the field, the flux is 0. Thus the current is produced by a changing flux through the armature windings produced in a slightly different manner. This principle is also used in some high-frequency generators in a somewhat different manner.

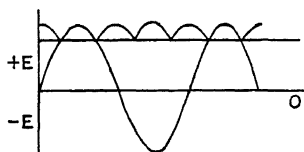


FIG. 141. Rectified alternating current using a split ring commutator and two coils at right angles.

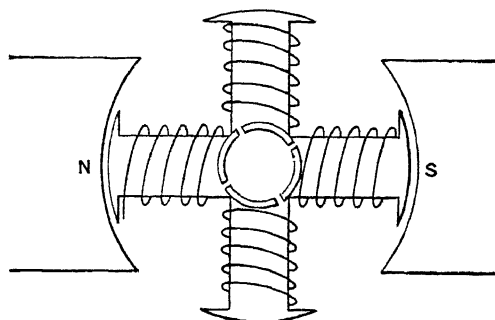


FIG. 142. Alternative method of winding an armature coil.

103. E.M.F. AND P.D. FROM A DYNAMO, EFFICIENCY OF A DYNAMO

If an e.m.f. of E volts is generated the current which comes out of the dynamo in accord with the same phenomenon discussed at the end of Chapter VII will be given by

$$i = \frac{E}{R_e + R_i}.$$

Here R_e is the external resistance of the circuit and R_i is the internal resistance of the windings. Again as in the case of batteries the potential difference V given across a certain external resistance R_e is related to the electromotive force by the equation

$$\frac{V}{R_e} = i = \frac{E}{R_e + R_i}.$$

If R_e is very large compared to R_i the V approaches E . For an open circuit ($R_e = \infty$), the V is equal to E . Furthermore, consider the expression

$$E = R_e i + R_i i,$$

and if V is equal to $R_e i$, therefore

$$V = E - R_i i.$$

It is thus seen that the potential V given depends on the e.m.f. E and on the internal resistance of the dynamo.

The power output of a dynamo is measured in watts. That is, it is measured in joules of energy per second. As will be remembered, the watt is the ampere multiplied by the volt, and the output of a dynamo would be V in volts times i in amperes. This is known as the *load on the dynamo*. The ideal efficiency of a dynamo is given by the expression

$$\begin{aligned}\text{Eff.} &= \frac{\text{Output in joules}}{\text{Input in joules of mechanical work}} \\ &= \frac{\text{Output in watts}}{\text{Mechanical input in joules per second}}\end{aligned}$$

Ideally then as Vi is the output, and as all the mechanical work put in should give rise to electrical energy, input then is given by Ei . Hence

$$\text{Eff.} = \frac{Vi}{Ei} = \frac{Vi}{Vi + R_i i^2}.$$

Thus the smaller the $R_i i^2$ term, the more efficient the dynamo. As the current is determined by practical requirements, R_i should be as low as possible. Practically one cannot write input = Ei , as there are other losses of energy besides the $i^2 R$ loss in the armature.

The difference between the output of a dynamo and the mechanical work performed on it is caused by certain losses. These losses are as follows:

1. Loss in the iron core of the armature due to eddy currents and hysteresis.
2. Loss from the iR drop in the field excitation due to R , the resistance of the magnet coils.
3. Loss in the copper wire of the armature due to the $i^2 R_i$ heat production in the armature resulting from the flow of current against the internal resistance. If a heavy current is being delivered this loss is considerable. It is endeavored to make the internal resistance of the armature winding as low as possible.
4. Loss in friction at the bearings of the armature and at the brushes.

These losses all appear as heat, and it will be observed that operating generators are warmer than the room. In fact, it may be noted that most good generators are rated for a given temperature at which they should normally run. If they heat up beyond this point trouble should be looked for.

104. RECIPROCAL RELATIONS BETWEEN DYNAMOS AND MOTORS *

The reciprocal relationship of the motor and dynamo now becomes of interest. If a source of electrical potential is put across a dynamo, the current flowing through the armature of the dynamo causes the armature to rotate. The dynamo thus acts as a motor. On the other hand, if the armature of a motor is turned by mechanical means and excites the field coils, the lines of force are cut and an e.m.f. is generated. Whether the dynamo is turned by mechanical means or is driven as a motor, the generated e.m.f. is picked up by the commutator. In general, a motor run as a dynamo is not particularly efficient, and the same can be said of the reverse. The reason for this lies in the conditions underlying the usage of the dynamo and motor. The dynamo requires, as was previously seen, that the internal resistance R_i be as small as possible in order that the potential V maintained be as great as possible for the external resistance applied. It is essential that the number of turns in the motor be high to give a large back e.m.f. This will naturally increase the value of R_i .

If an external potential V is now placed across a motor the force acting on the armature causes it to rotate, and as the force produced is constant the motor armature accelerates. If another action, which will be discussed presently, did not occur, acceleration would continue until the armature flew to pieces or until friction on the bearings equaled the force acting on the armature. However, a rotating armature in the magnetic field of the motor produces an electromotive force, and by Lenz's law the electromotive force opposes the applied potential causing the motion. It is thus to be expected that the electromotive force so produced will change the *net potential* and therefore cause the motor to cease accelerating. Therefore when the motor begins to rotate it generates a *back electromotive force* E' which causes a current to flow in a sense opposite to that of the imposed potential difference. Since, under these conditions, the only source of potential in the motor other than the back electromotive force is the iR drop $R_i i$ in the armature windings, the equation of the motor can be written in the simple form,

$$V - E' = R_i i.$$

Here V is the applied potential and E' is the back e.m.f. This is merely an application of the second of Kirchhoff's laws to the armature of a motor. Consequently it follows that

$$\frac{V - E'}{R_i} = i,$$

* As this course is intended to familiarize the engineering student with the *principles* of electricity, it has been considered expedient to discuss only those applications to engineering which clearly illustrate the fundamental principles involved. As a result, this discussion will be confined to only the simple case of motors with separate field excitation.

and that

$$V = R_i i + E'.$$

105. POWER CONSUMPTION, EFFICIENCY, AND TORQUE FOR MOTORS

If both sides of the equation can be multiplied by i we have

$$iV = R_i i^2 + E' i.$$

The first term iV is the power which is put into the motor. The second term $R_i i^2$ is the power loss in the armature of the motor which goes to heating the motor, if eddy currents and other losses are excluded. The term $E' i$ is the power consumed in forcing the current i against the potential E' . This is the electrical power consumed in running the motor.

Accordingly we must write that power P is given by

$$P = \frac{W}{T} = \frac{fs}{T} = E' i,$$

where W is the work, f is the force acting, and s is the distance through which f acts in a time T . If a prony brake, consisting of a belt working

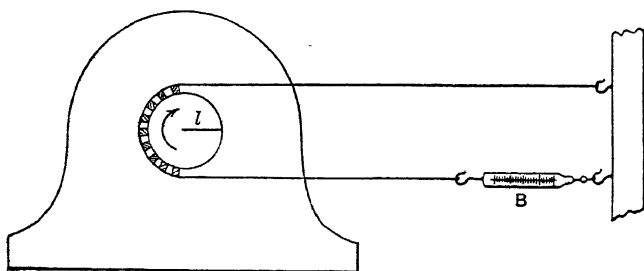


FIG. 143. The Prony brake.

on the pulley of radius l is used, the force f is given by the balance B of Fig. 143 and $\frac{s}{T} = 2\pi lN$, where l is the radius of the pulley and N is the number of revolutions per second. Hence

$$E' i = 2\pi N f l = 2\pi G N,$$

where G , the torque on the pulley, is given by $G = fl$. Since

$$E_{\text{vdt}} = \frac{n_1 NU}{10^8} \sin \theta,$$

the maximum value E_{max} , which is given in a d-c motor becomes

$$E' = E_{\text{max}} = \frac{n_1 NU}{10^8}.$$

Hence

$$E'i = \frac{n_1 NUi}{10^8} = 2 \pi GN,$$

and the torque $G = \frac{i n_1 U}{2 \pi 10^8}$. We may accordingly write

$$V = R_i i + \frac{n_1 NU}{10^8}.$$

From this it can be concluded that, as G decreases, i decreases. If then the applied potential V is kept constant and the torque G is reduced, decreasing the current i , the back e.m.f. $E' = \frac{n_1 NU}{10^8}$ must increase.

It can do so only by increasing the frequency of rotation N . Thus decreasing the load, or torque, on a motor reduces the current by increasing the back e.m.f., and the motor speeds up. In this action the motor efficiency is also increased. Regarding the expression for the ideal efficiency of the motor

$$\text{Eff.} = \frac{E'i}{Vi} = \frac{Vi - R_i i^2}{Vi} = \frac{V - R_i i}{V},$$

it is clear that as i decreases efficiency must increase.

It might be added that the *real* efficiency of the motor requires that the equation be written as follows

$$\text{Eff.} = \frac{E'i}{Vi_1},$$

in which i_1 is the total current fed into the motor, that is, $i_1 = i + i_f$, where i_f is the current taken to maintain the field, and i is the current supposedly consumed in useful work. If loss due to eddy currents, hysteresis, and friction occurs, the value E' will not be $E' = V - iR_i$, that which ideally would occur. Instead the motor will run more slowly, reducing N to give a lower value E'_1 of the back e.m.f. and thereby also increasing the current i drawn. E' would then be given by an equation of the form $E' = V - i(R_i + R_x)$, where R_x is an equivalent resistance such that the power loss to friction, hysteresis, and eddy currents is $P = i^2 R_x$, or $R_x = \frac{P}{i^2}$.

A motor must be so designed as to reduce the $R_i i^2$ loss and at the same time to get as high a back e.m.f., E' , as possible. In general, the practice is to make n_1 high, to produce a powerful field, U , and to run the motor at a high speed. The large number of turns n_1 make the resistance of the motor fairly high. The limitation on increasing the efficiency of motors by extending the quantities in the direction indicated above is set by the limit on the speed of the motor required

for practical purposes and the increase of R_i with too many turns. High-speed motors require gearing down for practical use.

106. STARTING BOXES

It is obvious that if we suddenly close a switch, putting a potential V on a motor, a current

$$i = \frac{V}{R_i}$$

flows through the armature windings. As the motor resistance is after all comparatively low the high voltage which is applied to the low resistance R_i before the motor begins to turn over may give a current great enough to melt the wires of the armature. When running at full speed the potential forcing current through the motor is $V - E'$. The current i is thus decreased, the $i^2 R_i$ heating is then very much less, and the motor operates safely. All heavy motors which are slow in gathering speed are therefore provided with a starting box. Such a starting box consists merely of a series of resistances which limit the maximum current i flowing through the motor armature. As the speed increases, each plug of the starting box gives a progressively lower series resistance and the motor is enabled to pick up speed without burning itself out.

CHAPTER XXI

INDUCTION

107. SELF-INDUCTION

Thus far in this book the student has been introduced to five essential electrical concepts, current, quantity, potential difference, resistance, and capacity. These, with a sixth — self-induction — now to be discussed, constitute the six basic concepts of electrical theory and practice. Of these the first three, current, quantity, and potential, are directly defined from experiment by force or work. The remaining three, resistance, capacity, and self-induction, result from ratios of the first three. They depend on the form or shape of the conductors and on properties of the materials from which the conductors and their surroundings are made. These will be considered at length at a later point and are mentioned on page 199 and discussed in section 153.

In 1831, while Faraday was engaged in the study of electromagnetic induction at his laboratory in the Royal Institution in London, an American school teacher in Syracuse, New York, was utilizing vacation periods when his classroom was not in use to carry on his own investigations. This teacher was Joseph Henry. Unaware of Faraday's work, he was studying the sparks drawn out of a coil of wire carrying a current when the current was broken. He found that these sparks were more powerful the more turns of wire he had and the more iron there was included in the circuit. A study of this phenomenon led him independently to the conception of electromagnetic induction and carried him further. The conclusion at which he arrived can best be formulated assuming that what we have learned about Faraday's work was known to Henry. A conductor or a circuit enclosing a space must, according to the laws of electromagnetic induction, yield an e.m.f. whenever there is a change in the number of lines of magnetic force in the space so as to cut the conductor. Furthermore, the direction of the induced e.m.f. must be such as to create a current opposing the change. Thus it can be expected that any conductor which carries a current producing a field within the area enclosed by it must *have an e.m.f. induced in itself of such a sense as to oppose any current change*. This can be called the *e.m.f. of self-induction*. It was the development of this concept that we owe to Joseph Henry.

The e.m.f. of self-induction has an exceedingly important property, for it acts at all times to produce currents which oppose any change in

the state of current in the system. Thus if a switch is closed to start a current flow in a coil, the growing field in the coil cuts the coil and produces an e.m.f. *opposing* the current. If there is a current in the coil and the connection is cut, the current as it dies out has its disappearing magnetic field cutting the coil. This gives an e.m.f. which would act to *maintain* the current. Thus the *self-induction of a circuit acts as an electrical inertia* striving to maintain the status quo of the circuit. As may at once be seen this action will exert a profound influence on all circuits where currents change with time.

The self-induction of a large electromagnet can most easily be shown by means of the arrangement given in Fig. 144. It is an experiment closely analogous to those of Henry with his sparks.

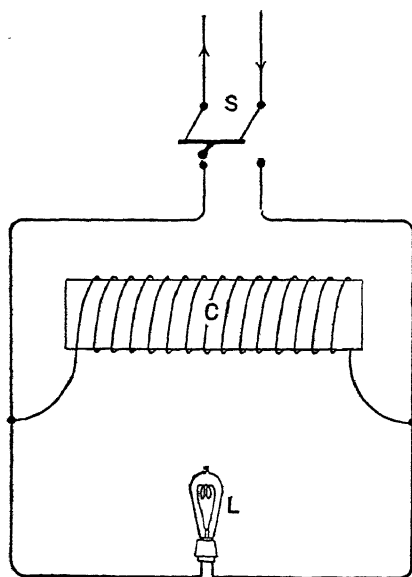


FIG. 144 Illustrating the effect of self-induction C on a lamp L .

A large iron-cored coil C of some thousand turns of low-resistance wire and having a continuous core of soft iron some 3 inches in diameter (not shown) is shunted by an incandescent lamp L . The switch S goes to the 110-volt d-c power mains. When the switch has been closed for some time the low resistance of C reduces the potential at S and across L , so that the lamp glows dimly. Suddenly *opening* S breaks the current in C and the e.m.f. of self-induction built up is sufficiently higher than the steady-state potential across L so that L flashes up brilliantly. A similar flash on *closing* S occurs in L for the reason that the e.m.f. of self-induction in C opposes the building up of the

current in C so that at first the e.m.f. of the 110-volt line acts in full force across L . Thus the lamp lights fairly brightly on *first* closing S . As the current builds up in C the lamp L dims. The flash on opening is much brighter than on closing since the rapidly collapsing field builds up a high e.m.f. of self-induction. The result of Henry's researches reached England in 1832 in time to prevent Faraday from making the discovery independently. Henry's claim to the discovery is recognized in naming the practical unit of self-induction for him.

It is now of interest to consider the nature of this new property of a circuit from a quantitative point of view and to define it precisely.

Assume a coil of wire of n turns about a magnetic circuit whose reluctance is Z . The flux ϕ in the circuit is, by Chapter XVIII, given

by $\phi = \frac{4 \pi n i}{10 Z} = \frac{4 \pi n i_a}{Z}$. If the circuit is broken in a time τ the flux ϕ disappears and cuts the n turns of the circuit, giving an e.m.f. of self-induction E which, according to Chapter XIX, has the average value $E = n \frac{\phi}{\tau}$. Actually the current dies out irregularly, and it is better to speak of the instantaneous value of E , E_i , defined by $E_i = n \frac{d\phi}{dt}$, where dt is so short that $d\phi$ changes at a uniform rate in the interval. Thus $\frac{d\phi}{dt} = \frac{4 \pi n}{Z} \frac{di_a}{dt}$, where di_a is the change in i_a causing the change $d\phi$ in ϕ . Accordingly we can write

$$E_i = n \left(\frac{4 \pi n}{Z} \frac{di_a}{dt} \right) = \frac{4 \pi n^2}{Z} \frac{di_a}{dt}.$$

Now $\frac{di_a}{dt}$ is a variable imposed from without, while $\frac{4 \pi n^2}{Z}$ represents a *constant* of the form, dimensions, and materials surrounding the circuit. It is at once seen that the ratio

$$\frac{E_i}{di_a/dt} = \frac{4 \pi n^2}{Z} = n \left[\frac{\frac{d\phi}{dt}}{\frac{di_a}{dt}} \right] = n \frac{d\phi}{di_a} = L,$$

a new *derived constant* of the circuit called the *coefficient of self-induction*. Its usefulness is manifest in that the e.m.f. E produced by *any* $\frac{di_a}{dt}$ is at once given by $E_i = L \frac{di_a}{dt}$. Hence L , the coefficient of self-induction, is defined as the ratio of $\frac{E_i}{di_a/dt}$, where E_i is the instantaneous induced e.m.f. and $\frac{di_a}{dt}$ is the rate of change of current producing it. Since

$$\frac{d\phi}{dt} = \frac{4 \pi}{Z} n \frac{di_a}{dt},$$

$$\frac{d\phi}{di_a} = \frac{4 \pi}{Z} n,$$

and

$$L = \frac{4 \pi}{Z} n^2 = n \frac{d\phi}{di_a}.$$

The expression $L = \frac{4 \pi}{Z} n^2$ applies to such forms of circuit in which Z

can be calculated and is thus somewhat restricted. It is, however, seen quite generally that $L = n \frac{d\phi}{di_a}$, where $\frac{d\phi}{di_a}$ represents the change in flux per unit current and can be used to compute L . If E_i and i_a , or Z , are in absolute e.m.u. then L will be in such units.

108. UNITS OF SELF-INDUCTION

It is at once clear from the equation $L = \frac{E_i}{di_a/dt}$ that the *absolute e.m.u. of self-induction is the self-induction which gives 1 absolute e.m.u. of potential when the current changes at the rate of 1 absolute e.m.u. of current per second*, or $L = 1$ e.m.u. if $E_i = 1$ e.m.u. and $\frac{di_a}{dt} = 1$ e.m.u. per second.

A dimensional analysis gives a clue to the dimensions of L in the absolute electromagnetic system. Now in the electromagnetic system $E = \frac{W}{q} = M^{1/2}L^{3/2}T^{-2}$, as seen on page 412. In the same system i has the dimensions $M^{1/2}L^{1/2}T^{-1}$, as seen on page 412, so that $\frac{di_a}{dt}$ has the dimensions $M^{1/2}L^{1/2}T^{-2}$. Hence the e.m.u. of self-induction is $L = \frac{M^{1/2}L^{3/2}T^{-2}}{M^{1/2}L^{1/2}T^{-2}} = L$ or a *length*. Thus the absolute e.m.u. of self-induction has the dimensions of a length, and hence in the c.g.s. system the e.m.u. is the *centimeter* of length.

The *practical* unit of self-induction is defined by

$$\frac{E}{di/dt} = \frac{\text{volts}}{\text{amperes/seconds}} = \text{henrys.}$$

Since a volt = 10^8 e.m.u., and the ampere/second = 10^{-1} e.m.u., the henry = $\frac{10^8}{10^{-1}} = 10^9$ e.m.u. = 10^9 cm.

The absolute e.s.u. of self-induction is given by

$$L = 1 \text{ e.s.u. if } E = 1 \text{ e.s.u. and } di/dt = 1 \text{ e.s.u./sec.}$$

Thus

$$\frac{E}{di/dt} = \frac{3 \times 10^{10} \text{ e.m.u.}}{1} = 9 \times 10^{20} \text{ e.m.u.} = 9 \times 10^{20} \text{ cm.}$$

$$\frac{3 \times 10^{10} \text{ e.m.u.}}{3 \times 10^{10} \text{ e.m.u.}}$$

To give an orienting idea as to magnitudes, it can be stated that an air-core coil of mean diameter roughly 10 cm (4 in.), length 5 cm (2 in.), and radial thickness some 5 cm (2 in.), having 3×10^3 turns of wire, will have about 0.7 of a henry self-induction. For a rectangular

iron core of $\mu = 1000$, with a length of 40 cm and an area of 20 cm² cross section with 400 turns, the self-induction is nearly 1 henry. For ordinary radio work inductances in the order of millihenrys, 10⁻³ henry, are used. For short-wave radio the inductances are of the order of 10⁻⁶ henry, i.e., microhenrys or 1000 cm, which are single-layer coils of a few turns and of 2 to 4 cm diameter.

109. NATURE OF SELF-INDUCTION

It is of interest to study further the character of self-induction with regard to the design of coils. The equation deduced indicates that L varies as the square of the number of turns of wire. L is inversely proportional to the magnetic reluctance of the circuit; thus it should be larger the larger the area of cross section of the core and hence coil, and the shorter the length of the circuit. It will, of course, vary directly with μ , the permeability. Thus, to get a high self-induction the number of turns must be large, their area must be large, and the coil should be compact and surrounded by a magnetic path of low reluctance. Since over any range of currents i_a , H , and thus μ vary

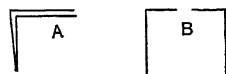


FIG. 145. Effect of shape on the value of self-induction.

appreciably, L must be specified for an iron-core circuit as $L = n \frac{d\phi}{di_a}$ at a specified i_a .

For the air-core coil the following analysis can be made. Consider Fig. 145, where the same length of wire is bent into circuits of two shapes, one a square and the other an inverted L. Now self-induction L depends on $\frac{d\phi}{di}$, the rate of change of flux produced by a

current change, or $\frac{di_a}{dt}$, i.e., L depends on $\frac{d\phi}{di_a}$. In the square circuit the flux ϕ enclosed represents a good share of the magnetic flux contributed by the wires surrounding the area and is all in the same sense. It is to be noted that the wires of an L-shaped figure are so close together that the field due to one wire on the outside of the area is nearly equal to and overlapping that of the other wire. Since these fields oppose each other and are superposed, the fields of the two wires will practically annihilate each other except throughout the very narrow space between wires. Since the area between the two wires in the L is negligible, $\phi = AH$ approaches 0. Hence in this case $\frac{d\phi}{di_a}$

is negligible and L is nearly 0. Such a winding is called a *noninductive* winding. It is used on nearly all good resistance coils. From this it is seen that for air-core coils the self-induction is dependent on the area enclosed by the wires.

For the single-layer air-core solenoid the self-induction is easily computed in an approximate fashion. Since $L = n \frac{d\phi}{di_a}$ and $\phi = AH$, where A is the area of cross section and H is the field in the solenoid, we can write $\frac{d\phi}{di_a} = A \frac{dH}{di_a}$, $H = \frac{4\pi ni_a}{l}$ so that

$$\frac{dH}{di_a} = \frac{4\pi n}{l}, \quad \frac{d\phi}{di_a} = \frac{4\pi nA}{l}, \quad \text{and} \quad L = \frac{4\pi n^2 A}{l}.$$

L will be in centimeters, if A and l are in square centimeters and centimeters. Actually there is for all coils a leakage of flux so that AH does not cut the whole n turns, especially if the coil is loosely wound and short. Nagaoka and others (see reference to Rosa and Grover) have calculated tables of the correction factor k to the equation above for a closely wound coil, as a function of the ratio of length to aperture. This allows L to be determined from $L = \frac{4\pi n^2 A}{l} k$, where k is a fraction.

A further study of $L = n \frac{d\phi}{di_a}$, where n is the number of turns, for air-core circuits is mathematically involved and beyond the scope of the book. The value of L can be computed very accurately for coils of certain definite forms, the calculations being as accurate as we care to make our mathematical approximations. These are given by Edward B. Rosa and F. W. Grover in *Scientific Papers*, U. S. Bureau of Standards S-169, Dec. 18, 1916, and by F. W. Grover in a book entitled *Inductance Calculations*, New York, D. Van Nostrand & Co., 1946.

A simple and useful equation for air-core coils, which is not very accurate, is expressed in the form

$$L = \frac{0.2 A^2 N^2}{3A + 9B + 10C},$$

where L is the coefficient of self-induction in microhenrys (10^{-6} henry), N = number of turns, A = diameter of the coil in inches, B = length of the coil in inches, and C = radial depth of the coil in inches.

110. MUTUAL INDUCTION

When two circuits, parts of a circuit, or two coils are near each other, and especially when the magnetic flux in one coil is linked to the other by means of a magnetic conductor, it is clear that a change of current in one coil will induce a change of current in the other coil or circuit, and vice versa. This interaction is ascribed to the *mutual inductance* of the two coils or circuits, and we speak of the *coefficient*

of *mutual induction* as linking them. The relations can again become clear by applying the simplifying assumption that the circuits are linked by a common iron core of reluctance Z to avoid a large loss of flux to stray fields, which happens in air-core coils. Consider a coil of n_1 turns, which will be called the primary, wound about the iron core, and have a secondary coil of n_2 turns also wound on the core. Then if a current change di_{a1} be made in the primary, an e.m.f. E_2 will be induced in the secondary coil n_2 . The change of current di_{a1} produces a change in flux $d\phi_1 = \frac{4 \pi n_1 di_{a1}}{Z}$ which threads n_2 , producing an e.m.f. of induction

$$E_2 = n_2 \frac{d\phi_1}{dt} = \frac{4 \pi n_1}{Z} n_2 \frac{di_{a1}}{dt}.$$

We now define

$$\frac{E_2}{di_{a1}/dt} = n_2 \frac{\frac{d\phi_1}{dt}}{\frac{di_{a1}}{dt}} = n_2 \frac{d\phi_1}{di_{a1}} = \frac{4 \pi n_1 n_2}{Z} = M,$$

as the coefficient of mutual inductance between the coils n_1 and n_2 .

That the inductance M is reciprocal and thus mutual can be shown as follows. Assume that the current in n_2 has been changed by di_{a2} , leaving n_1 open. Then the change in flux $d\phi_2$ produced would have been $d\phi_2 = \frac{4 \pi n_2 di_{a2}}{Z}$, and an e.m.f. E_1 would have been induced in n_1 given by

$$E_1 = n_1 \frac{d\phi_2}{dt} = \frac{4 \pi n_2 n_1}{Z} \frac{di_{a2}}{dt}.$$

Thus

$$\frac{E_1}{di_{a2}/dt} = n_1 \frac{d\phi_2}{di_{a2}} = \frac{4 \pi n_1 n_2}{Z} = n_2 \frac{d\phi_1}{di_{a1}} = \frac{E_2}{di_{a1}/dt} = M.$$

This means that the circuits are linked by a mutual interaction given by the coefficient of mutual induction M , in this example equal to $\frac{4 \pi n_1 n_2}{Z}$. Any change in i_1 induces an e.m.f. E_2 in circuit 2 and, vice versa, any change in i_2 induces an e.m.f. E_1 in circuit 1. The quantities $\frac{E_1}{di_{a2}/dt} = \frac{E_2}{di_{a1}/dt} = M$.

It is seen that in this case for a small value of Z the coefficient would be very large. For air-core coils with large Z the mutual induction is small and falls off very rapidly with the distance between the coils

as Z increases. Since the induced e.m.f. $E_2 = M \frac{di_{a1}}{dt}$, it is seen that, even if M is small, if $\frac{di_{a1}}{dt}$ is large, as is the case for the high-frequency radio currents (say 10^7 cycles per second), the value of E_2 may not be inappreciable. This requires that a high degree of shielding be resorted to for such currents if interference between nearby coils or wires is not desired.

The units of mutual inductance are the same as for self-induction, since the two coefficients are dimensionally the same. If E_1 and E_2 and $\frac{di_a}{dt}$ are in absolute e.m.u., M will be in centimeters. If they are in volts and amperes per second, M will be in henrys. If Z is in absolute units, then M will be in centimeters.

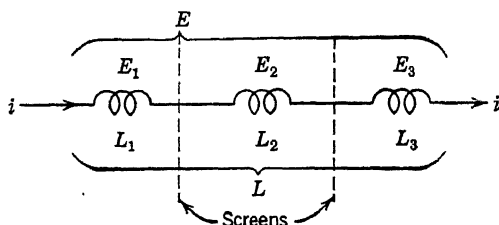


FIG. 146a. Noninteracting inductances in series.
Note that i is common to all inductances.

Inductances in Series and in Parallel. If a group of inductances L_1 , L_2 , and L_3 are placed in series (as in Fig. 146a) so that there is *no mutual interaction*, they are threaded by the same value of

$$\frac{di_1}{dt} = \frac{di_2}{dt} = \frac{di_3}{dt} = \frac{di}{dt}.$$

Here since the e.m.f.'s for each inductance are additive, it can be written that the total e.m.f. E is

$$E = E_1 + E_2 + E_3 = L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + L_3 \frac{di_3}{dt} = L \frac{di}{dt},$$

whence

$$L = L_1 + L_2 + L_3.$$

Inductances in series add directly.

If a group of inductances are placed in parallel with terminals connected together (as in Fig. 146b) so that there is *no mutual interaction* between coils, the induced e.m.f. is the same for each inductance

while the currents are in general different. Thus $E = E_1 = E_2 = E_3$, and hence

$$L \frac{di}{dt} = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} = L_3 \frac{di_3}{dt}.$$

Thus

$$\frac{L}{L_1} \frac{di}{dt} = \frac{di_1}{dt}, \quad \frac{L}{L_2} \frac{di}{dt} = \frac{di_2}{dt}, \quad \frac{L}{L_3} \frac{di}{dt} = \frac{di_3}{dt}.$$

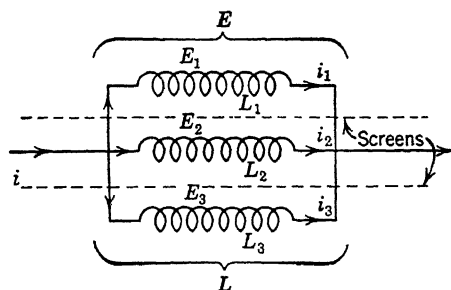


FIG. 146b. Noninteracting inductances in parallel.
Note that E is constant.

Now, in parallel, i , the total current, is given by

$$i = i_1 + i_2 + i_3,$$

where i_1 , i_2 , and i_3 are the currents in L_1 , L_2 , and L_3 , so that

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}.$$

Hence we can write

$$\frac{di}{dt} = \left(\frac{L}{L_1} + \frac{L}{L_2} + \frac{L}{L_3} \right) \frac{di}{dt}.$$

whence at once $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$. Thus in this respect inductances, when they are independent of each other, act like resistances in series and parallel combinations.

Next consider two coils, denoted by subscripts 1 and 2, that are connected in series and coupled by a mutual inductance M . Assume that the coils are *wound in the same sense* (as in Fig. 146c) and are placed in series in this sense. The coil 1 has an e.m.f. $E_1 = L_1 \frac{di}{dt}$ due to its self-induction. The mutual inductance M gives an e.m.f. in coil 1 due to the current change in coil 2 of $E_{M_1} = M \frac{di}{dt}$, and this

e.m.f. will act in the same sense as E_1 , for the field of coil 2 is in the same sense as that in coil 1. Likewise in coil 2 there will be induced a e.m.f. $E_2 = L_2 \frac{di}{dt}$ due to its self-induction and an e.m.f. $E_{M_1} = M \frac{di}{dt}$

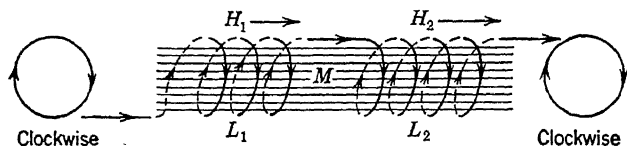


FIG. 146c. Coils in series, interacting and aiding.

due to the change of current in coil 1. Now, since the coils are wound in the same sense, the fields are in the same sense and the e.m.f.'s will be in the same sense, whence the total e.m.f.

$$E_a = E_1 + E_{M_1} + E_2 + E_{M_2} = L_1 \frac{di}{dt} + M \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt}.$$

Accordingly,

$$\frac{E_a}{\frac{di}{dt}} = L_a = L_1 + L_2 + 2M,$$



FIG. 146d. Coils in series, interacting and opposing.

where L_a is the self-induction for the coils in series and *aiding each other*.

If now the windings in coils 1 and 2 are in the *opposite sense* so that the current flows clockwise in coil 1 and counterclockwise in coil 2 (as in Fig. 146d) the induced e.m.f.'s due to self-induction continue in each coil to be in the same sense, each opposing the change of current di . Let us call this sense positive. Now, however, the field of coil 2 will thread coil 1 so as to make a field opposing that of the current i in coil 1. Hence the e.m.f. $E_{M_1} = M \frac{di}{dt}$ will be negative to E_1 . Likewise in coil 2, $E_{M_2} = M \frac{di}{dt}$ due to the field in coil 1 will oppose E_2 . Thus the total e.m.f. E_o for coils opposing each other will be

$$E_o = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} - 2M \frac{di}{dt},$$

so that

$$\frac{E_o}{di/dt} = L_o = L_1 + L_2 - 2M.$$

Here L_o is the self-induction of the coils in series with *fields opposed*.

Thus with coils *opposed* twice the mutual inductance M must be subtracted from $L_1 + L_2$, and with coils *aiding* twice the mutual inductance must be added to $L_1 + L_2$ to give the resultant inductance. That is, $L = L_1 + L_2 \mp 2M$, depending on the sense of winding of the coils, whether opposed or aiding. Hence in connecting two

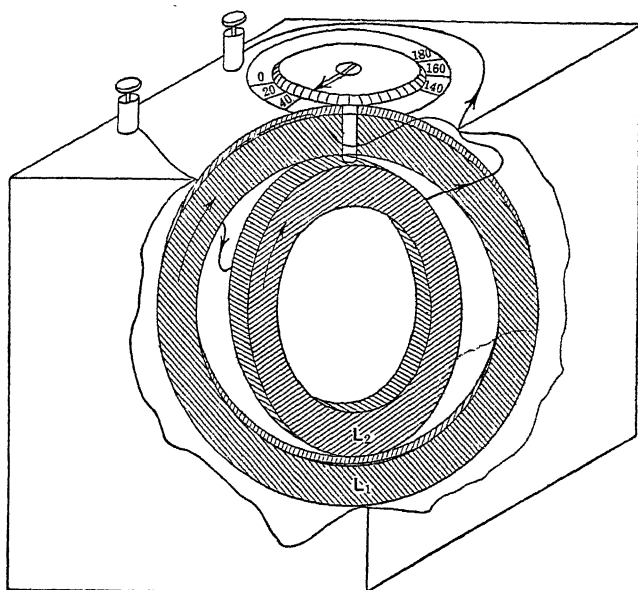


FIG. 147. The inductometer or variometer.

inductances in series *care* must be taken to note whether the coils are aiding or opposing. Otherwise, if M is large, much time will be lost wondering why induced currents that are not of the expected magnitudes appear. If the coils are far apart, M approaches 0 and $L = L_1 + L_2$. If $L_1 = L_2$ and the coils are very *closely* coupled, M approaches L_1 in value, for $n_1 = n_2$ and the two coils act as a single coil of inductance $L_a = 4 L_1 = 2^2 L_1$, with twice the number of windings. If the coils are opposed and $L_1 = L_2$, $M = L_1$, then $L_o = 0$, i.e., the self-induction of the system is zero, that is, it is noninductive. The ratio

of M to the square root of the product of L_1 and L_2 , is $k = \frac{M}{\sqrt{L_1 L_2}}$, which is called the *coupling coefficient*. It represents the relative value

of M and a sort of average value of L_1 and L_2 . The closer the coupling the larger the k . Thus k measures the degree of coupling. It is 1 when $M = L_1 = L_2$. Otherwise it is always less than 1.

By making two coils in series with L_1 nearly L_2 and mounting one inside the other, as shown in Fig. 147, it is possible, by rotating the inner coil about the common axis AA' relative to the outer coil by an angle going from 0 to π radians, to make the self-induction vary from that of the two coils aiding to that of the two coils opposing. The self-induction of the combination will thus vary from $L_a = L_1 + L_2 + 2M$ to $L_o = L_1 + L_2 - 2M$ continuously. Then if M is nearly L_1 , and L_1 and L_2 are nearly equal, L will vary from nearly $4L$ to nearly 0. This is the principle of the inductometer shown in Fig. 147. Its inductance can be calibrated in terms of the angular setting of the coils and furnishes a most useful variable inductance standard.

111. ENERGY OF THE ELECTROMAGNETIC FIELD OF A CURRENT

It was seen that an inductance of value L produces an e.m.f. E' in its n turns whenever there is a change of current di_a which produces a change of magnetic flux $d\phi$ within it. When the current di_a changes in a time dt this instantaneous e.m.f. E' was given as $E' = L \frac{di_a}{dt}$.

If at the time of the change in current di_a which produces the e.m.f. E' the current value is i_a for the short time dt , the changing current, does work in the amount $dW = E' i_a dt$. If, therefore, a current I_a is flowing initially at $t = 0$ and after a time t it has ceased to flow as a result of opening the circuit, there will have been a large number of small increments of work dW done as the current dies out. The total

work done, or energy liberated, is thus $W = \int_0^t dW = \int_0^t E' i_a dt$.

Here $\int_0^t dW$ is merely the symbolical way of writing the sum of all the small dw 's from $t = 0$ to $t = t$. Since $E' = L \frac{di_a}{dt}$ we can substitute in the equation for W and obtain,

$$W = \int_0^t L \frac{di_a}{dt} i_a dt = \int_{I_a}^0 L i_a di_a = \frac{1}{2} L I_a^2.$$

The limits of integration were changed from 0 to t for dt to limits ranging from I_a to 0 for di_a , since at $t = 0$, $i_a = I_a$ and at $t = t$, $i_a = 0$. The quantity I_a is any particular value of the current in absolute units flowing when the circuit is cut.

Thus it is seen that when a current I_a is flowing in an inductance L and this current is interrupted we get work, $\frac{1}{2} L I_a^2$, done by the current. If at the start, $t = 0$ there had been no current at $t = 0$ and

a final steady current I_a had been *created* in the inductance, a similar calculation would have shown that *electrical energy was consumed in building up the current in the wire*. It therefore appears that energy is stored in an inductance carrying a current. Since any circuit has some inductance it is seen that in establishing the current, energy, or work, is consumed which is restored when the current ceases.

If the current is looked on as being caused by a movement of electrons there might be some temptation to explain the energy as simply being spent in giving the myriad of flowing electrons the needed kinetic energy. That is, the energy is expended in overcoming the inertia of the electrons. When the electrons cease to flow they restore the energy of motion. While this viewpoint is perfectly correct, it is not nearly as useful or instructive as another view of the same situation. This is particularly true as one cannot be sure of the character of the *inertia* of an ultimate particle with an electrical field of its own.

Classic electrodynamics regards the apparent inertia or mass of the electron not as residing in the electron as a gravitational particle of matter, but in its electromagnetic field. This concept is facilitated by the fact that the quantitative calculation that made physicists conscious of the energy stored in an inductance by a current was given as $\frac{1}{2} LI_a^2$, i.e., in terms of self-induction and current. It is furthermore to be noted that it is known that this release of the energy stored in electronic motion is accomplished only by the mechanism involving the magnetic field which has been discussed. Since self-induction $L = n \frac{d\phi}{di_a}$ it is logical to regard the energy $\frac{1}{2} LI_a^2$ as related to the magnetic flux ϕ created by the current. It was this relationship, which will presently be discussed further, that led to the notion that the energy of a current stored in an inductance resides in its magnetic field. It is released on cessation of the current by the mechanism of collapse of the magnetic-field lines across the conductor giving the e.m.f. of self-induction.

Before any further discussion of the energy stored in the electromagnetic field, it is worth while to point out that, in essence, as a result of the creation of this field, any inductance has the properties of an electrical inertia, that is, it resists change of the electrical or magnetic state. In this respect its role in electrical circuits is quite analogous to the role played by mass in mechanical systems. Thus in the expressions $\frac{1}{2} mv^2$ and $\frac{1}{2} LI^2$ as related to moving matter and moving electricity there is a close *formal* analogy. In effect, many of the differential equations solved in relation to mechanical problems have aided in the solution of the analogous electrical problem. Thus in the inductance which has the properties of electrical inertia and in capacity that acts electrically like a coiled spring, or better, an elastic-restoring force, one has the basic properties entering into many electrical problems of which the analysis is facilitated by mechanical analogues.

The Energy of the Magnetic Field. It has been seen that the storage of the energy of a current in an inductance resided in the magnetic flux, that is the magnetic field which the current built up. It is now of interest to compute the energy stored in a magnetic field. This can be computed by electromagnetic-field theory in a very involved and general fashion. It can very easily be computed for more elementary students by choosing a special case and generalizing the result. Consider the toroidal solenoid of section 83. It has a

field H given by $H = \frac{4 \pi n i_a}{2 \pi r} = \frac{2 n i_a}{r}$, where r is the radius from the center to the axis of the toroid. This field H lies entirely within the solenoid of area of cross-section A and volume $V = 2 \pi r A$. The flux inside the toroid is $\phi = AH$. Now it has been found that the self-induction is $L = n \frac{d\phi}{di_a}$, where n is the number of turns in the solenoid.

Thus for the toroid

$$L = n \frac{d\phi}{di_a} = n \frac{d(2 n i_a A)}{r di_a} = \frac{2 n^2 A}{r} \frac{di_a}{di_a} = \frac{2 n^2 A}{r}.$$

The energy stored in the volume $2 \pi r A$ of empty space in the solenoid when the current i_a flows is thus given by

$$W = E = \frac{1}{2} L i_a^2 = \frac{2}{2} \frac{n^2 A}{r} i_a^2 = \frac{n^2 A i_a^2}{r}.$$

But the field H in the solenoid in which the energy resides is

$$H = \frac{2 n i_a}{r}, \text{ whence } i_a = \frac{r H}{2 n}.$$

Thus

$$E = \frac{n^2 A}{r} \frac{r^2 H^2}{4 n^2} = \frac{H^2}{4} r A.$$

But $V = 2 \pi r A$, so that $r A = \frac{V}{2 \pi}$. Hence $E = \frac{H^2 V}{8 \pi}$, or better, the

energy per unit volume of the field H is given by $\epsilon = \frac{E}{V} = \frac{H^2}{8 \pi}$. More

esoteric analysis would, for the general case of any given uniform magnetic field H , have given the same result. Thus when a uniform

magnetic field of strength H exists in space it took an energy of $\frac{H^2}{8 \pi}$

units per cubic centimeter to create it. If the field is destroyed by the cessation of the gross, or electronic, currents causing it, the energy is released in the process. The energy is in ergs if H is in absolute c.g.s. units.

It will be recalled that for the electrical field X , produced in the empty space between condenser plates, electrostatic energy in the

amount of $\frac{X^2}{8\pi}$ units per cubic centimeter of volume was stored. This storing of electrical and magnetic energy in space as a result of fields has great significance when study of the propagation of radiant energy in the form of electromagnetic waves is reached.

The relation $W = \frac{1}{2} LI^2$ can be utilized in many ways. It is particularly useful in that it is possible by this means to define and measure inductances. Thus since $W = \frac{1}{2} Li^2$, $L = \frac{2W}{i^2}$. If W , the energy necessary to create the current i , can be calculated or measured, L is at once defined.

The energy stored by the current in a line also plays very important roles in flashover and electrical breakdown in the switching of heavy currents. It is more important than line voltage in this respect as the stored energy $\frac{1}{2} LI^2$ can build up much higher potentials under some circumstances. This point is often forgotten.

The energy concept leads to another interesting application. For two circuits having inductances L_1 and L_2 and a mutual inductance M with currents i_1 and i_2 in them, we can write

$$W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2.$$

If the circuits are moved relative to each other along x , keeping the currents constant, the terms $\frac{1}{2} L_1 i_1^2$ and $\frac{1}{2} L_2 i_2^2$ remain constant.

Hence $\frac{dW}{dt} = i_1 i_2 \frac{dM}{dt}$. But the motion along x does mechanical work

fx at a rate $f \frac{dx}{dt}$. If a careful analysis is made of all changes in energy it turns out that it is justifiable to ascribe the whole work to the changes in energy in the electromagnetic field. Hence $f \frac{dx}{dt} = \frac{i_1 i_2 dM}{dt}$,

so that $f = i_1 i_2 \frac{dM}{dx}$. Thus Lord Kelvin has shown that, if $\frac{dM}{dx}$ can be calculated for two coils having the same current flowing through

them, $i_1 = i_2$ and therefore $i = \sqrt{\frac{f}{dM/dx}}$. This is the basis of Kel-

vin's absolute current balance and it gives an extremely accurate method of determining the absolute e.m.u. of current. The principle of the balance is illustrated in Fig. 148. Two identical coils A and A' have a second coil B coaxial and placed between them. The coils are precision-made to suitable specifications. They are so arranged that when the current i_a to be measured traverses them B is urged down to A' and away from A . B is rigidly fastened to the one end of a sensitive beam balance with the knife edge at F and a scalepan taking the weights mg . It is first carefully balanced with the pointer at 0 on the

scale S and the distances x between it and A and A' are exactly equal. Then the current is turned on and weights added to mg to restore it to 0. In the equation the value of $f = mg$ is obtained. With the coils

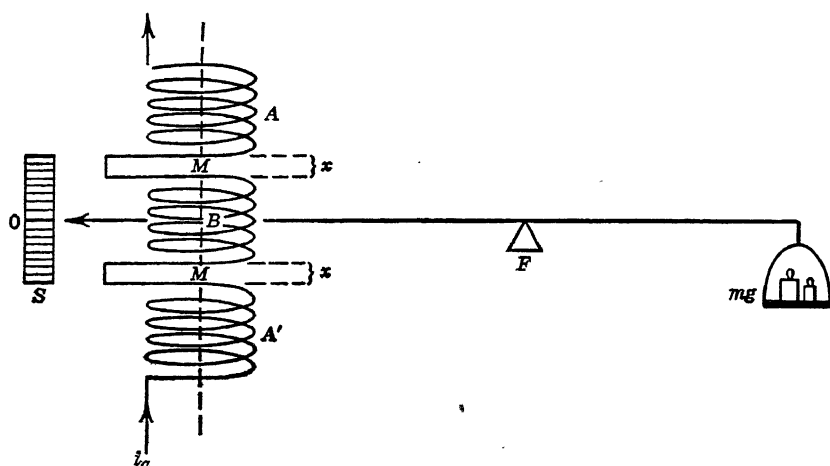


FIG. 148. Kelvin's absolute current balance.

accurately figured and measured and x accurately set it is merely a matter of tedious calculation to evaluate $\frac{dM}{dx}$ for the coil system. This can be done to as high a degree of precision as the coils can be manufactured and measured. Thus an absolute method of evaluating current far superior to the tangent galvanometer can be set up. More sophisticated arrangements than the one indicated are the basis of all the absolute electromagnetic system of standards.

112. SELF-INDUCTION AND ENERGY LOSSES, EDDY CURRENTS

Induction and self-induction effects lead to many phenomena. One of these is of great importance because it causes energy losses. If a continuous metal body is placed in the neighborhood of a conductor with an alternating electrical field, induced currents are set up in the solid conductor which, passing through the resistance, cause heat production in the solid body. They are termed *eddy currents*. The heat production caused may be very intense and results in a loss of energy to the circuit acting on it. In fact, these induction effects are used frequently to heat bodies to exceedingly high temperatures in vacuo, and they are known as induction furnaces. In such furnaces, high-potential high-frequency electrical oscillations are set up in a coil surrounding the body to be heated. The eddy currents in this body will raise it to enormous temperatures.

An excellent example of the effect of eddy currents is illustrated by the apparatus shown in Fig. 149. In this a pendulum, having a replaceable disk A which permits it to swing through a magnetic field H produced by a powerful electromagnet perpendicular to the plane of the drawing, oscillates for a considerable period in the absence of the field. The energy of the motion is then damped only by the friction in the bearing P . When the field is put on, the cutting of the magnetic force lines by the disk A as it swings through the field generates currents in A which are short-circuited in the highly conducting material.

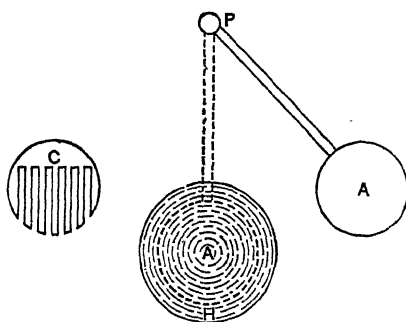


FIG. 149. Damping effect of eddy currents.

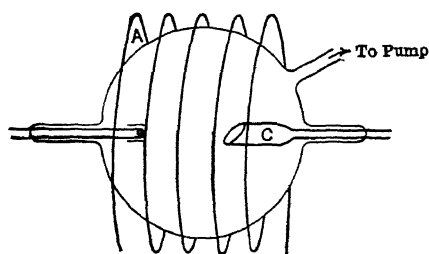


FIG. 150. Use of the induction furnace in outgassing an x-ray tube.

The energy of motion is thus completely consumed as A passes through the field H , and the disk acts as if it were cutting through a viscous liquid. If the disk A be replaced by the slotted disk C which reduces the scope of the eddy currents, the damping is much decreased and C will execute several oscillations before coming to rest. Electromagnetic damping can be used in mechanical systems to produce desirable damping effects.

In Fig. 150 is depicted an exhausted x-ray tube, fastened to a pump, having a metal anticathode C which is to be outgassed and heated to incandescence. About the tube is wound a coil A which is connected to a high-frequency source of oscillations having considerable energy. The high-frequency oscillations cause a rapid change in flux through the metal C , thereby generating numerous eddy currents which, being short-circuited, rapidly raise the metal to incandescence.

All iron cores in motor armatures and in transformers are subject to such losses as they permit the existence of small local eddy currents of self-induction. The losses may become quite great and reduce the efficiency of the transformer. It will be seen that the cores of transformers and armatures of most motors are made up of groups of wires, or thin laminae of iron, separated from each other by shellac or some insulating body in such a way that the main induced currents in the wire cannot be set up, at the same time permitting the lines of mag-

netic flux to pass through the iron without interruption. In Fig. 151*A* is shown the frame of a transformer; in Fig. 151*B* the end view of the same transformer shows the laminated structure of the iron core *C*. It is seen that the induced currents set up by the field in *I* would be such as to cause eddy currents to flow in the sense of the circular arrows *E* in *C*. Such flow is prevented by the insulated laminations. The side view of the same core indicates that at the same time the continuity of each iron lamina ensures a maximum of magnetic conductivity.

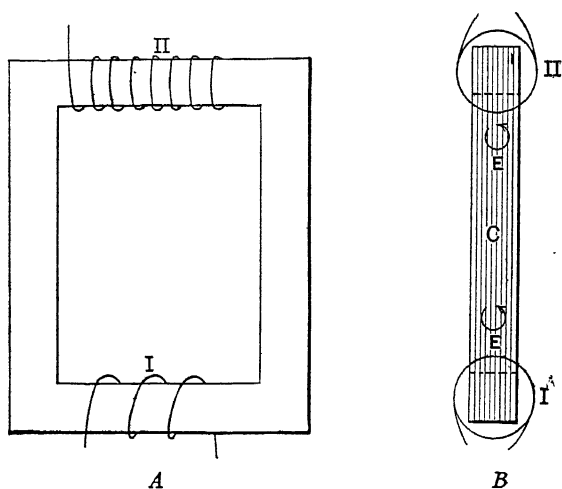


FIG. 151. Laminated structure of a transformer core.

Another manifestation of these eddy currents is the damping which they produce. If an instantaneous current is passed through a ballistic galvanometer, then upon opening the circuit the galvanometer coil will oscillate back and forth until the friction in its suspension exhausts the energy applied. This may take hours. If, however, the galvanometer is shunted by a resistance the oscillations rapidly cease. The reason for cessation is obvious. The coil oscillating in the magnetic field sets up induced currents which in the coil and resistance are changed to heat. The heat energy thus generated uses the energy of vibration of the galvanometer in the production of these electrical heating effects, and the oscillations cease.

113. THE INDUCTION COIL

Another use of the property of induction is in the induction coil. In this instrument primary coil *I* is connected to a fairly high current source of potential. This primary circuit *I*, Fig. 152, is interrupted by

some automatic breaking device B such that the current can be periodically made and broken. About the iron core C on which the primary is wound there are many turns of wire n_2 in another coil II called the secondary. The change of current and consequently of flux in the primary circuit results in the cutting of the secondary circuit by the magnetic field and causes multiplication to a high degree of the number of turns cut by the magnetic flux. On the basis of the analysis of mutual induction which has gone before, if this secondary coil II is endowed with many turns the electromotive force E_2 set up in it is equal to

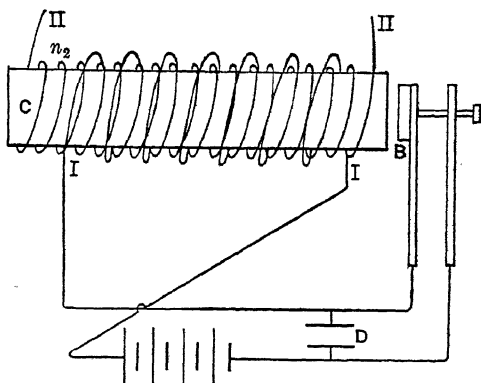


FIG. 152. Diagram of an induction coil.

$$\frac{4 \pi}{10^9} \frac{n_1 n_2}{Z} \frac{di_1}{dt} \text{ volts.}$$

If a small n_1 is used the potential necessary to drive a current i_1 through it need not be very large. On the other hand, the potential E_2 coming from the many turns n_2 of the secondary will be very high. It is, of course, necessary that Z be small in order to increase the magnetic flux. If n_1 is very great the self-induction of the primary is going to retard $\frac{di_1}{dt}$, and consequently reduce the value of E_2 . Thus, in the induction coil a small number of turns n_1 and a high current in the primary, a fairly high magnetic flux, a large number of turns n_2 in the secondary and a rapid break, high di_1/dt is used.

The primary current is very much more effective on the *breaking* of the circuit than on the making of the circuit, for in the breaking of the circuit, the collapse of the field is very rapid, while the growth of current is retarded by the *self-induction* of I and the mutual induction of I and II . The only factor which hinders the collapse of the current on breaking is the drawing out of an arc at the break. This can be reduced by placing a condenser D across the primary terminals which cuts off the flow through the arc, and therefore increases $\frac{di_1}{dt}$. Thus

the electromotive force obtained from an induction coil is almost unidirectional, for the break of the circuit causes the main increase in secondary potential. It is to be noted that in order to have a high

$\frac{di_1}{dt}$ by a rapid break, in the induction coil continuity of iron is sacrificed and thus relatively high Z is presented.

114. THE TRANSFORMER

The principle of a transformer is similar to that of the induction coil except that the make and break of the induction coil is done away with, and the current fed into the transformer is a sinusoidal alternating current. It thus has a low Z but $\frac{di_1}{dt}$ is now limited by the frequency and the amplitude of the a-c current in the primary.

The theory of the transformer, however, is very complicated owing to the effects of self-induction, mutual induction, and impedance on the oscillations. To increase efficiency in these transformers, the magnetic circuit is generally complete, the windings being on a continuous ring of metal. Since the power put into a transformer in the primary is given by $E_1 i_1$, where E_1 is the potential across the primary and i_1 is the current in the primary, and since the output of power in the secondary can never exceed the input of power in the primary, we have the *limiting condition* that $E_2 i_2$, the electromotive force and current in the secondary, are related to those in the primary by the equation $E_2 i_2 = E_1 i_1$, or $\frac{E_2}{E_1} = \frac{i_1}{i_2}$. If the efficiency of the transformer is

Eff., then $E_2 i_2 = \text{Eff.} (E_1 i_1)$, or $\frac{E_2}{E_1} = \text{Eff.} \frac{i_1}{i_2}$. The decrease in efficiency is the result of a small loss caused by hysteresis effects in the iron core of the transformer, and though the loss is small the equation to be correct must contain the factor for this energy loss.

It is loosely asserted in many elementary textbooks that the ratio of e.m.f.'s in secondary to primary in induction coils and transformers is in the ratio of the number of turns in secondary to primary. In general operation, this is not true if any appreciable current is being drawn from the secondary. It can, however, be shown from the theory that for *open-circuit work*, or with a *high resistance in the secondary*, when there is a low resistance in the primary, the ratio of E_2 to E_1 approaches the ratio n_2/n_1 . Here n_2 is the number of turns in the secondary and n_1 is the number of turns in the primary. This follows from the fact that the same flux passes through both primary and secondary coils.

The manner in which this occurs becomes clear from a brief quantitative discussion. In the treatment of the case of mutual inductance using a magnetic circuit of reluctance Z , it was shown that the e.m.f. E_2 of the secondary coil of n_2 turns due to a rate of change in current $\frac{di}{dt}$, in a primary of n_1 turns was $E_2 = \frac{4\pi}{10} \frac{n_1 n_2}{Z} \frac{di_1}{dt}$. If the current i_1

in the primary circuit causing $\frac{di_1}{dt}$ was a sinusoidal current of value $i_1 = \frac{E_1}{z} \sin 2 \pi Nt$, where z is the impedance (see section 138, page 373), E_1 is the applied e.m.f., and N is the frequency of the alternating current,

$$\frac{di_1}{dt} = \frac{E_1}{z} 2 \pi N \cos 2 \pi Nt.$$

The maximum value of $\frac{di_1}{dt}$ occurs at a value of $2 \pi Nt = 0$ or π and is

$$\left(\frac{di_1}{dt}\right)_{\max.} = \frac{E_1}{z} 2 \pi N.$$

Hence the maximum value of

$$E_2 = \frac{4 \pi}{10} \frac{n_1 n_2}{Z} \frac{E_1}{z} 2 \pi N,$$

or

$$\frac{E_2}{E_1} = \frac{4 \pi}{10} \frac{n_1 n_2}{Z z} 2 \pi N.$$

Now the value of z , the impedance, is $z = \sqrt{R^2 + L_1^2 (2 \pi N)^2}$. But in a primary circuit the reactance term $L_1^2 (2 \pi N)^2$ is very much larger than the assumed low ohmic resistance R squared. Thus z can be set as nearly equal to $z = L_1 2 \pi N = \frac{4 \pi}{10} \frac{n_1^2}{Z} 2 \pi N$. Accordingly,

$$\frac{E_2}{E_1} = \frac{4 \pi}{10} \frac{n_1 n_2}{Z} \frac{2 \pi N}{\frac{4 \pi}{10} \frac{n_1^2}{Z} 2 \pi N} = \frac{n_2}{n_1},$$

so that as long as R is small and as long as no current is drawn from the secondary so that the mutual inductance does not change the effective inductance L_1 and the e.m.f. in n_1 , we can write

$$\frac{E_2}{E_1} = \frac{n_2}{n_1}.$$

Transformers are used for many purposes aside from generating high potentials. They are of vast importance industrially in power transmission. Power is generated at low potential and high current. By transformer it is stepped up to high potential (about 11,000, 110,000, etc., volts) for transmission to population or distribution centers. There it is stepped down with the aid of transformers for house use at

110 to 220 volts. In radio circuits transformers step down the 110 volts house power to 6 volts to light the tube filaments. Transformers are widely used in connecting circuits with the alternating current used in radio, radio telephone, sound recording and broadcasting equipment. They are of step-up, step-down, or repeater types.

CHAPTER XXII

ELECTRICAL AND MAGNETIC MEASUREMENTS

115. INTRODUCTION

On the basis of fundamental electrical concepts, formulated in defining equations derived from experiment or theory, certain *absolute* electrical and magnetic *units* have been established that are directly measurable in terms of mechanical quantities such as force and work, in the absolute c.g.s. system of units. These units are determined either in terms of electrostatic measurements or electromagnetic measurements leading to the two fundamental c.g.s. systems of absolute units, the absolute electrostatic units and the absolute electromagnetic units. In addition, there is a series of practical units the magnitudes of which are conveniently commensurable with the values of the quantities met with in daily electrical operation. The instruments which are commonly used are calibrated in these units. This calibration is always achieved by comparison with standards otherwise established and eventually led back in comparison to the absolute measurements. This is necessary since the practical units are by definition derived as multiples or fractions of the absolute units.

In addition to absolute standards underlying the various practical units there have developed by international conventions in 1881, 1893, 1908, and subsequently the so-called international or legal standards. These are standards so easily reproducible experimentally that at any time when adequate primary or secondary standards are lacking instruments can still be calibrated. In recent years these standards have been rechecked carefully against the absolute standards. They have now been found to deviate very slightly from these in most cases.

The fundamental concepts, the measurement of which together with those of magnetic phenomena comprise the bulk of the electrical measurements made, are those of current, quantity, potential, resistance, capacity, and self induction. These concepts, their defining equations, their dimensions, their units in the electromagnetic electrostatic and practical systems can be found tabulated on pages 412 and 413 of the text. It will be noted that it was convenient to divide them into two groups, the "fundamental" and the "derived" concepts. The former are readily directly determinable in absolute measure by means of force and work measurements; the latter can logically be derived from ratios of the former as their defining equa-

tions indicate. Such a division though possibly arbitrary has certain simplifying advantages, such that in the fundamental concepts the electromagnetic and the electrostatic units are related by a factor equal to the velocity of light or its reciprocal, while the derived concepts in the two systems are similarly related by the square of the velocity of light.

It turns out that in each absolute system (e.m. or e.s.) two of the fundamental units potential and either quantity or current are derived from direct absolute measurements while the third fundamental unit current or quantity is indirectly derived from the directly determined one. Concerning the derived units it turns out that expediency makes a direct determination of the absolute ohm a convenient and useful measurement. Capacity can, with fair accuracy, be determined in absolute measure in both systems but self-induction can be directly determined in the electromagnetic system only. Standards of capacity in the electrostatic system and self-induction in the electromagnetic system can be achieved in an absolute fashion by careful construction and computation from the dimensions.

The two properties of matter involved in the electrical and magnetic Coulomb force equations as well as in the evaluation of capacity and self-induction, to wit, the dielectric constant D and the magnetic permeability μ , which are here treated as dimensionless constants, can be evaluated by measurement of the changes in capacity and self-induction produced by their presence.

116. MAGNETIC MEASUREMENTS

1. Measurement of Magnetic Fields and Magnetic Moments.

(a) Absolute measurements of a field and a magnetic moment were initially made by the magnetometer method outlined in Chapter III. These, though of limited precision, serve to evaluate a single magnetic moment and a uniform magnetic field. Once a standard of either sort is obtained further fields and moments can be determined either by direct comparison using a small compass needle, by torques on known magnetic moments in unknown fields, or by the period of oscillation of magnets of known moments and inertias in unknown fields. The range of fields and moments covered is large. The chief danger in such magnetic investigation lies in the finite sizes of the standard magnets where fields may not be uniform.

(b) As a result of electromagnetic theory it is possible to achieve uniform magnetic fields of high uniformity and widely varied strengths using carefully constructed coils or systems of coils (e.g., Helmholtz or Rubens' coils) with accurate absolutely measured currents in them. The precision obtained in the absence of iron or ferromagnetic materials is very high. Care must be used, however, in excluding extraneous effects such as the earth's field or fields from local currents or

steel construction in buildings, and deviations due to the wires or open ends of the coils.

(c) Fields may sometimes be evaluated by flip coils or by carefully constructed coils rotated at an accurately known speed in the field. By measurements of the electromotive force generated by the coil quite accurate values of the field can be determined. Such a device using a ballistic galvanometer for one-half turn or any galvanometer with a commutator for a large number of revolutions is called the earth inductor and is practically used for measuring the total intensity of the earth's magnetic field. A flip coil is a coil mechanically spun through 180° at a constant and repeatable speed. The e.m.f. generated can be determined by the throw of a ballistic galvanometer. The throw of the galvanometer is translated into field strength by calibration in a known field, sometimes integral with the coil mounting. The flip coils are not very accurate. Uniformly rotating coils can be used with high precision. Their drawback is that they require space for the transmission of the driving power. They would be very inconvenient submerged under water for measurement of ships' fields.

(d) The electrical resistances of certain substances are altered by magnetic fields. Bismuth in particular shows a very considerable effect. The effect is fourfold for a change in field from 0 to 30,000 oersteds at room temperature. At low temperatures the change is much greater. By making a very thin pancake spiral of bismuth wire, in which the wire is wound back on itself, and placing this normal to the field to be measured, the wire forming one arm of a Wheatstone bridge, the magnetic fields may be measured. The spiral is calibrated in known fields. In this fashion the fields may be determined in regions of the thickness of the wire in breadth. This method is particularly useful for the measurement of fields varying rapidly in space and is quite accurate.

(e) The exigencies that arose in World War II when it became necessary to protect ships against magnetic mines required sensitive, convenient, and rapid-reading instruments for measuring magnetic fields. One of these, the fluxgate magnetometer, achieved notable success and will be used hereafter in geophysical and other studies. It measures the component of the magnetic field parallel to its axis. It consists of a very easily magnetized core of material such as Mu metal or Permalloy. The core can be magnetized by a small constant uniform field applied by an outside solenoid coil giving an accurately known field. This field can be altered to compensate for any changes imposed from without by changing the current through it. The core is wrapped with another coil through which a 500-cycle a-c current is flowing. The amplitude of the a-c current is such as to be able to saturate the core on both half-cycles. A secondary coil on the same core picks up the 500-cycle alternating current by mutual inductive

action through the core. The cycle picked up by the secondary is, however, badly distorted when a magnetic field from outside or from the compensating coil is imposed since the phases of the primary a-c current adding to the field on the saturated side does not transmit much effect to the secondary. The other half-phase of the wave, acting against the d-c component of the outside field, gives notable induced currents. These "lopsided" a-c pulses are rectified by Thyrite rectifying crystals or by electron tubes and register on a meter as a d-c current. They can if necessary be amplified and filtered before rectification. The magnitude of this current is determined by the degree of saturation. The instrument is balanced in a known field to a certain current reading. In the unknown field a new current is registered. The compensating coil can then have its current adjusted to give the old reading. The compensating field change is accurately known from the compensating current change. This procedure utilizes the magnetometer as a null indicator and is the most accurate use. The reading of the d-c meter could also be used directly after calibration. Its readings are subject to the very complicated current relations and though rapid are thus subject to error. These instruments read vertical components of fields to 1 millioersted, i.e., to 1/500 of the earth's total field. They occupy only a small volume and record instantly. With automatic Speed-o-max recorders 100 instruments can have their readings registered on paper in 5 minutes. Such speed was imperative in determining operations in readying ships for sea.

(f) The use of search coils either with, or without, Permalloy cores. These record total changes in flux with great precision on the sensitive modifications of the Grassot fluxmeters, which are self-recording. They were developed during World War II by the General Electric Company and have revolutionized certain types of measurements involving changes of magnetic field. Thus the changes in the vertical component of the earth's field caused by the passage of a ship 80 feet above the search coil of such a combination gives a clear written record on a moving tape. This record can be directly interpreted in terms of changes of fields. With coils of large area such meters record the smallest changes in the earth's field.

2. The Measurement of Permeabilities. (a) Magnetic permeabilities can be determined by the magnetometer method using elongated ellipsoids of revolution made of the material to be tested instead of bar magnets of moment M . It happens that for such ellipsoids the moment $M = Im = VI$, where V is the volume and I is the intensity of magnetization. Thus, if M is measured, I and hence μ can be determined for each field. Incidentally the poles, i.e., centers of magnetization, are located at the foci of the ellipse. These ellipsoids are in general not convenient to make, though quite accurate.

(b) More frequent commercial recourse is had to the use of a primary field produced by a current in a coil, surrounded by a secondary

coil connected to a ballistic galvanometer. By observing the throw of the galvanometer in the absence of and in the presence of the material in the coil for known current changes the induction of the material can be determined. The procedure is discussed in section 99. The inaccuracies introduced by the demagnetizing fields due to having the ends of the specimen free in the air are avoided in accurate practice by using continuous toroidal rings of the material for cores or by placing the rod-shaped sample in a magnetic yoke of very low and known reluctance. The yoke method is standard practice for more accurate work and avoids the complications produced by the demagnetizing field in strongly magnetized samples. The most accurate studies use the continuous ring of the specimen, with windings about it. This obviates the loss by leakage which occurs in the yoke method.

117. SUMMARY OF METHODS OF MAGNETIC MEASUREMENT

1. Measurements of fields and moments.

- (a) Absolute fields and moments are accomplished by the magnetometer method. Comparison of M and H use the resultant angle of compass, the torques on suspended magnets, or the periods of oscillation in fields.
- (b) Uniform fields are produced by known currents in specially designed coils.
- (c) Flip coils, or the earth-inductor mechanisms, evaluate fields.
- (d) The change of resistance of a bismuth spiral measures fields.
- (e) Direct-reading fluxgate-type magnetometers measure fields.
- (f) Sensitive recording Grassot fluxmeters of the General Electric type together with a search coil measure and record small changes of field.

2. Measurement of permeabilities.

- (a) The measurement of the moment of an ellipsoid of revolution in a uniform field.
- (b) The use of electromagnetic induction with primary inducing field and secondary coil with ballistic galvanometer gives the permeability. Accuracy requires the use of a toroidal specimen or a yoke.

118. CURRENT MEASUREMENT

1. Absolute Standards. (a) The tangent galvanometer discussed in section 23 yields an absolute method applicable to the determination of the electromagnetic unit. Historically it is the first absolute method. Because the tangent galvanometer requires an accurate knowledge of the horizontal intensity of the earth's magnetic field, it is not convenient to use nor is its range of applicability great.

(b) The absolute standard of current measurement is today achieved by the absolute dynamometer or current balance ascribed to Kelvin. This consists of carefully and accurately calculated and constructed coils at a known distance apart, with the current to be measured flowing in both coils. The force of attraction is measured by placing the coils on the arms of a balance. Usually for the sake of symmetry two coils are placed at the ends of a beam balance. Each of the coils on the balance have a pair of identical coils fixed relative

to the balance arms coaxial with them, one above and the other below. The same current flows through all coils in such a sense as to rotate the scale arms in one sense. The force is determined by weights added to restore the balance to the zero position. This device is applicable to a large range of currents and gives absolute electromagnetic units. The current is given by

$$i_a = \frac{1}{2} \sqrt{\frac{F}{\partial M / \partial x}},$$

where F is the force in dynes and $\frac{\partial M}{\partial x}$ is the rate of change of mutual induction by displacement of the coil along its axis. This can be computed for properly made coils.

2. Common Methods. (a) Electrodeposition furnishes a method of high precision and convenient reproducibility. It serves as the international unit and also as a legal standard. It is as accurate as the electrochemical equivalent and the chemical techniques involved permit it to be. The international ampere = 0.99986 ± 0.00002 absolute ampere and corresponds to the deposition of 0.001118 gram of silver per second under standard conditions.

(b) Galvanometers of the moving-coil or magnet type, or sturdy forms of these instruments called ammeters for the higher current ranges, are most commonly used. The accuracy is not great, 0.1 of 1 per cent being good. These instruments have been adequately discussed in Chapter IX. Astatic galvanometers go from 10^{-12} ampere up, d'Arsonval galvanometers from 10^{-11} ampere up, microammeters from 10^{-6} ampere up, milliammeters from 10^{-3} ampere up, and ammeters from 1 ampere to 1000 amperes. Shunts are used to extend the ranges.

(c) Great precision can be achieved by measuring the potential drop across an accurately known resistance using a no-flow method involving the potentiometer. It is more tedious than most of the methods other than the current balance but it also is the most accurate next to that method.

3. Absolute Static and Small Currents. (a) Currents produced by the flow of small accumulations of static charges and in gases by various ionizing agents are in general too feeble to give measurable magnetic effects. Since capacity C is accurately determined in absolute units by properly designed capacities then by measuring the time rate of change of potential dV/dt by noting the rate of rise or fall of potential V in time using an electrometer or electroscopes, i can be obtained from the relation $i = C \frac{dV}{dt}$ and converted to the electromagnetic units from the known ratio of the units in the two systems.

The potential calibration of the indicating instruments can be made by means of a potentiometer. These methods make it possible for currents from about 10^{-12} to 10^{-18} ampere to be measured to about 0.2 per cent at best. The latter current corresponds to currents of about 6 electrons per second. Recently by using ionization by collision amplifiers thermionic currents of a few electrons per hour have been measured. In such measurements each electron by multiplication produces a current kick which can be counted. The difficulties in such work lie in reducing spurious or background counts.

4. Alternating Currents. (a) Alternating currents as well as direct currents can be measured by galvanometers or ammeters in which the current to be measured flows through the moving coil and through the field coils connected in series with it. The deflection is proportional to the square of the current, which makes an awkward scale of somewhat limited range when precision is required. This limited range necessitates shunts to extend the scale. For weak currents these instruments require that a soft iron of low permeability be used in the field coils to give a measurable effect. For radio-frequency alternating currents of more than 10^4 cycles per second and frequently below this value, the iron is not able to follow the rapid changes of the alternating current in its hysteresis cycle. Hence these meters cannot be used much above 10^4 cycles.

(b) If an alternating or direct current of any frequency flows through a series of fine wires alternately made up of two metals *A* and *B*, the junction of *A* to *B* being placed in a vacuum jacket or being air-insulated while the *B* to *A* junctions are kept cooled by contact with a solid surface, the thermoelectromotive forces due to the i^2R heating at the insulated junctions gives a direct current that causes a galvanometer to deflect. Instruments built on this pattern are known as thermogalvanometers and are widely used in high-frequency current measurement; see page 168. They have deflections proportional to the square of the current. Their precision is poor, being less than 1 per cent, and their readings are unsteady except in the vacuum type. For certain other purposes crude sturdy instruments apply the principle of the heating effect to give current. These depend on expansion of the metal or a bimetal strip on heating and are used in automatic current controls.

(c) It is now also possible to use a rectifying crystal, copper oxide-copper rectifiers, or electron or discharge-tube rectifiers to change alternating currents into direct currents and thus permit their galvanometric measurement. The accuracy is not known, but it is *not* high. In a circuit where the alternating current is not to be rectified, this principle is inapplicable. It is useful in a-c bridge circuits in series with galvanometers to replace the telephone.

119. SUMMARY OF CURRENT MEASUREMENT

1. Absolute electromagnetic methods.

- (a) By use of the tangent galvanometer. It is inconvenient and good to 0.5 per cent.
- (b) By use of the absolute current balance. It is the most accurate instrument and is the basis of the determination of the absolute ampere. It is good to nearly 0.001 per cent.

2. Common methods.

- (a) By electrodeposition using Faraday's laws. It serves as the international and legal standard. It is good to 0.005 per cent.
- (b) By the use of galvanometers and ammeters. The accuracy is about 0.1 per cent.

The instruments are classified as follows:

- 1. Astatic moving needle: $> 10^{-12}$ ampere.
- 2. Moving coil: $> 10^{-11}$ ampere.
- 3. Microammeter: $> 10^{-6}$ ampere.
- 4. Milliammeter: $> 10^{-3}$ ampere.
- 5. Ammeter: $1-10^4$ ampere with adequate shunts.

- (c) By the use of a potentiometer to measure the potential due to the iR drop in a standard resistance. As accurate as the potential and resistance standard calibration.

3. Absolute electrostatic method and very small currents.

- (a) By the use of the time rate of change in potential as given in e.s.u. in a capacity of calculated value in e.s.u. To get amperes the e.s.u. may be changed to e.m.u. The method is good to about 0.2 per cent. It measures down to 10^{-18} ampere with great precautions.

4. Alternating currents.

- (a) Galvanometers or ammeters having field coils and moving coils traversed by the same current. The deflections are proportional to i^2 . The instruments are good to 0.1 per cent. They cannot be used for low currents at frequencies above 10^4 cycles.
- (b) Hot wire ammeters are used where heating due to i^2R loss in a wire changes lengths or acts on a thermocouple. With thermocouples the thermogalvanometers read radio currents of 10^{-6} to 10^{-2} ampere to about 1 per cent.
- (c) For use on a-c bridges the alternating current can be rectified by crystals or thermionic rectifiers and then read on d-c meters.

120. POTENTIAL MEASUREMENT

1. **Absolute Measurements.** (a) The initial absolute measurement of potential difference following the definition $PD = W/q$ came from the heating effect produced by a known current in a calorimeter using the law that the heat in calories is $H = 0.239 Vit$, where V is in volts. Such measurements are cumbersome and not very accurate. Since more accurate methods replace it, the method is used to determine J , the mechanical equivalent of heat, from a knowledge of the value of Vit .

(b) The electromagnetic potential standard could today be set up in terms of electromagnetic induction. That is, by spinning at a fixed measured angular velocity a known number of turns of wire in a known magnetic field produced in a coil by a known current the induced e.m.f. could be compared by potentiometer with a constant

source of e.m.f. such as a standard cell. Then since from these data the e.m.f. given by the coil can be calculated the cell will be calibrated. This procedure has proved to be inconvenient and is not used. Instead, careful studies based on this general principle makes it possible once and for all to evaluate a standard resistance coil in absolute measure. A known current through the resistance gives a precision e.m.f. that is compared to a standard cell by potentiometer.

(c) Potentials can be evaluated in the electrostatic system in absolute measure by means of the attracted disk or absolute electrometer. It is applicable to a limited range of potentials only; see page 195.

2. Common Methods. (a) The international standard of e.m.f. and hence potential difference is that of the standard Weston cell. This cell has been compared to the absolute electromagnetic standard. Thus any potential can be evaluated to considerable precision by comparison with a standard cell, using the potentiometer. The standard Weston cell has an e.m.f. of 1.0183 international volts at 20° C and 1 international volt = 1.00034 ± 0.00003 absolute volt. Standard cells are temperature-sensitive. All measurements must be corrected for temperature.

(b) For many purposes potential differences can be measured conveniently and quickly using high-resistance galvanometers or voltmeters. Such measurements are good to about 0.1 per cent and the instruments require calibration. Care must be taken that the resistance of the voltmeter is large compared to the internal resistance of the potential source measured.

These matters are discussed in Chapter IX.

3. Alternating and Static Potentials. (a) Alternating potentials of frequencies below about 10^4 cycles can be measured on voltmeters having the field coils excited by the same current that flows through the moving coil. Thus the deflection proportional to the square of the potential is obtainable.

The same conditions apply as apply to a-c ammeters.

(b) Very-high-frequency potentials, as well as *low and high static potentials*, cannot be measured in this fashion. A static voltmeter built on the principle of the attracted disk electrometer, the gold-leaf electroscope, or the *quadrant* electrometer will measure static potentials exceptionally well. A quadrant electrometer idiosyncratically connected will measure a-c potentials of any frequency. The a-c potential will be the root mean square potential for pure sine-wave-form alternating currents. If the alternating current is rectified by means of an electron valve, the peak value is read and any one of the static instruments can be used. A large number of instruments of an indefinite range of values are at hand for static measurements. These must be calibrated directly by means of the potentiometer. The accuracy can be easily 0.1 per cent with proper instruments.

(c) Because of its practicability and handiness a new instrument

called the vacuum-tube voltmeter is being frequently used today for high-frequency potential measurement. It can be applied to d-c potentials and to very low d-c potentials as well. The instrument is more complicated for very low potentials. It depends for its action on the effect of a potential difference between the filament and the biased grid of a three-electrode thermionic vacuum tube in altering the current flowing in the tube. Thus if the potential variations are of the correct order of magnitude to lie within the linear part of the characteristic curve of the three-electrode tube this variation can by the change in the plate current be read on a current meter. It can be made to read either d-c or a-c potentials without drawing more than a fraction of a milliampere of current from the source. The defects of this instrument are that the sensitivity changes relatively rapidly and the voltmeter must almost continually be calibrated. It is more convenient than an electrometer for some work and can easily indicate potentials due to 10^{-15} ampere of current through a high resistance when properly amplified. The accuracy is not high, the error usually being greater than 1 per cent.

(d) Small potentials can be measured in a relative fashion by a phenomenon discovered by Quincke. A mercury column in a capillary tube of 0.5-mm diameter is connected to a large pool of mercury, and the tube above the mercury is filled with a dilute sulfuric acid. A difference of potential between the solution and the mercury causes a change in the interfacial tension between water and mercury. Hence the height of the mercury surface changes with its potential. This is read on a micrometer eyepiece. This device, called the capillary electrometer, is of limited range but is used by chemists because of its simplicity and adaptability to some of their measurements.

4. High Potentials. In the past, high potential measurement did not occupy much attention, but recent progress in many fields has made this measurement imperative.

(a) Up to 10,000 volts use can be made of voltmeters of high resistance, or else static voltmeters of the attracted disk type.

(b) Above 10,000 volts use can be made of the flow of current measured by milli- or micrommeter or galvanometer through precision high-resistance towers. Such towers good to 0.1 per cent and giving up to 10^4 megohms (10^{10} ohms) are now commercially sold. Instruments of this type giving potentials of the order of 10^6 volts to 1 per cent are now in use at the Department of Terrestrial Magnetism of the Carnegie Institution and at the National Bureau of Standards at Washington, D. C. Measurements of this type are generally good to 0.5 per cent.

(c) Spark gaps have in the past been used to determine potentials above 10,000 volts. The sparking potential in air between spheres of given radius at a given spacing is supposed to be known. Sparking potentials are hard to measure with any precision, and it was shown

not too long ago that the A.I.E.E. supposed standard spark-gap calibration was some 10 per cent off. Such measurements are certainly not good to 2 per cent, and above 10^5 volts are probably 5 per cent off. They are little used today by physicists.

(d) For very high potentials it is possible to use two air condensers of known capacity ratio in series. Then by placing a static voltmeter across the larger capacity the total potential difference can be evaluated in terms of the measured potential across the larger condenser. The condensers must not be conducting, otherwise bad results will ensue.

(e) For very high-potential work recent success has been achieved with static generating voltmeters. These consist in rotating a commutator with insulated segments in the field produced by the potential drop at a known rate of speed. Brushes then pick off the induced charges on the metal parts of the commutator produced by the field. The current generated as given by this commutator and appropriately measured can then be related to the potential difference across the plates. The device must be arbitrarily calibrated and is good to about 1 per cent. Devices of this type are used with the 2.5×10^6 volt Van de Graaff static x-ray machines. They have been developed for use in measuring the potentials of aircraft in flight and for the study of lightning potentials.

The appearance of very high energy particles from accelerators like cyclotrons, etc., as well as in nuclear reactions, have required evaluation of the particle energy in terms of the equivalent accelerating potentials on unit-charged particles in vacuum. These energies can be evaluated in numerous ways. Thus they can also be used to estimate the potentials which accelerate the particles in vacuum and can in emergency serve as a means of measuring high potentials. The term *electron volt* is defined as the energy given an electron in falling freely through a potential difference of 1 volt in vacuum. Since the charge on the electron is 4.8×10^{-10} e.s.u. and 1 volt is $1/300$ e.s.u. the energy equivalent to an electron volt is $W = ev = 1.60 \times 10^{-12}$ of an erg. One million electron volts is thus 1.6×10^{-6} erg. This is a small amount of energy, but when applied to as small an entity as an electron or a positively charged hydrogen nucleus, it is a great deal. The energy involved in the most powerful chemical reactions if evaluated per atom is not in excess of some 10 electron volts.

The ways in which the energies of particles imparted by fields or nuclear reactions can be found are as follows:

(a) If electrons are accelerated by such energies they can be allowed to impinge on matter and the energy of the resultant x-rays can be measured either by absorption studies, or by a measurement of x-ray wave lengths, using crystals or diffraction gratings. From the wave lengths the x-ray frequency is found and this, multiplied by the Planck constant h , gives the energy in ergs. The absorption coeffi-

cients of x-rays of known wave lengths in standard substances also furnish a fair estimate of their wave lengths.

(b) If the particles — electrons or ions — are available, some indication of their energy is possible, if their charge and mass are known, from a study of their range in various materials.

(c) If the particles are in a beam and can be subjected to a transverse magnetic field in vacuum, they are constrained to move in circular paths. If the radius of curvature, ρ , of the path can be obtained by proper geometrical arrangement of the experiment, or by use of cloud tracks of vapor condensed by adiabatic expansion, it is possible, knowing the charge and mass, to obtain the momentum, and hence the energy, from the relation $\frac{mv}{e} = H\rho$, where e is the charge, m the mass, H the magnetic field, and v the velocity.

(d) If the mass is not known then the energy can be obtained by simultaneous electrical and magnetic field deflections.

(e) Finally, where the charge is known, a study of the ionization of air or some known gas along the path and the radius of curvature of the track in a known magnetic field, will give the energy when proper cloud track pictures can be obtained.

By this means, for instance, the energy of such obscure particles as some of the cosmic ray constituents can be obtained. Here energies range as high as 1000 million electron volts.

121. SUMMARY OF POTENTIAL MEASUREMENTS

1. Absolute measurements.

- (a) Heating effect of a current using a calorimeter gives an accuracy of about 0.1 per cent, but is inconvenient.
- (b) A standard current through an absolute standard ohm gives a potential standard as accurate as the other standards used. Accuracies up to 0.005 per cent have been achieved.
- (c) An absolute electrometer gives potential in electrostatic units. It is good only between some hundreds of volts and 10^4 volts and is accurate to 0.1 per cent.

2. Common methods.

- (a) Comparisons can be made with an international standard cell using the potentiometer. It is good to about 0.01 per cent, depending on the cell used.
- (b) Galvanometers and voltmeters of high resistance are most frequently used. These do not give the electromotive forces accurately. They are good to 0.1 per cent with good instruments only.

3. Alternating and static potentials.

- (a) Voltmeters with the current from the potential difference going through both the field coils and the moving coil measure a-c potentials to 0.1 per cent from 10^{-3} to 10^3 volts below 10^4 cycles.
- (b) (1) Alternating potentials $>10^4$ cycles can be measured using quadrant electrometers idiostatically connected and calibrated with direct current. This method is good to 0.1 per cent. The meaning of the potential measured depends on the form of the alternating current. If it is sinusoidal the root mean square value is given.

- (2) Peak value of rectified a-c potentials given by thermionic rectifiers as well as static potentials can be measured to 0.1 per cent from 10^{-4} to 10^4 volts using quadrant electrometers, absolute electrometers, or gold-leaf electroscopes.
- (c) Vacuum-tube voltmeters measure a-c and d-c potentials to about 1 per cent conveniently with small current loss from 10^{-3} volt on up to 10^3 volts.
- (d) Small potentials of the order of 10^{-2} volt are measured by the capillary electrometer.
4. High potentials. These are measured as follows:
- (a) By voltmeters of the current type or absolute electrometers to 10^4 volts.
- (b) By galvanometers in series with high resistance towers, from 10^4 to 10^6 volts to 0.5 to 1 per cent.
- (c) By spark gaps for direct and alternating currents. This method is good to less than 10 per cent.
- (d) By the fall of potential across two *nonleaking* condensers in series.
- (e) By the static generating voltmeters, 10^4 volts up.
- (f) By the determination of energy of known ionic or electronic particles accelerated by these potentials in vacuum, using one of several techniques.

122. RESISTANCE MEASUREMENT

1. Absolute Measurements. Absolute measurements of the ohm have been made by making a coil to be used as a standard. This coil is mounted on a nonmagnetic shaft perpendicular to its axis and spun in a known homogeneous magnetic field produced in a coil by an absolutely determined current. The field produced by the current from the induced e.m.f. in the ohm acting through its resistance combined with the uniform field acts on a small compass needle. From the deflection obtained the value of the resistance can be calculated from

$$\tan \alpha = \frac{\pi^2 n^2 a \omega R}{R^2 + (L^2 - \pi^2 n^2 a L) \omega^2}.$$

Here R is the resistance, α the angle of deflection of the compass needle, a the radius of the resistance coil, n the number of turns, ω the angular velocity of coil, and L its self-induction. This gives the absolute ohm but is not used except for absolute standards.

2. Common Methods. (a) With the absolute standard thus determined comparison of resistance can be made to 1 part in 10^6 by the Wheatstone bridge if temperature effects and thermal e.m.f.'s at junctions are avoided; see page 93. Thus the Wheatstone bridge or the many modifications such as Kelvin's bridge for low resistances suffices for comparison. The standard absolute ohm has been related to the international or legal ohm which is the resistance of a uniform column of mercury 106.3 cm long with a mass of 14.4521 grams at 0°C , and 1 international ohm = 1.00078 ± 0.00002 absolute ohms.

(b) A nearly equally accurate method for measuring a resistance in the condition in which it operates when carrying a given current i is to measure i with an accurate instrument, and to put a potentiometer

across the resistance to measure V , the iR drop. Hence R is obtained from the relation

$$R = V/i.$$

This is good to 1 part in 10^5 . It is useful for measuring the resistance of hot filaments, etc.

(c) Resistances can sometimes be measured by the heating effect H in calories produced in a time t by a known current i from the relation $H = 0.239 i^2 R t$. The method is not accurate, but there are circumstances where it is applicable.

(d) Quick and rough measurements of resistance can be made by reading the current through a resistance with an ammeter in series and measuring the potential with a voltmeter across the resistance in question, the voltmeter having a high resistance compared to the resistance in question. These are good only to a few per cent as the voltmeter diverts the current when the resistance is appreciable.

(e) Finding enough adequate resistance boxes to make a Wheatstone bridge and having a galvanometer handy it is possible to measure resistances in comparison to a known resistance of the same order plus a variable resistance box using the method of substitution. This is especially useful for resistances of 1 megohm or more. It is *not accurate* to better than 0.5 per cent, and bridges are always more convenient. It is *not* to be used except in emergency when bridges are not available.

3. High Resistances. Very high resistances are mostly measured by measuring the static leakage currents produced, or effects resulting from them.

(a) Apply a constant known potential to a resistance and measure the current by galvanometer. This is good to about 10^7 ohms and gives 1 per cent accuracy.

(b) Charge a condenser of capacity C and observe the rate of discharge through the resistor for small changes in applied potential. An electroscope has its leaf fall measurable amounts for a change in potential of a fraction of a volt, for large deflections of the leaf, i.e., some 300 to 1000 volts. Then, if the capacity C is known by measuring dV , the change in V , in a time t , one has $i = C \frac{dV}{dt}$ and $R = \frac{V}{i}$. This is good only where the measured fall is small compared to V . It is good to 1 or 2 per cent.

(c) Again a condenser C can be charged to a potential V_1 in which it has a quantity $Q_1 = CV_1$. This quantity discharged through a ballistic galvanometer gives a deflection θ_1 . Next the condenser is again charged to V_1 and discharged for a time t through the resistor R until its potential has fallen to V_2 . It is then discharged, and the deflection θ_2 given by the ballistic galvanometer for the residual charge

$Q_2 = CV_2$ is noted. Then by the theory given on page 362, which reads

$$\frac{Q_2}{Q_1} = \frac{\theta_2}{\theta_1} = e^{-\frac{t}{RC}},$$

R can be found if C is known. The method is good to 0.5 per cent and applies only to resistances above 10^8 ohms and capacities of microfarads.

(d) With the demand today for accurate high resistances for potential measurement high resistance standards are being made up by making coils commercially of 10^6 to 10^7 ohms in units and placing these in series. These units can then be measured accurately and excellent standards are now at hand. They are good to tenths of 1 per cent. Great care must be used in calibration since the temperature coefficients of resistance are not negligible and a current flow causes heating which changes R .

123. SUMMARY OF RESISTANCE MEASUREMENTS

1. Absolute measurements.

(a) The absolute ohm is determined by electromagnetic induction, to 0.002 per cent.

2. Common methods.

(a) The best method is the Wheatstone bridge, Kelvin bridge, or other appropriate type of bridge. Its reliability as generally used is of the order of 0.05 per cent. The accuracy is limited by the accuracy of the resistances involved.

(b) The measurement of a current and the measurement of the potential across the resistance due to the iR drop by means of the potentiometer gives a method useful for parts of a circuit carrying a current, e.g., hot filaments. The accuracy is of the order of 0.1 per cent or better.

(c) By utilizing the heating effect of the current in a calorimeter. The accuracy is not high, but at times the method is valuable.

(d) By means of a voltmeter and an ammeter. The accuracy is of the order of 1 per cent.

(e) By the method of substitution. Substitution is accurate to 0.5 per cent, but if bridges can be used it is not convenient nor advisable except for high resistances.

3. High resistances.

(a) Apply a known high potential and measure the current through the resistance by a galvanometer. This gives about 1 per cent accuracy.

(b) Measure the leakage current through the resistance by fall of potential across a known charged condenser in parallel. The method is good to 1 or 2 per cent.

(c) Use a condenser and ballistic galvanometer. The accuracy is about 0.5 per cent.

124. CAPACITY MEASUREMENT

1. Absolute Measurements. (a) A capacity in terms of absolute electrostatic units can be built according to specifications in the form

of spherical, cylindrical, or parallel-plate condensers. To ensure accuracy it must be remembered that near the open ends of condensers the fields are not uniform and hence the peripheral portions do not add as much to the capacity as they should. Also care must be taken that the outer parts of a small-area parallel-plate condenser are not too near other objects leading to auxiliary capacities which are not included in the computation. For this reason the spherical and cylindrical condensers with the outside conductors grounded are the more accurate. The edge effects can be reduced with guard rings. Correction must be made for the dielectric constant of insulating spacers. Hence it is possible to have standard capacities made to order and, knowing the ratio of the absolute electromagnetic to the absolute electrostatic units, express the capacity in either units. The method can be accurate to 0.1 per cent, and standards up to 10^4 -cm capacity can be made.

(b) In electromagnetic units the absolute value of a capacity can be determined as follows: A capacity C is charged to a potential V and discharged through a ballistic galvanometer. Then a deflection θ_1 is observed such that $Q = CV = \frac{Tk}{2\pi} \rho \theta_1$, where T is the galvanometer

period, ρ the damping factor, and k the figure of merit (when θ_1 is in millimeters on a scale 1 meter distant). If the period T , the potential V , and the figure of merit k are determined in absolute electromagnetic units, C will be given in absolute e.m.u. This is accurate to perhaps 0.1 per cent. This method, with a standard condenser from (a), can be used to determine the ratio of e.m.u. and e.s.u. and applies to capacities above 1000 cm only.

2. Comparison Methods. (a) Actually a capacity of any magnitude cannot be readily evaluated in absolute measure, though for very small values, standard capacities calculated in centimeters can be made ($10 - 10^4$ cm). Capacities are in general standardized against inductances which can be accurately manufactured to specifications. There are many bridges analogous to the Wheatstone bridge using alternating currents, where a capacity replaces or acts in conjunction with a self-induction such that if all the resistances and inductances are known the capacity is determined accurately in terms of the self-induction. This is good to 0.1 per cent or better.

(b) Capacities, once standards are available, can be compared very conveniently by replacing the unknown and the standard *resistances* in the arms of a Wheatstone bridge by the unknown and the standard *capacity*. Balance is determined as in the Wheatstone bridge by absence of an *alternating* current in the galvanometer arm of the bridge. This can be tested for by silence in a telephone when an audible-frequency alternating current activates the bridge, or by rectifying the alternating current and using a galvanometer. It is accurate only when the capacities are of the same order of magnitude. Comparison to 0.1 per cent is possible.

(c) Either a capacity or a self-induction can be determined if the frequency of electrical oscillation can be determined accurately. This has become possible today to a very high degree of precision by means of quartz or piezoelectric oscillators. Using a known variable self-induction across the unknown capacity and varying the self-induction until the frequency of oscillation of the capacity-inductance system corresponds exactly to the crystal gives the capacity if the other variables are known. The frequency match can be determined from the stationary character of the Lissajous figure in a cathode-ray oscillograph invoked by the two as the frequency changes, or by

the method of beats. Then from the expression $N = \frac{1}{2\pi} \sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}}$

C can be calculated from known values of N , R , and L . Here N is the frequency, L is the self-induction, and R is the resistance. This method is capable of very great precision since frequencies can be measured today to better than 1 in 10^7 .

3. Small Capacities. (a) Very small capacities of the order of 500 cm down to 10 may be compared by the following processes.

If a known small capacity C_1 be charged to a potential V_1 by some means and V_1 be measured by an electrometer or electroscope and if this be placed in parallel with an unknown capacity C_2 the potential will fall to V_2 . Then

$$\frac{V_1}{V_2} = \frac{C_2 + C_1}{C_1}.$$

This method may be good to 1 per cent or better, depending on conditions.

4. Other Methods. An unknown capacity C of large value can be charged to a potential V_1 and the potential recorded. Then the capacity can be discharged for a measured time t through a known high resistance R . Next the condenser can be discharged through a ballistic galvanometer giving a deflection θ_2 . A discharge of the fully charged capacity gives a deflection θ_1 . As in the measurement of high resistances we get $\frac{\theta_2}{\theta_1} = e^{-\frac{t}{RC}}$, and C can be determined. The method is good to a few tenths of 1 per cent.

125. SUMMARY OF CAPACITY MEASUREMENT

1. Absolute measurements.

(a) By building suitable cylindrical condensers with guard rings to specification standard capacities of small value in absolute electrostatic units or centimeters are obtained. These are accurate to 0.1 per cent or better.

(b) Discharging a condenser charged to a known potential in e.m.u. and discharging it through a ballistic galvanometer giving Q in e.m.u. gives C from

$$C = \frac{Q}{V}. \quad \text{This is good to 0.1 per cent or better.}$$

2. Comparison methods.

- (a) By the comparison of capacity with a standard inductance made to specifications by means of bridge methods. The precision is 0.1 per cent or better.
- (b) By direct comparison with a known capacity of the same order of magnitude using a Wheatstone bridge with alternating current. This method is good to 0.1 per cent.
- (c) By measuring the frequency of oscillation of a circuit with the capacity and a known variable inductance at resonance with an accurately known frequency. This is as accurate as the variable inductance permits.

3. Small capacities.

Statically by the method of mixtures comparing the capacity with a known capacity using a static potential-measuring device. This method is accurate to about 1 per cent.

4. Other methods.

By discharging the condenser for a known time through a known high resistance R and discharging it through a ballistic galvanometer. The method is good to about 0.5 per cent.

126. SELF-INDUCTION MEASUREMENT

1. Absolute Measurement. (a) The absolute coefficient of self-induction in the electromagnetic system can be computed accurately for an air-core coil carefully made to predetermined specifications. Hence absolute electromagnetic standards can be made to any desired degree of precision. (See F. W. Grover, *Inductance Calculations*, New York, D. Van Nostrand & Co., 1946.)

(b) Self-induction as well as mutual induction can be *measured* in the absolute electromagnetic system by means of a standard resistance R . Only the treatment of self-induction will be given. A Wheatstone bridge is made up of noninductive resistances with a ballistic galvanometer in place of a damped one. In the arm R_x the unknown self-induction L is placed with resistance R_x and an accurately known small resistance R calculated in advance, to be of a size appropriate to the value of L . R must be very small compared to R_x . The bridge is balanced for direct current with R shunted by a very low resistance. Next the battery is cut off suddenly, and an e.m.f.

$$E = L \frac{di}{dt}$$

sends a current through the network, part of which goes through the galvanometer giving a throw θ_1 corrected for damping. Then the battery is connected again, and the resistance R inserted by opening its shunt. A steady current now flows equivalent to that produced by an e.m.f. $= Ri_x$: For since the bridge was in balance with i_x in that arm and R cut out, putting R in does not *materially* change i_x but introduces an e.m.f. $= Ri_x$ which causes a steady deflection of the galvanometer θ_2 . Then since the ballistic throw θ_1 was due to the quantity aLi_1 passing through the galvanometer where a is a factor of proportionality depending on the resistances in the network,

$aLi_1 = \frac{Tk}{2\pi} \theta_1$. But the current through the galvanometer with R in is

$aRi_1 = k\theta_2$; thus $ai_1 = \frac{k\theta_2}{R}$ and $\frac{L}{R} = \frac{T}{2\pi} \frac{\theta_2}{\theta_1}$. This method is limited in accuracy to some tenths of 1 per cent. It is absolute, however.

2. Comparison Methods. (a) If a standardized variable inductance inductometer (see section 110), made by rotating one of two coils in series inside the other one, from a position of addition of e.m.f.'s to one of subtraction, which can be calibrated for dial marked settings, is available, a very convenient a-c bridge method of comparison is at hand. Placing the unknown inductance at R_x with a variable resistance in series and the variable inductance at R_s and replacing the galvanometer either by a galvanometer for d-c balance and by telephone or a galvanometer and rectifier for a-c balance the first step is to adjust the bridge with direct current for a balance by altering the ratio arms and R_x . Then the telephone is inserted and the battery is replaced by a single-frequency vibrator to give alternating current. The variable inductance is then adjusted for silence, or no current if rectifier and galvanometer are used. The precision is as good as the standard. In general, methods like this are good to about 0.5 per cent.

(b) Other a-c bridge methods compare the inductance with standard capacities such as in Anderson's, Maxwell's, or Owen's bridge. These can in their sensitive ranges give results to 0.1 per cent.

(c) Mutual inductances can be determined by bridge methods, and in simple geometrical arrangements of conductors they can be calculated with high precision.

(d) If a standard variable capacity is at hand the value of the self-induction can be determined accurately by adjusting the system to resonate to a standard accurately known frequency.

127. SUMMARY OF MEASUREMENTS OF SELF-INDUCTION

1. Absolute methods.

(a) The coefficient of self-induction of coils made to specified dimensions can be calculated to a high degree of precision in absolute e.m.u. or centimeters.

(b) Inductances can be measured in absolute units in terms of resistance by means of a Wheatstone bridge and a ballistic galvanometer. The precision is about 0.1 per cent or less.

2. Comparison methods.

(a) Comparison with a standardized variable inductance in a Wheatstone bridge with alternating current gives an accuracy of about 0.5 per cent.

(b) Comparisons with accurately calibrated condensers can be made by a-c bridges such as Anderson's, Owen's, and Maxwell's bridges to 0.1 per cent.

(c) Mutual inductances can be computed as under 1 (a) above or they can be compared with self-inductions or other mutual inductances by bridge methods.

(d) By combining the inductance with a standard variable condenser and tuning to resonance with an accurately known frequency the self-induction can be determined as accurately as the capacity is known.

128. POWER MEASUREMENT

(a) For *direct currents* the power may be obtained by the product of an independent reading of a voltmeter across the load and an ammeter in series with the load. With an alternating current these instruments yield, in the product of their readings, the *apparent power consumption*, the value of which divided into the *true power consumption* in watts defines the cosine of the angle of phase lag or advance in a circuit. This quantity is known as the *power factor*.

(b) By a wattmeter the true power consumption is at all times obtained for alternating and direct currents. The accuracy is of the order of 0.5 to 0.1 per cent.

CHAPTER XXIII

THE BALLISTIC GALVANOMETER — TRANSIENT PHENOMENA

129. THE BALLISTIC GALVANOMETER

An instrument of great importance in the measurement of electrical phenomena where currents change rapidly with time merits special consideration. This is the ballistic galvanometer. In Chapters XIX and XXI, where induced currents were discussed, it was seen that the induced electromotive force generated was a function of the time rate of change of the current. Most often this is not constant and is too rapid to follow by any instruments readily available (it can be studied by the oscillograph). The difficulty is frequently overcome, as stated in Chapter XIX, by letting the electromotive force generated cause a current to flow through the resistance coil of a galvanometer which is *little damped*, whose period is long compared to the duration of the current. Such a galvanometer is called a *ballistic* galvanometer. The instantaneous current i_{dt} flowing gives an instantaneous force f_{dt} which acts on the galvanometer coil for the time dt during which it flows. The product of f_{dt} times the time interval dt gives the impulse communicated to the coil at that instant. The sum of all the impulses over the time the current flows gives the total impulse given the coil, or the momentum communicated to it.

The sum of all the currents times the same time intervals dt gives the total quantity of electricity passing through the system resulting from the electromotive force generated. Thus one can write that

$Q = \int_0^t i_{dt} dt$ and this is proportional to $\int_0^t f_{dt} dt$ or the momentum given the coil. The momentum causes the coil to swing to a point where the kinetic energy of rotation given to it by the field is changed into the potential energy of the twisted suspension. This deflection, or "throw," of the galvanometer is related in a definite way to the momentum given and therefore to the quantity passing.

Thus by measuring the "throw" of the galvanometer, it is possible to measure Q and therefore determine some of the constants of the circuits studied in Chapters XIX and XXI. In fact, the ballistic galvanometer is often used in determining the coefficient of self-induction and in studies of induced currents.

130. DERIVATION OF THE EQUATION OF THE BALLISTIC GALVANOMETER

In Chapter IX, the couple acting on the galvanometer coil of effective area A (area multiplied by the number of turns of wire), when a current i_{dt} flows through it in a field H , was given by $G_{dt} = i_{dt}AH$. The force moment times the time dt over which the current has the value i_{dt} (the instantaneous impulse), is then given by

$$G_{dt} dt = i_{dt}HA dt.$$

Since the total impulse given the coil from the time 0 when the current starts to the time t when it has ceased is the sum of all the elementary impulses times their time of duration, one may write for the impulse

$$\overline{Gt} = \int_0^t G_{dt} dt = AH \int_0^t i_{dt} dt.$$

But $\int_0^t i_{dt} dt$ is nothing other than the total quantity of electricity which has passed, namely, Q : whence

$$\overline{Gt} = HAQ.$$

Now by Newton's second law, force times time is $ft = mat = mv$, mass times velocity, or momentum. By analogy, where the force acts on a lever arm r , it is possible to write $frt = mart = mvr$. But $v = \frac{ds}{dt}$, $\frac{ds}{dt} = \frac{rd\theta}{dt}$, and $vr = \frac{r^2d\theta}{dt}$, or $r^2\omega$, where ω is the angular velocity. This may be written $frt = mr^2\omega$. Now mr^2 for the simple case above, representing a mass m at a distance r , is the moment of inertia I . It accordingly follows that the impulse is $frt = I\omega$.

By analogy to this the couple times the time, \overline{Gt} , is the impulse and must equal the moment of inertia of the coil times the angular velocity produced. Thus one may write

$$\overline{Gt} = I\omega = AHQ.$$

Again by Chapter IX, there is the galvanometer constant K' given by $K' = T_0/AH$, whence $AH = T_0/K'$,

$$I\omega = \frac{T_0Q}{K'}$$

and

$$Q = K'\omega \frac{I}{T_0}.$$

Now ω cannot be measured directly. But the quantity $\frac{1}{2} I\omega^2$ measures the kinetic energy of the coil just after the impulse has been given it.

This causes the coil to deflect until the potential energy of the system equals the kinetic. The potential energy of the twisted suspension is the average torque times the angular displacement. At the rest point the torque is 0. At an angle θ it is $T_0\theta$ by Hooke's law. The average torque is then $\frac{T_0\theta + 0}{2} = \frac{1}{2} T_0\theta$. The displacement is θ . Thus the potential energy is $\frac{1}{2} T_0\theta^2$ and this must equal $\frac{1}{2} I\omega^2$.* Accordingly one has

$$\frac{1}{2} I\omega^2 = \frac{1}{2} T_0\theta^2$$

and

$$\omega = \theta \sqrt{\frac{T_0}{I}}.$$

Therefore

$$Q = K'\theta \sqrt{\frac{I}{T_0}}.$$

To get the ratio $\sqrt{\frac{I}{T_0}}$ it is only necessary to remember that the period of an oscillating system $T_1 = 2\pi \sqrt{\frac{I}{T_0}}$. (See Chapter IV.)

Thus the quantity Q of electricity which passed is given by

$$Q = \frac{K'\theta T_1}{2\pi}.$$

Thus from the "throw" θ of the ballistic galvanometer, its period of oscillation T_1 , and its constant K' , it is possible to determine Q , the quantity which passed through its coil in a time short compared to its period. The quantity Q will be in coulombs if K' is measured in amperes per unit deflection. If K' is measured in absolute units, Q will be in absolute units. Since the product of the figure of merit k and the deflection θ_m in millimeters on a scale 1 meter distant, $k\theta_m$, gives current in amperes, the equation can be written

$$Q = \frac{k\theta_m T_1}{2\pi}.$$

Q will then be given in coulombs.

131. THE FLUXMETER

The more recent developments in engineering and physics have indicated a need for a better knowledge of magnetic measurements than in the past. Thus it is important that students of today be

* The equality stated applies to the maximum potential and maximum kinetic energies in oscillatory motion which occur at maximum and 0 deflection, respectively.

acquainted with the principle of the Grassot fluxmeter, which in its modern variants will frequently be encountered in physical practice.

This instrument measures the change in magnetic flux through any coil placed where such measurements are desired. This change can take place slowly or rapidly and can be recorded. The coil used is termed the search coil. It may be of any desired dimensions, with the requisite number of turns and may or may not have a Permalloy or other core. It can be calibrated by subjecting it to known changes of flux.

The search coil is continuously connected to the terminals of a special type of galvanometer. Whereas with the ballistic galvanometer the quantity of electricity must have flowed in a time so short that the galvanometer had not deflected appreciably before it had ceased, the fluxmeter galvanometer must respond to the quantity of electricity over any finite time of delivery. This is accomplished by reducing all *mechanical damping*, due to air resistance and friction in the suspension, to a very small value. On the other hand, the electromagnetic damping caused by the movement of the galvanometer coil in its own magnetic field in the short-circuited system must be quite prominent. Its mechanical suspension must be very light, so that it exerts as nearly zero torque on the coil as possible. With appropriate suspensions the natural mechanical period of an open circuit coil can be of the order of several minutes.

The theory of its action is as follows. When a change of flux in the search coil occurs, an e.m.f. is induced in the search coil of value E_s . As the galvanometer deflects it also cuts its own field H with its M_c windings and produces an e.m.f., E_c . This opposes E_s . As a result, a net e.m.f., $E = E_s - E_c$, acts in the circuit at any instant.

If the search coil plus its leads and the galvanometer have a total resistance R a net current i will flow of value $E/R = i$. This current acts on the galvanometer coil to produce a torque, $G = iAH$, as in all galvanometers. Now a torque acting on a system with a moment of inertia I will produce an angular acceleration $I \frac{d\omega}{dt}$, where ω is the angular velocity. Thus it is possible to write

$$G = iAH = I \frac{d\omega}{dt} = \frac{E}{R} AH.$$

Now E_s , the e.m.f. in the n_s turns of the search coil, is $n_s \frac{d\phi}{dt}$. The e.m.f. in the galvanometer coil is produced by the rotation of the coil with angular velocity, $\omega = \frac{d\theta}{dt}$, across the uniform radial field H of the galvanometer. It is $E_c = 2 \pi r n_c \frac{d\theta}{dt} = AH\omega$, with $2 \pi r n_c$ as the total

area A of the M_c turns of the galvanometer coil. It follows that $E = E_s - E_c = \left(n_s \frac{d\phi}{dt} - AH\omega \right)$. Accordingly

$$I \frac{d\omega}{dt} = \frac{AH}{r} \left(n_s \frac{d\phi}{dt} - AH\omega \right).$$

This must be integrated over the time from 0 to t during which the flux changes, so that

$$\frac{AH}{r} \int_0^t \left(n_s \frac{d\phi}{dt} - AH \frac{d\theta}{dt} \right) dt = I \int_0^t \frac{d\omega}{dt} dt.$$

Now the effect of the torsionless suspension and of the electromagnetic damping is that both at $t = 0$ and at $t = t$ there is a steady deflection. That is, there is no angular velocity ω at $t = 0$ and $t = t$. Accordingly,

$$\int_0^t \frac{d\omega}{dt} dt = \int_0^t d\omega = [\omega]_{t=0}^{t=t} = 0.$$

Thus

$$\int_0^t \left(n_s \frac{d\phi}{dt} - AH \frac{d\theta}{dt} \right) dt = 0,$$

whence it is seen that

$$n_s \int_0^t d\phi = AH \int_0^t d\theta, \quad \text{or} \quad n_s [\phi]_{t=0}^{t=t} = AH [\theta]_{t=0}^{t=t}.$$

The first term is merely the number of turns in the search coil n_s times $\Delta\phi$, the total change in flux, and the second term is just AH times θ , the change in angular deflection resulting from the change in flux $\Delta\phi$ in n_s . Hence it is seen that for the fluxmeter we have $n_s \Delta\phi = AH\theta$. The deflection of the galvanometer θ is thus directly proportional to the change in flux, i.e., $\Delta\phi = K\theta$, where K is the constant of the fluxmeter. The instrument must be directly calibrated with known changes in flux. Modern electronics techniques make it possible to record even the smallest deflections so that a trace due to a continuously varying flux can be obtained. Sensitivity is very high if an adequate search coil can be used. The General Electric fluxmeter accomplishes the difficult task of obtaining a torsionless suspension and yet obtaining enough torque from the deflections to move a pen on a recording tape. The mechanical torque of the necessarily sturdy suspension for the galvanometer coil and mirror is compensated by a small magnet suspended in an appropriately designed magnetic field. Thus as the coil deflects the small bar magnet attached to the lower end of the coil suspension also deflects. The deflection of the small compensating bar magnet of moment M in its special *auxiliary* field H' produces a torque $MH' \sin \theta$ resisting or compensating the torque $T_0\theta$

of the twisted suspension. While $T_0\theta = MH' \sin \theta$ can be made only for small values of θ , say less than 10° , by judicious adjustment of M and H' a sufficient compensation for torsion over 20° can be obtained.

To convert the motion of the mirror to the mechanical motion of a recording pen the device is provided with an angular- or wedge-shaped plane mirror having its sharp edge lying parallel to the common axis of the pen and the galvanometer mirror and the wedge directed at the suspension. Just above the galvanometer mirror and coaxial with it on an *independent* mounting is the *pen mirror* on the writing pen. Light is focused on the galvanometer mirror and from that is reflected by a curved fixed mirror onto the pen mirror and comes to a fine focus on the sharp edge of the wedge-shaped mirror. From there the beam splits on the wedge. If galvanometer and pen mirror are exactly parallel and at zero setting, the light beam is exactly split in two by the wedge. Each equal half goes to one of the two balanced photoelectric cells to the right and left of the axis defined by light source, fixed mirrors, and suspensions. If now the galvanometer mirror deflects, light is deflected on reflection from the curved fixed mirror so that it leaves the pen mirror with a greater area going to one photoelectric cell. The increased photocell current is amplified and acts to give a current deflecting the pen by electromagnet in such a way as to rotate its mirror parallel to the pen mirror. Once these are again parallel the light intensity on both photocells is equal and no current flows. Thus by unbalancing the photo currents in the two photocells one has an electromechanically operated optical follow-up system that causes the pen to follow accurately the movement of the fluxmeter mirror. It is possible to alter the range of sensitivity as needed.

132. DAMPING

The equation for the ballistic galvanometer is subject to one correction. The coil does not oscillate perfectly as a simple harmonic oscillator. That is, the second swing in the direction of the initial deflection is not quite as great, and each succeeding swing to one side is less than the preceding one. This is caused by a frictional loss of energy in the suspension and to the action of the viscous drag of the air on the coil. If the coil remains short-circuited after closing the key to pass the current, damping is caused by the i^2R loss in the resistances in the circuit resulting from the generation of the e.m.f.'s as the coil oscillates in its own magnetic field. The energy of oscillation is gradually converted into heat and the oscillation is said to be damped. Under some circumstances this damping can be quite large.

The result of this damping is that even the first throw of the galvanometer has not the true value which it would have had in the absence of damping. To calculate Q , the damping of the system must be known and the θ observed corrected for it.

An undamped simple harmonic oscillator has a displacement given by

$$\theta = \theta_0 \sin \frac{2\pi t}{T},$$

where T is its period, t the time at which the deflection is measured, θ_0 its maximum deflection, i.e., that when $t = T/4$ or an odd multiple of $T/4$. If friction damps the oscillation the equation becomes

$$\theta = \theta_0 e^{-\rho t} \sin \frac{2\pi t}{T}.$$

That is, the deflection θ at a time t is changed from the value it would have in an undamped oscillation by the quantity $e^{-\rho t}$ where e is the

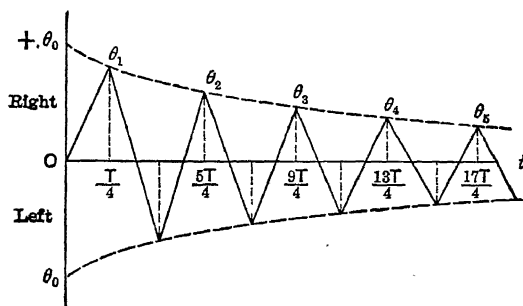


FIG. 153. Characteristics of a damped oscillation.

base of the natural system of logarithms, t is the time, and ρ is a constant. $e^{-\rho t}$ is called the *damping factor*. The curve for a damped oscillation is plotted in Fig. 153; see also section 142, page 388.

It is seen at once that θ is a maximum θ_1 at the end of the first quarter swing occurring at $t = \frac{T}{4}$. At this time $\sin \frac{2\pi}{T} t$ is unity and

$\theta_1 = \theta_0 e^{-\frac{\rho T}{4}}$. When the ballistic galvanometer is used it is usually θ_1 which is observed. Now what is required in calculation is the actual throw θ_0 which *would* have occurred had the instrument been free from damping. It is seen that $\theta_0 = \theta_1 e^{\frac{\rho T}{4}} = \theta_1 \sqrt{d}$, where $d = e^{\frac{\rho T}{2}}$ is called the *decrement* of the motion and $\log d = \log e^{\frac{\rho T}{2}} = \frac{\rho T}{2}$ is called the *logarithmic decrement*. Thus to correct the observed θ_1 it must be multiplied by $e^{\frac{\rho T}{4}} = \sqrt{d}$.

This quantity can be obtained by observing the damping as given by *successive* maxima of deflection to the *same side* as θ_1 which corre-

sponded to the first quarter swing. These occur a whole swing (i.e., four quarter-swings) later, that is, at the fifth, ninth, thirteenth, seventeenth, twenty-first, etc., quarter swings. Hence if *successive swings to the same side* are counted as $\theta_1, \theta_2, \theta_3, \theta_4 \dots \theta_n$, these occur at times $t_1 = \frac{T}{4}, t_2 = \frac{5T}{4}, t_3 = \frac{9T}{4}, t_4 = \frac{13T}{4} \dots t_n = \frac{4(n-1)+1}{4} T$.

Accordingly one can write

$$\frac{\theta_1}{\theta_2} = \frac{\theta_0 e^{-\frac{\rho T}{4}}}{\theta_0 e^{-\frac{5\rho T}{4}}} \text{ and } \theta_1 = \theta_2 e^{\frac{4\rho T}{4}},$$

whence $e^{\frac{\rho T}{4}} = \sqrt[4]{\frac{\theta_1}{\theta_2}}$. Then by observing θ_1 and θ_2 and taking the

fourth root of the ratio, $e^{\frac{\rho T}{4}} = \sqrt[4]{d}$ is evaluated so that $\theta_0 = \theta_1 \sqrt[4]{\frac{\theta_1}{\theta_2}}$.

Now for a good ballistic galvanometer θ_1 and θ_2 differ too little for an accurate determination of the ratio. In general, θ_1 and θ_n are observed, where n may be relatively large. In this case the equations above yield

$$\frac{\theta_1}{\theta_n} = \frac{\theta_0 e^{-\frac{\rho T}{4}}}{\theta_0 e^{-\frac{4(n-1)+1}{4} \rho T}} = e^{\frac{4(n-1)}{4} \rho T}$$

and

$$e^{\frac{\rho T}{4}} = \sqrt[4]{d} = \frac{4(n-1)}{4} \sqrt[4]{\frac{\theta_1}{\theta_n}}.$$

If the deflection θ_1 had been missed two deflections θ_p , say, the second or third could have been taken and later θ_n , the eighth or ninth, could have been taken. In this event reasoning analogous to that above gives

$$\frac{\theta_p}{\theta_n} = e^{4(n-p)\frac{\rho T}{4}} \quad \text{and} \quad e^{\frac{\rho T}{4}} = \frac{4(n-p)}{4} \sqrt[4]{\frac{\theta_p}{\theta_n}}.$$

Thus the quantity Q as given by an observation of θ_{m1} , the deflection in millimeters, is

$$Q = \frac{k\theta_{m1}T_1}{2\pi} \frac{4(n-p)}{4} \sqrt[4]{\frac{\theta_p}{\theta_n}}.$$

These high roots of the ratios $\frac{\theta_p}{\theta_n}$ can at once be found by dividing the

logarithm of the ratio by 4 ($n - p$) and looking up the antilogarithm of the result of the division. In general the correction should not run much above 2 per cent.

133. TRANSIENT PHENOMENA

As noted in section 111 the behavior of capacity and self-induction in electrical circuits is analogous to a coiled spring and an inertia in mechanics. If a mechanical system of weights connected by springs be imagined it presents a fair mechanical analogue of a section of an electrical circuit. Perhaps a good practical picture is a long row of freight cars with their masses and spring coupling. When the locomotive starts to accelerate at one end it takes some time for the whole system to get into motion. A wave of acceleration passes up the line. Thus before the whole system has achieved the uniform velocity of the locomotive there will be a transient or transitory phase in which the motion builds up.

Quite analogously, while previous chapters have discussed only steady state electric currents equivalent to the uniform state of motion of the train, it must be realized that there is a transient phase from the time an electrical switch is thrown until the currents reach their steady state. In this period the following sequence takes place. At $t = 0$ there is no current and no e.m.f. As a switch is closed an e.m.f. is applied at one point. This starts a current flow. However, resistance may retard the flow as it governs the stream of flow. Since there are always capacities distributed along the line which are initially at zero potential, these must charge up to their equilibrium potentials before they can effectively transmit the steady flow. The rate of charging will depend on the capacity and the flow. Finally, since there always is some inductance in the line this acts like an inertia to retard the flow by virtue of the back e.m.f.'s. As a result, step by step down a circuit the building up of the current is retarded by the opposing action of inductance and the need for charging up the capacities through resistances. Thus depending on R , L , and C the establishment of a steady current in a circuit will take time. Hence the rise of current after a switch is closed, or its decline as the circuit is broken or changed, will take time. These transition events are called *transient phenomena*. It is necessary to study them.

Initially little was known about them since with common values of R , L , and C they occur in intervals of time which are relatively short, i.e., in the order of 10^{-3} second. However, with the advent of longer transmission lines both for communications and power transmission these phenomena began to assume greater and greater importance. Thus today no engineer can consider his education complete without some knowledge of the fundamental processes at work.

In this field of study theory was able to solve many of the problems

which were inaccessible to experiment with existing techniques. In fact, it will presently be seen how theory allows the accurate prediction of what will happen. This was fortunate for it was many years before suitable instruments became available for the study of transients. The instruments eventually developed were called oscillographs. The successful commercial development of the cathode-ray oscillograph beginning in the 1930's has contributed vastly to the advance of electrical techniques. It is today as much a part of the equipment of the physical or engineering laboratory as is the voltmeter or ammeter. Improvements have gone so far in the last years that resolving times of 10^{-6} second in stock instruments are common and more complicated experimental instruments give down to 10^{-8} second. The technique of radar owes much of its success to the cathode-ray oscillograph. Thus before the theory of transient effects is discussed it is worth while to give a fundamental description of the oscillograph.

The Cathode-Ray or Braun-Tube Oscillograph. To study any transient phenomena, that is, either a potential or a current that varies with time, it is necessary to have an instrument that draws out the potential or current on, say, a vertical axis, while the elapsed time is run out along the corresponding horizontal axis. This was initially achieved by mechanical devices which operated on the following principle, and are still in use. A very light galvanometer coil with mirror was mounted about a vertical axis. The potential or current to be measured was applied to this galvanometer. The galvanometer was aperiodic, i.e., its period was such that it deflected in proportion to the current or potential at all times without imposing the characteristics of its own periodic motion on the motion to be measured. This was at times hard to achieve. A fine beam of light from an arc was reflected from the mirror onto a piece of photographic paper which could move vertically at right angles to the motion of the light beam at a uniform speed. Connections and timing were so arranged that when the key initiating the phenomenon was closed the paper was set into motion on its rolls at uniform speed, the light shutter opened, and the galvanometer coil was set in motion by the transient. A timing scale could be traced on the paper by a second galvanometer actuated by a precisely timed alternating current or by any one of a myriad of possible devices for putting a periodic mark on the paper. Such oscillographs were not capable of going beyond 10^{-3} to 10^{-4} second because of the limitations placed on the mechanical devices by inertia of galvanometer and paper, the limitations of aperiodic motion, etc.

The fact that beams of electrons can be deflected by both transverse electric and magnetic fields and that electrons have very little inertia suggested that these could be used in oscillographs. The recording of the deflection was at first a problem. Such beams could be recorded by deflecting the beam along one axis and by moving photographic film in vacuum at right angles to it. They were initially so operated, but

were limited in speed by the film speed. Later the improvement of fluorescent screens permitted the trace of the moving electron beam to be viewed on these. The timing axis had then to be furnished by some potential that could be placed on plates giving, say, a horizontal sweep with a uniform decay or growth of potential with time against vertical transient deflections. Means for achieving this were available by 1931 and steady improvement in this sense has gone on since then. The next problem to be met was the improvement of definition by getting finer and more intense electron beams and yet a good amplitude in deflection. These improvements came in the late 1930's with the development of sharp beam focusing by means of the new science of electron optics. It required the development of convenient d-c power packs (electronic high-voltage power sources) to yield more intense beams giving good luminosity on impact on the screen. It also required the development of high-fidelity amplifying circuits to amplify the currents or potentials in such a way as to be able to deflect the higher energy beams. With these improvements, production line commercial cathode-ray oscillographs or oscilloscopes capable of going down to 10^{-6} second or better are now available. Limitations to further reduction of time scales depends on the development of proper sweep circuits and higher-intensity fluorescent screens to yield enough light on fast sweeps. Photographic plate in vacuum can be used for the sweeps down to 10^{-8} second or below. They are awkward, as they require opening up the vacuum to develop the plates.

The device in principle consists of an electron gun, *FGO* in Fig. 154*a*. This is made up of an incandescent filament thermionic emitter source *F* of electrons which are focused by the arrangement of the cylindrical shield *G* surrounding the filament and the perforated diaphragm *D*. A battery lights the filament. Between *F* and *D* there is a high potential source of from 3000 to 10,000 volts, now delivered by power pack instead of battery as shown in Fig. 154*a*. The filament *F* is negative to *D*. The electrons speeded up to 10,000 volts energy (see page 423) in vacuum in *FD* move so rapidly that they pass through the pinhole *O* in *D* as a fine beam. Additional diaphragms or rings about the beam with appropriate potentials such as *R* to give an intense well-focused beam impinging on the fluorescent screen at *S* may be used. Between *D* and *S* there are a system of deflecting plates. These consist of two small pairs of metal plates *EE* and *E'E'*. In Fig. 154*a* they are shown separated along the beam axis, though in many oscillographs they are coterminal. An end-on view of the tube is shown in Fig. 154*b*. *EE* are horizontal and represent a parallel-plate condenser system with a field *X* determined by the plate configuration and the potential *V* applied. This potential *V* causes the vertical deflection and comes from an amplifier connected to the transient which it is desired to study. The two vertical plates of which the axis is parallel to the beam marked *E'E'* give an electrical field *X'* between them.

when a potential V' is provided across them. V' comes from an amplifier that is activated by a sawtooth wave shown in Fig. 154c. This is a device which produces an increasing deflection of the beam to the right as time progresses. Such action is accomplished by charging a condenser through an appropriate gas tube (e.g., a neon lamp), resistor in such a way that the potential V' increases from 0 to some value V_m' during a given time t of the sweep. Sweep time can be varied by changing the capacity of the charging condenser. That is, the potential on $E'E'$ or V' varies as shown in Fig. 154c, though the

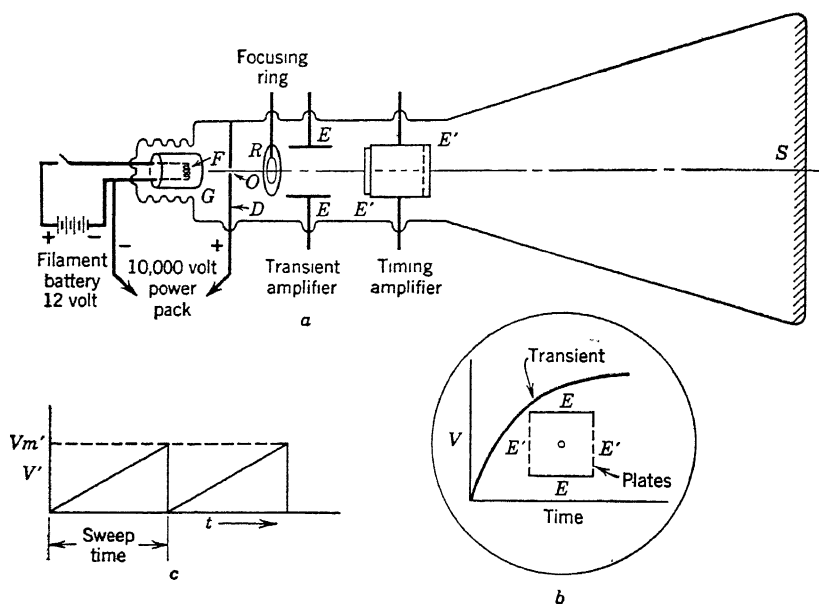


FIG. 154. The cathode-ray oscillograph as used for single sweep. *a*. Cross section lengthwise through the tube. *b*. End-on view of filament, deflecting plates, and transient curve with potential and timing axes. *c*. Sawtooth wave for timing axis.

decline in t is usually less abrupt than in the figure. Various circuits have been developed for achieving a linear rise in potential V' with adjustable timing, which it is unnecessary to go into. At any rate, these plates and their associated circuits are called the sweep circuit. They are usually adjustable to different sweep rates within limits.

The operation is relatively simple. The filament and high potential source are turned on and the position of the point of impact of the electron beam on the screen S is noted. The focusing control rheostats are turned until the beam gives a sharp fine image on the screen. Then the bias on the deflecting plate circuits EE and $E'E'$ are adjusted to place the point of impact on the lower left-hand corner of the screen,

or, if desired, on the left-hand center of the screen. If desired, the sweep circuit can be tested by putting on a repeat sweep so that it sweeps across the screen from left to right in a rapid succession of sweeps. This gives one a chance to see that the time axis is clearly defined and straight. Connections to the source to be tested are then made and the switches thrown so that when the switch giving the transient is thrown, both V and V' are applied to the plates EE and $E'E'$ simultaneously. In other words, the start of the transient triggers the sweep circuit for a single sweep. The transient can then be viewed visually or photographed with a high-speed camera.

If it is desired to study a transient potential, the two points to be studied are connected to the voltage amplifying system, if the transient potential is low. The amplified potential varying in time in proportion to the original signal will then be impressed on the plates such that the maximum voltage V_m will give a suitable vertical deflection on the screen. If the potential is too high then a noninductive resistance is placed across the source and a small fraction of the iR drop is picked up on the oscilloscope plates EE to give the desired deflection, no amplification being needed.

For *current* transients the problem was more difficult, especially for higher currents in less than 10^{-4} second. The current to be measured if small flows through a fixed noninductive resistor. Across this the iR drop is picked up by the contacts to the voltage amplifier of the plates EE . For smaller currents and longer times it is not difficult to get suitable resistors. In some studies, however, these noninductive resistors are hard to obtain. The exigencies of the World War II devices have now led to the development of suitable resistors for this work, especially for high-speed transients.

If desired, magnetic fields can be used to cause the vertical deflections. Pole pieces wound with wires through which the transient currents are to pass are made with the pole faces parallel to the plates $E'E'$, such that the field is parallel with the timing field X' . This gives a vertical deflection proportional to the magnetic field H .

For the study of alternating currents or similar phenomena the two parts of the a-c phenomena, e.g., the current and potential, can be placed across the two sets of plates EE and $E'E'$. If the amplitudes, frequencies, and phases of the two e.m.f.'s so applied are the same and amplifiers are set for equal gain, the resulting figure on the screen will be a diagonal trace at 45° with the axes. If the e.m.f.'s are $\frac{\pi}{2}$ radians out of phase the trace will be a circle. If frequency is the same, but amplitudes are not the same the situation with $\frac{\pi}{2}$ phase lag will be an ellipse, and that with 0 phase lag will be a line but not inclined at 45° . With different frequencies and phases different Lissajous figures will appear of which the characteristic forms will give much information

concerning the variables in question. With this particular use of the oscillograph the time sweep is accomplished by one a-c source and the transient effect by the other. It does not require the elaborate sawtooth wave, nor does it require the triggering that the single sweep requires. The variations in the employment of these devices are so manifold that it is necessary merely to note the various uses to which it can be put.

The Theory of Transient Phenomena Caused by Inductance and Capacity.

Case I. A Circuit with Self-Induction and Resistance. Regard the circuit pictured in Fig. 155 with a resistance R , a self-induction L , and an electromotive force E . Let the key be closed at A . The current begins to flow in the circuit. As it is building up, an e.m.f. of self-induction $-L \frac{di}{dt}$ is built up to oppose the growth of the current. The current was built up through the action of the impressed e.m.f., E .

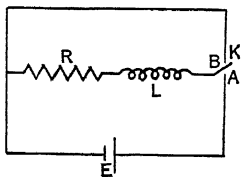


FIG. 155. Circuit with self-induction and resistance.

Thus we have $E - L \frac{di}{dt}$ as the e.m.f. in the circuit. By Kirchhoff's second law this at any instant must equal the iR drop in the resistance. Hence one can write

$$L \frac{di}{dt} + Ri = E.$$

This is what is known as a differential equation. As the interest focuses not on the relation between di and dt , but on i as a function of t , the equation must be integrated to get rid of the differentials. To integrate it may be rearranged as follows:

$$\frac{\frac{di}{dt}}{\frac{E - Ri}{L}} = dt.$$

This may be transformed to

$$\frac{-\frac{L}{R} d\left(\frac{E - Ri}{L}\right)}{\frac{E - Ri}{L}} = dt, \text{ for } di = -\frac{L}{R} d\left(\frac{E - Ri}{L}\right).$$

The integral of this is

$$\frac{L}{R} \log \left(\frac{E - Ri}{L} \right) = -t + K,$$

in analogy with the operation

$$\int \frac{dx}{x} = \log x + K,$$

where K is a constant of integration. This K can be evaluated as follows. When $t = 0$, the key was open and $i = 0$. Putting these values into the equation,

$$\frac{L}{R} \log \left(\frac{E}{L} \right) = K.$$

Therefore

$$\frac{L}{R} \log \left(\frac{E - Ri}{L} \right) - \frac{L}{R} \log \frac{E}{L} = -t,$$

$$\log \frac{E - Ri}{E} = \frac{-Rt}{L},$$

or

$$\frac{E - Ri}{E} = e^{\frac{-Rt}{L}},$$

and

$$i = \frac{E}{R} \left(1 - e^{\frac{-Rt}{L}} \right).$$

Since $\frac{E}{R} = i_0$, the steady state current, the equation becomes, $i = i_0 (1 - e^{\frac{-Rt}{L}})$. The interpretation of such an equation is the following. It states that at any time t the current i is the maximum steady state current, i_0 , multiplied by a *diminishing* factor $(1 - e^{\frac{-Rt}{L}})$, which runs from 0 to 1.

To understand its character it should be noted that the diminishing factor is 1 less a quantity $e^{\frac{-Rt}{L}} = \frac{1}{e^{\frac{Rt}{L}}}$. This quantity $\frac{1}{e^{\frac{Rt}{L}}}$ is a fraction.

If t is indefinitely great the quantity $e^{\frac{Rt}{L}}$ is indefinitely great and its reciprocal approaches zero. As t gets indefinitely small the quantity approaches $\frac{1}{e^{\frac{Rt}{L}}} = \frac{1}{1} = 1$. Between these extremes the quantity $e^{\frac{-Rt}{L}}$ has the shape shown in Fig. 157. It starts at 1 and falls off to lower values with increasing t , at first rapidly, then more and more slowly approaching zero asymptotically. The rate at which this falls off depends on the value of $\frac{R}{L}$. It is to be noted that $e^{\frac{-Rt}{L}}$ is an exponen-

tial. Now a number can only be raised to a *numerical power*, that is, *all exponents must be dimensionless*. Since the exponent $-\frac{R}{L}t$ has a time, t , in it, in order that it be dimensionless, $\frac{R}{L}$ must have the dimensions of $\frac{1}{T}$. Thus $\frac{R}{L} = \frac{1}{T}$, or $\frac{L}{R} = T$. We can accordingly replace $\frac{L}{R}$ by a time, T , characteristic of the values of the ratio, and write for $e^{-\frac{R}{L}t}$ the expression $e^{-\frac{t}{T}}$. If T is large it will take a large value of t to reduce the quantity $e^{-\frac{t}{T}}$ to some fractional value such as $\frac{1}{2}$, $\frac{1}{e}$, or any other fraction of its initial value. If T is small then it will take a smaller value of t to make $e^{-\frac{t}{T}}$ reach the same value. Usually one compares various exponentials of the form $e^{-\frac{t}{T}}$ with different characteristic values of T , by the value of the time t to make $t/T = 1$, or to make $e^{-\frac{t}{T}} = e^{-1} = \frac{1}{e}$. Now e has a value 2.7183 and $\frac{1}{e} = 0.36788$, or about 36.8 per cent. Thus T is the time t takes for the quantity $e^{-\frac{t}{T}}$ to fall from 100 to 36.8 per cent of its value.

In view of this relationship $\frac{L}{R} = T$ is called the *time constant of the circuit* and it must be noted that it controls the rate of decline of the function $e^{-\frac{R}{L}t}$. The larger $T = \frac{L}{R}$ the more slowly will the exponential term decline. It is now possible to discuss the relation $i = i_0 (1 - e^{-\frac{R}{L}t})$. It is seen that the term in parentheses starts from 0 as $t = 0$ and rises rapidly and then more slowly as $e^{-\frac{R}{L}t}$ declines. At an indefinitely long time t it reaches 1. Practically it reaches $1 - 0.367 = 0.633$ when $t = T = \frac{L}{R}$. It reaches about 0.99 at $\frac{t}{T} = \frac{9}{2}$, or at $t = 4.5 \frac{L}{R}$. Thus the current i grows from 0 at $t = 0$ as indicated in Fig. 156, a being the curve when $\frac{L}{R}$ is small and b being the curve when $\frac{L}{R}$ is large.

Therefore it can be seen that the self-induction L acts as a sort of electrical inertia, delaying the arrival at the steady state current

in the measure that it is large compared to the resistance. In using this equation care must be taken to express L and R in *consistent* units. They must both be practical units, i.e., henrys and ohms or absolute electromagnetic units. If $L = 1$ henry and $R = 1$ ohm, $T = 1$ second. The current with these constants rises to 63.2 per cent of its value in 1 second.

If the key k had been closed at A and the current was flowing, breaking the key at A and connecting it to B would produce the fol-

lowing situation: at $t = 0$, i would be $\frac{E}{R} = i_0$. Then i would begin to die out, but an e.m.f. of self-induction $L \frac{di}{dt}$ would be generated trying to maintain the current. As opening the key removes the impressed e.m.f. one has $E = 0$. The current i flowing at any instant and the e.m.f. of self-induction would both act in the same sense, as $L \frac{di}{dt}$ is trying to *maintain* the current. Thus Kirchhoff's second law gives

$$L \frac{di}{dt} + Ri = 0, \quad \text{and} \quad \frac{-L}{R} \frac{di}{i} = dt,$$

whence integrating

$$\frac{L}{R} \log i = -t + K.$$

Now when

$$t = 0, i = \frac{E}{R} = i_0.$$

Thus

$$K = \frac{L}{R} \log i_0,$$

and hence

$$\log \frac{i}{i_0} = \frac{-R}{L} t, \quad \text{or} \quad i = i_0 e^{\frac{-R}{L} t} = \frac{E}{R} e^{\frac{-R}{L} t}.$$

From what was said about the function $e^{\frac{-R}{L} t}$ in the case of *making* the current it is seen that $e^{\frac{-R}{L} t} = e^{\frac{-t}{T}}$ falls off exponentially from 1 at $t = 0$ to zero as t gets indefinitely great as seen in Fig. 157. It is also seen that the rate of falling off, i.e., the time taken to fall to

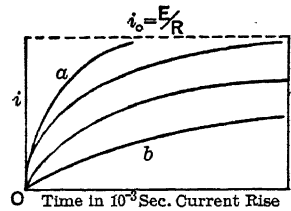


FIG. 156. Rise of the current in a circuit containing self-induction and resistance.

$\frac{1}{e}$, or 36.8 per cent of its value, is directly proportional to the *time constant* $\frac{L}{R} = T$. This decline of i from i_0 to 0 is shown for different values of $T = \frac{L}{R}$ in Fig. 157. It is seen that for small $\frac{L}{R}$ the curve a falls off quickly while with large $\frac{L}{R}$ the current in curve b decays slowly. If L is 1 henry and R is 1 ohm, T is 1 second and it would take the current 1 second to fall to 36.8 per cent of its initial value.

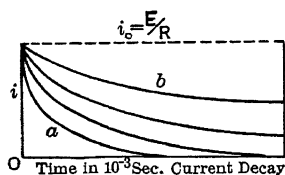


FIG. 157. Decay of a current in a circuit containing self-induction and resistance.

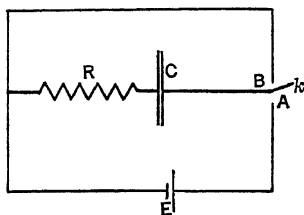


FIG. 158. Circuit for charging and discharging a condenser through a resistance.

Case II. Charging a Condenser. Another simple case merits treatment at this point. It does not involve the use of self-induction, but indicates in an interesting fashion the action of a condenser on transient currents. Consider the circuit pictured in Fig. 158. The condenser C is in series with a resistance R and a battery E . k is a key which, to begin with, can be considered open. If it makes contact at A , the battery E charges the condenser. If it makes contact at B , the condenser discharges through R . The charge on the condenser is 0 at the instant the key k is closed at A . Then a current begins to flow charging the condenser up. In charging up the condenser, an e.m.f. opposing the flow of the current is built up. The back e.m.f. due to the charge on a condenser is given by the relation $E' = \frac{Q}{C}$, where Q is the quantity and C is the capacity.

Thus Kirchhoff's second law may be expressed as

$$E - E' = E - \frac{Q}{C} = Ri.$$

Now

$$Q = \int i dt, \text{ or } i = \frac{dQ}{dt}.$$

It is possible to substitute $\int i dt$ for Q in the equation. Such an integral equation is hard to solve. What is done then is to convert the equation into a differential equation in Q and solve for Q . Then, from Q , i can be determined by $i = \frac{dQ}{dt}$.

The equation above thus becomes

$$R \frac{dQ}{dt} + \frac{Q}{C} = E.$$

Algebraic transformation permits us to write

$$\frac{-CR d\left(\frac{E - Q/C}{R}\right)}{\frac{E - Q/C}{R}} = dt$$

and integration yields

$$CR \log \left(\frac{E - Q/C}{R} \right) = -t + K.$$

As $Q = 0$ at $t = 0$,

$$CR \log \frac{E}{R} = K$$

and

$$\log \frac{E - Q/C}{E} = -t/CR$$

or

$$Q = EC \left(1 - e^{-\frac{t}{CR}} \right).$$

But $EC = Q_0$, the final charge on the condenser; and we can write

$$Q = Q_0 \left(1 - e^{-\frac{t}{CR}} \right).$$

This says that the charge on the condenser at any time t is Q_0 , the final charge when $t = \infty$, less $Q_0 e^{-\frac{t}{CR}}$. That is, the greater C and the greater R , the more slowly does the condenser charge up.

Where the condenser is charged and the battery is removed, letting the condenser discharge through a resistance R , k is thrown from A to B . In this case E is 0 and one has the e.m.f. $E' = \frac{Q}{C}$. Then, as the only other e.m.f. in the circuit is the iR drop, the equation

is the same as before with $E = 0$. Hence

$$Ri + \frac{Q}{C} = 0, \quad \text{or} \quad R \frac{dQ}{dt} + \frac{Q}{C} = 0,$$

whence

$$\log Q = -\frac{t}{CR} + K,$$

and as $Q = Q_0$ at $t = 0$, $K = \log Q_0$. Thus

$$\log \frac{Q}{Q_0} = \frac{-t}{CR} \quad \text{and} \quad Q = Q_0 e^{\frac{-t}{CR}}.$$

The charge therefore falls off exponentially with the time as the condenser discharges and falls the more slowly the larger C and R . Thus the condenser acts to *delay the arrival of a steady state where quantity is involved*. It causes *potential to build up slowly* and to disappear slowly.

It is of interest to see how the *current* varies in this case. The current equations may be derived at once from the quantity equations above. Since $i = \frac{dQ}{dt}$, the case of *charging* a condenser yields,

$$i = Q_0 \frac{d}{dt} \left(1 - e^{\frac{-t}{CR}} \right) = \frac{Q_0}{CR} e^{\frac{-t}{CR}},$$

and for *discharge*,

$$i = Q_0 \frac{d}{dt} \left(e^{\frac{-t}{CR}} \right) = -\frac{Q_0}{CR} e^{\frac{-t}{CR}}.$$

As $E = \frac{Q_0}{C}$, then

$$\frac{Q_0}{CR} = \frac{E}{R} = i_0.$$

Hence for charge one has

$$i = i_0 e^{\frac{-t}{CR}},$$

and for discharge,

$$i = -i_0 e^{\frac{-t}{CR}}.$$

Thus a circuit with capacity and resistance gives rise to these four equations:

$$\text{For quantity on charging} \quad Q = Q_0(1 - e^{\frac{-t}{RC}})$$

$$\text{For quantity on discharging} \quad Q = Q_0 e^{\frac{-t}{RC}}$$

For current charging $i = i_0 e^{-\frac{t}{RC}}$

For current discharging $i = -i_0 e^{-\frac{t}{RC}}$

For quantity the charging equation varies as $(1 - e^{-\frac{t}{RC}})$, in analogy to the growth of current with inductance. It is seen that in this case, however, the *time constant* is $T = RC$. Thus the charge on the condenser will rise in a growth curve analogous to the growth of current with self-induction at a rate governed by RC . The charging curve for Q is shown in Fig. 159A. The rate of rise will be the more rapid the smaller $T = RC$. For a large resistance playing into a large capacity the time of charging will be long.

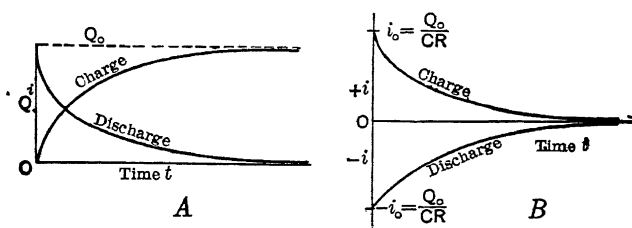


FIG. 159. Curves of charge and discharge of a condenser through a resistance. A. Variation of *quantity* on the condenser with time. B. Variation of the charging or discharging *current* with time.

For discharge the quantity Q on the condenser falls off exponentially with time as seen in Fig. 159A. The rate is the more rapid the smaller the time constant $T = RC$.

In contrast to the effect on the quantity of charge Q it will be noted that both for charge and discharge the *current decreases exponentially with time*, as seen in Fig. 159B. The difference in sign of the currents on charge and discharge, i.e., the negative sign of i on discharge, merely means that on charging the current flows *into* the condenser, and on discharge it flows *out*. The time constant for the current decline is the same as for the quantity Q . The units of C and R used must be consistent. If R is made 1 ohm and C 1 farad $T = 1$ second. However, even 1 microfarad of capacity is a large capacity. Thus unless R is very high capacitive time constants are usually small. If R is 1000 ohms and C is 1 microfarad $T = 10^{-3}$ second. With 10 microfarads and 10^5 ohms, T becomes 1 second.

The important thing to note, however, is that with capacity the current is always a *maximum* at $t = 0$. The reason for this is clear. If the condenser is empty the electrical pressure causing flow *into* the empty condenser is then the greatest. If it is charged the electrical pressure for discharge is then the greatest. Hence capacity *stimulates flow of current*. Thus when the current varies periodically with time

it can be expected that the effect of capacity will be to cause an *advance of phase in current*. With *inductance*, however, the inertial effect of this property is to cause a *delay in current or a phase lag*. Such behavior will become clear at a later point in the book.

134. MEASUREMENT OF HIGH RESISTANCES

In the discharge of the condenser, the equation found was

$$\frac{Q}{Q_0} = e^{\frac{-t}{CR}}.$$

If C and R are very large, $\frac{Q}{Q_0}$ will vary with time in a measurable amount. For a C of 1 microfarad and an R of 10^8 ohms, the time of discharge would lie in the neighborhood of a minute or two. As C

can easily be measured, and as resistances of 10^8 ohms cannot easily be measured, this decay is used as a means of measurement of high resistances. The circuit is pictured in Fig. 160. E is a battery for charging the condenser C of known capacity which has little dielectric absorption. R is the unknown resistance and G is a galvanometer of the ballistic type. k_2 is a charging key and k_1 is the key used in measurement. To charge, k_1 is open and separated from both A and B , while k_2 is closed. k_2 is then opened and k_1 thrown to B . The condenser discharges through G and the ballistic throw measures Q_0 , the charge on the condenser. Then the condenser is charged again. This time the key k_1 is closed to A for a suitable time t , say 30 seconds. Then it is rapidly switched to B and the deflection of G noted. This gives Q the quantity left on the condenser after a time t . Having Q , Q_0 , t and C , one can at once solve for R .

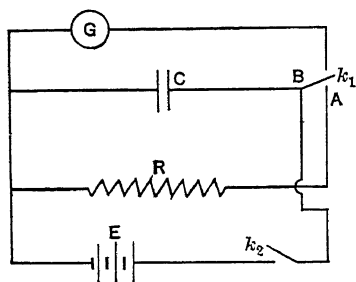


FIG. 160. Measurement of high resistance by the discharge of a condenser through it.

CHAPTER XXIV

ALTERNATING CURRENTS I: DEFINITIONS, ROOT MEAN SQUARE AVERAGES, AND EFFECT OF SELF-INDUCTION

135. THE IMPORTANCE OF ALTERNATING CURRENT

In the last chapter it was shown that self-induction became of importance when currents vary with time, that is, self-induction causes an electromotive force only when there is a time rate of change of current, $\frac{di}{dt}$. Besides the transient effects of the last chapter, which

emphasizes this property, there is another important group of currents which also produce such changes, those altering *periodically* with time.

Now it seems strange that it should be necessary to deal with alternating currents when direct currents can be generated quite simply. However, in modern industrial life efficiency demands that electrical power be generated in central plants. In these plants the distribution of power to outlying points is a great problem. The loss in power distribution comes almost entirely in the i^2R loss in the line. Where electrical power is transmitted over 20 miles of line and as it is today over hundreds of miles, R is quite a factor and the currents range in the thousands of amperes. Thus the i^2R loss in heating is very great, and for transmission of direct currents the loss would be dependent on this factor. However, in alternating currents it is possible to transmit the alternating current at a very high voltage, that is, E of the term Ei which expresses the power sent is large, and i correspondingly small. By means of transformers (which have been discussed in Chapter XXI), it is possible with very little loss of energy to convert the high potential alternating current to a low potential at a substation. Thus today practically all power transmission depends on alternating currents. Again, in all telephony and especially in radio, the changes produced in the electrical system by periodic changes in currents are being dealt with. It is consequently essential to study the alternating current.

136. DEFINITIONS OF ALTERNATING-CURRENT TERMS

In Chapter XX on the simple a-c dynamo it was seen that the e.m.f. was given by an equation $E = \frac{2\pi HAN}{10^8} \sin \theta$. Here N repre-

sents the number of revolutions per second and θ represents an instantaneous angle made by the plane of the armature coil with the perpendicular to the field, which is the axis of reference. It is seen that

for N constant the term $\frac{2 \pi H A N}{10^8} = E_0$ is a constant. E_0 then represents

a value of E which occurs when $\sin \theta = 1$ and thus when $\theta = \frac{\pi}{2}$,

$\frac{3\pi}{2}, \frac{5\pi}{2}$, or an odd number of $\frac{\pi}{2}$ radians. This is the maximum or *peak* value of the e.m.f., and it is generated when the plane of the armature coil is *parallel* to the field H and is cutting lines of force at its maximum rate. Thus we may write $E = E_0 \sin \theta$ for the e.m.f. of the dynamo where E_0 is the maximum or *peak* value given.

Now since the armature is rotating N times a second where N is called the *frequency*, the time of one rotation $T = \frac{1}{N}$. The time T of one complete *cycle* or *rotation* is called the *period*. In this same time θ has undergone a change in value of 2π radians. In a time t therefore the angular change in θ will be 2π times the ratio $\frac{t}{T}$ radians change.

Hence we may write that the *change* in angle $\theta = \frac{2\pi}{T} t = 2\pi Nt$, so that the angular change in armature position is a function of the time t .

Now the time t representing the angular change of θ can be measured as *starting with* $t = 0$ *at any point in the rotation of the armature*. Thus suppose as in Fig. 139 we choose to start our stop watch, or counting of time, at the instant when the plane of the coil is not at the reference position C , the point where no e.m.f. is being generated, but at some angle ϕ with this direction, such as B . At an instant t later the armature coil will have moved through $\theta = \frac{2\pi}{T} t$ so that the angle will

be $\phi + \frac{2\pi}{T} t$. Hence instead of writing the equation for the e.m.f. as

$E = E_0 \sin \theta$, where θ represents an angle swept out in time t starting when the coil is at the point C , we will write the more general equation

$E = E_0 \sin \left(\frac{2\pi t}{T} + \phi \right)$. This makes it possible for us to start our

stop watch at $t = 0$ at any instant when the armature coil makes an angle ϕ with our zero position C . This angle ϕ is called the *phase* angle. If for a clockwise rotation ϕ is measured to the right of the zero point C , the armature is ahead of its initial position, ϕ is positive, and we have a *phase advance*. If the angle ϕ is taken to the left of C , it is behind the zero point, ϕ is negative, and we have a *phase lag*.

It is now possible to plot the curves of E as a function of t and θ .

In Fig. 161 ϕ is made 0. Thus at $t = 0, E = 0$. At $t = \frac{T}{4}, \theta = \frac{\pi}{2}$ and $E = +E_0$, its peak value. At $t = \frac{T}{2}, \theta = \pi, E = 0$ and is passing from positive to negative values. At $t = \frac{3T}{4}, \theta = \frac{3\pi}{2}, E = -E_0$, and we pass through a negative peak of potential. At $t = \frac{4T}{4} = T, \theta = 2\pi, E = 0$, and we are at the end of the first cycle and ready to begin with the next one. It is seen that the curve traced out is a sine

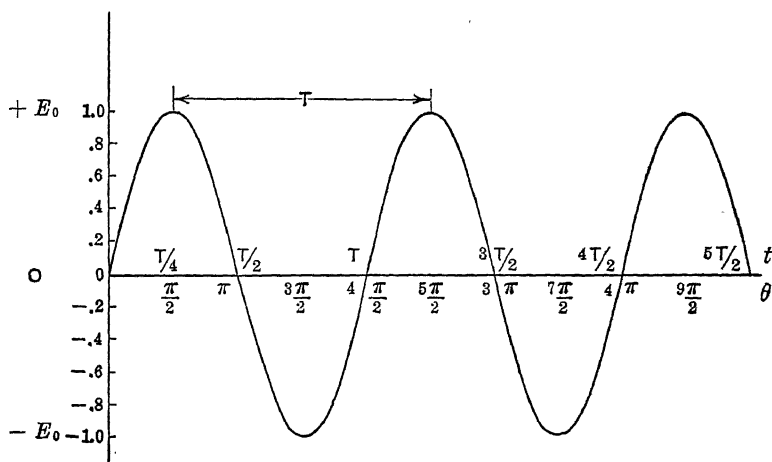


FIG. 161. The sinusoidal alternating potential.

curve of total amplitude from $+E_0$ to $-E_0$. It is seen therefore that the curve for E may be plotted either in terms of θ or of time t . If p is set equal to $\frac{2\pi}{T}$ and it be remembered that θ varies as $\frac{2\pi t}{T}$, then $\theta = pt$.

If ϕ is not zero but $+\frac{T}{8}, \theta = +\frac{\pi}{4}$, then E at $t = 0$ has already risen to $0.707 E_0$. This represents a phase *advance* of 45° , as seen in Fig. 162. In the same figure the dotted curve represents a curve for $\phi = -\frac{T}{8}, \theta = -\frac{\pi}{4}$, and E at $t = 0$ is at $-0.707 E_0$ and has not yet reached its zero value. This represents a phase *lag* of 45° .

In order to clarify discussion the new terms in the equations may be summarized and identified. $E = E_0 \sin (\theta + \phi) = E_0 \sin \left(\frac{2\pi t}{T} + \phi \right) = E_0 \sin (2\pi Nt + \phi) = E_0 \sin (pt + \phi)$, as follows:

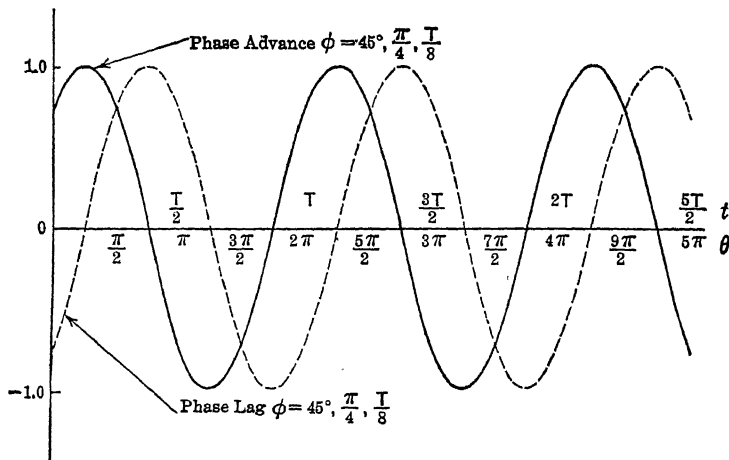


FIG. 162. Phase lag and advance in a sinusoidal alternating potential.

E is the *instantaneous* value of the e.m.f.

E_0 is the *maximum* or *peak* value; it is half the *total amplitude* of oscillation.

θ is the change in the angle of the armature since the zero of time, $t = 0$.

ϕ is the *angle of phase lag or advance*; it is the angle of the armature with the zero position at the instant when t was 0.

T is the *period* of the alternating potential, i.e., the time of a complete revolution of the armature or the time to complete one cycle of the sine wave.

N is the *frequency* of the oscillation and of rotation of the armature.

t is the *instantaneous value of the time*, the independent variable.

$p = \frac{2\pi}{T} = 2\pi N$ is the *period factor*; it is really the *angular velocity* of the armature. It is used hereafter in equations to avoid rewriting $\frac{2\pi}{T}$. Note also that if ϕ is 0, $t = \frac{\theta}{p}$.

It might be added that exactly similar equations may be written for the alternating current. In this case i replaces E and i_0 replaces E_0 , so that we write $i = i_0 \sin\left(\frac{2\pi t}{T} + \phi\right)$.

137. VALUES OF CURRENTS GIVEN BY INSTRUMENTS, ROOT MEAN SQUARE CURRENT

The next question which arises in these considerations is the following: Suppose that we have an e.m.f. of the type discussed above,

having, as in lighting circuits, the frequency of 60 cycles per second; what will the measuring instruments record with such a current? In the first place, if alternating currents are going to be measured such currents or potentials cannot be measured with ammeters and voltmeters having fixed magnetic fields. For as indicated in Chapter IX, every time that the current or potential reverses the force acting on the moving coil of the instrument will reverse. If the instrument could not follow the rapid alternations of the current or potential, no reading would be obtained. If, however, the current which flows through the moving coil is also allowed to excite the magnetic field, the direction of the torque on the moving coil will always be the same. Then a *varying deflection* proportional to current or potential squared will be obtained provided that the coil has so small an inertia that it can follow the rapid oscillations. Actually the inertia is too great and the coil takes up a position which is intermediate between that of the zero position and the maximum position caused by i^2 . That is, *all alternating current potential and current measuring devices give a deflection which is neither the peak, or maximum, amplitude of the quantity nor the zero value.* In fact, in such instruments an average e.m.f. or current is being dealt with. This must be related in some fashion to the maximum value, and the relationship between the average and the maximum value of E must be known in order to apply the equation for the e.m.f. at any instant.

In deducing this value, the average e.m.f. or current must be computed. To do this, it must be remembered, however, that since the same current flowing through the coil and through the field is twice acting to cause the deflection, *the deflection is proportional to the product of the current by itself, or to i^2 .* The same rule applies to the e.m.f. The problem before us thus is to calculate the average square of the current for a current which varies sinusoidally with the time. The deduction to be given for the current is the same as would be given for the e.m.f., for the noninductive high resistance R introduced to make the instrument read potential permits one to write $E = Ri$.

Let us now assume a sinusoidally varying current $i = i_0 \sin(pt + \phi)$ and for simplicity let us assume that $\phi = 0$, i.e., that we count time from the instant the coil passed through C in Fig. 139. Now the deflection of the ammeter is proportional to i^2 , so that the instantaneous torque acting on the ammeter coil varies as $i^2 = i_0^2 \sin^2 pt$. A plot of the variation of i^2 with t or, even more simply, with angle θ since $\theta = pt$, is shown in Fig. 163 as a bell-shaped curve that starts from 0, rises to a maximum i_0^2 , and returns to zero in the interval 0 to π or from $t = 0$ to $t = \frac{\pi}{p}$. At this point it again rises to a positive maximum at $\theta = \frac{3\pi}{2}$ and falls to zero at $\theta = 2\pi$, since i^2 is al

ways positive even when i is negative. Now the height of the curve as a function of t would represent just the movement of the coil and its needle, did it not have too much inertia. Instead of this, the needle takes on a steady position at some value \bar{i}^2 and stays there. It is required to evaluate \bar{i}^2 in terms of i_0^2 .

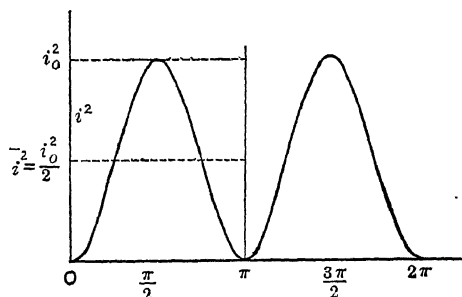


FIG. 163. Determination of the average squared current for a sinusoidal alternating current.

This is accomplished as follows. The torque G on the coil of the meter at any instant when the current of value i flows is given by $G = Bi^2$. Here B is a constant that depends on coil area, the number of turns in the field coil windings, reluctance of the magnetic field circuit, etc. This torque lasts for a short time dt , at which the current has changed to a new value. Thus in the interval dt the coil gets an impulse $G dt = Bi^2 dt$. Now during one half-cycle, $t = 0$ to $t = \frac{\pi}{p}$, the current squared goes from zero up to a value i_0^2 and back to zero. In this time the coil has received a whole series of small varying impulses $G dt$ running from 0 to a maximum, $Bi_0^2 dt$ and back to zero. In the next interval $\frac{\pi}{p}$ to $\frac{2\pi}{p}$, while i is reversed the impulse is the same and positive as i is squared. This gives a total impulse reached by adding up all the separate $G dt$'s as indicated by $\int_{t=0}^{t=\frac{\pi}{p}} G dt = \int_{t=0}^{t=\frac{\pi}{p}} Bi^2 dt$, which persists during all the cycles. The effect of this total summed-up impulse is the same as that of a steady torque \bar{G} acting continually over the time interval from $t = 0$ to $t = \frac{\pi}{p}$ since after this interval the cycle repeats with the same effect. Thus it is possible to write that $\bar{G} \frac{\pi}{p} = \int_{t=0}^{t=\frac{\pi}{p}} G dt$. Now such a steady torque \bar{G} acting

over a time interval of $\frac{\pi}{p}$ and repeating each interval of $\frac{\pi}{p}$ could be produced by a steady current of average value \bar{i}^2 such that $\bar{G} \frac{\pi}{p} = B \bar{i}^2 \frac{\pi}{p}$. Thus it follows that

$$\begin{aligned} B \bar{i}^2 \frac{\pi}{p} &= \bar{G} \frac{\pi}{p} = \int_0^{\frac{\pi}{p}} G dt = \int_0^{\frac{\pi}{p}} B i^2 dt = \int_0^{\frac{\pi}{p}} B i_0^2 \sin^2 pt dt \\ &= B i_0^2 \int_0^{\frac{\pi}{p}} \sin^2 pt dt = \frac{i_0^2}{p} B \left[\frac{1}{2} pt - \frac{1}{4} \sin^2 2pt \right]_0^{\frac{\pi}{p}} = \frac{i_0^2}{p} B \left[0 + \frac{\pi}{2} \right] \\ &= \frac{\pi}{2} \frac{i_0^2}{p} B. \end{aligned}$$

Accordingly it is seen that $\frac{\pi}{2} \frac{i_0^2}{p} B = B \bar{i}^2 \frac{\pi}{p}$, or that $\bar{i}^2 = \frac{i_0^2}{2}$. Thus the average steady current squared that would produce the same deflection as produced by an i^2 varying as in Fig. 163 will be just half the *peak* value, i_0^2 , of i^2 . Accordingly one sets $\bar{i} = \frac{i_0}{\sqrt{2}}$.

Thus the deflection given by the a-c ammeter for an alternating current reads as if a direct current \bar{i} were acting, the value of which was the *peak* value of the alternating current divided by the square root of 2.

The calculation just made can be regarded geometrically as follows. The total effective impulse for half a period due to the alternating current is the area of a bell-shaped curve and represents the effect of a fluctuating squared current following the curve. On an instrument it produces a fixed deflection acting steadily for half a period. The height \bar{i}^2 of a rectangle of base θ equal to π radians, or in time to $\frac{\pi}{p}$, whose area is equal to that under the bell-shaped curve must be found.

That is, $B \bar{i}^2 = \frac{\bar{G} \frac{\pi}{p}}{\pi/p}$. As seen, this equals $B \frac{i_0^2}{2}$. Thus the height of the squared alternating current, and thus of the rectangle equivalent in area to the total a-c impulse, is one-half the peak value of the bell-shaped curve.

On exactly similar reasoning the current $i = i_0 \sin pt$ through an a-c voltmeter will give a deflection reading $\bar{i} = \frac{i_0}{\sqrt{2}}$, which is proportional to the e.m.f. impressed. Thus, since for the noninductive

resistance coil $E = iR$, we can write $\sqrt{\bar{E}^2} = \bar{E} = iR = \frac{i_0}{\sqrt{2}} R = \frac{E_0}{\sqrt{2}}$;

whence $\bar{E} = \frac{E_0}{\sqrt{2}}$; The potential and current read on an a-c instrument are thus average values, termed the *root mean square* currents and potentials. In value they are the peak values* of the potentials and currents divided by $\sqrt{2}$.

138. EFFECT OF SELF-INDUCTION ON AN ALTERNATING E.M.F.

It is next important to see what occurs in a circuit when an alternating current is impressed on it. Assume an alternating potential given by $E = E_0 \sin pt$ with the phase angle 0. Let this be placed across a pure noninductive resistance R . Then at any instant the e.m.f. is E ,

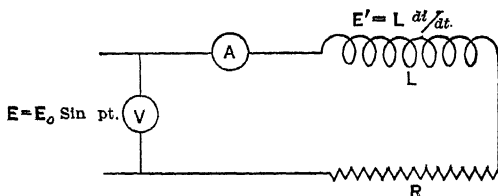


FIG. 164. Circuit for the study of the effect of self-induction on an alternating e.m.f.,

and this, by Kirchhoff's second law, must be equal to the iR drop.

Hence $E = E_0 \sin pt = iR$ and $i = \frac{E_0}{R} \sin pt$, whence $i = i_0 \sin pt$ with $i_0 = \frac{E_0}{R}$.

If now a self-induction L is placed in series with the resistance R , as shown in Fig. 164, the situation becomes more complicated. For since the e.m.f. E is varying with time, it is obvious that the current will vary with time. Such a time-varying current will at once interact with the self-induction L to produce a back e.m.f. equal to $-L \frac{di}{dt}$, where $\frac{di}{dt}$ is the time variation of the current the form of which is to be found. Thus Kirchhoff's second law for the circuit must now be written as $E - L \frac{di}{dt} = Ri$, and since $E = E_0 \sin pt$, we have $L \frac{di}{dt} + Ri = E_0 \sin pt$ or $\frac{di}{dt} + \frac{R}{L} i = \frac{E_0}{L} \sin pt$. An equa-

* Note that this implies that the alternating currents are of the form $i = i_0 \sin pt$. If they deviate from this form these relations no longer hold and new averages must be sought.

tion of this type is in itself of little help to us for we are interested in knowing *how* i varies with E , R , L , and t , while this equation contains a rate term $\frac{di}{dt}$. Because it contains the differential quotient $\frac{di}{dt}$, it is known as a differential equation. The equation is thus not as simple to solve as were the equations for the transient effects in Chapter XXIII, as it is now no longer possible to separate the variables for integration. The equation is termed a linear differential equation of the first order and first degree with constant coefficients. This means that it contains no second or higher derivatives such as $\frac{d^2i}{dt^2}$,

that it contains no squared terms such as $\left(\frac{di}{dt}\right)^2$, and that $\frac{R}{L}$ and $\frac{E_0}{L}$ are constants. A solution for an equation of this type can be found in books dealing with the solution of differential equations. On integration it takes the following form:

$$i = \frac{E_0}{L} \frac{\left(\frac{R}{L} \sin pt - p \cos pt\right)}{\left(\frac{R}{L}\right)^2 + p^2} + ce^{-\frac{Rt}{L}}$$

This equation appears complicated, but it is not as bad as it seems for it can be transformed to a more convenient form. The only unknown term is c , a constant of integration. This could be evaluated if desired, but this is not necessary. The term $ce^{-\frac{Rt}{L}}$ is one that already has been encountered in the study of the transient currents taking place when the switch is opened or closed. The transient currents are of no particular interest here; what is wanted is the *steady-state* alternating current long after the switch is closed. Thus, if t is sufficiently large, the term $ce^{-\frac{Rt}{L}}$ vanishes, while the periodic terms in $\sin pt$ and $\cos pt$ have finite values. To simplify the equation further the numerator and the denominator of the remaining equation may next be multiplied by L . Then for the steady state it is seen that the current i is

$$i = E_0 \frac{(R \sin pt - Lp \cos pt)}{R^2 + L^2p^2},$$

a current that varies as the sine plus the cosine of the time.

It may be stated at once that the presence of two terms varying with $\sin pt$ and with $\cos pt$, which is $\frac{\pi}{2}$ radians or 90° out of phase with $\sin pt$, comes from the effect of self-induction and an alternating current. Since the sum of the two terms which are 90° out of phase

must be dealt with, a transformation can be made which will throw the results into a more useful form. Set up the rectangular coordinate system of Fig. 165 and lay off the value of the coefficient of the sine term R along the X -axis. The value of Lp , the coefficient of the cosine

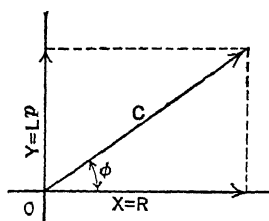


FIG. 165. Diagram for transformation of the equation for the instantaneous value of an alternating current in a circuit with self-induction and resistance.

term, may be laid off along the Y -axis. At a point Lp , R on these axes, i.e., at the corner of the rectangle Lp high and R long, a diagonal to the origin may be drawn. Call this line C and note at once that it makes an angle ϕ with the X -axis. It at once follows that

$$R = C \cos \phi$$

$$Lp = C \sin \phi$$

$$L^2 p^2 + R^2 = C^2$$

$$\tan \phi = \frac{Lp}{R}$$

It is now possible to replace R and Lp by their equivalents in terms of C and ϕ . Thus i becomes

$$\begin{aligned} i &= E_0 \frac{(C \cos \phi \sin pt - C \sin \phi \cos pt)}{C^2 \cos^2 \phi + C^2 \sin^2 \phi} \\ &= E_0 \frac{C(\cos \phi \sin pt - \sin \phi \cos pt)}{C^2} \end{aligned}$$

Now a well-known trigonometric relation says that $\sin (pt - \phi) = \cos \phi \sin pt - \sin \phi \cos pt$. The equation for the current i accordingly becomes simply

$$i = \frac{E_0 \sin (pt - \phi)}{C} = \frac{E_0}{\sqrt{R^2 + L^2 p^2}} \sin (pt - \phi).$$

Thus when an e.m.f. $E = E_0 \sin pt$ is impressed on a circuit with self-induction and resistance, the current which results is

$$i = \frac{E_0}{\sqrt{R^2 + L^2 p^2}} \sin (pt - \phi) = i_0 \sin (pt - \phi)$$

with
$$i_0 = \frac{E_0}{\sqrt{R^2 + L^2 p^2}} \quad \text{and} \quad i_v = \frac{E_v}{\sqrt{R^2 + L^2 p^2}},$$

$$\tan \phi = \frac{Lp}{R}, \quad E_v = \frac{E_0}{\sqrt{2}}, \quad i_v = \frac{i_0}{\sqrt{2}}.$$

The result shows the following:

1. The current i , like the impressed e.m.f., varies sinusoidally with time with the same period factor p .

2. The current *lags behind* the e.m.f. by a phase angle ϕ , for $E = E_0 \sin pt$ and $i = i_0 \sin (pt - \phi)$.

3. The value of the angle of phase lag is obtained from $-\tan \phi = -\frac{Lp}{R}$. Thus the greater the time constant $\frac{L}{R}$, the greater is the phase lag. The greater $p = 2\pi N = \frac{2\pi}{T}$ and thus the higher the frequency, the greater the phase lag. The maximum value of ϕ as $\frac{Lp}{R}$ approaches infinity is $\tan^{-1} \infty = \frac{\pi}{2}$ or 90° . Thus angles of phase lag run from 0° to 90° as a limit.

4. The current i_0 is not $\frac{E_0}{R}$. Instead it is less than $\frac{E_0}{R}$ owing to the presence of a term L^2p^2 under the radical. This is a consequence of inductive action and of the angle of phase lag. Hence *Ohm's law does not hold for a circuit with self-induction when an alternating current flows*. This is caused by the introduction of the e.m.f. of self-induction which is out of phase with the impressed e.m.f. The current is reduced but unlike that due to the ohmic resistance R this reduction does not result in heat dissipation.

5. Ohm's law must be modified for such circuits by replacing $R = \sqrt{R^2}$ by a term $z = \sqrt{R^2 + L^2p^2}$, called the *impedance*. This contains the *ohmic resistance* R and the *inductive reactance* Lp .

6. It is seen that, if $L = 0$, $z = R$ and Ohm's law holds. The same holds if $p = 2\pi N = 0$, i.e., for direct currents. The greater L and the greater N , the greater the impedance. The primary of a transformer has a very low resistance. Its impedance is very much larger.

7. The average current i_v read by an a-c ammeter in such a circuit is the average impressed e.m.f. E_v divided by the impedance. Thus these quantities can be read by the a-c meters with the significance attached to such readings. Since Lp and R are additive the product Lp has the dimensions of a resistance and we speak of the impedance z in equivalent ohms. Again, as in all these equations, the results come out in the system of units used as long as the systems are consistently used in an equation.

Call the instantaneous e.m.f., E , and i the instantaneous current.

Then from $E = i\sqrt{R^2 + L^2p^2}$ and from $i = E_0 \frac{(R \sin pt - Lp \cos pt)}{(R^2 + L^2p^2)}$

it follows that,

$$E = i\sqrt{R^2 + L^2p^2} = \frac{E_0(R \sin pt - Lp \cos pt)}{(R^2 + L^2p^2)} \sqrt{R^2 + L^2p^2}$$

$$= \frac{E_0}{\sqrt{R^2 + L^2p^2}} (R \sin pt - Lp \cos pt) = i_0R \sin pt - i_0Lp \cos pt.$$

It is seen that the instantaneous e.m.f. E is the sum of an $i_0R \sin pt$ term and an $Li_0p \cos pt$ term. The $i_0R \sin pt$ is the instantaneous value of the e.m.f. due to ohmic resistance. The $Li_0p \cos pt$ is the term which is 90° out of phase with the e.m.f. due to inductive reactance. It is an *instantaneous value of the e.m.f. due to self-induction*, for if the instantaneous current equation $i = i_0 \sin (pt + \phi)$ is considered, it is seen that $L \frac{di}{dt} = Li_0p \cos pt$. Thus the instantaneous

impressed e.m.f. E can be plotted on a vector diagram as the diagonal made up of component parts Li_0p , the e.m.f. of self-induction on the axis of ordinates, and Ri_0 , in e.m.f. due to iR drop, plotted along the axis of abscissas. They define the angle of phase lag ϕ . Such a diagram is shown in Fig. 166 for the virtual e.m.f. and currents. This device is extremely useful in the analysis of circuits of more complex form.

It was noted that in the diagram of Fig. 166 the virtual or average currents and potentials were used instead of the peak values. This is correct for a-c terms. It is necessary, however, to see that the general relation $E_v = \frac{E_0}{\sqrt{2}}$ and $i_v = i_0\sqrt{2}$ is not altered by the use of self-induction. To do this, proceed to square the equation $E = i_0R \sin pt - Li_0p \cos pt$. Multiply the terms by dt and integrate them between 0 and $\frac{\pi}{p}$, i.e., over the half-period. Division by $\frac{\pi}{p}$ as in section 137 then should yield the averaged square e.m.f. E_v^2 in the circuit. When this is done the equation takes the form

$$E_v^2 = R^2 \frac{i_0^2}{2} + L^2 \frac{i_0^2 p^2}{2} = R^2 i_v^2 + L^2 i_v^2 p^2.$$

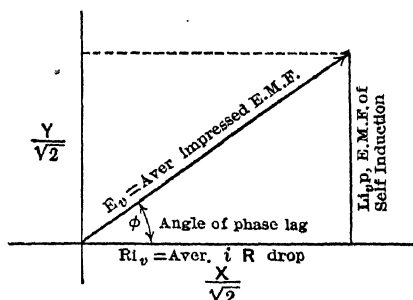


FIG. 166. Vector diagram for an a-c circuit with self-induction and resistance.

CHAPTER XXV

ALTERNATING CURRENTS II: EFFECT OF CAPACITY ON ALTERNATING CURRENT AND THE EFFECT OF CAPACITY AND SELF-INDUCTION COMBINED — ELECTRICAL OSCILLATIONS

139. EFFECT OF CAPACITY ON THE IMPEDANCE OF AN ALTERNATING CURRENT

It has been asserted at various points (cf. page 218) that whereas a direct current cannot flow through a condenser, an alternating current can. As shown in section 358, when a potential E_0 is applied to a condenser a current flows charging the condenser to E_0 and then ceases. If now one considers a condenser connected by a resistance to an alternating e.m.f., as in Fig.

167, one can reason as follows. Consider the instant when the positive phase puts $+E_0$ on the ammeter side of the condenser C . This tries to force a current charging the upper plate of C positively. By induction through the dielectric a negative charge is drawn to the lower place of C . Owing to the

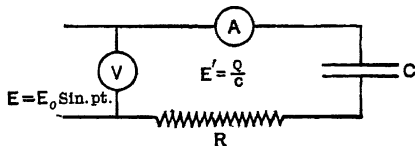


FIG. 167. Circuit for the study of the effect of capacity and resistance on a sinusoidal alternating current.

resistance there is a limited current flow and before the condenser can charge to Q_0 at E_0 , if alternations are rapid, E has fallen below $+E_0$. By the time E has reached 0 the current is flowing *from* the upper plate of C . This releases bound negative charges from the lower plate. As the potential E on the upper side goes more and more negative the upper plate of the condenser C *discharges* enough positive electricity to go negative. A positive charge by induction flows into the lower plate of C . Thus, unlike a direct current, an *alternating current can, by electrostatic induction, flow through a capacity*. It is seen that the flow will be governed by the capacity, the resistance, and the frequency since all these factors will affect the extent to which the capacity can charge before the current reverses. The theory of the current flow in such a circuit merits study.

In the last chapter the effect of self-induction on an alternating current was studied. When it is attempted, however, to carry the same mode of approach to the action of capacity on a circuit having an impressed alternating sine-wave-form e.m.f. difficulties are en-

countered which make the mode of approach used for self-induction impossible. Since E' , the back e.m.f. produced by capacity, is given by $E' = \frac{Q}{C}$, where Q is quantity and C is capacity, and since $i = \frac{dQ}{dt}$,

the expression for E' becomes $E' = \frac{\int i dt}{C}$. In choosing the limits of integration in order to compare the equation with the current equation, a troublesome constant of integration is found to enter in, justification for the dropping of which cannot be made in this form. It is necessary therefore to proceed in a different manner, and this procedure is quite parallel to that used in studying the effect of capacity on transient phenomena.

Assume a circuit of the form shown in Fig. 167 and that the impressed e.m.f. has the magnitude $E = E_0 \sin pt$. Then Kirchhoff's second law leads to the expression $E - E' = Ri$, where E' is the back e.m.f. due to C , and R is the resistance in the circuit, while i is the current. Now $E' = Q/C$, where Q is the quantity of electricity on C at any time and $i = \frac{dQ}{dt}$. Hence it follows that,

$$R \frac{dQ}{dt} + \frac{Q}{C} = E_0 \sin pt,$$

whence

$$\frac{dQ}{dt} + \frac{1}{RC} Q = \frac{E_0}{R} \sin pt.$$

This equation is a linear differential equation of the first order and first degree with constant coefficients. The general solution of this is found in any textbook on differential equations and is as follows:

$$Q = \frac{E_0}{R} \frac{\left(\frac{1}{RC} \sin pt - p \cos pt \right)}{\left(\frac{1}{RC} \right)^2 + p^2} + ce^{-t/RC}.$$

The first member on the right-hand side of the equation is a term that varies periodically with t and is the one of importance in this problem. The second term multiplied by the undetermined constant of integration c represents a term that approaches 0 as t becomes large. It is the term that represents the *transient* effect when the switch is first thrown and is, in fact, quite similar to the transient term that was found for capacity in Chapter XXIII. Currently the situation of interest is that existing in the steady state after the transient has died out (i.e., currents are measured some seconds after the switch has been closed). Thus t may be set as very great and this term approaches 0. The equation may now be thrown into a

more convenient form for study. To do so write, as in Chapter XXIV, that $1/RC = B \sin \varphi$ and $p = B \cos \varphi$, where $B^2 = \frac{1}{R^2 C^2} + p^2$.

Then

$$Q = \frac{E_0}{RB} \frac{(\sin \varphi \sin pt - \cos \varphi \cos pt)}{\sin^2 \varphi + \cos^2 \varphi},$$

whence

$$Q = \frac{-E_0}{RB} \cos (pt + \varphi).$$

Now

$$i = \frac{dQ}{dt} = \frac{pE_0}{RB} \sin (pt + \varphi),$$

or

$$i = \frac{E_0}{\frac{R}{p} \sqrt{\frac{1}{R^2 C^2} + p^2}} \sin (pt + \varphi),$$

therefore

$$i = \frac{E_0}{\sqrt{R^2 + \frac{1}{p^2 C^2}}} \sin (pt + \varphi) = i_0 \sin (pt + \phi),$$

which is the equation sought for the current in this circuit. From this we can also write, since $i = i_0$ when $\sin (pt + \varphi) = 1$,

$$i_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{p^2 C^2}}}, \quad \text{and} \quad i_v = \frac{E_v}{\sqrt{R^2 + \frac{1}{p^2 C^2}}}.$$

Since $\frac{1}{RC} = B \sin \varphi$ and $p = B \cos \varphi$,

$$\frac{\sin \varphi}{\cos \varphi} = \tan \varphi = \frac{1}{RpC}.$$

1. As in the case of self-induction, it can be concluded that the current with capacity is a sinusoidal current of the same period as the e.m.f.

2. Since we chose $E = E_0 \sin pt$ while $i = i_0 \sin (pt + \phi)$, we see that the current i is ahead of E by a phase angle ϕ . In the case of self-induction the phase angle φ was the angle by which the current was behind the e.m.f. Thus it can be concluded that self-induction causes a lag in the current behind the impressed e.m.f., while with capacity the current leads or is advanced in phase ahead of the impressed e.m.f.

3. Again as before the angle of phase advance ϕ can be found at once from the relation $\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{1}{RpC}$. It is the greater the smaller R^2 , C , and p .

4. It is also noted that in this circuit the current does not follow Ohm's law, but that

$$i_v = \frac{E_v}{\sqrt{R^2 + \frac{1}{p^2 C^2}}}.$$

5. The effect of capacity through the phase difference between E' and E is to reduce the current below that to be expected when it is absent. The capacity impedance z is $z = \sqrt{R^2 + \frac{1}{p^2 C^2}}$ and the capacity reactance is $\frac{1}{pC}$. It should be noted that $\frac{1}{pC}$ has the dimensions of a resistance, so that the impedance is expressed in equivalent ohms. In so doing C must be in practical units, that is, *farads*. If *microfarads* are used for C then C must be written as $10^{-6} C$ in the equation above.

6. The greater C and p , the more nearly is Ohm's law obeyed, while the smaller C and p , the greater is the departure. This can be readily shown by placing either a small capacity or a large one in an a-c circuit with an electric light in series. While 10 microfarads at 60 cycles lights the lamp, 1 microfarad does not do this.

7. The use of $\sin \phi$ and $\cos \phi$ for $1/RC$ and p of the equation makes it possible to make the same sort of a vector diagram for the effect of capacity as for self-inductions in which i_v/Cp is plotted as ordinate and $i_v R$ is plotted as abscissa, with the impressed e.m.f. E as diagonal at an angle ϕ with $i_v R$.

140. WHERE SELF-INDUCTION, CAPACITY, AND RESISTANCE ARE PRESENT

Where self-induction, capacity, and resistance are in the circuit, the differential equation becomes

$$E - (E' + E'') = Ri,$$

where E' is the e.m.f. of self-induction and E'' is the e.m.f. due to capacity. On setting $E = E_0 \sin pt$, we obtain

$$L \frac{di}{dt} + Ri + \frac{Q}{C} = E_0 \sin pt,$$

and

$$\frac{d^2 Q}{dt^2} + \frac{R}{L} \frac{dQ}{dt} + \frac{Q}{LC} = \frac{E_0}{L} \sin pt.$$

The resulting differential equation is a linear one of the second order, first degree, with constant coefficients. It is more difficult to solve, but it leads to a solution of the same form as for capacity alone with,

$$i = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{pC} - Lp\right)^2}} \sin (pt + \varphi).$$

Here φ is now defined by the equation

$$\tan \varphi = \frac{\frac{1}{Cp} - Lp}{R}.$$

This equation is quite analogous to the one for capacity and self-induction and by elimination of C or of L reduces to the forms encountered before for equations with L and R , and C and R , respectively.

The impedance is $\sqrt{R^2 + \left(\frac{1}{pC} - Lp\right)^2}$ and the reactance is $\frac{1}{pC} - Lp$.

The nature of the phase effect, whether lag or advance, depends on the relative magnitudes of Lp and $1/Cp$. If Lp is the greater $\tan \varphi$ is negative and φ is negative; that is, *the current lags behind the impressed e.m.f.* Otherwise there is a phase advance.

Again care must be exercised in maintaining the units in the same system when using these equations. It is best to use ohms, farads, and henrys throughout.

It must be emphasized that the decrease in current due to the reactance term of the impedance is not caused by a loss of power to heat as with the ohmic resistance term. It is a decrease in resultant current due to the phase differences of the impressed, inductive, and capacitive e.m.f.'s. This will become clearer when the power loss in such circuits is discussed.

141. POWER FACTOR

The fact that there are phase changes in a circuit upon which an outside e.m.f. is applied when either inductance or capacity is present in the circuit leads to an important correction in the question of power measurement. It will be remembered that power was defined as electromotive force times current. Thus the power in watts = volts \times amperes.

In the case of alternating currents we measure virtual volts, that is,

$$E_v = \frac{E_0}{\sqrt{2}},$$

and we measure virtual amperes

$$i_v = \frac{i_0}{\sqrt{2}}.$$

The power in watts consumed would be therefore

$$E_v i_v = \frac{E_0 i_0}{2}.$$

These measurements could be made independently and the proper watts computed if the current was not varying, or if no phase lag was involved. Suppose, however, that instead of such conditions the e.m.f. of a circuit is nearly 90° out of phase with the current flowing in it. This would be an extreme occurrence but it is closely approached with high inductances or small capacities. The result would be that although the voltmeter correctly read the voltage and the ammeter correctly read the current the watts computed would be incorrect, for when the voltage was a maximum the current would be nearly 0, and the actual power would be nearly 0, while a larger amount was computed. This can best be understood by studying the equations. Suppose the instantaneous electromotive force to be given by

$$E = E_0 \sin (pt + \varphi),$$

and the instantaneous current by

$$i = i_0 \sin pt.$$

The instantaneous power P will be

$$\begin{aligned} P &= [E_0 \sin (pt + \varphi)] i_0 \sin pt \\ &= E_0 i_0 \sin pt (\sin pt \cos \varphi + \cos pt \sin \varphi) \\ &= E_0 i_0 \sin^2 pt \cos \varphi + \frac{1}{2} E_0 i_0 \sin 2 pt \sin \varphi. \end{aligned}$$

If the average value \bar{P} of P be taken, previous procedure leads at once to the relations,

$$\begin{aligned} \bar{P} &= \frac{\int_0^{\frac{\pi}{p}} P dt}{\int_0^{\frac{\pi}{p}} dt} \\ &= \frac{E_0 i_0}{\pi/p} \left[\left(-\frac{1}{2} \cos pt \sin pt + \frac{1}{2} pt \right) \cos \varphi - \frac{1}{2p} \cos 2 pt \sin \varphi \right]_0^{\frac{\pi}{p}} \\ &= \frac{E_0 i_0}{2} \cos \varphi \end{aligned}$$

and hence

$$\bar{P} = E_v i_v \cos \varphi.$$

Accordingly the average watts of power consumed, or \bar{P} , is given by

$$\bar{P} = \frac{1}{2} E_0 i_0 \cos \varphi.$$

If $\varphi = 90^\circ$, $\cos \varphi = 0$, and the average power consumption \bar{P} is 0. If $\varphi = 0$, $\bar{P} = \frac{1}{2} E_0 i_0$. In between, $\cos \varphi$ has all values and the power consumed may have any value between

$$\frac{1}{2} E_0 i_0 = E_v i_v, \text{ and } 0.$$

Thus, it is seen that if the voltmeter and the ammeter readings of a circuit with self-induction and capacity are taken there is serious danger of not reading the true power consumption. What is read is called the *apparent watts*. The apparent watts times $\cos \varphi$ gives the *true watts*. The *power factor*, as $\cos \varphi$ is called, gives the ratio of

$$\frac{\text{True watts}}{\text{Apparent watts}} = \frac{\bar{P}}{E_v i_v} = \cos \varphi.$$

Hence the *power factor equals cos φ* .

A wattmeter which has its fields excited by the current in the circuit, and its moving coil excited by the potential difference, will give the true power consumption because the phase differences in the two parts of the instrument give a simultaneous reading of the product of the true current and voltage combined in their right phase relationship. The power factor can be determined very easily by taking the wattmeter reading on a given circuit, and *then* taking the voltmeter-ammeter reading. The ratio of the wattmeter reading to the voltmeter-ammeter reading gives the power factor, and *so gives the cosine of the angle of phase lag*. Consequently the phase lag can be calculated at once.

142. ELECTRICAL OSCILLATIONS

The property of capacities of storing electricity and the inertial effects of self-induction lead in combination to a most important phenomenon — that of electrical oscillation. In order to visualize what happens, recourse can be had to an analogy to the behavior of fluids.

Two tanks of water are connected by a stopcock with a hole in it. If the stopcock has a small hole of large resistance and one tank is full and the other empty, on opening the stopcock water will gradually flow from the full tank to the empty tank and when the two reach the same level the flow will stop. This is because the potential energy of the water which was initially higher than the final level was converted into kinetic energy in the pipe and this because of friction was converted to heat. Again, suppose the stopcock to have a large hole with very little friction. Then on opening the stopcock there will be a sudden rush of water from the full tank to the empty one. In the

empty one the water, owing to its inertia, will rise up to a height nearly as great as the height in the initial tank. It then rushes back into the initial tank, and its inertia carries it up to nearly its former position. Thus the water will oscillate back and forth, eventually coming to rest owing to the losses of kinetic energy in friction through the tube.

In electricity almost a complete analogy is presented. A capacity, it can be supposed, is equivalent to the water tank. Assume that the capacity is charged but the circuit open. If the circuit is closed the electricity will flow from one plate to the other, but it lacks one factor to make oscillation complete. The water, besides having a capacity present, had to have an inertia to carry it past its resting point. In electricity, the inertia is supplied by a self-induction. Thus, assume that a capacity is connected across a self-induction as indicated in Fig. 168. If now the capacity C is charged up and the switch S closed the positive electricity will flow through the inductance L to the other side of the circuit. Owing to its inertia it will charge the other side of the condenser to a higher potential than the equilibrium position of its charge. The electricity will then flow back again and oscillation

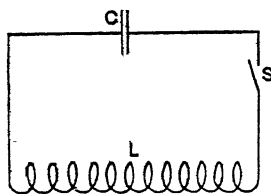


FIG. 168. Simple oscillating circuit with self-induction and capacity.

will continue until the resistance damps out all oscillations. If resistance is high the same situation will result as in the two tanks of water with a small hole, that is, a gradual flow and no oscillation. The condition for oscillation then is that the capacity and self-induction be sufficient, but that the resistance be small enough relative to the two to prevent damping out the oscillations. Again, it is perfectly obvious from the water analogy that the period of the oscillation will depend

on the capacity and self-induction present. As a matter of fact the period is

$$T = 2\pi\sqrt{LC},$$

to a good degree of approximation. A further study of such oscillating circuits belongs in a more advanced course. The study of such oscillations is absolutely essential to all investigation of radio engineering.

143. DERIVATION OF THE EQUATION FOR OSCILLATORY DISCHARGE OF A CONDENSER WITH RESISTANCE CAPACITY AND SELF-INDUCTION

For those students who wish to delve into the nature of electrical oscillations a little more deeply than the time in such a course as this permits, the equations for the discharge of a condenser of capacity C through a self-induction L and a resistance R are given in a compact

form below. Consider the circuit shown in Fig. 169 containing C , L , and R as indicated. Assume the condenser C charged to a potential E by closing the switch K to the battery. Suppose that now the switch K is opened, and the switch K' to the self-induction and capacity, which was previously open, is closed. Current then starts to flow through the circuit and we can quantitatively consider the conditions which follow as outlined qualitatively in the preceding section. In the mesh CRL the e.m.f. applied is now zero. At any instant, however, there are an e.m.f. E' due to the charge on the condenser, and an e.m.f. E'' of self-induction and the iR drop in the line. Had the e.m.f. E been applied in the mesh the complete Kirchhoff law equation for the mesh would have been $E - (E' + E'') = Ri$ as indicated earlier in the chapter for the case of an alternating e.m.f., E , applied in series with self-induction, resistance, and capacity.

In the diagram of Fig. 169 the circuit depicted has E placed across C , with R and L in parallel with C . If we had written $E = E_0 \sin pt$, this would present a very different problem from that previously solved in section 140. To study the conditions in such a circuit, where the applied e.m.f., E , may be constant as indicated in the diagram of Fig. 169, or varying as $E_0 \sin pt$, one must write two Kirchhoff law equations for the two branches with the current i divided into two currents, i_L for the section with inductance and i_c for the section with capacity. The branch of the circuit with the battery E must then contain a resistance R_E to yield a proper problem. In this case the two Kirchhoff law equations are,

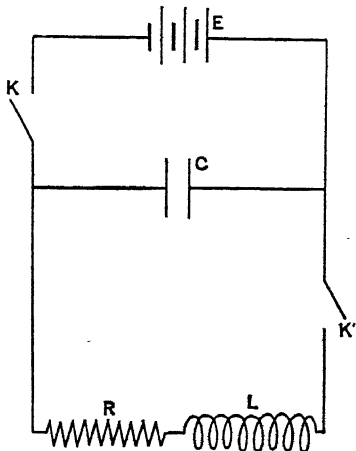


FIG. 169. Circuit for the study of the discharge of a condenser through an inductance and a resistance.

$$L \frac{di_L}{dt} + R_E i + Ri_L = E, \quad \text{and}$$

$$R_E i + \frac{Q_c}{C} = E, \quad \text{where} \quad \frac{dQ_c}{dt} = i_c.$$

These two simultaneous equations can then be combined by differentiating and an appropriate manipulation of symbols. The resulting differential equation is quite complicated and is beyond the scope of this book.

For the purpose of this discussion, which will be confined to the case where K is closed with K' open, thus charging C and then opening K and closing K' to discharge C through R and L in the mesh, one may write for the mesh $0 = Ri + E' + E''$, with $E = 0$, $E' = Q/C$ and $E'' = L \frac{di}{dt}$. Then since $i = \frac{dQ}{dt}$, the equation for the simple mesh system becomes

$$L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{Q}{C} = 0.$$

This is called a linear differential equation of the second order and first degree in Q with constant coefficients. For convenience we may write,

$$\frac{d^2Q}{dt^2} + \frac{R}{L} \frac{dQ}{dt} + \frac{Q}{CL} = 0.$$

Such an equation may be solved by setting $Q = e^{\alpha t}$ as a solution and studying the effect on the equation. This treatment yields

$$\alpha^2 e^{\alpha t} + \frac{R}{L} \alpha e^{\alpha t} + \frac{1}{CL} e^{\alpha t} = 0,$$

whence

$$\alpha^2 + \frac{R\alpha}{L} + \frac{1}{CL} = 0.$$

The fact that this substitution results in an equation between the constants only indicates that in fact it is a solution. On this basis the resulting equations may be studied. This is a second-degree equation in the constant α algebraic solution of which takes the form

$$\alpha = -\frac{R}{2L} \pm \frac{\sqrt{\frac{R^2}{L^2} - \frac{4}{CL}}}{2}.$$

This follows from the general relation that an equation $ax^2 + bx + c$ has a solution $x = \frac{-b}{2} \pm \frac{\sqrt{b^2 - 4ac}}{2}$. This gives us at once two solutions

$$Q = A'e^{\left(-\frac{R}{2L} + \frac{1}{2}\sqrt{\frac{R^2}{L^2} - \frac{4}{CL}}\right)t}$$

$$Q = B'e^{\left(-\frac{R}{2L} - \frac{1}{2}\sqrt{\frac{R^2}{L^2} - \frac{4}{CL}}\right)t}.$$

Here A' and B' are arbitrary constants of integration to be evaluated later. For convenience in handling one can call $\frac{R}{L} = 2b$ and $\frac{1}{LC} = k^2$. The equations become

$$Q = A'e^{(-b+\sqrt{b^2-k^2})t}$$

and

$$Q = B'e^{(-b-\sqrt{b^2-k^2})t}$$

Each of these is a possible solution. Thus the more general solution must contain both possibilities. As

$$Q - A'e^{(-b+\sqrt{b^2-k^2})t} = 0$$

and

$$Q - B'e^{(-b-\sqrt{b^2-k^2})t} = 0$$

the complete solution can be written as

$$2Q - A'e^{(-b+\sqrt{b^2-k^2})t} - B'e^{(-b-\sqrt{b^2-k^2})t} = 0$$

and calling $\frac{A'}{2} = A$, and $\frac{B'}{2} = B$, we have

$$Q = Ae^{(-b+\sqrt{b^2-k^2})t} + Be^{(-b-\sqrt{b^2-k^2})t}.$$

We can now find A and B . For since $Q = Q_0 = EC$ at $t = 0$

$$A + B = Q_0 = EC.$$

Again, since when $t = 0$, $\frac{dQ}{dt} = i = 0$, we have

$$(-b + \sqrt{b^2 - k^2})A + (-b - \sqrt{b^2 - k^2})B = 0.$$

These two equations for A and B may now be solved, and we obtain for Q the equation

$$Q = Q_0 \left\{ \frac{1}{2} \left(1 + \frac{b}{\sqrt{b^2 - k^2}} \right) e^{(-b+\sqrt{b^2-k^2})t} + \frac{1}{2} \left(1 - \frac{b}{\sqrt{b^2 - k^2}} \right) e^{(-b-\sqrt{b^2-k^2})t} \right\}.$$

It is seen that Q at any time t is a function of $Q_0 = EC$ and the constants $b = \frac{R}{2L}$ and $k^2 = \frac{1}{CL}$. When $b^2 - k^2$ is positive the exponents are negative and we have the case of a decline in Q which is the initial charge on the condenser multiplied by the sum of two exponential terms which decrease as t increases. Characteristic curves are shown

in Fig. 170a. The quantity and hence the current which is $\frac{dQ}{dt}$ thus decreases exponentially with time. This condition of b^2 greater than k^2 corresponds to a high resistance, for unless $b^2 = \frac{R^2}{4L^2}$ is greater than $k^2 = \frac{1}{CL}$, the condition above does not hold. Hence for values of C, R, L such that $\frac{R^2}{4L^2}$ is greater than $\frac{1}{CL}$ the flow of current is the

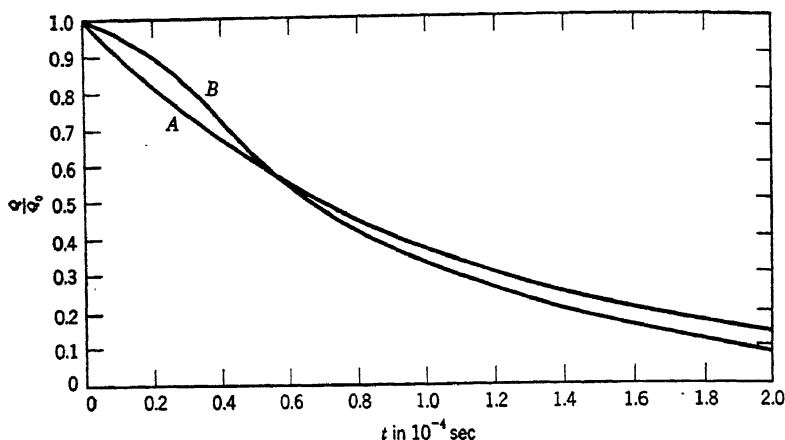


FIG. 170a. Damped discharge of a condenser with C, L , and R . Compare with Fig. 170b. Curve A: $L = 0.1$ henry; $C = 0.01$ microfarad $= 10^{-8}$ farad; $R = 1 \times 10^5$ ohms; $b = 5 \times 10^5$; $k = 3.16 \times 10^4$. Curve B: $L = 0.1$ henry; $C = 0.01$ microfarad; $R = 8.92 \times 10^3$ ohms; $b = 4.46 \times 10^4$; $k = 3.16 \times 10^4$; $b^2 = 2 \times 10^9$; $k^2 = 1 \times 10^9$.

exponential decrease characteristic of a water connection with high resistance where the energy of current flow is consumed in heating the tube as it flows through.

If now b is less than k , or $\frac{R^2}{4L^2}$ is less than $\frac{1}{CL}$, the situation is entirely different. The two exponential terms as well as the quantities A and B become imaginary, that is, they contain the square root of -1 . Thus $b^2 - k^2$ is now negative, for k^2 is greater than b^2 , and should be written $\sqrt{-1} \sqrt{k^2 - b^2}$. The $\sqrt{k^2 - b^2}$ is then real. Represent the $\sqrt{-1}$ by the symbol j . In writing the equations for this case the parts of the terms

$$e^{-(b \pm j\sqrt{k^2 - b^2})t}$$

can be written in the form

$$e^{-bt} e^{\pm(\sqrt{k^2-b^2})t}.$$

The equation deduced for Q then becomes, if we collect the terms multiplied by j and by $\frac{b}{\sqrt{b^2-k^2}}$ separately,

$$Q = Q_0 e^{-bt} \left\{ \frac{e^{j\sqrt{k^2-b^2}t} + e^{-j\sqrt{k^2-b^2}t}}{2} + \frac{b}{j\sqrt{k^2-b^2}} \frac{e^{j\sqrt{k^2-b^2}t} - e^{-j\sqrt{k^2-b^2}t}}{2} \right\}.$$

Now by Euler's theorem the exponentials $\frac{e^{jat} + e^{-jat}}{2} = \cos at$, and

$$\frac{e^{jat} - e^{-jat}}{2j} = \sin at. \quad \text{If } a = \sqrt{k^2 - b^2} \text{ then the above equation for } Q$$

at once simplifies to the form

$$Q = Q_0 e^{-bt} \left\{ \cos \sqrt{k^2 - b^2} t + \frac{b}{\sqrt{k^2 - b^2}} \sin \sqrt{k^2 - b^2} t \right\},$$

and by a well-known theorem in trigonometry similar to that applied in deducing the current relations on page 472,

$$Q = \frac{Q_0 k e^{-bt}}{\sqrt{k^2 - b^2}} \cos (\sqrt{k^2 - b^2} t - \varphi),$$

where

$$\tan \varphi = \frac{b}{\sqrt{k^2 - b^2}}.$$

In terms of R , L , and C the equations become

$$Q = Q_0 \frac{e^{-\frac{R}{2L}t}}{\sqrt{LC} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} \cos \left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} t - \varphi \right)$$

and

$$\tan \varphi = \frac{R}{2L \sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}}}.$$

Accordingly it is seen that: (1) a discharge takes place which varies periodically with time, for $\cos \sqrt{k^2 - b^2} t$ varies periodically with time; in other words when k^2 is greater than b^2 there is an oscillatory discharge; (2) the oscillation is one of decreasing amplitude; that is, it is a *damped* oscillation, for the cosine term is multiplied by the

exponential $e^{-\frac{R}{2L}t}$ which decreases with time, and this represents a damping factor (see Chapter XXIII); and (3) a phase factor φ is introduced into the equation where φ can be evaluated by

$$\tan \varphi = \frac{b}{\sqrt{k^2 - b^2}}.$$

The period of the oscillation is determined by the coefficient of the t in the cosine term. Since by definition of an oscillatory phenomenon (see Chapter XIX) the period T is given by the T in the transcendental term, sine or cosine, in the form $\sin \frac{2\pi}{T}t$, it is seen at once that

$$\frac{2\pi}{T} = \sqrt{k^2 - b^2}$$

or

$$T = \frac{2\pi}{\sqrt{k^2 - b^2}} = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}.$$

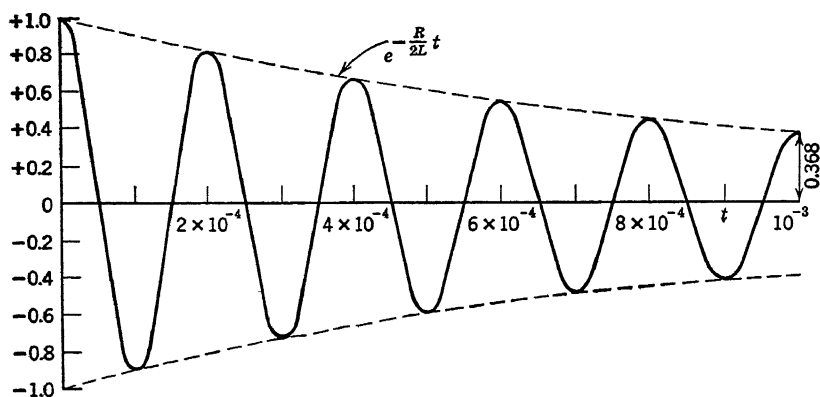


FIG. 170b. Highly damped oscillation of charge with L , C , and R . $L = 0.1$ henry; $C = 0.01$ microfarad $= 10^{-8}$ farad; $R = 500$ ohms; T is closely 2×10^{-4} ; $N = 5000$ cycles; $\tan \phi = 1.6 \times 10^{-4}$; $\phi = 0$.

Thus the oscillatory discharge will have a damped oscillation of period T defined by

$$T = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}.$$

Such a curve for a highly damped oscillation is shown in Fig. 170b.

Now where sustained oscillations are desired R is reduced to a mini-

mum and the term $\frac{R^2}{4L^2}$ becomes negligible. Thus the terms containing $\frac{R^2}{4L^2}$ are simplified by putting $\frac{R^2}{4L^2} = 0$ and the elaborate equation for Q above becomes $Q = Q_0 \cos \frac{t}{\sqrt{LC}}$, for φ is 0 and $e^{-\frac{R}{2L}t}$ becomes unity. Hence for this case the damping is negligible, and the period is $\frac{2\pi}{T} = \frac{1}{\sqrt{LC}}$ or $T = 2\pi\sqrt{LC}$ as stated in the simplified equation given in the preceding qualitative discussion.

CHAPTER XXVI

THE RATIO OF ELECTROMAGNETIC AND ELECTROSTATIC UNITS, MAXWELL'S EQUATIONS AND ELECTROMAGNETIC WAVES, AND THE CHARACTER OF ELECTRICAL UNITS AND THEIR DIMENSIONS

A. THE RATIO OF THE UNITS

144. INTRODUCTION

In 1854 in the immediate pre-systematic period in which the unification of electrical concepts was, through the efforts of mathematical physicists and the improved techniques of the experimental physicists, approaching its logical termination, a young genius, James Clerk Maxwell, was completing his education at Trinity College, Cambridge. While Maxwell was finishing his studies and entering his first position as professor of natural philosophy at Aberdeen University, an experimentalist to whom electricity owes a great debt of gratitude, Wilhelm Weber, was completing a most important series of investigations. Weber, influenced by the great mathematical thinkers of his period, especially Gauss, had started a systematic determination of the absolute values of the units in the electromagnetic and the electrostatic systems. In 1856 Weber, together with Kohlrausch, established the ratio of the unit of current in the absolute electromagnetic and electrostatic systems as being 3.107×10^{10} , a value close to the accepted value for the velocity of light.

Such a mere coincidence of numerical values has necessarily little meaning. This discovery was, however, distinctly more significant. For it can be shown by dimensional reasoning in the light of later-day experience that in fact the ratio of the absolute units of current should have the dimensions of a velocity. That the velocity should have been that of light requires more elucidation, which will be given at a later point in the discussion. In the meantime it is of interest to analyze the ratio of the units dimensionally in order to establish this point. In doing so the system of definitions and dimensions developed earlier in this book will be used. The system is essentially what is known as the Gaussian system of units which will be discussed in more detail at the end of this chapter.

145. THE RATIO OF THE ELECTROSTATIC AND ELECTROMAGNETIC SYSTEMS OF UNITS

In the electrostatic system quantity of electricity q is defined as $q = \sqrt{fr^2k}$, where the symbol k replaces D as the symbol for dielectric

constant. k is a constant chosen in this book as dimensionless. Hence $q_{es} = k^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$. In the electromagnetic system q_{em} is defined by $q_{em} = i_{em} t$, where i_{em} is current defined electromagnetically and t is time. But i_{em} is defined by $f = \frac{i_{em} ds m}{r^2}$, where ds and r are lengths

and m is magnetic pole strength. Now m is defined by $f = \frac{mm'}{\mu r^2}$, where

r is a distance. Here μ again is a constant of the surrounding material, the magnetic permeability, which has also been chosen as dimensionless

in this book. Thus $m = \mu^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$, so that $i_{em} = \frac{ML^2 T^{-2}}{\mu^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}} = \mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$, and accordingly $q_{em} = i_{em} t = \mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}}$. Now let us

assume that the observed ratio $\frac{q_{em}}{q_{es}} = \frac{1}{\eta}$, where η is a quantity as yet

undetermined in magnitude and dimensions. Then $\frac{q_{em}}{q_{es}} = \frac{1}{\eta} =$

$\frac{\mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}}}{k^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}} = \frac{1}{LT^{-1} \sqrt{\mu k}}$, or $\frac{\eta}{\sqrt{\mu k}} = LT^{-1}$. This means that the

quantity $\frac{\eta}{\sqrt{\mu k}}$ has the dimensions of a velocity.

Thus $v = \frac{\eta}{\sqrt{\mu k}}$, and it remains for an assignment of dimensions to μ

and k to determine the dimensions of η . In the Gaussian system which has been adopted in this book μ and k are dimensionless and $\mu k = \mu_0 k_0 = 1$ for empty space. Now in empty space the velocity v will be designated as equal to c , whence $v = c = \eta$ and η is a velocity for empty

space when $\mu k = 1$. Thus as $q_{em} = \frac{q_{es}}{\eta}$, a quantity *evaluated* in electrostatic units must be divided by a velocity $c = \eta$ to give the quantity

in electromagnetic units. That is, $q_{em} = \frac{q_{es}}{\eta} = \frac{q_{es}}{c}$.

Now the results of measurements of this ratio for *currents*, but *equally applicable to quantity*, published by Weber and Kohlrausch in 1856, gave the ratio as 3.107×10^{10} . Thus $\eta = c = 3 \times 10^{10}$ cm per second is the value of this velocity which is needed to convert a quantity measured in electrostatic units to a quantity as it would be

measured in electromagnetic units, i.e., $q_{em} = \frac{q_{es}}{c} = \frac{q_{es}}{3 \times 10^{10}}$. Hence

the quantity *expressed in electrostatic units is numerically* 3×10^{10} *greater than the same quantity expressed in electromagnetic units*, and must dimensionally be divided by a velocity to make it equivalent. The *electromagnetic unit is accordingly* 3×10^{10} *times as great as the electrostatic unit*. This ratio has since 1856 been determined by a number of different investigators with the following results:

Weber and Kohlrausch	1856	$\eta = 3.107 \times 10^{10}$ cm per second
Lord Kelvin	1869	$\eta = 2.82 \times 10^{10}$ "
Rowland	1889	$\eta = 2.98 \times 10^{10}$ "
Abraham	1890	$\eta = 2.991 \times 10^{10}$ "
Rosa and Dorsey corrected by Birge 1929		$\eta = 2.9978 \pm 0.0001 \times 10^{10}$

It can further be shown by dimensional reasoning that this ratio c is also the ratio between the three "fundamental" units, current, quantity, and potential determined in the two systems of units, and appears in squared form, i.e., 9×10^{20} in the ratios of the "derived" units, resistance, capacity, and self-induction. This can be seen in the table of pages 412–413. That this velocity was identical with that of light which was historically determined first and is now given by Birge as $(2.99776 \pm 4) \times 10^{10}$ cm per second aroused a great deal of interest. It was this circumstance that started Maxwell among others on an intensive study of the electromagnetic equations to see whether there was a reason for this numerical coincidence.

B. MAXWELL'S EQUATIONS AND ELECTRO- MAGNETIC WAVES

146. INTRODUCTION

In order to understand the character of the reasoning which led Maxwell to the discovery of the relation between the ratio of the units and the velocity of light, one may proceed as follows. An electrical charge at rest sends out its hypothetical electrical lines of force throughout all space to the number of $4 \pi q$ for q charges, as assumed in section 69, Chapter XIII. If there are ρ charges in a cubic centimeter of space, then a volume $dx dy dz$ will send out $4 \pi \rho dx dy dz$ such lines. It is these lines ending on other charges and manifesting forces that enable us to recognize the presence of electricity and to define quantity of electricity in the electrostatic system.

When a charge is in motion it is in fact a current of electricity. Its detection as such comes from the fact that the current is surrounded by a magnetic field. This current by means of magnetic lines of force, interacts with magnet poles (probably the result of other electricity in motion) and gives the definition of current in terms of magnetic forces, i.e., Ampère's equation, serving to define current magnetically. Now these systems of units are defined by electrical phenomena due to electricity in two different states, at *rest* and *in uniform motion*. It is likely that in seeking an understanding of the relation between the two systems we will have to consider what takes place in changing the state of a charged body from one at rest to one in motion.

As a static charge is removed from its state of rest to one of uniform motion there is a period in which it is being accelerated. In this period its lines of electrostatic force are being deranged; i.e., kinks travel outward in space along them, adapting the condition of the

static lines to their new rôle of motion. At the same time *as motion begins, a magnetic field is being created which spreads outward* through all space ultimately to give the magnetic field of the moving particle. Thus an accelerated electrical charge is sending throughout all space an electrical and a magnetic disturbance. The electrical pulse is a displacement along the lines of electrical force parallel to the direction of motion of the charge moving outward along the lines of force. The magnetic field spreads outward in a direction at right angles to the direction of motion of the charge and radially away from the direction of motion, as would be expected from the right-hand rule. It is now a question as to how fast, i.e., with what velocity, this disturbance propagates itself in space. As will presently be seen, it is possible from the electromagnetic equations to calculate this speed directly from the values of the dielectric constant and permeability of the medium, as was evident from dimensional reasoning.

Hence it need not appear strange to see that the ratios of the units in the electrostatic and electromagnetic systems are such as to be related to the velocity of propagation of the disturbance created when a charge is changing its status from one detected statically to one detected magnetically.

That the velocity should be the velocity of light need not be a surprise. Light is merely the periodic disturbance created in space when an atomic electron is accelerated in executing a simple harmonic motion, for there is no restriction on the size of the charge which is accelerated.

To understand how the dielectric constant, hereafter designated as k , and magnetic permeability μ enter into these relations, certain simple laws which have already been encountered will have to be *generalized*. We shall go one step further than we have so far gone in enlarging our concepts of electrical current, an enlargement already almost unconsciously made in discussing alternating currents and transient phenomena. This step is the famous one that Maxwell made in deducing the electromagnetic theory long before the facts we used in Chapters XV and XXIII were known.

147. THE EQUATIONS OF CONTINUITY

Assume a very small parallelepiped of volume $dx\ dy\ dz$ in a space defined by the coordinate axes x , y , and z . See Fig. 171. Let it contain electricity with a *volume density* of ρ charges per cubic centimeter. Let the dielectric constant be k . Then into the face $dydz$ along the x -axis, if the field be F_x along x , there will enter $kF_x\ dy\ dz$ lines of force. For if F_x be the field in the absence of a dielectric, it will, in analogy to the case of magnetic induction, have kF_x lines per centimeter entering (and/or emerging from) it. Since there are charges inside, the lines of force may increase in number from kF_x at the left-hand face

to $k \left(F_x + \frac{\partial F_x}{\partial x} dx \right)$ at the other end of $dx dy dz$, dx centimeter away, for the rate of increase of field is $\frac{\partial F_x}{\partial x}$. Hence the increase in number of lines of force running along the x direction between the near and far sides of the volume will be

$$k \left(F_x + \frac{\partial F_x}{\partial x} dx \right) dy dz - k F_x dy dz = k \frac{\partial F_x}{\partial x} dx dy dz.$$

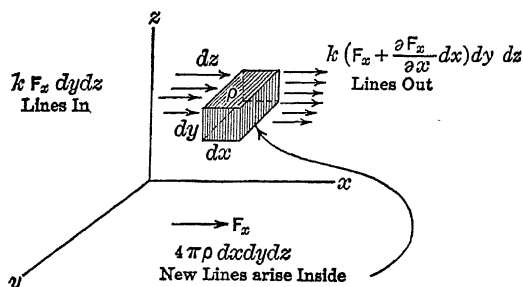


FIG. 171. Effect of a volume density of charge ρ .

Likewise along the y -axis we can write this change as $k \frac{\partial F_y}{\partial y} dx dy dz$, and similarly along the z -axis it is $k \frac{\partial F_z}{\partial z} dx dy dz$. Hence the change in the number of lines entering and leaving the volume along all three axes is

$$k \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) dx dy dz.$$

Now this must equal the number of new lines created in $dx dy dz$ as the result of the charge density ρ , to wit, $4 \pi \rho dx dy dz$. Hence we can write

$$\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = \frac{4 \pi \rho}{k}, \quad (1)$$

which is called the divergence of F , written $\text{div } F = \frac{4 \pi \rho}{k}$ for short. The equation merely states that the lines of force emerging from a region in space can come only from charges, and if there is any change in field intensity along x , y , and z in $dx dy dz$ it is called forth by charges in the measure given.

Now in a magnetic field in a volume element $dx dy dz$ carrying ρ poles per cubic centimeter, since the magnetic induction $B = \mu H$ where μ is the permeability, it can be shown in complete analogy with the above dielectric case, if H_x , H_y , and H_z are the components of the magnetic

field along x , y , and z , that

$$\left(\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} \right) = \frac{4 \pi \rho}{\mu} \text{ or that } \operatorname{div} H = \frac{4 \pi \rho}{\mu}, \quad (2)$$

where ρ is now the density of magnetic poles. This is known as Poisson's equation. This equation is merely the magnetic analogue of the electrical field equation and states that a magnetic field is changed only by the presence of poles. In a space free from matter there are no poles and $\rho = 0$. The equation then states that as many lines leave as enter a region. Where no poles exist $\operatorname{div} H = \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$, and similarly if no free charges exist in space $\operatorname{div} F = 0$. This is known as the equation of continuity.

148. THE GENERALIZATION OF AMPÈRE'S LAW

It is now necessary to turn from the field equations relating to charges at rest, which have just been written in a more general form, to a more general description of Ampère's law of the magnetic field H produced by a current, i.e., by moving charges. The problem specifically is to calculate the fields H_x , H_y , and H_z in the xy , yz , and zx planes produced by current flows $i_x dy dz$, $i_y dx dz$, and $i_z dx dy$ along the x -, y -, and z -axes; i_x , i_y , and i_z being current densities (i.e., absolute units per square centimeter) and $dy dz$, $dx dz$, and $dx dy$ the very small area in the flow. To get these three fields, H_x , H_y , and H_z from the currents i_x , i_y , and i_z , it is not found convenient to use Ampère's law in its simple forms. Instead a trick will be resorted to similar to one used in the derivation of the field produced in a long solenoid in Chapter XVII. There the work of carrying a pole m around the whole field was equated to that of the pole m circling each wire once. If ϕ lines of magnetic force are cut when a pole moves about a current, the work $W = i\phi$, and for a pole of strength m , $W = 4 \pi mi$. If the field produced by the current i is H , then $W =$

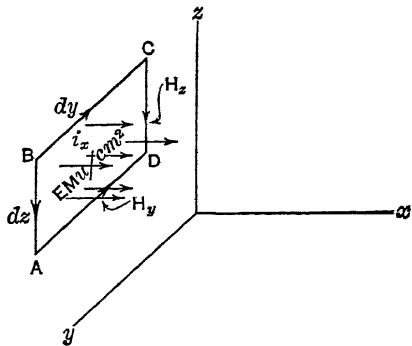


FIG. 172. Magnetic field due to a current.

$\oint mH dl$, where \oint is the integral around the magnetic circuit. What we do then is to take a small area $dy dz$ bounding, for example, the current $i_x dy dz$ and calculate the work W to move a pole m once around the rectangle $ABCD$ of area $dy dz$ in Fig. 172. Then by equating this to $4 \pi mi$, we can get H . The case is complicated in that the field H along z and along y may not

be constant. This must be done for each of the areas $dx dy$, $dy dz$, $dx dz$ in turn and will give the three field equations.

Now the work in going around $ABCD$, the contour of the area $dy dz$, for a unit pole $m = 1$ can be computed as follows. Let the value of the field H along DC be H_z , as it is parallel to Z , and let it be downward.

Along AB the field is $H_z + \frac{\partial H_z}{\partial y} dy$ and also downward. Along DA let the field be inward and of value H_y and along BC let the field be inward and of value $H_y + \frac{\partial H_y}{\partial z} dz$. Then per unit pole the work W taken with due regard to sign, with work done *on the pole by the field as negative*, will be

$$\begin{aligned} W = \oint H dl &= -H_y dy - \left(H_z + \frac{\partial H_z}{\partial y} dy \right) dz + \left(H_y + \frac{\partial H_y}{\partial z} dz \right) dy \\ &\quad + H_z dz = - \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) dy dz = 4 \pi i_x dy dz; \\ \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= -4 \pi i_x. \end{aligned}$$

Similarly one can write for i_y and i_z ;

$$\begin{aligned} \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= -4 \pi i_y; \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= -4 \pi i_z. \end{aligned}$$

The relations on the left-hand side of these equations give the components of a vector known as the curl of H . It is merely a statement in three dimensions of the value of the field components H_x , H_y , and H_z resulting from a current density with components i_x , i_y , and i_z .

149. MAXWELL'S DISPLACEMENT CURRENT

Now Maxwell arrived at the conclusion that the *current density* expressed in these equations was not simply the electrical current flowing through a wire or in space due to a stream of electrons and ions, i.e., to the movement of a space charge ρ of electrons or protons, with a velocity v_x , v_y , and v_z along x , y , and z . His reason for this conclusion was the following. Consider a wire leading to a parallel-plate condenser filled with a dielectric of constant k . Now while this represents an open circuit to a direct current, i.e., to a continuous mass motion of electrons, it is not impervious to currents which vary with time; witness the passage of an alternating current through a condenser. In fact, there will always be currents of the form $i = C \frac{dV}{dt}$ through the

condenser whenever the potential is changing with time. Thus as one plate of a condenser charges up there is in the surrounding dielectric or empty space what *Maxwell termed a displacement current*. This likewise holds for any region surrounding the conductor. The magnitude of this current can be evaluated as follows. Assume a condenser of unit surface area. The surface charge density on it is σ (see Chapter XIV). Then the σ charges causing the displacement current change

with time and give a current $i = \frac{d\sigma}{dt}$. Now in analogy to magnetic

induction in which B , the number of lines leaving the surface of a body in a field H is written as $B = \mu H$, the uniform (isotropic) dielectric can be regarded as a conductor of lines of electric force and can be written $N = kF$, where N is the number of lines of electrical force per square centimeter leaving the conductor and F is the electrical field. Then $\frac{N}{4\pi}$ is the equivalent number of surface charges on the dielectric in the

field F produced in the space. Hence $\sigma = \frac{N}{4\pi} = \frac{kF}{4\pi}$, so that the dis-

placement current densities i_x , i_y , and i_z are given by:

$$i_x = \frac{d\sigma_x}{dt} = \frac{k}{4\pi} \frac{dF_x}{dt},$$

$$i_y = \frac{d\sigma_y}{dt} = \frac{k}{4\pi} \frac{dF_y}{dt},$$

$$i_z = \frac{d\sigma_z}{dt} = \frac{k}{4\pi} \frac{dF_z}{dt};$$

and the equations, including movement of volume densities of charge, become:

$$\left. \begin{aligned} \frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial z} &= -\frac{4\pi}{\eta} \left(\frac{k}{4\pi} \frac{\partial F_x}{\partial t} + \rho v_x \right), \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= -\frac{4\pi}{\eta} \left(\frac{k}{4\pi} \frac{\partial F_y}{\partial t} + \rho v_y \right), \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= -\frac{4\pi}{\eta} \left(\frac{k}{4\pi} \frac{\partial F_z}{\partial t} + \rho v_z \right). \end{aligned} \right\} \quad (3)$$

For $i'_x = \rho v_x$, $i'_y = \rho v_y$, $i'_z = \rho v_z$ where i'_x , i'_y , i'_z are the parts of the currents due to a mass motion of the volume density of charge ρ along x , y , and z with velocities v_x , v_y , and v_z . However, when relations in magnetic units, e.g., H units, are equated with strictly electrostatic quantities such as F , it is necessary to insert a constant of proportionality η of unknown dimensions which may be determined. As the ratio of units shows, this constant η will probably be related to the

velocity of light. This explains the presence of a constant η in the equations. This set of equations is merely an expression of Ampère's law for the field produced in any kind of space in three dimensions due to a current composed either of a dielectric displacement in which the field F changes with time or a mass motion of charges with a velocity v or both. If there are no charges, as in empty space, the currents are all the result of changes in the field F (i.e., displacement currents); and, vice versa, if there is a *steady* current, the field will be the result of the mass motion of charges with velocity v , and $\frac{dF}{dt}$ will be zero, i.e., there will be no displacement current.

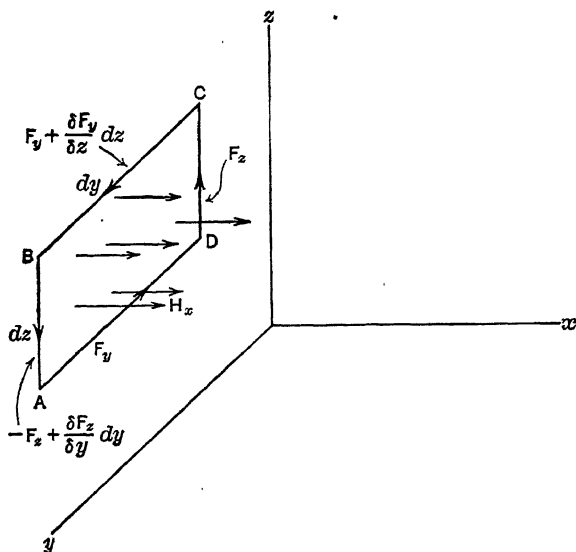


FIG. 173. The e.m.f. generated by an alternating or changing magnetic field H_x .

150. THE GENERALIZATIONS OF FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

A fourth set of general relations which express Faraday's law of electromagnetic induction for a material in space in three dimensions can now be derived. As currents produce magnetic fields, so also changing magnetic fields produce currents and electrical fields. That is, if a changing magnetic field exists, causing a change of flux in a region, a conductor in this region has an e.m.f. induced in the region, and this in turn gives magnetic fields. Let us consider the circuit $ABCD$ of Fig. 173 lying in the YZ plane as before, and let there be a flux $\phi_x = B_x dydz$ flowing through the area, where B_x is the magnetic induction along x . Hence $\phi_x = B_x dydz = \mu H_x dydz$, and an e.m.f. is

generated around $ABCD$ of value $e = -\frac{d\phi_z}{dt} = -\mu \frac{dH_z}{dt} dydz$. The electromotive force e is composed of the sum of the potential gradients along AB , BC , CD , and DA , that is, by the electrical fields along AB , BC , CD , and DA , multiplied by the lengths of the elements, to wit, dz , dy , dz , and dy . Along CD , the field F_z is upward, and along AB it is downward and is $-\left(F_z + \frac{\partial F_z}{\partial y} dy\right)$ in magnitude. Along AD the field is inward $-F_y$, and along BC it is outward and equal to $+\left(F_y + \frac{\partial F_y}{\partial z} dz\right)$. Hence the e.m.f.

$$\begin{aligned} e &= +F_z dz - \left(F_z + \frac{\partial F_z}{\partial y} dy\right) dz - F_y dy + \left(F_y + \frac{\partial F_y}{\partial z} dz\right) dy \\ &= \left(\frac{\partial F_y}{\partial z} - \frac{\partial F_z}{\partial y}\right) dydz = -\frac{\mu}{\eta} \frac{dH_z}{dt} dydz, \end{aligned}$$

and similarly for circuits in the yx and zx planes. This yields the equations

$$\left. \begin{aligned} \frac{\mu}{\eta} \frac{dH_x}{dt} &= \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \\ \frac{\mu}{\eta} \frac{dH_y}{dt} &= \frac{\partial F_z}{\partial z} - \frac{\partial F_z}{\partial x} \\ \frac{\mu}{\eta} \frac{dH_z}{dt} &= \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \end{aligned} \right\} \quad (4)$$

Again, as in the equations for Ampère's law, a constant η taking care of the ratio of the units must be inserted.

It is seen that this set of three equations merely represents the values of the induced electrical fields produced as a result of a change of magnetic flux in a small region of space resolved along the x , y , and z axes. As might have been expected from the closely reversible relations observed experimentally in Chapters XVI and XIX, where magnetic fields and currents caused motion and where motion in a field caused a current, the resulting general equations are very symmetrical in form.

It is further to be noted that time changes in electrical fields produce magnetic fields, and, vice versa, time changes in magnetic fields in space produce electrical fields interrelated in a very symmetrical form. It is essential to remark at this point that in this general form the four sets of equations have a wide range of applicability for the solution of different problems and that they are free from the limited applicability of the more specific equations obtained from special experiments used in their derivation. An analogous situation exists in the relation between the experimental law for flow

of heat in an infinite plane slab which reads $Q = -KA \frac{d\theta}{dx} t$, where θ is the temperature and Q is the heat quantity which flowed in a time t , and the Fourier heat equation which reads

$$\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \frac{K}{\rho C} = \frac{d\theta}{dt},$$

which can be applied to the solution of any problem of heat flow.

151. DERIVATION OF THE WAVE EQUATION

The task is now to see how these general equations are of value in considering the effect produced by changing the electrical field strength at a point in space with time. For this purpose let us choose a point in empty space devoid of charges ($\rho = 0$) and magnetic poles ($\rho = 0$) having a dielectric constant k and a permeability μ , and let us by moving a charge somewhere produce a change of the field F parallel to some axis, say the z -axis. That is, let us give a value to the quantity $\frac{\partial F_z}{\partial t}$ along the Z direction and see what will happen, for it is clear from equations 3 and 4 that changes in F_z will change H_x and H_y and this will in turn react to produce further electrical field changes. In order to analyze the problem the equations 1, 2, 3, and 4 can be written imposing the conditions of the problem that $\rho = 0$ and no mass motion of current results.

$$\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 0, \text{ as } \rho = 0. \quad (1)'$$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0, \text{ as } \rho = 0. \quad (2)'$$

$$\left. \begin{aligned} \frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial z} &= -\frac{k}{\eta} \frac{\partial F_z}{\partial t} \\ \frac{\partial H_z}{\partial z} - \frac{\partial H_x}{\partial x} &= -\frac{k}{\eta} \frac{\partial F_y}{\partial t} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_z}{\partial y} &= -\frac{k}{\eta} \frac{\partial F_x}{\partial t} \end{aligned} \right\} (3)'$$

$$\left. \begin{aligned} \frac{\mu}{\eta} \frac{\partial H_x}{\partial t} &= \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \\ \frac{\mu}{\eta} \frac{\partial H_y}{\partial t} &= \frac{\partial F_z}{\partial z} - \frac{\partial F_z}{\partial x} \\ \frac{\mu}{\eta} \frac{\partial H_z}{\partial t} &= \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \end{aligned} \right\} (4)'$$

It is next convenient to eliminate *either* H or F from the equations in H , F , x , y , z , and t and get equations either in F , t , and x , y , z , or in H , t , and x , y , z . To accomplish this simplification differentiate equation 3' with respect to time, giving for the first equation

$$\frac{\partial^2 H_x}{\partial y \partial t} - \frac{\partial^2 H_y}{\partial z \partial t} = -\frac{k}{\eta} \frac{\partial^2 F_z}{\partial t^2}. \quad (5)$$

Then differentiate the second equation of 4' with respect to z and the third with respect to y . These give

$$\frac{\mu}{\eta} \frac{\partial^2 H_y}{\partial t \partial z} = \frac{\partial^2 F_z}{\partial z^2} - \frac{\partial^2 F_z}{\partial z \partial x}. \quad (6)$$

$$\frac{\mu}{\eta} \frac{\partial^2 H_z}{\partial t \partial y} = \frac{\partial^2 F_y}{\partial x \partial y} - \frac{\partial^2 F_x}{\partial y^2}. \quad (7)$$

Add 6 to 5, subtract 7, and remember that 1' can be rewritten to read

$$\frac{\partial}{\partial x} \left(\frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) = - \frac{\partial^2 F_x}{\partial x^2}.$$

This at once gives

$$\frac{\partial^2 F_x}{\partial x^2} + \frac{\partial^2 F_x}{\partial y^2} + \frac{\partial^2 F_x}{\partial z^2} = \frac{k\mu}{\eta^2} \frac{\partial^2 F_x}{\partial t^2}. \quad (8a)$$

Similar equations can be deduced for the y and z components by manipulation, to wit

$$\frac{\partial^2 F_y}{\partial x^2} + \frac{\partial^2 F_y}{\partial y^2} + \frac{\partial^2 F_y}{\partial z^2} = \frac{k\mu}{\eta^2} \frac{\partial^2 F_y}{\partial t^2}. \quad (8b)$$

$$\frac{\partial^2 F_z}{\partial x^2} + \frac{\partial^2 F_z}{\partial y^2} + \frac{\partial^2 F_z}{\partial z^2} = \frac{k\mu}{\eta^2} \frac{\partial^2 F_z}{\partial t^2}. \quad (8c)$$

Similar operations can be used to eliminate F_x , F_y , and F_z and to give relations for the magnetic fields H_x , H_y , and H_z of this form

$$\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} = \frac{k\mu}{\eta^2} \frac{\partial^2 H_x}{\partial t^2}, \quad (8d)$$

with analogous expressions for H_y and H_z . What we see before us are equations representing the changes in F_x , F_y , and F_z , H_x , H_y , and H_z at various places when at some point chosen F_x , F_y , or F_z or H_x , H_y , or H_z varies with time. To simplify discussion let us consider the case where the components of F and H are functions of x and t only. That is, assume that F and H do not vary with y and z . Then equations 8a, 8b, and 8c become

$$\frac{\partial^2 F_x}{\partial x^2} = \frac{k\mu}{\eta^2} \frac{\partial^2 F_x}{\partial t^2}, \quad (9a)$$

$$\frac{\partial^2 F_y}{\partial x^2} = \frac{k\mu}{\eta^2} \frac{\partial^2 F_y}{\partial t^2}, \quad (9b)$$

$$\frac{\partial^2 F_z}{\partial x^2} = \frac{k\mu}{\eta^2} \frac{\partial^2 F_z}{\partial t^2}; \quad (9c)$$

and $8d$, and analogous expressions for H_y and H_z become:

$$\frac{\partial^2 H_x}{\partial x^2} = \frac{k\mu}{\eta^2} \frac{\partial^2 H_x}{\partial t^2}, \quad (9d)$$

$$\frac{\partial^2 H_y}{\partial y^2} = \frac{k\mu}{\eta^2} \frac{\partial^2 H_y}{\partial t^2}, \quad (9e)$$

$$\frac{\partial^2 H_z}{\partial z^2} = \frac{k\mu}{\eta^2} \frac{\partial^2 H_z}{\partial t^2}. \quad (9f)$$

These are partial differential equations of the second order and the first degree with constant coefficients.

152. SOLUTION OF THE WAVE EQUATION

A solution of this equation for one component is at once possible by several methods for the solution of partial differential equations. The solution takes the very general form of

$$F_x = f_1 \left(x - \frac{\eta t}{\sqrt{\mu k}} \right) + f_2 \left(x + \frac{\eta t}{\sqrt{\mu k}} \right),$$

for the case of the x component from equation $9a$. Here f_1 and f_2 are mathematical functions, the form of which depends on the initial conditions imposed on F_x , F_y , and F_z , i.e., the nature of the cause of the disturbance. That is, if F_x , F_y , or F_z are varied with time in some definite fashion given by a mathematical relation, the form which the functions f_1 and f_2 will take can at once be determined. Thus if $F_x = F_{x0}e^{-at}$ as for a transient field produced by the discharge of a condenser through a resistance, or if $F_x = F_{x0} \sin \frac{2\pi}{T} t$, which is the field produced by a charge moving sinusoidally with time along z , the form of f_1 and f_2 can at once be established.

However, before going on to the discussion of particular forms of the functions f_1 and f_2 determined by a particular case of assumed form for the time variation of F with t , we can learn some more about the general equation

$$F_x = f_1 \left(x - \frac{\eta t}{\sqrt{\mu k}} \right) + f_2 \left(x + \frac{\eta t}{\sqrt{\mu k}} \right). \quad (10a)$$

The correctness of this general solution can be tested by differentiating F_x twice with respect to x and twice with respect to t , when it will be seen to yield the differential equation $9a$. Suppose next that t is increased from a value t to one $t + \tau$. At the same time x is increased

by adding to it $\frac{\eta\tau}{\sqrt{\mu k}}$. Then for f_1 of $10a$ it is found that

$$f_1 \left[\left(x + \frac{\eta\tau}{\sqrt{\mu k}} \right) - \frac{\eta}{\sqrt{\mu k}} (t + \tau) \right] = f_1 \left(x - \frac{\eta t}{\sqrt{\mu k}} \right).$$

This means that at a point $x + \frac{\eta\tau}{\sqrt{\mu k}}$ the quantity f_1 of equation $10a$ has the same value at $t = t + \tau$ that it had at the point x when $t = t$. In other words the particular value of F_x at a time $t = t$ has propagated itself along *positive* values of x such that it covers a distance $\Delta x = \frac{\eta\tau}{\sqrt{\mu k}}$

in a time $\Delta t = \tau$. Hence it is clear that in its indefinite functional form the equation indicates that a particular value of the field along x *propagates itself along positive values of x from the origin $x = 0$ at*

a velocity $v = \frac{\Delta x}{\Delta t} = \frac{\frac{\eta\tau}{\sqrt{\mu k}}}{\tau} = \frac{\eta}{\sqrt{\mu k}}$. Had a similar procedure been carried out for a time $t + \tau$ with f_2 it would have been found that the function had the same value at $x - \frac{t}{\sqrt{\mu k}}$, i.e., that f_2 represents

a wave moving in the opposite direction along its x -axis. Hence this indicates that a disturbance due to a time variation of F propagates itself from $x = 0$ in both directions along x with a velocity $v = \frac{\eta}{\sqrt{\mu k}}$,

f_1 representing the wave moving in the positive, and f_2 in the negative, x direction. Similar results can be found for the y - and z -axes and for the fields H_x , H_y , and H_z , as well as for F_x , F_y , and F_z . Thus it is seen that in the most general case of an electrical field disturbance in empty space this disturbance propagates itself with a velocity $v = \frac{\eta}{\sqrt{\mu k}}$. The disturbance in the case considered, where variations

of F along y and z were omitted in deriving equations $9a$, $9b$, and $9c$, has the same value at a given instant at all points on a plane passing normal to x at a given value of x . Such a disturbance is designated as a plane wave of infinite extent progressing along x .

It is thus seen that the generalized electromagnetic equations of Maxwell lead to the conclusion that a time variation of an electrical or magnetic field propagates itself in space of dielectric constant k and permeability μ with a velocity $v = \frac{\eta}{\sqrt{\mu k}}$. Here η is a constant

expressing the ratio of the electrical and magnetic units used in writing equations 3 and 4 or 3' and 4' in order to make the equations consistent

and also encountered in obtaining the ratio of the units. Depending on the dimensions arbitrarily assigned μ and k , η can take on any dimensions desired. Throughout this book we have adhered to the Gaussian system of units which gives μ and k no dimensions, as will later be seen. In this case η , the ratio of the units, becomes a velocity v . Thus since an accelerated electron in an atom will also emit electromagnetic waves which are known as light waves, it is seen that the ratio of the units $\eta =$ the velocity of light. This confirms and justifies the relations deduced in the first part of this chapter.

For the purpose of further discussion, consider only the wave progressing in the positive x direction and write

$$F_x = f_1(x - vt).$$

In a similar way for the solution of equations 9b, 9c, 9d, 9e, and 9f it may be written that

$$F_y = g_1(x - vt). \quad (10b) \quad H_x = i_1(x - vt). \quad (10d)$$

$$F_z = h_1(x - vt). \quad (10c) \quad H_y = j_1(x - vt). \quad (10e)$$

$$H_z = k_1(x - vt). \quad (10f)$$

These may be substituted at once into the equations

$$\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 0, \quad (1')$$

and

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0. \quad (2')$$

Substitution shows that since F_y and F_z , H_y and H_z are functions of x and t only, one has

$$\frac{\partial F_x}{\partial x} = \frac{\partial f_1(x - vt)}{\partial x} = 0 \text{ and } \frac{\partial H_x}{\partial x} = \frac{\partial i_1(x - vt)}{\partial x} = 0,$$

which means that F_x and H_x are constant with regard to x .

Similarly substitution of these quantities F_x , F_y , F_z and H_x , H_y , H_z in equations 3' and 4' yields the facts that

$$\frac{k}{\eta} \frac{\partial F_x}{\partial t} = \frac{k}{\eta} \frac{\partial f_1(x - vt)}{\partial t} = 0,$$

and

$$\frac{\mu}{\eta} \frac{\partial H_x}{\partial t} = \frac{\mu}{\eta} \frac{\partial i_1(x - vt)}{\partial t} = 0,$$

while all the partials of the functions g_1 , h_1 , j_1 , and k_1 have values. Thus since the derivatives in x and t are both zero, F_x and H_x are constant in x and t , while both F_y , F_z and H_y , H_z vary in time and

in x . Hence the fields along the line of travel of the wave are constant or zero, while at *right angles* to the line of travel the *magnetic and electrical fields vary with x and t* , that is, the electrical and magnetic field variations are in the $x = \text{constant}$ or yz plane and are *perpendicular to the direction of propagation*. That is to say, the electromagnetic wave motion is transverse, and not longitudinal as in sound waves.

To discuss the equation further one may consider the disturbance initiating as an electrical variation along the z -axis which is *periodic with time* and completes a whole cycle in T seconds, that is, it can be produced by charges moving in an atom or an oscillating circuit.

Let us assume that $F_z = F_{z_0} \sin \frac{2\pi}{T} t$ at $x = 0$. Then it follows

that the disturbance travels along x producing an electrical field displacement parallel to the z -axis periodic in t which moves along x as a train of waves according to $F_z = h_1(x - vt)$. Thus the expression $h_1(x - vt)$ at $x = 0$ must vary as $F_{z_0} \sin \frac{2\pi}{T} t$. That is, at

$x = 0$, $h_1(-vt) = F_{z_0} \sin \frac{2\pi}{T} t$. In order that this be true, $h_1(-vt) =$

$F_{z_0} \sin \left(-\frac{2\pi v}{\lambda} t \right)$ where the quantity $\frac{2\pi v}{\lambda} = \frac{2\pi}{T}$. That is, the argument of the sine function must be divided by a length λ such that $vT = \lambda$. Since v is the velocity and T the period, λ is the distance between successive similar phases of the waves, i.e., λ is the wave length. Since at $t = 0$ the quantity $h_1(x - vt)$ has values varying with x , the quantity x will also be multiplied by $\frac{2\pi}{\lambda}$. Hence $F_z =$

$F_{z_0} \sin \left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t \right)$. This is the equation for an infinite undamped wave train of amplitude F_{z_0} , of wavelength λ , period T , frequency $\frac{1}{T} = N$, and wave velocity $v = \frac{\eta}{\sqrt{\mu k}}$. In this case the

progress is towards the positive x direction, the electrical field disturbance being perpendicular to x along z . The direction of the magnetic disturbance accompanying the electrical disturbance, a vector along z , can at once be found. Since in this case H_z , F_z , and F_y are constant, the equation for $\frac{\partial H_z}{\partial t}$ in equation 4' gives $\frac{\mu}{\eta} \frac{\partial H_z}{\partial t} = \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = 0$, which means that H_z is constant. On the other hand, from the expression for $\frac{\partial H_y}{\partial t}$ from 4', it is seen that

$$\frac{\mu}{\eta} \frac{\partial H_y}{\partial t} = \frac{\partial F_z}{\partial z} - \frac{\partial F_x}{\partial x},$$

which gives on substitution

$$\frac{\partial H_y}{\partial t} = -\frac{\eta}{\mu} \frac{\partial \left[F_{z_0} \sin \left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} \right) \right]}{\partial x} = -\frac{\eta}{\mu\lambda} F_{z_0} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right).$$

This equation may be integrated for t and yields

$$H_y = \frac{\eta T}{\mu\lambda} F_{z_0} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right).$$

Since

$$\frac{T}{\lambda} = \frac{1}{v} = \frac{\sqrt{k\mu}}{\eta},$$

$$H_y = \sqrt{\frac{k}{\mu}} F_{z_0} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right).$$

This equation states that a magnetic disturbance accompanies the electrical disturbance F_{z_0} moving along x . This magnetic disturbance H_y is along the y -axis, i.e., perpendicular to x and F_z . It is *in phase*

with F_z but has an amplitude $H_{y_0} = \sqrt{\frac{k}{\mu}} F_{z_0}$. Its period is obviously the same as that of the electrical component along z as is the case with the velocities. In empty space where $\mu = k$ the amplitude $H_{y_0} = H_{z_0}$, i.e., the amplitudes will be equal. If the motion along y had been studied it would have been found that the periodic disturbance of the field along z would have propagated an exactly similar wave along the y -axis with the electrical vector along z and the magnetic vector along x .

Thus a simple periodic electrical disturbance along z propagates along the xy -plane, the electrical vector being parallel to the original disturbance along the z -axis and the magnetic vector being normal to the direction of propagation and to z . There is no disturbance propagated parallel to z . This is illustrated in Figs. 174a and b.

It must also follow from the form of the equations that if a magnetic displacement along y had been used to propagate a magnetic disturbance along the x -axis with magnetic vector parallel to y , this would also have been accompanied by an electrical vector along z , that is, either a magnetic or an electrical disturbance is propagated as an electromagnetic wave.

Thus it is clear, from the general expressions for the laws of Ampère and Faraday as applied to empty space, that an electrical disturbance periodic in time along an axis, say the z -axis, will propagate itself along the x -axis, having an electrical component along z parallel to the original disturbance and a magnetic component along the y -axis in phase but possibly of different amplitude. The velocity

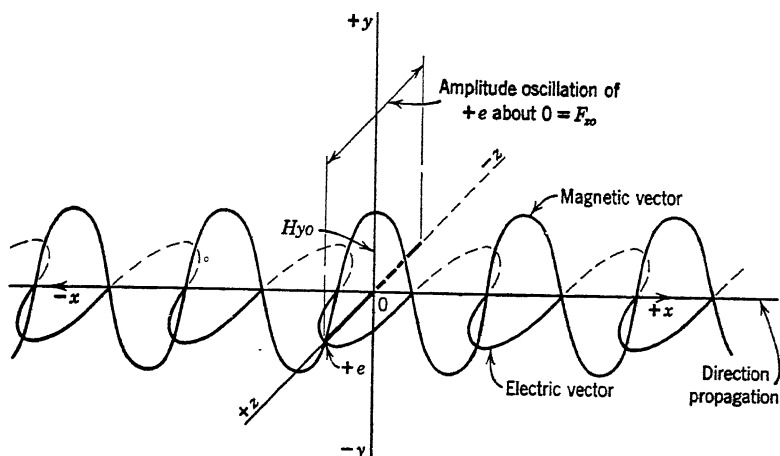


FIG. 174a. Propagation of plane electromagnetic wave in empty space, caused by oscillation of $+e$ back and forth along z through 0, with $F_z = F_{zo} \sin\left(\frac{2\pi t}{T}\right)$.

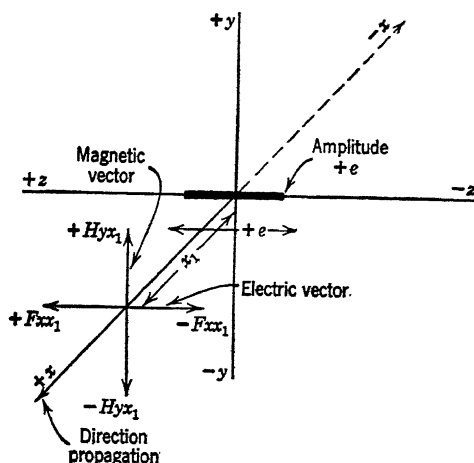


FIG. 174b.

of these waves is given by $v = \frac{\eta}{\sqrt{\mu k}}$, and the ratio of the amplitudes by $\sqrt{\frac{k}{\mu}}$. Where k and μ are equal, as in empty space, the amplitudes are equal.

It was these electromagnetic waves which Maxwell predicted in 1865. At the time neither the techniques for producing nor for detecting such waves existed, and Maxwell's ideas met strong op-

position. Light had long been believed to be a wave motion. Its possible electromagnetic character had early been suspected by Faraday. As a result Faraday in 1846 discovered the rotation of the plane of polarization by crystals and other substances placed in a magnetic field. In 1862 Faraday actually put a flame between magnetic poles and observed the spectrum with a spectroscope. His fields were weak and his resolving power was low, so that he failed to observe a change in the wave length of light magnetically produced. It was discovered with the improved techniques of the year 1896 by Pieter Zeeman. Thus when Maxwell published his theory in 1865, while there was some suspicion that light waves might be electromagnetic in character, Maxwell had neither electromagnetic waves nor light as a support of his theory.

Still Maxwell's theory of 1865 definitely predicted that the velocity of electromagnetic waves was given by the expression $v = \frac{\eta}{\sqrt{\mu k}}$.

Now different colorless transparent substances, such as gases and glass, have a μ which is very nearly unity. The dielectric constant k , however, varies through considerable limits; e.g., it is around 2 for paraffin and 6 for some glass relative to empty space. If μ for empty space is designated by μ_0 and k for empty space by k_0 , the velocity of electromagnetic waves and hence light, if it is such a wave motion,

for empty space is $c = \frac{\eta}{\sqrt{\mu_0 k_0}}$. For any other substance in which

the values of dielectric constant and permeability are different, k and μ can be written for these. Then while k , k_0 , μ , and μ_0 have the arbitrary dimensions, $\frac{\mu}{\mu_0} = \mu_1$ and $\frac{k}{k_0} = k_1$, the specific magnetic inductive capacity and the specific electrical inductive capacity are merely ratios, i.e., pure numbers, which result from measurement. Thus, for a light beam traveling from empty space where its velocity

is $c = \frac{\eta}{\sqrt{\mu_0 k_0}}$ into a plane surface of a pure colorless transparent

substance where its velocity is $v = \frac{\eta}{\sqrt{\mu k}}$, the angles of incidence i and refraction r are related by the well-known Snell law. This says that

$$\frac{\sin i}{\sin r} = n = \frac{c}{v} = \frac{\sqrt{\mu k}}{\sqrt{\mu_0 k_0}}.$$

Hence n , the index of refraction, should be given by the simple relation

$n^2 = \frac{\mu k}{\mu_0 k_0} = \mu_1 k_1$. Since μ_1 is so nearly unity that its deviations

from unity can be neglected, this says that $n^2 = k_1$. Maxwell in 1873 applied this test to his theory for the only substance for which

k_1 was well known. His value of k_1 taken from n^2 was in fair, but not good, agreement with the k_1 then observed. Shortly afterward in 1874 Boltzmann determined the values of k_1 for gases with the following striking results.

GAS	n	$\sqrt{k_1}$
Air	1.000294	1.000295
H ₂	1.000138	1.000132
CO ₂	1.000449	1.000473
CO	1.000346	1.000345
N ₂ O	1.000503	1.000497

The law has been shown to hold for liquids and solids that contain no permanent dipoles and for which the wave length of light used is far from any absorption band. For example:

SUBSTANCE	STATE	n	$\sqrt{k_1}$
C ₆ H ₆	Liquid	1.50	1.51
Petroleum	"	1.39	1.39
CO ₂	"	1.19	1.23
N ₂ O	"	1.19	1.23
CS ₂	"	1.64	1.62
Se	Solid	2.93	2.73

The character of the deviations observed due to polar molecules led Debye to the theory by which dipole moments can now be measured (see section 79). The effect of wave length and of absorption bands later through the theoretical work of Drude, Lorentz, and others led to complete agreement with the electromagnetic theory. However, the electromagnetic theory of Maxwell was making relatively slow progress in being established at the time of his early death at the age of 48, in 1879.

In the period 1887 to 1888 a brilliant young German physicist, Heinrich Hertz, conceived of and achieved a means of proving the correctness of Maxwell's assertions. In the systematic period the unification of electrical theory made it possible for mathematical physicists to develop the theory of electrical circuits. This had been done in 1855 for a circuit containing self-induction, resistance, and capacity by Lord Kelvin, to whom we owe the derivation at the end of the last chapter. Feddersen, by means of a rotating mirror, in 1859-1861 had shown that the spark of a discharging condenser was oscillatory as predicted by Kelvin's theory, and the complete solution was given in 1864 by Kirchhoff. Now the difficulty with electromagnetic disturbances is that they fall off in intensity inversely as the square of the distance from the source, for the electrostatic and the magnetic fields are of the Coulomb type. Hence it is very hard to detect the minute disturbances caused by a change in field at any distance away, and detecting instruments were then not what they are today. Hertz, however, realized that if a periodic disturbance, even though weak, can fall on a

system which resonates, the energy transfer can become appreciable; witness a tuning fork set into vibration by the air-borne impulses of a neighboring fork. He first used a spark from an induction coil and a resonant gap circuit. Next he set up, near each other, two Feddersen oscillators, with capacity, which were in resonance. He then caused one to spark and thus radiate damped a-c electromagnetic pulses. The second oscillator was provided with a pilot spark gap. He found that if the first jar were set into oscillatory discharge, the second would show a spark in its gap. Later experiments in which frequencies were varied showed that this sparking took place only for tuned circuits and that electrical shielding prevented the transfer of energy. He next showed that the waves could be reflected from mirrors, got standing waves, refracted them, and measured the velocity and wave length, finding the velocity roughly that of light. Thus the electromagnetic waves were discovered and the foundation for modern radio communication was laid. The velocity of these waves in empty space has been determined with the following results:

OBSERVER	DATE	VELOCITY
Hertz-Poincaré	1890	$3 \times \sqrt{2} \times 10^{10}$
Trowbridge and Duane	1895	3.003×10^{10}
Saunders	1897	2.997×10^{10}
MacLean	1899	2.991×10^{10}
Mercier	1924	2.9978×10^{10}

This, however, refers to waves in empty space. Near the ground there is evidence that the velocity is considerably less, as with conductors. In other words, when waves propagate near media where there are values of μ and k differing from those for empty space, and the wave lengths are of the magnitude of the distances involved, the group velocity will be altered.

C. THE IMPORTANT ELECTRICAL CONCEPTS, THEIR UNITS AND INTERRELATIONS, AND THE PROBLEM OF THEIR DIMENSIONS

153. IMPORTANT CONCEPTS

For the purposes of this course and in the interests of simplicity a system for definition and classification of the units has been built up. Aside from the definition of the magnetic units and those of concepts such as field strength and charge density, etc., the major interest of the student of electricity or the engineer who will apply it lies in six important concepts: current, quantity, potential, resistance, capacity, and self-induction. These six concepts can be defined as units in terms of their electrostatic effects. This gives the absolute electrostatic system of units. Or they can be defined in terms of their electromagnetic effects. This gives the absolute electromagnetic sys-

tem of units. For practical purposes they may be defined in terms of arbitrarily chosen convenient units commensurable with the everyday values encountered. These and their interrelations are presented in convenient tabular form which has been derived from a knowledge of the six defining equations. It is the purpose of this discussion to present the table and indicate its origin. There is included in it for reference a table of dimensions using the Gaussian system. The dimensions are not vital to the table but are given for the convenience of the reader. More will be said about the Gaussian system and dimensions later.

First it is found convenient to divide the table into an upper part and a lower part set off by a double dividing line. Above the line are what the author has chosen to call the *fundamental* concepts in contrast to those below, which are termed the *derived* concepts. If the concepts of current, quantity, and potential derivable from experiment directly in terms of force and work are taken as *fundamental*, the division though purely arbitrary has the advantage that the ratios of e.m.u. and e.s.u. above the line are all in the ratio of the velocity of light, i.e., $c = 3 \times 10^{10}$. The three derived concepts below the line, viz., resistance, capacity, and inductance, are ratios of the fundamental concepts above the line. Below the line the ratio of the derived e.m.u. and e.s.u. is the square of the velocity of light, i.e., $c^2 = 9 \times 10^{20}$. It is then necessary in the second column to write down the defining equations of the concepts in the first vertical column in the order: current, quantity, potential, and below the line, resistance, capacity, and self-induction. In doing so, note that the current definition is derived from the electromagnetic action of a current, and the quantity definition comes from static experiments. Ignoring the columns on equations and dimensions, it is next necessary to head the fourth column "Units in Electromagnetic System," and the adjoining or fifth column, "Units in Electrostatic System." One can then go down the electromagnetic column in the following manner: $i = 1$ e.m.u. if (referring to the equation for current), $m =$ unit pole, $r = 1$ cm, $ds = 1$ cm. perpendicular to r , and $f = 1$ dyne. In the second square, opposite quantity, write $q = it$ and $q = 1$ e.m.u. if $i = 1$ e.m.u. and $t = 1$ second. In the third square, opposite potential, write P. D. = 1 e.m.u. if $q = 1$ e.m.u. and $W = 1$ erg. A similar procedure down the column below the line defines the electromagnetic units.

In the column under "Electrostatic System" one begins with $i = \frac{q}{t}$ for current, stating $i = 1$ e.s.u. if $q = 1$ e.s.u. and $t = 1$ second. It is to be noted that *current* in the *electrostatic* system is defined as quantity (defined directly) per unit time, while *quantity* in the *electromagnetic* system is current (defined directly) times time. That is the reason for writing electromagnetic and electrostatic under the definitions of current and quantity in column 1.

TABLE OF UNITS

ENTITY Fundamental or Derived	Defined by the Following Equation	Dimensions	Units in Electromagnetic System
CURRENT FUNDAMENTAL: Comes from electromag- netic system	$f = \frac{idsm}{r^2}$ ds = Normal to r r = Distance i = Current f = Force on magnet pole m m = Strength magnet pole	In e. m. system, $i = M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$ In e. s. system, $i = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	i = 1 e.m.u. if m = unit pole r = 1 cm f = 1 dyne on pole m ds = 1 cm normal Also $f = iH$ i = 1 when l = 1 cm f = 1 dyne H = 1 oersted
			Large
QUANTITY FUNDAMENTAL: Comes from electrostatic system	$f = \frac{qq'}{Dr^2}$ q and q' = Quantities f = Force be- tween them r = Distance D = Dielectric constant	In e. m. system, $q = M^{\frac{1}{2}}L^{\frac{1}{2}}$ In e. s. system, $q = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	q = i q = 1 e.m.u. if i = 1 e.m.u. t = 1 second
			Large
POTENTIAL FUNDAMENTAL: Comes from work in either system	$W = q \times \text{P.D.}$ or W = P.D. i W = Work q = Quantity i = Current t = Time P.D. = Potential	In e. m. system, P.D. = $M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}$ In e. s. system, P.D. = $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	P.D. = 1 e.m.u. if q = 1 e.m.u. W = 1 erg P.D. = $\frac{d\phi}{dt}$, $\frac{d\phi}{di}$ = no. lines force cut by conductor per second P.D. = 1 when $\frac{d\phi}{dt} = 1$
			Small
RESISTANCE DERIVED: From e. m. system. A constant of the shape and circuit material	$R = \frac{\text{P.D.}}{i}$ R = Resistance P.D. = Potential i = Current	In e. m. system, $R = LT^{-1}$ a velocity In e. s. system, $R = L^{-1}T$ a reciprocal of velocity	$R = \text{P.D.}/i$ R = 1 if P.D. = 1 e.m.u. i = 1 e.m.u.
			Very small
CAPACITY DERIVED: From e. s. system. A constant of the shape and the material surrounding the circuit	$C = \frac{q}{\text{P.D.}}$ C = Capacity q = Quantity P.D. = Potential	In e. m. system, $C = L^{-1}T^2$ In e. s. system, $C = L$ Unit is the cm	$C = \frac{q}{\text{P.D.}}$ C = 1 if q = 1 e.m.u. P.D. = 1 e.m.u.
			Very large
SELF-INDUCTION DERIVED: Defined from e. m. sys- tem. A constant of a cir- cuit depending on the shape and magnetic medi- ums involved	$\text{P.D.} = L \frac{di}{dt}$ P.D. = Potential generated $\frac{di}{dt}$ = Rate of change of current L = Coefficient of self-in- duction	In e. m. system, $L = L$ Unit is the cm In e. s. system, $L = L^{-1}T^2$	P.D. = $L \frac{di}{dt}$ L = 1 if P.D. = 1 e.m.u. $\frac{di}{dt} = 1$ e.m.u./sec.
			Very small the cm

AND DIMENSIONS

Units in Electrostatic System	Practical Unit	Relation Between E.M.U. and E.S.U.	Relation Between E.M.U. and Practical	Relation Between E.S.U. and Practical
$i = \frac{q}{t}$ $i = 1$ e.s.u. if $q = 1$ e.s.u. $t = 1$ second Small	Ampere defined as 0.1 of e.m.u.	1 e.m.u. = 3×10^{10} e.s.u.	1 e.m.u. = 10 amperes	1 ampere = 3×10^9 e.s.u.
$f = \frac{qq'}{Dr^2}$ $q = 1$ e.s.u. if $q' = 1$ e.s.u. $r = 1$ cm $f = 1$ dyne $D = 1$ Small	Coulomb defined as 1 ampere for 1 second or as 0.1 e.m.u. of q	1 e.m.u. = 3×10^{10} e.s.u.	1 e.m.u. = 10 coulombs	1 coulomb = 3×10^9 e.s.u.
$W = \text{P.D.} \times q$ $\text{P.D.} = 1$ when $W = 1$ erg $q = 1$ e.s.u. Large	Volt defined as 10^8 abs. e.m.u.	1 e.s.u. = 3×10^{10} e.m.u.	1 volt = 10^8 e.m.u.	1 e.s.u. = 300 volts
$R = \text{P.D.}/i$ $R = 1$ when $\text{P.D.} = 1$ e.s.u. $i = 1$ e.s.u. Very large	Ohm defined as $\frac{\text{volts}}{\text{amperes}} = \text{ohms}$	1 e.s.u. = 9×10^{10} e.m.u.	1 ohm = 10^9 e.m.u.	1 e.s.u. = 9×10^{11} ohms
$C = \frac{q}{\text{P.D.}}$ $C = 1$ when $\text{P.D.} = 1$ e.s.u. $q = 1$ e.s.u. Very small the cm	Farad defined as $\frac{\text{coulombs}}{\text{volts}} = \text{farads}$. Common unit the microfarad = 10^{-6} farad	1 e.m.u. = 9×10^{20} e.s.u.	1 e.m.u. = 10^9 farads = 10^{18} microfarads	1 farad = 9×10^{11} e.s.u. = 9×10^{11} cm 1 microfarad = 9×10^8 cm 1 micromicrofarad = 0.9 cm
$\text{P.D.} = L \frac{di}{dt}$ $L = 1$ when $\text{P.D.} = 1$ e.s.u. $\frac{di}{dt} = 1$ e.s.u./sec Very large	Henry defined as $\frac{\text{volts}}{\text{amp/sec}} = \text{henrys}$	1 e.s.u. = 9×10^{20} e.m.u.	1 henry = 10^9 e.m.u. = 10^9 cm 1 millihenry = 10^{-3} henry = 10^6 cm 1 microhenry = 10^{-6} henry = 1000 cm	1 e.s.u. = 9×10^{11} henrys

Returning to the static units, one proceeds to define quantity $q = 1$ e.s.u. if $q = q'$, $r = 1$ cm, $f = 1$ dyne, and $D = \text{unity}$. One then proceeds similarly down the "Static Unit" column. The sixth column is headed "Practical Unit." In this one writes as follows:

$$\text{Ampere} = 0.1 \text{ e.m.u.}$$

$$\text{Coulomb} = 1 \text{ ampere} \times \text{second or } 0.1 \text{ e.m.u.}$$

$$\text{Volt} = 10^8 \text{ e.m.u.}$$

$$\text{Ohm} = \frac{\text{volt}}{\text{ampere}} - \text{a ratio}$$

$$\text{Farad} = \frac{\text{coulomb}}{\text{volt}} - \text{a ratio.}$$

$$\text{Common unit } 10^{-6} \text{ farad} = 1 \text{ microfarad.}$$

$$\text{Henry} = \frac{\text{volts}}{\text{amperes/second}} - \text{a ratio}$$

Note also that the e.s.u. of capacity is a length = 1 cm and the e.m.u. of self-induction is a length = 1 cm. These may be inserted in the appropriate places in columns 4 and 5. It should be noted that each *practical* unit is named for the scientist responsible for the discovery of the concept or most closely related to the establishment of the unit.

The next three columns are to be labeled "Relation between E.M.U. and E.S.U.," "Relation between E.M.U. and Practical," and "Relation between E.S.U. and Practical." They are the most *useful* columns of the table, as the interconversion of units is continually necessary in practical work. The filling in of these columns in a systematic fashion is relatively simple if just one fact is remembered — that *it takes a large current to produce an appreciable magnetic effect*, and that *a very small quantity of static charge gives a dyne of force*. Hence one writes in column 4 opposite Current (large), opposite

Quantity (large), opposite Potential = $\frac{1 \text{ erg}}{q}$ (small); for to move 10

coulombs against a potential difference and get only 1 erg of work means a small potential difference. With this, remembering that above the dividing line the ratio of the units is 3×10^{10} , for the ratio of e.m.u. and e.s.u. it can be written that 1 e.m.u. of current = 3×10^{10} e.s.u., 1 e.m.u. of quantity equals 3×10^{10} e.s.u., 1 e.s.u. of potential equals 3×10^{10} e.m.u. Using these data the columns below the line can at once be filled in for the ratio of e.m.u. and e.s.u., remembering that the ratio involves a factor of 9×10^{20} in one sense or another. Then with the ratio of the practical and e.m.u. as given in column 6, the ratio of e.m.u. to practical in column 8 can be filled in. Column 9 then is easily derived from column 7 and column 8.

In addition to this table perhaps the following more or less important definitions should be listed.

154. TABLE OF AUXILIARY CONCEPTS

Magnetic pole strength:

$$f = \frac{mm'}{\mu r^2}, \quad m \text{ and } m' \text{ poles interacting at distance } r.$$

 f = force. μ = magnetic permeability.

Unit magnetic pole:

$$m = 1 \text{ if } m = m', r = 1 \text{ cm, } \mu = 1 \text{ and } f = 1 \text{ dyne.}$$

Magnetic field strength:

$$H = \frac{f}{m}, \text{ i.e., force per unit pole, direction, that of motion of an N pole.}$$

Unit field strength:

$H = 1$ oersted if $m = \text{unit pole}$ (also equal to 1 maxwell per square centimeter normal to the lines).

$$f = 1 \text{ dyne.}$$

Magnetic moment:

$$M = ml. \quad m = \text{pole strength of a magnet.}$$

 l = distance between poles.

A characteristic constant of a magnet that defines its magnetic behavior.

Flux:

$$\phi = AH \text{ or } \phi = AB, \text{ total number of lines passing through an area } A.$$

 H is magnetic field. B is magnetic induction if matter is present.

Unit is the line of force called the maxwell.

Magnetic induction B :

This is the analog of the magnetic field existing in empty space in the case where matter is present. It is the *total number of lines of magnetic force entering a surface of magnetic material per unit area taken normal to the lines*. Unit is the gauss.

Magnetic permeability $\mu = \frac{B}{H}$, applied to the induction caused by a field. It is related to the intensity of magnetization I , which is the magnetic moment per unit volume $I = \frac{M}{V}$, and to the magnetic susceptibility κ by the relation

$$B = \mu H = H + 4\pi \frac{M}{V} = H + 4\pi I = H(1 + 4\pi\kappa).$$

Analogous expressions hold for dielectric phenomena.

Magnetic potential difference $PD_m = \frac{W}{m}$, i.e., the work per unit pole to move a pole from one point to another in the field. For current i_a cutting ϕ lines of force, $W = i_a\phi$. For a pole of strength m , $\phi = 4\pi m$. Thus $W = 4\pi mi_a$. If a field H exists $W_{ab} = \int_a^b mH dx$, where x is the distance parallel to H .

Electric field strength:

$$X = \frac{f}{q} = \text{force on unit quantity.}$$

Direction is that in which a positive pole would move.

Unit field:

$$X = \text{unit electrostatic field if} \\ f = 1 \text{ dyne, } q = 1 \text{ e.s.u.}$$

$$X = \text{unit electromagnetic field if} \\ f = 1 \text{ dyne, } q = 1 \text{ e.m.u.}$$

Practical unit of field strength comes from the relation:

$$\int X ds = V \text{ or } X = \frac{dV^*}{ds}, \text{ where } V \text{ is the potential difference and } ds \text{ is a length}$$

along the field. For a uniform field of length d , $X = \frac{V}{d}$, and $V = Xd$.

Practical unit

$$X = \text{unit field if } V = 1 \text{ volt,} \\ d = 1 \text{ cm. It is the volt per centimeter.}$$

Electrical conductivity $\sigma = \frac{1}{R}$, where R is the resistance. The specific conductivity

is $\sigma_s = \frac{1}{R_s}$, where R_s is the specific resistance. The unit of conductivity is the mho, or reciprocal ohm.

Electrical dipole moment:

$$\bar{\mu} = ql \quad \begin{array}{l} q = \text{charge,} \\ l = \text{distance between } + \text{ and } - \text{ charges.} \end{array}$$

The number of lines of force emerging from a dielectric, the dielectric induction D , is related to the field X by

$$D = X + 4\pi \frac{\bar{\mu}}{V} = X + 4\pi P = X(1 + 4\pi\gamma) = kX.$$

Here V is the volume, P is the polarizability, γ is the electrical susceptibility, and k is the specific inductive capacity. k is related to the dielectric constant, defined by

$$f = \frac{qq'}{kr^2}.$$

The Faraday constant or the faraday = the quantity of electricity required to liberate 1 *gram-atom equivalent* of substance by electrolysis, i.e., $96,514 \pm 10$ absolute coulombs per gram equivalent.†

Volume charge density of electrification $\rho = \frac{q}{V}$, number of charges per cubic centimeter.

* Strictly speaking, $X = -\frac{dV}{ds}$, since X is a vector quantity. For these purposes the negative sign may be neglected as we are interested in the values only.

† This value of F is on the physical scale with the choice of the atomic weight of the isotope of oxygen ^{16}O as having the value 16.000000. The chemical scale uses atmospheric oxygen having isotopes ^{16}O and ^{18}O present in small amounts as exactly 16.000000. If this scale is used the faraday F is $96,487 \pm 10$ absolute coulombs per gram equivalent.

Surface charge density of electrification $\sigma = \frac{q}{A}$, number of charges per square centimeter.

Electrical power consumption is $P = P.D.i_a$. Where PD and i_a are in absolute units power is in ergs/sec. If PD is in volts and i is in amperes, $P = Vi$, with P in joules/sec = watts. When a-c power is measured V and i must be measured in the proper phase relation to get true power consumption.

155. SYSTEMS OF ELECTRICAL AND MAGNETIC UNITS

As has been stated under the short note on units and dimensions in Chapter III, the choice of assignment of dimensions is quite arbitrary. As long as any assignment chosen is consistent no one can challenge a system of assignments except on grounds of individual taste or of convenience. The fact that there are two fundamental defining equations in electricity and magnetism, $f = \frac{qq'}{kr^2}$ and $f = \frac{mm'}{\mu r^2}$, in which both

the concepts q and m as well as two constants k and μ are new allows a wide latitude of choice in the dimensional analysis applied to these equations. In practice, however, for the most part the assignment of dimensions is limited to three general schemes or systems. These have been extensively discussed in the *American Physics Teacher* from the viewpoint of pedagogical uniformity and utility. Attention of those interested in a more profound discussion is called to three excellent articles by R. T. Birge, *American Physics Teacher*, 2, 41, 1934; 3, 102, 171, 1935, dealing not only with the use of the units but with their establishment as well. Briefly, the three systems referred to are called the electrostatic system, the electromagnetic system, and the Gaussian system. To differentiate these we must recall that in section 79 and in section 152 of this chapter k was defined as the dielectric constant, k_0

that for empty space, and k_1 the ratio of $\frac{k}{k_0} = k_1$, a pure number called the specific inductive capacity. For the magnetic equation μ the permeability and μ_0 the permeability of empty space were used and a new ratio $\frac{\mu}{\mu_0} = \mu_1$ named the "magnetic specific inductive capacity,"

following Birge. These quantities μ_1 and k_1 are dimensionless ratios whereas μ and k have their magnitudes and dimensions dependent on the system of units. Now since in $f = \frac{qq'}{kr^2}$ and q and k are new, the dimensions of k or q may be chosen arbitrarily. In the electrostatic system k_0 is assumed to be unity and dimensionless. Then $f = \frac{qq'}{kr^2}$

defines q as having dimensions $M^{1/2}L^{3/2}T^{-1}$, and thus k_0 and k_1 are identical in magnitude and dimensions. On the other hand, $f = \frac{mm'}{\mu r^2}$

leaves the units and dimensions of m and μ unassigned. In the electromagnetic system μ_0 is assumed to be unity and dimensionless. Then unit pole on the electromagnetic system has the dimensions $M^{1/2}L^{3/2}T^{-1}$. Accordingly also μ_0 and μ_1 are in this electromagnetic system identical in magnitude and dimensions. In the Gaussian system k is taken as dimensionless and $k_0 = 1$, while likewise μ is taken as dimensionless and $\mu_0 = 1$, leading to the assignment of dimensions to q and m as given in the text

Since, however, there is constant need for the conversion of units and since in electromagnetic theory both μ and k appear together, it is clear that some study of the effect of the assignment of dimensions must be made. When the ratio of the units and also the generalized equations for Ampère's and Faraday's laws were studied, it was found necessary to introduce a ratio constant η which expressed the ratios of the units numerically and dimensionally. This was actually done because of the unknown effect produced by the arbitrary assignment of dimensions to μ and k . This constant η appears in the wave equation and, as derived from the ratios of the units, is related to μ and k in the expression $v = \frac{\eta}{\sqrt{\mu k}}$, and for empty space, where $\mu = \mu_0$ and

$k = k_0$, in the relation $c = \frac{\eta}{\sqrt{\mu_0 k_0}}$. It is these conditions which will

then determine the effect of the assignment of dimensions to μ and k . It is at once clear that a wide latitude of choice is possible in the dimensions assigned to μ and k as long as the dimension of η is unrestricted. It further follows that the dimensions assigned to μ and k fix those of η . Fortunately there are in common practice only three ways in which dimensions are assigned to k and μ . These lead to the systems of units described by Birge as the electrostatic system, the electromagnetic system, and the Gaussian system.

Since $\mu_0 = \frac{\mu}{\mu_1}$ and $k_0 = \frac{k}{k_1}$, where μ_1 and k_1 are pure numbers, considerable simplification results if it is remembered that $v = \frac{\eta}{\sqrt{\mu k}}$, $c = \frac{\eta}{\sqrt{\mu_0 k_0}}$, and $\frac{c}{v} = \sqrt{\mu_1 k_1}$, and the values are tabulated for the dimensions and magnitudes of k_0 , μ_0 , and η for the different systems as follows:

	DIMENSIONS			MAGNITUDE		
System.....	k_0	μ_0	η	k_0	μ_0	η
Electrostatic.....	0	$L^{-2}T^2$	0	1	c^{-2}	1
Electromagnetic..	$L^{-2}T^2$	0	0	c^{-2}	1	1
Gaussian.....	0	0	LT^{-1}	1	1	c

Thus in the Gaussian system η has the velocity of light in empty space while k_0 and μ_0 are unity and dimensionless. This does not

preclude k and μ having magnitudes different from unity and η having a velocity less than c . The index of refraction is then

$$n = \frac{c}{v} = \sqrt{\frac{k\mu}{\mu_0 k_0}} = c \sqrt{k\mu/\eta} = \sqrt{\mu_1 k_1}.$$

It is to be noted that in the Gaussian system the *electrical* quantities are identical in magnitude with those of the *electrostatic system*, while the *magnetic* quantities are identical with those in the *electromagnetic system*. In the large majority of textbooks the Gaussian system is explicitly or implicitly used. A variant of the Gaussian system is frequently encountered in mathematical physical textbooks. This is the so-called Heaviside-Lorentz system. In writing the fundamental equations in the Gaussian system the factor 4π appears in the equations, making the form of the equations unsymmetrical, as seen in the equations of page 397. The Heaviside-Lorentz system, written H.L., by a proper sequence of transformations with what amounts to a redefinition of the meaning of some of the classic symbols, suppresses the appearance of the factor 4π . In this system the fundamental equations appear in a symmetrical form which is easy to remember. The use of such a system is a matter of aesthetic taste and convenience. In carrying out computations the 4π is, of course, ultimately brought in, as it must be. It is suppressed only in the form of the equations. It is important, however, to realize the fact that the H.L. system exists and to verify carefully in which system an equation in a textbook is given before applying it.

CHAPTER XXVII

DISCHARGE THROUGH GASES AND ATOMIC STRUCTURE

156. DISCOVERY OF X-RAYS

In 1895 a German physicist, W. K. Roentgen, was studying the electrical discharge of an induction coil in an evacuated tube made possible by the advance in the development of vacuum technique. He noticed a peculiar greenish fluorescence when the vacuum in the tube became low enough. This greenish fluorescence was accompanied by a stream of bluish rays moving in straight lines from the negative electrode. By accident, he observed that a screen of a material called barium platinocyanide lying on the table near the apparatus became phosphorescent whenever the greenish fluorescence appeared. Investigation showed that this phenomenon resulted from a new type of radiation which passed through the glass walls of the vessel and through air. The remarkable properties of these radiations, which were called x-rays because of their unknown origin, led to feverish experimentation in this field of work. These rays were found to cause fluorescence in certain substances, they were found to affect a photographic plate, and they were found to make the air through which they passed conducting. It was the conductivity of air, which had been considered almost a perfect insulator from the time of Coulomb, that enabled further fundamental researches in the study of x-rays and carriers of electricity to be made. The x-rays were found to travel in a straight line from the source and to cast shadows when thick opaque objects were placed in their path.

157. CATHODE RAYS

Immediately following the discovery of the x-rays came the investigation of the peculiar bluish streamers emitted from the negative electrode in the evacuated tube whose presence simultaneously with the x-rays was always observed and to whose presence the x-rays were ascribed. It was very quickly found by different observers that these rays were apparently negatively charged; that they were deflected in a magnetic field as if they were a current of negative electricity; that they moved in straight lines; and that shadows cast by objects placed in the tube showed that where they did not strike the tube there was no fluorescence due to the x-rays.

It was J. J. Thomson who succeeded in showing that these stream-

There is one more point that must be mentioned. It is not possible to determine the facts about an electron or any other particle of matter with complete accuracy. The *uncertainty principle* of Heisenberg says that product $\Delta p \Delta q = h$. That is, the uncertainty Δp in evaluating the *momentum* of an electron or particle, i.e., its velocity times mass, multiplied by the uncertainty Δq in locating its *position*, is of the order of magnitude of h and cannot be less. If an experiment is devised to make the error in momentum measurement, Δp , small the error in any measurement of position Δq cannot be less than $\Delta q = \frac{h}{\Delta p}$. Since h has simultaneously the dimensions of moment of momentum pq , $p = mv$, $q = r$, and of energy times time, $E = h\nu = \frac{h}{T}$, $h = ET$, the uncertainty principle can take another form, this is, that $\Delta E \Delta t = h$. Here ΔE is the uncertainty in the evaluation of the energy, i.e., the width of an energy level, and Δt is a characteristic time, the average lifetime of a particle like the electron, in that energy level or state (see page 447).

The negative electron was observed to come from all metals and was early identified as being responsible for metallic reflection of light, for metallic electrical and heat conduction, and for the metallic state in general. Tolman and Stewart of the Chemistry Department of the University of California in 1917 showed that it could be thrown to the edge of a copper disk by centrifugal forces when the disk was rotated at high speed. It was also knocked out of nonmetallic substances by x-rays and fast-moving particles. In 1896 Pieter Zeeman in Holland successfully repeated an experiment tried earlier by Michael Faraday. He placed a flame with a characteristic salt spectrum in a strong magnetic field and observed that the bright spectral lines were shifted and multiplied relative to their positions in the absence of the field. H. A. Lorentz supplied the necessary theory. By accident the particular lines studied exactly fitted the theory of Lorentz in the character and magnitude of the shifts. Thus it was possible to evaluate the quantity $\frac{e}{m}$ for the agency emitting light in atoms and to show

that this was just the value of $\frac{e}{m}$ for J. J. Thomson's newly discovered electrons.

Further studies of J. J. Thomson and others of the positive ions liberated in a gas when electrons were removed from atoms and molecules convinced the physicists that the electron was a universal constituent of all atoms and that aside from an intrinsic positive charge associated with the atom the atom was composed of electrons. This led Thomson in 1906 to publish a book entitled *The Corpuscular Nature of Matter*. In this from x-ray studies he estimated the number of elec-

trons in an atom to be about half its atomic weight. This value was later found nearly correct for the lighter elements. Thus, as indicated in Chapter X, atoms contain negative electrons as one of their two constituents. The negative electron, therefore, is a stable ultimate entity in the external atom, in metals, and in the free state in vacuum and in gases. It was in 1899 observed to come from the nuclei of atoms in the form of β rays and was thus supposed to be a part of atomic nuclei when these were discovered. Today it is realized that electrons do not *exist free* in nuclei. In restoring the balance of charge in unstable nuclei electrons *may be created* but shortly leave the nuclei as β rays. They do *not exist free* within nuclei.

Positive Electrons. In contrast to the stable negative electron Carl D. Anderson in 1932 in a study of cosmic rays discovered a *positive electron*. This particle has been extensively studied since then. It is created together with a *negative* electron when hard x-rays of more than 1.02 m.e.v. of energy pass close to the nucleus of a heavy element like Pb. This is called *pair production*, and the energy balance is discussed on page 426. Positive electrons or *positrons* are also emitted from the nuclei of many unstable isotopes in the process of gaining stability. They exist only while in rapid motion. As observed they have most of the properties of the negative electron having the same value of e , but positive in sign and the same $\frac{e}{m}$. However, as soon as positive electrons lose their energy, or speed, they are very readily united to a negative electron to give two gamma rays of 0.51 m.e.v. each, as seen on page 426. Thus while positive electrons exist, their transient existence makes them of little interest to the student of electricity. In contrast the *negative* electron is a fundamental entity in all electrical phenomena.

162. POSITIVE RAYS AND MASS SPECTROGRAPHS

Concerning the character of positive electricity, J. J. Thomson again led the way to an understanding. In a partially evacuated glow-discharge tube, such as gave place to cathode rays and x-rays, a new phenomenon was discovered. In 1886 Eugen Goldstein observed that in such a discharge tube with a perforated cathode (negative electrode), there were streamers going *backward through* the cathode in contrast to the cathode rays proceeding forward toward the positive electrode. They were called *canal rays* since they passed through canals or channels in the cathode. In 1898 Wilhelm Wien examined these rays to see how they were deflected in a magnetic field. He found that they were deflected by *strong* magnetic fields as if positively charged and that the deflection was much less than for cathode rays. Now both canal rays and cathode rays are accelerated by the same potential difference, the cathode fall of potential, V_c , in the glow-dis-

charge tube. Thus, since for low speeds $V_c e = \frac{1}{2} m v^2$, $v = \sqrt{\frac{2 V_c e}{m}}$, and unless $\frac{e}{m}$ for these rays was equal to that of an electron it must be less and hence v is less. But for a given observed ρ , the field H required to deflect canal rays was very much greater. That is, $\frac{v}{e/m} = H\rho$ for these particles is much greater than for cathode rays. Hence with a reduced v the $\frac{e}{m}$ for these rays is far less than for electrons.

If e is the same order of magnitude for both the electrons of the cathode rays and the canal rays the only meaning can be that m for canal rays is much greater than for electrons. Accordingly, it was believed that the canal rays were atoms which had lost their electrons in the negative glow just beyond the Crookes dark space in front of the cathode (see page 436). They were then accelerated to the cathode by the cathode fall of potential V_c . They thus acquired energies of the order of 50 to 10,000 volts, depending on V_c . On reaching the cathode those headed for holes in the cathode went right through because of their high velocity, despite the deflecting action of the negative field near the hole. The positively charged atom beams then continued on back of the cathode. They were luminous because of the excitation and ionization of themselves and the residual gas atoms they encountered back of the cathode. Gradually they lost their speed, became neutralized, and disappeared.

By 1911-1912 J. J. Thomson had succeeded in getting magnetic and electrical fields to act simultaneously to deflect the canal rays. The rays were reduced to a single fine beam going through one hole in the cathode shown in Fig. 180a. The magnetic and the electrical fields were parallel. Thus the magnetic field caused, say, a vertical deflection y of the beam of Fig. 180b and the electrical field caused a horizontal deflection, z , of Fig. 180b. If the vertical magnetic deflection is called y and the horizontal electrical deflection z , then the equations indicated on page 422 as applied to Thomson's apparatus take on values,

$$y = \frac{e}{m} \frac{H}{v} A, \quad z = \frac{eX}{mv^2} B.$$

Here A and B are constants of the apparatus such as lengths of the deflecting plates, and length of path to the screen. These equations

further allow v and $\frac{e}{m}$ to be solved for in the following forms, $v = \frac{y}{z} \frac{BX}{AH}$, $\frac{e}{m} = \frac{y^2}{z} \frac{BX}{A^2 H^2}$. Thus for v constant but $\frac{e}{m}$ varying a series of

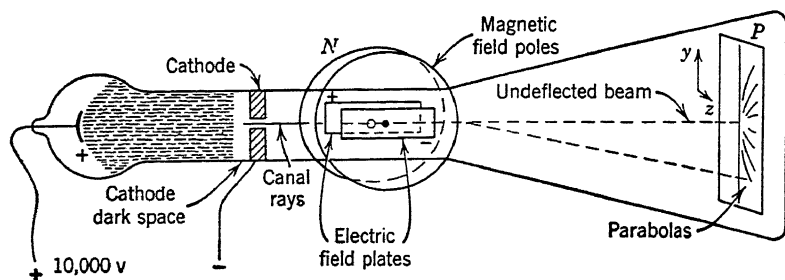


FIG. 180a. J. J. Thomson's mass spectrograph.

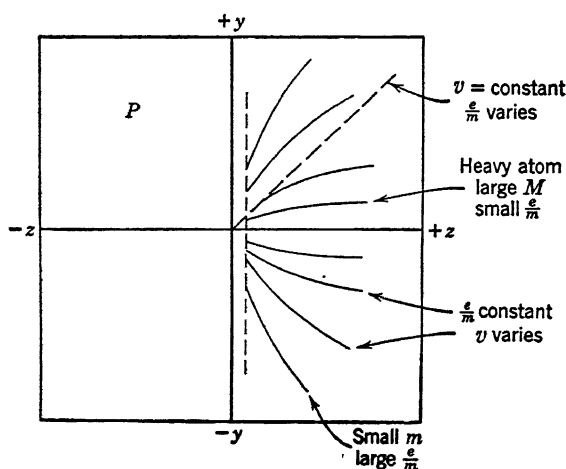


FIG. 180b. Parabolas yielded by Thomson's mass spectrograph.

spots would be obtained for each $\frac{e}{m}$ present lying on a straight line passing through the origin, $y = 0, z = 0$ (see Fig. 180b). For a given $\frac{e}{m}$ with v varying a parabola would be obtained passing through $y = 0, z = 0$, which enclosed the z or electric axis; see Fig. 180b. The greater $\frac{e}{m}$, i.e., the lighter the atom, the more open the parabola (greater y deflections).

Now in such a tube as Thomson's, while the cathode fall V_c is constant, the positive ions, or canal rays, are formed at different points by chance fast electron impacts which ionizes the atoms. Thus v will vary. Again, in a gas not only atoms of the gas but plenty of impurities were present. Thus many atoms and molecules were present, and thus many ions. In addition, there were atoms and molecules

that had lost 1, 2, 3, and more electrons so that e could have various values. Thus even in *relatively pure* atomic gases usually more than one $\frac{e}{m}$ was present. Thomson caught these positive rays on photographic plates, P , placed normal to the axis of the undeflected beam shown in Figs. 180a and 180b. The magnetic field was reversed during exposure to give both sides of the parabola and to eliminate dissymmetry in the apparatus from the measurements.

Since the velocities varied in his tube Thomson observed the parabolas and, using certain other relations resulting from the equations, was able to evaluate $\frac{e}{m}$ for the gases present and identify the gaseous impurities recorded.

The method was difficult and exposures of many hours were required because of the feeble beams and the spread of velocities along the parabolas. The accuracy of the $\frac{e}{m}$ determination was not high.

Thomson nevertheless established a number of important facts. The positive charge on the H atom was definitely 1 electron in magnitude and that on the He atom was 2 and no more. Heavier atoms could be charged up by losing 1, 2, 3, or more electrons and as many as 8 electrons could be removed from Hg. In addition, he indicated that in Ne gas there were 2 atoms of Ne, one of atomic weight 20 and the other of atomic weight 22. This was the first definite evidence of the existence of *isotopes*, that is, the existence of elements having the same chemical behavior, occupying the same position in Mendeleev's table but having different atomic weights. In 1913 Soddy and Fajans arrived at the same conclusion from a study of the radioactive disintegration series. Today there are some 260 known *stable isotopes* and again as many known *unstable isotopes* of the 96 elements.

The importance of the method for determining m , if e was known, and its usefulness in detecting isotopes, led to a large variety of improvements. F. W. Aston, at one time Thomson's research assistant, in 1917 discovered that by opposing magnetic and electric deflections along the same axis instead of at right angles separation could still be obtained. This follows since the magnetic deflection varies as $\frac{1}{mv}$, whereas the electrical deflection varies as $\frac{1}{mv^2}$. He was thus

able to get deflections in which all ions for the same $\frac{e}{m}$ fell on one spot with sharp focus irrespective of v . This reduced exposures from 24 hours to minutes. With his method he could get quite precise relative values of $\frac{e}{m}$ and thus of m . At about the same time A. J.

Dempster used the simple device of Fig. 177 for his mass spectrograph.

Instead of a hot filament for electrons he used the positive metal ions known to be emitted by heated salts. With the precise energy given the ions by a known applied positive potential (in place of the negative potential used, as V for electrons in Fig. 177), and an accurately constructed system of slits so that ρ was accurately determined the precision achieved was very high. By varying H , different values of

$\frac{e}{m}$ could be brought to focus on the collector. A plot of the ion current to the collector as a function of H then gives a peak of current when H is just right for the ρ required by that given $\frac{e}{m}$. In later years

the source of ions was improved. The hot salt-coated filament was replaced by a slit in the wall of a high current density arc-discharge tube in the gas in question. The ions produced by the intense arc-discharge were drawn out of the arc column through the slit and accelerated in known high fields. Such an ion source is shown in the linear accelerator of Fig. 188.

Very many variants of the *mass spectrograph* have been designed since Aston's time and have served their purposes very well. However, the need for isotope separation on a large scale in the atomic bomb researches led, at the Radiation Laboratory of the University of California, to the development of the calutron. This device is in essence merely an improvement on the design of the original Dempster mass spectrograph. With batteries of such devices enough of the light isotope of uranium, U^{235} , was separated from U^{238} to make the first atomic bombs fired. Such devices are now in constant use for isotope separation for scientific purposes, and for analysis in chemical reactions. Recording in commercial instruments is electrical and automatic.

163. THE THOMSON ATOM AND THE NUCLEAR ATOM

When J. J. Thomson attempted to construct an atom containing his newly found corpuscles he concluded that the atom must consist of a sphere of positive electricity of the order of magnitude of the observed kinetic theory radii of atoms and molecules, i.e., of about 10^{-8} cm, for he had observed that positive electricity was always associated with the mass of the atom remaining when electrons were removed. He thus postulated that the atom consisted of a sphere of positive electricity of about 10^{-8} cm radius with about $\frac{A}{2}$ electrons studding its interior in regular geometrical configurations. Such a picture was very convenient, for electrons can be held in stable configurations in such a sphere by the positive charge and their mutually repulsive forces. The electrons when disturbed from their resting positions in this atom could then oscillate with simple harmonic

motions, giving all of the well-known optical effects as well as the Zeeman effect.

This picture was doomed to be shattered by the discoveries of Lord Rutherford. He had proved that the alpha particles emitted by radioactive elements in their spontaneous disintegration were atoms of helium which had lost both of their electrons. They were traveling with energies between 2 and 10 m.e.v., and at velocities in the neighborhood of 2×10^8 to 2×10^9 cm per second. Now such charged particles of high energy in going through a gas tear electrons out of the outer portions of atoms quite freely. Thus such particles traveling through air at atmospheric pressure and within a radius of some 10^{-5} of a centimeter from their path remove electrons from the atoms or molecules. They will remove about 10^3 electrons per millimeter of path. Owing to their mass of about 7000 times that of an electron and their high energy they are seldom diverted from their straight-line trajectories by electronic encounters. Their paths are very straight. In studying the motion of these particles through matter Rutherford and his pupils in 1911 observed, however, that an unexpectedly large number of α particles were deflected through large angles by thin metal foils. In 1912 C. T. R. Wilson succeeded in making the tracks of ions left by rapidly moving electrons and alpha particles visible by condensation of water vapor on them by adiabatic expansion of air supersaturated with water vapor. The device used for this purpose was the famous C. T. R. Wilson *cloud chamber*. The character of these tracks confirmed Rutherford's findings with scattering from thin metal foils.

Rutherford concluded that such sharp bends could occur only if an occasional encounter took place between the alpha particle and something in the atom that was very small, quite massive, and very hard. The soft electron-studded sphere of positive electrification in Thomson's atomic picture could cause no such bends.

Rutherford therefore postulated that the *positive charge in the atom was concentrated in a single minute central region in the atom*, i.e., a virtual *point positive charge*, which he called the *nucleus*. He further assumed that this charge had associated with it all the mass of the atom, beyond that of the Z electrons present outside, required to make an atom neutral. He next postulated that the positively charged α particle of charge E , mass M , and velocity V was repelled by the nuclear charge $+Ze$ of an atom according to Coulomb's law of force, i.e., that $f = \frac{ZeE}{r^2}$. Here r is the distance between the

nucleus and the α particle. He used atoms of heavier elements like gold for scattering such that he could assume that the heavy nucleus was set into motion relatively little by impact of the light α particle. With such assumptions it follows that α particles will execute hyperbolic orbits about a point. The point in question is the conjugate

focus f' of the hyperbolic system, if the nucleus Z is placed at the real focus f , as in Fig. 181. From the analytic geometry of the hyperbola and applying the mechanical laws of conservation of momentum and of energy in the encounter, with $f = \frac{ZeE}{v^2}$, Rutherford

was able to calculate the number of α particles scattered through any given angle ϕ . He used a beam giving Q alpha particles per second incident on a foil of metal, nuclear charge Ze , of thickness t , where

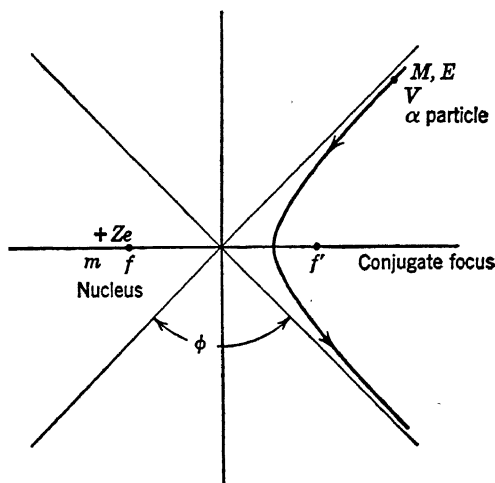


FIG. 181. Track of α particle. ϕ = angle of scattering.

t is quite thin so as not to slow down the α particles, as shown in Fig. 182. He counted the number of α particles impinging on a small area S of phosphorescent ZnS screen under a microscope per second, shown in Fig. 182. The screen was at an angle ϕ with the initial path of the α particles and distant r cm, as in Fig. 182. The quantity $N_{d\phi s}$ thus counted could be compared with his theory, which predicted that

$$N_{d\phi s} = \frac{Q N t s}{4 r^2} \frac{Z^2 e^2 E^2}{M^2 V^4} \csc^4 \frac{\phi}{2}.$$

The theory was completely verified for angles ranging from $\phi = 5^\circ$ to $\phi = 150^\circ$, though $N_{d\phi s}$ varied over a range of the order of 200,000 to 1. In later years Z was measured for a number of elements by Chadwick, who confirmed the value of Z predicted by Mosely in 1914, within the limits of experimental error of 1 per cent. The most vital element in the law is the variation of $N_{d\phi s}$ with $\frac{1}{V^4}$. This variation

is critically dependent on the exponent of r in the law of force, $f = \frac{ZeE}{r^2}$.

This was established to be 2.0 within 2 per cent, the accuracy of the measurements. Such agreement may be wondered at because of the atomic electrons surrounding the nucleus Z . However, long before the α particle had penetrated close enough to the atomic nucleus to be appreciably deflected, i.e., within 10^{-11} to 10^{-10} cm, the electrons had all been sucked away from the atom by the field of the rapidly approaching α particle.

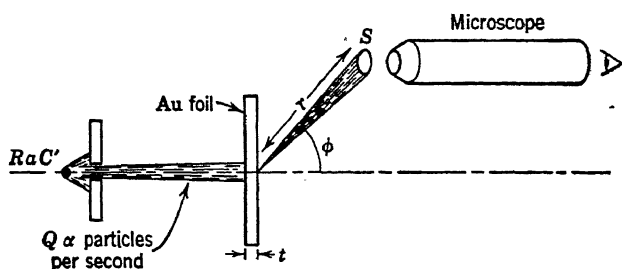


FIG. 182. Rutherford's method of studying α particle deflections in their passage through thin gold foils.

The theory was later extended by C. G. Darwin to include nuclei of mass m commensurate with M . With this theory deflections with closer approach between α particle and nuclei were studied. It was then observed that deflections began to deviate from the predictions of the theory when α particle and nuclei got to within about 2×10^{-12} cm or less of each other. The scattering obtained under such conditions was called *anomalous scattering*.

The conclusion that could be drawn was that at distances between α particle and nucleus of the order of $1 - 2 \times 10^{-12}$ cm the force laws changed from the inverse square law repulsive force to either higher repulsive forces or, in some cases, to very powerful but short-range attractive forces. These distances were thus assumed roughly to represent the *dimensions* of the nuclei.

With the advent of wave mechanics an attempt was made to interpret the scattering of α particles in terms of *plane wave* α particles of wave length $\lambda = \frac{h}{MV}$. These were supposed to be *diffracted* by

a nuclear potential energy hill of form $P = \frac{ZeE}{r}$, with a crater of very low potential energy in the center at a distance r_0 . This analysis agreed with the normal Rutherford scattering with the same precision as the original Rutherford calculations. It did better than this, in that it accounted for close encounter scattering, such as between α

particles and He atoms, with greater precision than could the Rutherford theory. If the encounters between α particles and nuclei succeeded in placing the α particles within the crater of the nuclear potential hill, under appropriate energy conditions, it was observed that the α particle was absorbed and the nucleus transformed to a nucleus of different Z . This was often accompanied by the somewhat later emission of the nucleus of an H atom, or a proton.

The result of Rutherford's investigations was thus to prove beyond the shadow of a doubt that the positive electricity in an atom is concentrated in a minute massive region of space and is not a large diffuse sphere. This concentration of positive charge and mass in a small region of the atom Rutherford called the *nucleus* of the atom. Thus by the year 1913 it was generally acknowledged that the atom was composed of a nucleus with positive charge of $+Z$ electronic units, with most of the atomic mass. This charge from 2×10^{-12} cm outward acts on electrical charges e with the Coulomb law $f = \frac{Ze^2}{r^2}$.

The Z electrons required to make an atom neutral thus must reside in the space *about* the nucleus. This situation at once introduced serious difficulties compared to those presented by Thomson's simple atom.

164. THE BOHR ATOM

If the Z electrons are bound outside a positive nucleus of charge Ze by central forces they *cannot* be in *stable equilibrium at rest*, according to basic mechanical postulates. Thus electrons must be assumed to be in orbital motion in elliptical orbits about the nucleus as the focus of the ellipse.* An electron in such an orbit is constrained by

the attractive force $f = \frac{Ze^2}{r^2}$ from flying off the atom because of the centrifugal force $\frac{mv^2}{r}$. This means that the electron is being ac-

celerated toward the nucleus with an acceleration $a = \frac{f}{m}$. But *accelerated electrons* radiate electromagnetic waves according to Maxwell's equations. Thus the radiated energy from an electron orbit removes orbital energy. This energy can only come from a decrease of electronic potential energy $-\frac{Ze^2}{r}$ through the electron orbit, decreasing its radius r and the electron falling into the nucleus. This would destroy the atom. But atoms are notoriously stable, having existed some 10^9 years.

*The inverse square law sets the elliptical form. In some cases the special ellipse called the circle can be used to simplify calculations.

The solution of this dilemma began when Niels Bohr postulated that there are certain orbits in which electrons can remain and *not* radiate. Electrons remain in these unless given energy and raised to orbits of larger r . From there they can return to the original orbits. In going from a higher to a lower orbit they will radiate energy and hence light. Using the quantum theory of 1913 Bohr arbitrarily fixed on the criterion for stable orbits as being given by the condition that for a simple circular orbit the moment of momentum p of the electron in the stable orbit is $p = mvr = \frac{nh}{2\pi}$. Here n is an integer, i.e., 1, 2, 3, 4, etc., and h is Planck's constant of quantum action. If this relation be combined with the condition that $\frac{mv^2}{r} = f = \frac{Ze^2}{r^2}$, the properties and characteristics of the stable circular orbits can at once be described. The resulting relations are as follows:

$$\text{Energy of electron in an orbit in equilibrium } E = -\frac{1}{2} \frac{Ze^2}{r}.$$

$$\text{Radius of the orbits} \quad r = \frac{n^2 h^2}{4 \pi^2 Z e^2 m}.$$

$$\text{Velocity in the orbits} \quad v = \frac{2 \pi Z e^2}{nh}.$$

$$\text{Energy in an orbit resulting from Bohr's theory } E = -\frac{2 \pi^2 m Z^2 e^4}{h^2 n^2}.$$

It appears that more significant than the radii of the orbits are the values of energy in the orbits. These have a *negative sign*, indicating that it takes work to remove the electron *from* the orbit, or that energy is radiated in falling into the orbit from higher orbits or from infinity.

It is noted that only orbits for which n has integral values occur. These increase in radius proportional to n^2 , as n increases. They have radii that vary inversely with the nuclear charge Z , or *effective* nuclear charge Z' as screened off by inner electrons. The negative energy in the orbits is greatest for $n = 1$, and decreases in *negative value* as $\frac{1}{n^2}$, for increasing n . As n approaches infinity the energy in the orbit approaches zero, i.e., r is so large that the electron is then no longer bound by the atom. It will be noted that the negative energy increases as Z^2 , i.e., as the square of the effective nuclear charge. A diagram of energy levels which clarifies this description is given in Fig. 183.

It should be noted that for the substance H, $Z = 1$ with 1 electron in an orbit $n = 1$, there exists a sort of an atomic measuring stick.

It is called *unit Bohr orbit*. Its value of $p = mvr = \frac{h}{2\pi}$. It has a radius of 0.528×10^{-8} cm. Its magnetic moment is $\mu = \frac{e}{m} \frac{h}{4\pi}$, and it is called the *Bohr magneton*; see page 253.

If there is an electron in an outer orbit (larger r) with greater n and if there is a vacant orbit of lower n (smaller r), the electron will fall from the larger orbit to the smaller one.

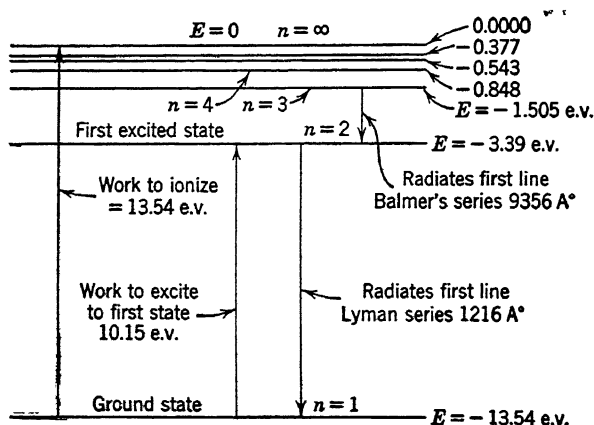


FIG. 183. Bohr energy level diagram for circular H orbits.

In general, it will fall to such an orbit within an average time τ , emitting electromagnetic radiation (light) during the period. If the starting point was N_0 atoms having electrons in a given type of orbit, $n = x$ at $t = 0$, after a time t there would be only N atoms left, given by $N = N_0 e^{-t/\tau}$ with electrons in the $n = x$ orbit. The rest of the electrons would have fallen from $n = x$ to the vacant lower level $n = y$, where x and y are integers, $x > y$. The value of τ will depend on the character of the orbits $n = x$ and $n = y$. It will range from 10^{-10} second to about 10^{-7} second for commonly observed transitions. Some transitions are not allowed and for these τ is theoretically indefinitely long. Where such levels are next to the ground or lowest state they are called *metastable* states. Electrons can enter such states from higher levels but cannot reach the ground state. These long-lived, activated, metastable atoms can transfer this energy to other atoms and to surfaces liberating electrons and what-not.

As the electron falls from an outer to an inner orbit it will radiate light. Bohr postulated that the frequency of light radiated would be governed by the Planck radiation law. This says that $E_x - E_y = h\nu_{x-y}$. Here E_x and E_y are the energies in the states $n = x$ (outer

orbit) and $n = y$ (inner orbit) given by

$$E_x = -\frac{2\pi^2 m Z^2 e^4}{x^2 h^2}$$

and

$$E_y = -\frac{2\pi^2 m Z^2 e^4}{y^2 h^2}.$$

The quantity h is the Planck constant and ν_{x-y} is the *frequency* of the light radiated, that is, $\nu_{x-y} = c/\lambda$, where λ is the wave length of light. Thus the frequency of the light radiated in the example above is

$$\nu_{x-y} = \frac{2\pi^2 m Z^2 e^4}{h^3} \left(\frac{1}{y^2} - \frac{1}{x^2} \right).$$

The predictions of these Bohr equations about the form of the expressions for the light which is radiated by the simple hydrogen-like atoms were remarkably close to the observed values. The analysis of the frequencies of observed spectral lines in general had indicated that they followed laws akin to those derived. The Bohr theory went one better and gave a general basis for the type of relations observed. The constant term outside of the parentheses multiplied by $\frac{c}{Z^2}$ in the expressions for ν_{x-y} is called the *Rydberg constant*. It coincided numerically, within the limits of experimental accuracy, with the value of a very important constant deduced by Rydberg from the study of spectra. It was, in fact, the close general agreement of Bohr's simple theory with observed optical spectra that led to most of the unraveling of the nature of the structure of the atom.

The theory of Bohr was extended to elliptical orbits by Sommerfeld, Bohr, and many others. With the more accurate data on spectra and on the Zeeman effect, as well as many other new facts, the modern wave mechanical theory of the atom evolved. The discovery of the electron spin led to what is known as the vector model of the atom.

Final clarification resulted from an initial suggestion by L. V. de Broglie, in 1921, that Bohr's arbitrary assumption of stable non-radiating orbits for electrons could be explained in another fashion. Since mechanical mass point electrons circulating about a nucleus must either violate Maxwell's laws if in stable orbits, or else must radiate energy, it occurred to de Broglie that *electrons might not be mass point electrons at all*. He indicated that if electrons were some

sort of complex wave motion, of wave length $\lambda = \frac{h}{mv}$, the orbit conditions of Bohr could be accounted for in another way. Unless such wave electrons could get whole multiples of a wave length into

an orbit circumference, such waves would not be stable, i.e., they would radiate energy. If, however, in the length $2\pi r$ about a nucleus n whole wave lengths could be accommodated, where n can be 1, 2, 3, etc., then the wave-length orbit of the electron would be stable. Thus if

$$n\lambda = 2\pi r = \frac{nh}{mv}, \text{ stable orbits will exist. But this relation at once}$$

gives Bohr's 1913 criterion for stable orbits, namely, that $mvr = \frac{nh}{2\pi}$.

This suggestion, together with the theoretical work of Heisenberg, Schroedinger, and Dirac, together with the experimental investigations of A. H. Compton on x-rays, and of Davisson and Germer and G. P. Thomson, finally clarified all the main difficulties in the wave mechanical descriptions of electron and atom. The results of these studies in the organization of the atoms is discussed under the appropriate sections 45 and 89.

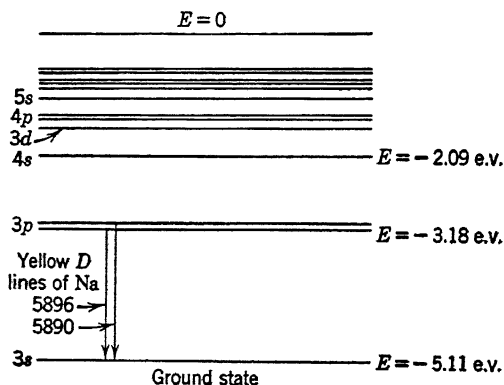


FIG. 184. Energy level diagram for Na.

There it is seen that the state of an atom, or an electron in an atom, is described by the energy of the electron which is determined by the four quantum numbers, n , l , m_l , and s . In essence, the energy levels are no different from the orbital energies indicated above for the simple hydrogen-like circular Bohr orbits. In detail, they differ in magnitude and in a greater complexity. This is seen in Fig. 184, which shows the energy-level diagram for Na.

165. CHARACTER OF EXCITATION AND IONIZATION IN ATOMS

It is, therefore, possible to characterize the lowest energy state (orbit) occupied by an electron in an atom and to designate and evaluate other higher states corresponding to the various quantum numbers to which it could be raised. When raised to these levels the

electron will return to its lower energy state in time, τ , as indicated, and in so doing it will radiate light. To cause an electron to emit light it must *receive* just enough energy to raise it from its lowest level to some higher allowed excited level. This energy can be imparted in several ways. If a quantum of radiant energy (light waves) of frequency ν_{y-x} is allowed to fall on the atom with an electron in the state y , there is some chance that the energy will be absorbed if $h\nu_{y-x}$ just equals $E_x - E_y$ for the states. Not every quantum will be absorbed by a receptive atom, though some few will. This is called *photoelectric excitation* of the atom. If the excited atom in state x returns directly to y , then ν_{x-y} will be reradiated. This is called *resonance radiation*. If the excited atom returns from x to y by an intermediary step to a level z , lying between x and y , two frequencies in succession will be radiated, given by

$$h\nu_{x-y} = h\nu_{x-z} + h\nu_{z-y}.$$

This is called fluorescent radiation.

The electron in state y can also be raised to state x by the impact of an external electron of energy $\frac{1}{2}mv^2$ in excess of $h\nu_{x-y} = E_x - E_y$. If such an electron approaches the atom it will occasionally raise an electron from the state E_y to the excited state E_x . The electron doing this work then escapes with a residual energy $\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 - h\nu_{x-y}$. The excited electron after τ second then returns to its initial state giving radiation. This is called an *exciting impact* or a *radiating impact*. If the impacting electron has an initial energy *less* than $E_x - E_y$ it may be *deflected* by the atom but cannot excite it or lose very much energy. Such a deflecting impact of electron and atom is called an *elastic electron impact*. Classic atomic theory and the Thomson atom do *not* permit of such elastic impacts. The Bohr theory does. It was thus very important for the Bohr theory that in the same year that the theory was proposed, 1913, Franck and Hertz experimentally demonstrated the existence of elastic and exciting impacts at just the electron energies predicted by Bohr's theory. It can also happen that if *two atoms* collide with an energy in excess of the energy $E_x - E_y$ an electron in one of the atoms may be raised to the excited state. This is known as *temperature excitation* or *thermal excitation* and has been observed in ovens. The laws governing excitation and ionization by fast thermal impacts have been developed by M. N. Saha in his now famous equation.

If the electron in an energy level $-E_y$ receives energy, either as a quantum $h\nu_x$, or energy of impact from a fast outside electron *in excess* of $E_y - E_\infty$ (that is, E_y , as E_∞ for the orbit $n = \infty$ is 0), the electron at E_y will be *removed from the atom*. This is called *ionization*. The electron escapes with a kinetic energy, theoretically at infinity, of $\frac{1}{2}mv_1^2 = h\nu_x - E_y$, when it is removed by a high-energy light quantum or photon. This is called *photoelectric ionization*. It does not occur

very readily. If the electron is removed from the atom by an incoming fast outside electron, of energy $\frac{1}{2}mv^2$, then the *atomic electron* at E_y and the *outside electron* escape from the atomic neighborhood, with energies $\frac{1}{2}mv_1^2$ and $\frac{1}{2}mv_2^2$, given by $\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mv^2 - E_y$. If the impacting electron had a very high energy one of the two energies $\frac{1}{2}mv_1^2$ or $\frac{1}{2}mv_2^2$ will also be high. After the impact, wave mechanics says that the impacting electron cannot be distinguished from the atomic electron. This process is called *ionization by electron impact*. Under some conditions fast electrons may remove as many as one, two, or more electrons from the same atom at once. Such occurrences are rare and the probability goes down roughly by a power of 10 for each electron removed. The energy E_y just needed to remove an electron from an atom is called the *ionization potential*. This was also first measured by the Franck and Hertz techniques. Ionization potentials for various atoms and molecules range between 24.5 e.v. for the first outer electron in normal He to around 3.88 e.v. for the outer electron in Cs, while the first excitation potentials in these atoms range from 19.75 e.v. to 1.3 e.v. To remove the electrons from the inner shells of the various atoms, such as the *K* and the *L* shells, requires very much higher energies, running up to 10^5 volts for the *K* shell in element 92, or *U*.

When an electron is removed from an atom, i.e., when the atom is *ionized*, the electron escapes and moves around freely in the gas, dissipating its energy. As it slows down it may be caught by the walls of the vessel and thus eventually run off to earth. It could also be picked up by some molecules, such as O_2 or some atoms, as Cl, to give negative molecular, or atomic, ions such as O_2^- and Cl^- . On very rare occasions, unless the density of positive ions is very great, it could be captured by a positive ion. It would then radiate light of a frequency $h\nu_{vy} = \frac{1}{2}mv^2 + E_y$. Here v is its velocity before capture and E_y is the level into which it was captured. It is the more readily captured the slower its motion. If a *high field* is present the electron could gain more energy and perhaps do a bit of ionizing on its own account. Since its new electron could do the same, such ionization would increase rapidly and then is called *cumulative ionization*.

The positively charged remainder *A* of the atom left by the electron can do one of several things. It can collide with atoms or molecules, *B*, the ionization potential E_{yB} of which is less than its own E_{yA} . There is a very good chance in this situation that *A* will steal one of *B*'s electrons. It is then neutral and *B* is positively charged. The difference in energies, $E_{yA} - E_{yB}$, goes to kinetic energy of separation of ion and atom. Such change of charge was discovered by Kallman and Rosen and is called the *Kallman-Rosen effect*. The positive ion can wander, i.e., diffuse, to the walls, ultimately pick up an electron there, and become neutralized. It can, if energetic enough, liberate an electron from the metal. It can in the gas recombine with an electron by

picking the electron from a negative *ion* that comes close enough to it. This is called ion-ion recombination and is frequently observed. The energy of neutralization is given out as kinetic energy of separation of the neutralized ions. A positive ion can rarely pick up an electron and radiate the appropriate frequency as it returns to normal.

166. X-RAY LEVELS AND X-RAYS

It was stated that the energy levels of inner electrons represent very large negative values of E . It was noted in the simple Bohr theory that the value of E was numerically the greater the smaller the n and the larger the value of Z . For the innermost electrons, which feel the full value of Z without any screening by inner electron shells, the value of $-E$ is very large. In addition for $n = 1$, the E is not diminished by n^2 . Thus in such shells it should not be surprising to find $-E$ in element 92, U, to be of the order of 92^2 times what it is for the innermost orbit $n = 1$ for H, where $Z = 1$, or *unit* Bohr orbit. This means that there are energy levels in the *interior* of heavy atoms of the order of -10^5 e.v. The electrons in the intermediate levels have all orders of magnitude of $-E$ between values of 10^5 e.v. and the some 10 e.v. of the outer levels.

Notable among the inner levels are the $n = 1$ and $n = 2$ levels in all atoms. These are called, respectively, the *K* and the *L* shells, or levels. The *K* shell is the innermost level and the *L* the next innermost level. See page 123. Electrons in these levels, in most atoms, do not have immediately vacant outer levels to which they can be raised. Thus usually for such electrons to be excited they *must be removed to the vacant outer* levels beyond the last filled shell. If energy enough in the form of a "photon, quantum of radiant energy, $E_K = h\nu_K$, $E_L = h\nu_L$, or by an energetic impacting external electron is given the electron in this shell, it will usually be enough to *ionize* the atom from this level, instead of raising it to one of the very outermost levels. Thus by very fast electron impact or very energetic photon the *K* or the *L* shell electrons can be removed.

The atom in this condition is unstable. If a *K* electron is removed an electron from the next level, the *L* level, takes up the vacant place in the *K* shell. In this *transition*, radiation of frequency $h\nu_K = E_K - E_L$ is liberated. This frequency is very high. For the uranium *K* radiation it is of the order of 3×10^{19} cycles per second in contrast to 2.5×10^{17} cycles for Na *K* radiation. These frequencies correspond to wave lengths of light of 1.3×10^{-9} and 1.19×10^{-7} cm. The ordinary yellow light from the D lines of Na coming from transitions in the outer valence orbit amounts to 5.800×10^{-5} cm. The energies in volts corresponding to excitation of these lines is 95,500 e.v., 1050 e.v., and 2.14 e.v.

These anticipated radiations on the basis of Bohr's theory belong

in part of a very large expanse of the electromagnetic radiations predicted by Maxwell in his electromagnetic theory. At the present writing a very large section of such radiations have been created and studied. They range in wave lengths from kilometers for slow electrical oscillations into the tens of meters for short-wave radio and down to 1 cm or less for the currently used microwave radar. Below about 1 mm and down to 10^{-3} cm the gap is bridged between radio waves and very long heat waves. From 10^{-3} cm to 10^{-5} cm extend the infra-red wave lengths. These come from the oscillations or rotations of charged molecular configurations due to atomic heat vibrations. Thus the oscillation of the N atom through the triangle of 3 protons in NH_3 corresponds to wave lengths of 1.6 cm. From wave lengths of 9×10^{-5} cm to 4.5×10^{-5} cm the spectrum of *visible* light exists. From 4×10^{-5} cm on down to 10^{-6} cm extend the ultra-violet and extreme ultra-violet light waves. These radiations come from the *outer* levels of *multiple ionized* atoms and from the shells next, inside the surface shells. Extending below 10^{-6} cm is the soft x-ray region. Thus the radiations emitted, when electrons fall into ionized *K* and *L* shells from the next outer shells, are in the x-ray region of electromagnetic waves. Beyond the x-ray region the waves are caused by charge oscillations in the nuclear mass. These are called γ rays and have energies ranging from 10^5 e.v. up to tens of m.e.v. In cosmic rays there are even more energetic photons. Ten m.e.v. γ -rays correspond to frequencies of the order of 3×10^{21} and wave lengths of the order of 10^{-11} cm or less. A summary of the electromagnetic wave-length spectrum is given on the facing page.

It will be noted that x-rays were discovered when electrons of 1000 to 10,000 volts in glow-discharge tubes at low pressure impinged on the glass walls, or heavy metal targets, called *anticathodes*. It was thus suspected that x-rays were short electromagnetic waves. There was a reason for suspecting this nature of the x-ray. It was recognized that if a charge is accelerated or decelerated, it will radiate electromagnetic waves. Thus if the electrons in the cathode beam are suddenly stopped, or decelerated, in the anticathode, they will radiate electromagnetic waves. From the rate of deceleration J. J. Thomson estimated that the frequency radiated would be very high. However, with all the possible rates of deceleration occurring in a target the wave lengths radiated should not be very homogeneous. Thus it was thought that such x-rays would be a sort of general white radiation of all wave lengths.

For many years, from 1896 to 1913, there were no ways of studying x-rays other than by their ionization in gases and absorption in various materials. Even with this crude technique C. G. Barkla was able to show the presence of certain radiations that were characteristic of the various elements by their absorption coefficients. He developed techniques for segregating these out from the white radiations of the tubes

ELECTROMAGNETIC WAVE-LENGTH SPECTRUM

Wave Length	Range	Source	Name	How and by Whom Studied
∞	to 10 cm	Movements of electricity in large systems with capacity and self-induction.	Radio waves.	Predicted by Maxwell. Discovered by Hertz.
10 cm	to .01 cm	Electrical oscillations of minute systems, metal filings. Includes radar wave lengths.	Very short electromagnetic waves.	Nichols and Tear. Large gratings. Radar.
.01 cm	to 7×10^{-5} cm	Oscillations or vibrations of charged atomic or molecular systems, ions in crystals, or gas molecules. Rotation of dipoles.	Infra-red or heat waves.	Rubens Paschen. Gratings and residual rays.
7×10^{-5}	to 4×10^{-5} cm	Loosely bound outer valence electrons.	Visible light.	Prisms, gratings, interferometers.
4×10^{-5}	to 1.6×10^{-5} cm	Outer electrons more tightly bound.	Ultra violet.	Schumann and Lyman. Gratings.
$1.6 - 10^{-5}$	to 1.2×10^{-5} cm	Inner electrons of light atoms, or electrons in stripped light atoms, or shells next to valence shells of heavy atoms.	Extreme ultra-violet rays or soft x-rays.	Millikan and Bowen. Vacuum spectrograph, gratings.
1.2×10^{-5}	to 1.6×10^{-6} cm	Interior electrons of elements.	Soft x-rays.	Thibaud reflected from glass gratings at grazing incidence.
1.6×10^{-6}	to $1.25 - 10^{-9}$ cm	Innermost electrons of atoms, shorter waves apply to heaviest atoms.	X-rays hard and soft, K and L series.	Laue, Bragg. Crystal gratings, also from $Ve = \frac{1}{2}mv^2 = h\nu$, $\nu = c/\lambda$.
1.25×10^{-9}	to 10^{-12} cm	Oscillations of charge in nuclei.	γ -rays.	Robinson, de Broglie, Ellis, Meitner from $\frac{1}{2}mv^2 = h\nu$, ν from magnetic field.
10^{-12}	to —	The cosmic rays also contain γ rays of a very penetrating character which are probably of secondary origin. The wave lengths, judging from the energies of cosmic-ray showers produced, may extend below 10^{-12} cm.	Cosmic rays.	Estimated by curvature in magnetic fields of electrons' tracks liberated by these γ rays.

and for differentiating among those from different elements. He observed that for metals beyond Al there were two characteristic series of x-rays for each element, penetrating ones which he called *K* x-rays and less penetrating ones that he called *L* x-rays. They were the more

penetrating the higher the atomic weight. They also depended on a certain minimum potential across the x-ray tube for their appearance.

If x-rays are electromagnetic waves like light waves and radar waves they should be capable of interference and diffraction effects and of reflection. Early attempts at detecting these effects had resulted in failure. This was, first, because the wave lengths of x-rays were too short and, second, because all matter acts like a metallic conductor to the high-energy x-rays. Thus x-rays needed very closely ruled gratings and to be studied assuming the *negative* indices of refraction characteristic of metals.

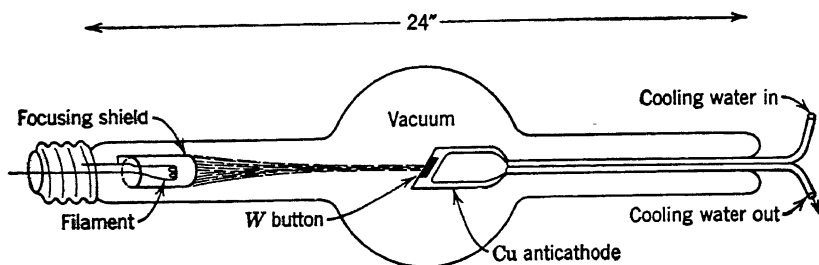


FIG. 185. 100-kilovolt Coolidge-type x-ray tube.

X-Ray Tubes. By 1913 numerous advances in techniques facilitated the study of x-rays. The hot-tungsten and oxide-coated filament sources of electrons allowed the investigators to get rid of the old gaseous-discharge x-ray tubes of Roentgen's time. In place of these a hot filament was introduced into a highly evacuated bulb, as shown in Fig. 185. It was surrounded by a focusing shield. The electrons were accelerated in very good vacuum to the anticathode, which was attached to the positive potential of a rectifier set. The anticathode was usually a copper block, provided with water cooling, with a button of W, Mo, or U set in at the focal point of impact of the electrons. In this x-ray tube developed by W. D. Coolidge the potentials up to 250,000 volts could be applied with milliamperes of electron currents. The quality of the x-rays produced depended on the P.D. applied and the nature of the target material, as will be seen. Such a tube is shown in Fig. 185. Since then, no great improvements in tube design have followed. The focusing is better and tubes can dissipate more power from the anticathode as a result of water cooling and better glass seals. High voltage requires long tubes. Vacuum techniques have improved. The requirements for high-potential x-rays for industrial and medical purposes have resulted in some improvements of power sources. With the development of electronic rectifiers such as the Kenotron (see page 535), steady d-c potentials of up to 250 kv have been achieved. If needed, more power could be obtained by mercury-arc rectifiers.

With potentials above 250 kv, however, the tube design becomes a problem. While sparks require 30,000 volts to traverse 1 cm in open air, high voltage tends to cause corona and brush discharges that creep over insulating surfaces and build up local fields causing sparks and puncture. Thus if potentials of 1 to 3 million volts are applied to a tube the tube must be of the order of 5 meters or more long. It must be built up of sections of insulator with evenly spaced conducting rings between sections to distribute leakage stresses. Such high-potential resistant tubes have been built up for atomic disintegration experiments as well as for high-energy x-ray. They were used with transformers in cascade at 60 cycles, 10^6 volts, with the Cockcroft and Walton generator at 10^6 volts, and with the Van de Graaff machine for x-radiography by the Navy in World War II, as well as by Westinghouse Research Laboratory at 2.5 to 3.5×10^6 volts, for nuclear studies. They must, however, be pumped continually as leakage is a problem. For potentials above 2.5×10^6 volts the *outside* of the tube must be at high pressure with some spark-suppressing gas like "Freon" to prevent flashover. All such tubes are continuously attached to high-speed diffusion pumps.

The problem of high-energy x-rays was solved in another way, initially by David Sloan at Berkeley in 1936. What was essentially a Tesla coil was mounted inside a vacuum and set into oscillation over one-half turn by 20-kv-amplitude high-frequency oscillations of the order of 10^7 cycles. The end of the coil of some forty turns then swung some 10^6 volts positive and negative. At its high-potential end was a water-cooled anticathode, as shown in Fig. 186. A hot-filament source on the ground side opposite the anticathode furnished the electrons. This general principle was also incorporated into the 10^6 volt General Electric x-ray generator. The high frequency was introduced into an air-core transformer in vacuum. The high-potential end of this transformer was likewise opposite the electron gun. These devices appear to work very well for industrial radiography up to 10^6 volts. So far the Van de Graaff tube is the only one successfully to give in excess of 2×10^6 volt x-rays for commercial radiography.* All these devices for high-voltage x-rays except the 1000-kv General Electric generator require pumps to be maintained in operation to ensure good vacuum. Recently it was claimed that a successful *sealed-off* x-ray tube for the 2×10^6 volt Van de Graaff generator had been perfected. Since a tube of this character costs on the order of \$10,000, it will for some time probably be cheaper to operate with the troublesome vacuum pumps, since filaments have the unfortunate habit of burning out and a \$10,000 tube is a pretty large item.

With the Coolidge x-ray tube giving powerful and controlled x-ray sources by 1913 the stage was set for the other great advances in the

* There is some probability that the 20-m.e.v. betatron* will eventually be of great service in radiography.

study of x-rays. In 1913 it occurred to Max von Laue that the distance between atoms in a crystal lattice was of the right order of magnitude to diffract x-rays. Accordingly, under his direction his students Friederich and Knipping sent a beam of x-rays through a rock-salt crystal. They at once observed the predicted diffraction images of the pinhole x-ray beam on a photographic plate placed behind the crystal.

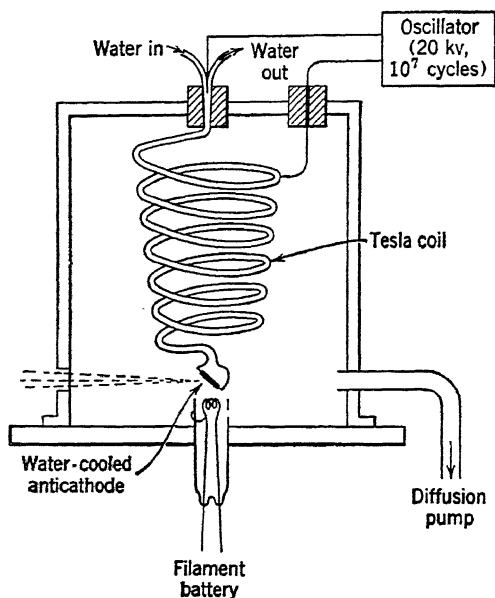


FIG. 186. Sloan 1-m.e.v. x-ray generator.

The technique of using the atomic planes of a crystal lattice for convenient measurement of the wave lengths of x-rays was immediately developed by W. H. and W. L. Bragg. In their technique they indicated that in a crystal the atoms lay along certain planes. The distance, d , between the planes could be calculated from the crystal shape, the density, and the Avogadro number, then just fairly accurately computed from Millikan's value of e . Then by measuring the angles θ with the plane at which the maxima of the diffracted x-ray beam occurred the wave length, λ , of x-rays could be deduced from the simple equation $n\lambda = 2d \sin \theta$. Here n is an integer (1, 2, 3) representing the "order" of the x-ray spectrum. With this method great advances both in the knowledge of x-rays and of crystal structure were made possible.

Moseley's Laws. Using the Bragg method, H. G.-J. Moseley, a student in Rutherford's laboratory, at once set to work to study the

characteristic x-rays from the various elements ranging from Al to Au. He discovered that the so-called characteristic K and L series of x-rays for each element observed by Barkla constituted two groups of x-ray wave lengths. The K series had three characteristically spaced lines $K_{\alpha 1}$, $K_{\alpha 2}$, and K_{β} , in which K_{β} had the shortest wave length, i.e., the highest frequency, and $K_{\alpha 2}$ the lowest frequency. These appeared for each element and the wave lengths decreased (and the frequencies increased) in going up in atomic weight. The L series of x-rays consisted of five characteristically grouped lines the wave lengths of which were of the order of some eight times as great as the K series. These first appeared as x-rays in heavier elements. Later, still longer wave-lengths series of x-rays were found, both in absorption and by wave length, for the heavier atoms, i.e., the M and the N series.

Now, Moseley was working in Rutherford's laboratory where Bohr had just the year before proposed his theory of spectral lines. Moseley observed a progressive change in frequency of lines of the K and the L series as the atomic weight of the elements increased. He measured the frequency, ν , of these characteristic x-ray lines, say $K_{\alpha 1}$, and found that they could be accurately given by an equation

which read: $Q = \sqrt{\frac{\nu}{3/4 \nu_0}}$. Here ν_0 is a fundamentally important constant and Q was a number that changed by unity in going from element to element in succession in the Mendeleeev periodic table. If instead of frequency $\nu = \frac{c}{\lambda}$ is replaced by wave number, $\nu' = \frac{1}{\lambda}$, then $Q =$

$\sqrt{\frac{\nu'}{(3/4) R}}$, where R is the *Rydberg spectroscopic* constant evaluated in terms of e , m , h , and c in Bohr's equation. With Bohr's theory to guide him, the significance of the experimental relation above became quite clear to Moseley. For if it is realized that $\frac{3}{4} = \frac{1}{1^2} - \frac{1}{2^2}$, and Q be expressed as $Q = (Z - z)^2$, squaring Moseley's experimental law, it is seen that

$$\nu' = Q^2 R \left(\frac{3}{4} \right) = (Z - z)^2 R \left(\frac{1}{1^2} - \frac{1}{2^2} \right).$$

This can be compared with Bohr's equation for hydrogen-like spectra, which says that $\nu' = Z^2 2 \frac{\pi^2 m e^4}{h^3 c} \left(\frac{1}{n_y^2} - \frac{1}{n_z^2} \right) = Z^2 R \left(\frac{1}{n_y^2} - \frac{1}{n_z^2} \right)$.

The analogy is striking.

Thus the characteristic x-rays fall into Bohr's theory for hydrogen-like atoms. It is seen that for the K and the L series of x-ray spectral lines the Bohr theory for hydrogen-like spectra must only be modified by replacing Z , the nuclear charge, by $(Z - z)$ and putting in the

appropriate values for n_x and n_y for the particular line of the series. But in Bohr's theory Z was the charge on the nucleus. It is noted that with these x-rays ($Z - z$) is not the nuclear charge, but *the nuclear charge diminished by z* . For the K series $z = 1$, for the L series it is 7.4.

Now, the K series as indicated at the beginning came from the inner layer or shell of electrons, near the nucleus. This has 2 electrons in it, as indicated on page 123. If one electron is removed by electron impact, the incoming electron from the L shell that radiates the K x-ray frequency will be acted on by a nuclear charge Z less $z = 1$. This is because of the one remaining electron already in the K shell. For the L shell with 2 K electrons inside the L shell and 7 electrons well distributed over the L shell, the M shell electron falling into the vacancy left in the L shell by ionization feels the nuclear charge Z screened effectively by 7.4 inner charges only.

Moseley's study thus revealed a number of very important facts. The first was that the characteristic x-ray frequencies of atoms came from electronic configurations which are *the same and invariant in form in all atoms above a certain place in the periodic tables*. See page 123. The second was that these frequencies are governed by a constant Q , *which changes by unity from place to succeeding place in the periodic table*. That quantity Q according to Bohr's theory must be related to the *nuclear charge*. Thus the nuclear charge $+Z$ *corresponds to the position of the element in the periodic table* and fixes that position. This was confirmed by changes in chemical character observed in radioactive decay series, and later by direct measurement in nuclear studies. Finally the study indicated that characteristic x-rays were in accord with an extension of the Bohr theory of optical spectra to the peculiar conditions inside the atom.

The Compton Effect. As stated before, x-rays can be produced in two ways. If an electron has energy enough to remove a K , L , or other electron from an inner atomic shell, the subsequent emission of characteristic x-rays of that element will be possible. The incident electron must have an energy greater than that to remove an electron from the appropriate shell in the atom, i.e., $\frac{1}{2}mv^2 = E \geq h\nu_\infty$, for the shell in question. It must also be fortunate enough to interact with such an electron. On the other hand, an electron which is suddenly decelerated will also radiate electromagnetic waves. Thus radiation from an x-ray tube will be a mixture of its characteristic K and L series lines together with a general background of deceleration radiation in all wave lengths. If such x radiation is allowed to fall on a block of material of lower atomic weight than the target material it will photoionize the atoms of that block. These will radiate their characteristic x-rays in all directions. This excitation will be done both by the white deceleration radiation and the characteristic lines

of the target, if their energy is high enough. It will be more effective the nearer the two frequencies of x-rays.

In addition, there will be a certain amount of all the radiation scattered by the block of the material by a different process. It had long been known that light atoms, such as C, Al, and so forth, would scatter x-rays from elements like W as if the electrons in Al were virtually free. This was known as "white" x-ray scattering and was used before Moseley's day by J. J. Thomson to estimate the number of electrons per atom in the elements. In 1921 to 1922 A. H. Compton had investigated this "white" scattering with more effective techniques. He discovered that the white scattered x-rays had suffered a *slight shift in wave length on scattering*. This shift in wave length changed with the angle of scattering of the x-rays. His measurements led him to a rather radical picture.

He considered the x-rays as *photons*, i.e., *particulate* quanta of radiant energy, which, on the basis of quantum theory and relativity, have the following properties. They have an energy $h\nu$, where ν is the frequency. They have a momentum of $\frac{h\nu}{c}$, a mass equivalent

to $\frac{h\nu}{c^2}$ and a velocity c , with c the velocity of light. He then assumed that these photons collided with the outer loosely bound, i.e., essentially free, electrons. As a result of such collisions the photons bounced off at an angle θ with their original direction. The electron received energy, was readily removed from the atom, and was projected with a velocity v at an angle ϕ with the photon's original path. From the laws of conservation of momentum and energy it is then possible to show that the change in wave length, the "Compton shift," is given by the relation,

$$\delta\lambda = \lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta).$$

The energy of the projected electron is $E = h\nu \frac{h\nu/mc^2 (1 - \cos \theta)}{1 + h\nu/mc^2 (1 - \cos \theta)}$,

and the angle of the electron is given by $\cot \phi = - \left(1 + \frac{h\nu}{mc^2} \right) \tan \frac{\theta}{2}$.

These equations accurately fit the observations. This result *nearly* led to the *abandonment* of the wave theory of light. It was clarified through the peculiar dualistic nature of light and matter as expressed by the wave mechanics in later years. The Compton scattering has also been actually visualized in the C. T. R. Wilson cloud tracks of x-rays.

The fluorescent scattering is at a maximum at just about the

threshold frequency for its occurrence. It thereafter declines rapidly as frequency increases. Thus as the frequency of x-rays is increased fluorescent scattering from lower atomic weight elements and outer shells is decreased. Eventually as the frequency gets very large fluorescent scattering becomes small indeed. The intensity of Compton's scattering and absorption of energy in the Compton process falls off far less rapidly with frequency. In Pb, for instance, the characteristic or photoelectric absorption predominates from 100 kv up to about 1 m.e.v.-energy x-rays. From 1 to 3 m.e.v. of energy the absorption and scattering are mostly Compton scattering. At about 1 m.e.v. a new process of x-ray energy absorption or scattering begins. This is the *pair production*, or the creation near heavy nuclei of fast positive and negative electrons. These quickly lose their energy to heat in the metal, or produce x-rays of much softer character. They thus scatter, absorb, and degrade energy very rapidly. The positrons produced also degrade to two 0.51 m.e.v. rays on slowing down and uniting with negative electrons. Hence with 4 m.e.v. energy x-rays the absorption by pair production in Pb is rapidly increasing and outweighs the other losses by a large factor. Other elements and metals show the decreases and increases at different values of energy than for Pb. This has some industrial importance. Today x-radiographic inspection of castings, etc., is becoming of greater and greater importance. Since radiography depends on differential x-ray absorption, and since much of x-ray absorption is caused by scattering, scattering is of importance to radiography. For the radiography of thick castings, therefore, while high-energy x-rays may be desirable in order to penetrate, there is an optimum value of the frequency for each metal at which absorption is at a minimum and beyond which it is not worth while to go. Thus whereas the x-rays from the 10- to 20-m.e.v. betatron may be excellent for the heaviest castings it will not pay to go to more energetic x-rays, and with Pb the best value is 4 m.e.v., but for brass it may be 7 m.e.v.

Refraction of X-Rays — Grating Wave-Length Measurement.

The fact that penetrating x-rays were scattered by matter as if the outer electrons in those atoms were free indicated why *refraction* of x-rays had not been successfully measured. If an attempt is made to *refract*, i.e., bend, light waves by a metal prism as normally done for light it would not succeed. The reason is that on interaction of light with a free electron surface there is a phase shift on reflection which gives a phase *advance*. Thus such waves *appear* to have a velocity *greater than light in the metal*. This means its index of re-

fraction is less than unity, i.e., $n = \frac{c}{v}$ is less than unity. Thus the relation, $n = \frac{\sin i}{\sin r}$, is less than 1. This means that x-rays incident

on a surface are *totally reflected* at certain angles, just as light moving in water is totally reflected at certain angles when it strikes an air-water interface, where its velocity is greater than in water. In optics total reflection occurs at angles when $\sin r = \frac{1}{n} \sin i$ is greater than 1.

If θ , the glancing angle, is used instead of its complement i , there is a total reflection at $\frac{1}{n} \cos \theta > 1$, or at $\cos \theta > n$. Now the value of

n is given by classic electromagnetic electron theory as $n = 1 - \frac{n^1 e^2}{2 \pi m \nu^2}$.

Here n^1 is the number of free electrons per cubic centimeter, e is the charge of an electron, m its mass, and ν is the frequency of the x-rays. Thus n will have values of the order of $1 - 5 \times 10^{-6}$ for substances like glass, using K x-rays of tungsten. These give angles for total reflection of the order of tens of minutes of arc.

Once this fact was realized the reflection of x-rays was looked for at small angles and observed. It was then realized that at small angles ruled gratings, such as used for optical spectra, act essentially as if they were ruled with very much closer spacing. Hence N. Carrara attempted to use a diffraction grating on x-rays at grazing incidence. In 1925 Compton and Doan, using some very fine Michelson gratings, succeeded in diffracting x-rays from ruled gratings.

With improving techniques over the years the accuracy of wavelength measurement of x-rays in the K region of tungsten has reached a point where the precision reaches, or exceeds, five significant figures. Thus while not comparable to the precision of optical wave-length measurements the grating x-ray wave lengths are among the best-known quantities.

Gamma Rays. It should be noted that in discussion here the term *x-rays* has been applied to short wave-length electromagnetic waves of wave lengths below 10^{-7} cm and extending from energies of 1000 e.v. to 4 m.e.v. Actually, with the discovery of radioactivity, Rutherford distinguished, besides the α rays (fast, doubly-charged helium atoms) and the β rays (fast electrons), what he called γ rays. These were rays more penetrating than the ordinary x-rays, though some radioactive γ rays were recognized as soft x-rays. When in 1916 Rutherford and Robinson succeeded in measuring the energies of γ rays by measuring the energies of fast electrons liberated by them from metals in a magnetic field, it was observed that γ ray energies from natural radioactive sources were, in general, in the neighborhood of 10^5 e.v. and above, going up to about 2 m.e.v. It is thus probably best that the word *x-ray* be used for the radiations characteristic of the K shells of the heaviest atoms and below, i.e., below perhaps 0.15 m.e.v., and that above this the radiations be called γ rays. There is a further justification for this, in that beyond the K shells of atoms there is no proper mechanism for generation of shorter

x-rays, and thus nuclear processes and electron deceleration *near nuclei* to give shorter wave lengths must be resorted to. In the nuclei the agencies giving rise to frequencies of the order of 1 m.e.v. or more are no longer electronic transitions.

167. APPROACH TO A DETERMINATION OF NUCLEAR CONSTITUENTS

It is hardly possible within the scope of this textbook to expect to cover the vast field of nuclear structure. Since some of the nuclear particles are electrical it is perhaps fitting that something be said concerning nuclei. It has been shown that the mass of the atom and its positive charge resides in the nucleus. It has been shown that up to within about 2×10^{-12} cm of a nucleus the Coulomb law of force predominates; that inside of this, however, very powerful and little-known forces of attraction occur which bind the ultimate nuclear particles together.

For a long time not much was known about nuclear forces or the ultimate particles. The reason for this lay primarily in the fact that the only probes available for penetrating into the nucleus were the α particles of radioactive substances. In fact, it was only the observations of these few unstable radioactive elements of high Z which gave us any information about the nucleus at all. Although radioactivity was discovered by Becquerel in 1896, later progress in its study was slow. The α , β , and γ rays were discovered and their character and properties were studied. Great advance became possible when Rutherford and Soddy put out the *theory of successive transformation* to account for the rise and decay of radioactivity in chemical separations and time studies of activity. Thus with the later work around 1913 it was indicated that unstable nuclei such as U^{238} , U^{235} (later identified), and Th^{232} broke down and emitted *either* He^{++} atoms (α particles) or electrons (β rays) from their nuclei in, respectively, three progressive decay series. In the process of any step in the series the value of Z changed by $-2e$ for α rays, or $+1e$ for β rays, and with it their chemical behavior changed progressively down the periodic table. When an α particle came out in a change the mass was reduced by approximately 4 atomic weight units with no notable change of mass on emission of a β particle. This allowed virtually all the radioactive substances discovered to be assigned to one of these three series.

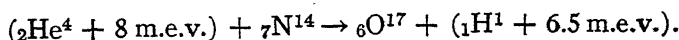
More progress resulted from a study of the emission and energy of γ rays as related to the associated α and β ray changes. With the scattering experiments of Rutherford and his pupils and the discovery of the nucleus real advance became possible. The application of these scattering measurement techniques to light nuclei for the first time permitted probing of the nuclei to proceed. For to get close to,

or inside, a nucleus the projectile particle, in this case α particles, must not only be aimed properly but they must have an energy great enough to approach within less than 2×10^{-12} cm of a nucleus. Now the nuclear charge repels the α particle with a force proportional to $\frac{ZeE}{r^2}$. Unless the α particle has an energy $\frac{1}{2} MV^2 \geq \frac{ZeE}{r_0}$ it cannot under the best conditions get closer than within r_0 of the nucleus. In fact, since the nucleus struck can also move the closest distance of approach is given by

$$r_0 = \frac{ZeE}{V} \left(\frac{1}{M} + \frac{1}{m} \right) (1 + \sec \theta).$$

Here V is the α particle velocity, M is its mass, m is the mass of the nucleus, and θ is the angle of deflection of the α particle. For the only fast natural radioactive particles available, $V = 1.922 \times 10^9$ cm per second, and, using He atoms as targets with $\theta = 180^\circ$, i.e., head-on impact, r_0 could reach 1.5×10^{-13} cm. But for atoms like Na with $Z = 11$ the best that could be hoped for was values of r_0 of the order of 2×10^{-12} cm.

Even with such particles Rutherford in 1919 succeeded in causing the N atom to be transformed on α particle impact. The reaction was the following:



The energies indicated are kinetic energies of impacting α particle and receding hydrogen nucleus. Following this discovery several of the lighter elements besides N were disrupted by α particle bombardment. This was the first indication that nuclei might contain the *elementary massive positive unit, the proton*, i.e., the nucleus of the hydrogen atom. It had always been believed that the proton was an ultimate building-stone of nuclei, but it had not been in evidence before.

The α particle and electron were observed to be emitted by radioactive nuclei and had thus been assumed to be nuclear constituents. Today it is realized that *neither α particles nor electrons exist in nuclei*. They are *created* in the nuclei just before they are emitted. As to *how* they are created is not known.

These early disintegration studies and the inadequacy of the projectiles available stimulated physicists to try to accelerate nuclei to high energies for further studies. These endeavors have resulted in the development of the cyclotron, the linear accelerator, the Van de Graaff static machine, the Cockcroft and Walton accelerator, the betatron, and now the synchrocyclotron and the synchrotron. These devices are discussed in section 175 following.

168. THE NEUTRON

In the meanwhile, α ray bombardment of Be and B nuclei yielded some very peculiar radiations. They had the penetrating power of some 10 m.e.v. γ rays. However, when they passed through materials containing H in an ionization chamber they *increased* the observed ionization instead of reducing it by absorption. Through a series of brilliantly conceived experiments Sir James Chadwick proved that these things liberated from Be by α particles were *neutral particles* of about the mass of H atoms. Chadwick called them *neutrons*. They were emitted from Be with some 7 m.e.v. of energy. Now a neutral particle can go right through the external electron shells of atoms and not disturb them. Both the radius of the neutron, $\sim 10^{-12}$ cm and that of the electron, $\sim 10^{-13}$ cm, make direct collisions between them unlikely. On the other hand, having no charge, the fast neutrons unlike electrons or α particles do not transfer energy to atomic electrons. Hence unless neutrons meet other nuclei they travel through what to us is solid matter as if it were not there. When, however, they strike nuclei, e.g., protons, He^{++} , etc., they can impart momentum and energy to them. If a proton were hit centrally head-on by a neutron the proton would be projected with all the velocity of the neutron while the neutron remained at rest. This sort of action accounted for the increased ionization observed when neutrons struck H containing materials near an ionization chamber. Using the tracks left in cloud chambers by atoms of H, He, N, O, etc., struck by neutrons, Chadwick was able to estimate the mass of the neutron as being very close to that of the H atom. Its more accurate mass evaluation will come later.

It was later found that neutrons were also produced when nuclei of heavy hydrogen, ${}_1\text{H}^2$ (deuterons), accelerated in the cyclotron struck atoms of heavy hydrogen with more than 2 m.e.v. of energy. Later, with the new nuclear projectiles neutrons were found to be produced in large quantities in quite a number of nuclear reactions. Since 1939 a new source of neutrons produced by the *fission* of heavy nuclei is available. In the chain-reacting uranium pile the mass production of neutrons goes on.

It was then possible to produce and study neutrons and neutron reactions at will. The experiments with neutrons are not easy. It is not possible to produce neutrons of unique velocities or energies as with charged particles, nor is it possible to confine them in directed beams. They can be detected by the ionization that fast neutrons cause light atoms like H or He to produce. They can also be detected by the fact that they are very readily absorbed by certain atoms, in some of which they produce nuclear instability and artificial radio-activity. They are absorbed by other atoms with emission of γ rays.

As neutrons have no nuclear charge they experience no repulsive

forces as they approach other nuclei. In fact, if they come sufficiently close they are strongly attracted and absorbed by the nuclei. Thus neutrons sooner or later will encounter a nucleus and if they have the right energy will react with it. In fact, the neutrons will react with most nuclei. The character of the reaction as well as its frequency or probability will depend on the speed or energy of the neutrons.

To describe nuclear reactions on matter produced by projectiles an experimental equation with an interpretation of it could be used. If a neutron or nuclear beam traversing a thickness of material is considered, the attenuation of the beam caused by absorbing reactions will follow a law $n = n_0 e^{-x/L}$. Here x is the thickness traversed, and n is the number of neutrons in the beam after traversing x out of the n_0 that started. The quantity L has the dimensions of a length characteristic of the absorption process. It represents the distance in which $\frac{n}{n_0}$ has been reduced to e^{-1} or $\frac{1}{e} = 0.368$.

For if $x = L$, $\frac{n}{n_0} = e^{-1}$. Now in the kinetic theory of gases the expression for the number of molecules out of n_0 starting that proceed x cm without an impact is given by precisely the same expression. L is there called the *mean free path*. It can be shown from a kinetic theory reasoning that $L = \frac{1}{\pi N \sigma^2}$. Here N is the number of molecules per cubic centimeter and σ is the extreme distance between molecular centers in collision or, in a nucleus, the extreme distance for reaction. Thus $\pi \sigma^2$ is the *molecular target area for collision*, or interaction. In a similar fashion, with N nuclei of material per cubic centimeter, the L in the equation makes it possible to evaluate $\pi \sigma^2$ and hence the *cross-sectional nuclear reaction areas*, or σ , the diameters for nuclear interaction.

For ordinary nuclear encounters the distance for interaction was given as 2×10^{-12} cm. Hence the cross section would be $4 \pi \times 10^{-24}$, or of the order of 12.5×10^{-24} cm². Nuclear and neutron cross sections for reaction will vary from about 10^{-27} cm², for very infrequent reactions, to 10^{-20} cm² for the very large cross sections exhibited by such elements as Cd, Gd, Sm, Eu, Hg, and B in reaction with neutrons. In general, cross sections for nuclear reactions are of the order of 10^{-24} cm² or less, except for resonance capture, where the values will be 10^{-22} cm². The values for resonance capture of neutrons can go as high as 10^{-20} cm² in elements like Cd.

Neutron reactions may be considered as: (1) *Fast neutron reactions*, $E \sim 1$ m.e.v. (2) *Resonance neutron reactions*, in which neutrons react with large σ within narrow range for certain speeds. These resonance neutrons have effective de Broglie wave lengths that

resonate with characteristic frequencies within the nuclei. The resonance speeds may vary from values corresponding to energies of 0.1 m.e.v. down to some tens of e.v. For example, ${}_{92}\text{U}^{238}$ captures 35 e.v. neutrons to give ${}_{93}\text{Np}^{239}$. (3) *Thermal neutron reactions.* In these the neutrons lose their energy by frequent collisions with light nuclei, e.g., in water or paraffin, until they have energies of the order of 0.03 e.v. This is the energy of thermal agitation at room temperatures. The paraffin can be cooled to liquid air temperatures at which thermal neutron energies are correspondingly lower. Some nuclear reactions go at high neutron energies whereas the same nucleus with thermal neutrons will exhibit entirely different behavior.

Since heavy nuclei will reflect most fast neutrons with little loss of neutron energy in collision, heavy elements act as *neutron reflectors*. If, on the other hand, it is wished to slow neutrons down to thermal velocities rapidly they would be allowed to traverse matter with light atoms, e.g. ${}^1\text{H}^1$, ${}^1\text{H}^2$, ${}^2\text{He}^4$, ${}^4\text{Be}^9$, ${}^6\text{C}^{12}$, etc. Then the neutrons will *share* their energy with each of the light nuclei struck. Such elements are known as *moderators*. To get an idea of how tenuous matter is for chargeless particles it is only necessary to say that the average free path for a neutron of 1 m.e.v. in Pb is of the order of 10 cm magnitude.

169. NUCLEAR CONSTITUTION

The discovery of the neutron plus the many new nuclear reactions produced when particles such as protons, deuterons, and helium nuclei of up to 30 m.e.v. energy could be hurled against atomic targets has given a fairly clear insight into nuclear constitution.

It is recognized today that all nuclei are composed of Z protons, p (hydrogen nuclei of charge $+e$ and mass 1.00813 atomic mass units), and $A - Z$ neutrons, n , of charge 0 and mass 1.00893 atomic mass units. Here A is the *atomic mass number*, that is, the atomic weight to the nearest whole number. The masses of proton and neutron have been derived from the energies involved in nuclear reactions, the isotopic atomic weights as determined by mass spectrograph, and the Einstein law of conservation of mass and energy, i.e., $E = mc^2$. It is seen that the *number of protons fixes the atomic number of the atom*, i.e., its place in the periodic table and its properties. *The number of protons plus the number of neutrons fixes the atomic weight, or, better, the atomic mass number.*

What binds protons and neutrons together is not known. The binding is ascribed to "*exchange forces*." These are great at 1×10^{-13} cm and may become negligible at $3 - 5 \times 10^{-13}$ cm. Attempts have been made to associate these forces with electrons, or with heavy electrons which hop across from neutron to proton, but with little success. All we know is that at close distances the forces are powerful and that they drop off rapidly with distance.

With the few particles, n and p , in a small nucleus there appear to be fairly clearly defined energy levels or states not unlike those in the extranuclear atom. In contrast, however, the nuclear energies are reckoned in the tens of m.e.v., whereas *extra nuclear energies* are in the tens of e.v. As the nuclei get larger and more particles are present the nucleus appears to have many more energy states closely spaced. In fact, the energies in the heavy nuclei are so closely spaced as to make nearly a continuum. They thus resemble more closely the energy distribution among the individual molecules in a small drop of water. The thermodynamics of such assemblies can almost be applied to the heavy nuclei. It must be added that nuclear γ rays come from *oscillations of the nuclear mass*, which with its protons constitutes a charged assemblage. The radiation is described as dipole, quadripole, or octopole radiation, indicating the form of electrical asymmetry responsible for the radiation. In general, in stable nuclei there are about as many protons as neutrons, e.g., in He, Li, C, etc. The heavier nuclei, however, have a considerable excess of neutrons. Thus ${}_{92}\text{U}^{238}$ has 92 protons and 146 neutrons.

It will be noted that in nuclear physics a notation is used in which a subscript *preceding* a chemical symbol fixes the nuclear charge or Z , while the superscript following the chemical symbol gives A , the atomic mass number.

Isotopes. Returning to the constitution of nuclei it is noted that a given element is at once fixed by the number of nuclear protons, Z . There is no stated limitation on the number of neutrons. Hence an element like H with $Z = 1$ could have 0, 1, 2, 3, 4, etc., neutrons, thus acquiring *mass numbers* $A = 1, 2, 3, 4, 5$, etc. By atomic mass number A , is meant the atomic mass to the nearest whole number. This possibility readily accounts for the isotopes. In fact, there are in existence the elements ${}_1\text{H}^1$, ${}_1\text{H}^2$, the latter being the deuteron discovered by Urey, Brickwedde, and Murphy in 1932. The hydrogen isotope ${}_1\text{H}^3$ has been created but is unstable with a half-life of thirty-one years, emitting an electron and becoming ${}_2\text{He}^3$. All other isotopes of ${}_1\text{H}$ are unstable. Likewise, ${}_2\text{He}^3$ and ${}_2\text{He}^4$ are stable isotopes of helium. The isotopes ${}_2\text{He}^2$ and ${}_2\text{He}^5$ are so unstable that they have never been created, while ${}_2\text{He}^6$ has been made but has a half-life of 0.8 second going to a ${}_3\text{Li}^6$ isotope with beta-ray emission. It thus appears that there are certain limits to the ratios of protons and neutrons in stable isotopes of the various elements.

With the aid of nuclear reactions nuclei of atomic numbers from 1 to 96 have been obtained today. Thus it is known that a few elements such as atomic numbers 61, 85, and 87 have *no* stable isotopes, though the unstable elements have been created. Some elements like ${}_9\text{F}^{19}$, ${}_{11}\text{Na}^{23}$, ${}_{13}\text{Al}^{27}$, and ${}_{15}\text{P}^{31}$ have only 1 stable isotope. Other elements like ${}_{50}\text{Sn}$ have 10 isotopes ranging in mass from $A = 114$ to $A = 124$, whereas ${}_{54}\text{Xe}$ has 7, ${}_{60}\text{Nd}$ has 7, and ${}_{80}\text{Hg}$ has 6. The other

elements fall in between these extremes. Beyond ${}_{83}\text{Bi}^{209}$, which has 1 stable isotope, no element is stable. All are radioactive and many of them belong to one of four progressive disintegration series which begin with curium ${}_{96}\text{U}^{238}$, ${}_{92}\text{U}^{235}$, or ${}_{90}\text{Th}^{232}$, and end in Bi, ${}_{82}\text{Pb}^{206}$, ${}_{82}\text{Pb}^{207}$, or ${}_{82}\text{Pb}^{208}$, respectively.

170. ARTIFICIAL RADIOACTIVITY

The stable isotopes of an element may be next each other in atomic weight or may be interspersed with atomic weights having unstable configurations. In any case, above and below the atomic weights of the stable group of isotopes the isotopes are unstable ones. Many of these have been created. Assume an unstable nucleus, ZA , with Z protons and $A - Z$ neutrons. If element $(Z + 1)A$ is stable a neutron in ZA will emit a *negative electron* and yield element $(Z + 1)A$, with $(Z + 1)$ protons and $[A - (Z + 1)]$ neutrons. If element $(Z - 1)A$ is stable with $(Z - 1)$ protons and $[A - (Z - 1)]$ neutrons ZA can change to this by changing a proton to a neutron. It can do this by *emitting a positive electron* from a proton. It can also do this by *absorbing a negative electron* from the extranuclear K shell (K capture) into a proton. The two processes are alternative with different probabilities in different cases. If ZA is unstable while both $(Z + 1)A$ and $(Z - 1)A$ are stable, stability can be achieved by any one of three reactions. Thus there would be *competing nuclear reactions*. This is in essence the nature of the *artificial radioactivity* first observed by the Curie-Joliot in 1934. It is widespread in products of nuclear transformation among elements of atomic numbers below 83. Achievement of nuclear stability by neutron emission is relatively rare for elements of low A but becomes frequent for heavier nuclei with a larger n to p ratio. Elements above 83 are naturally radioactive. Some of these can by nuclear reactions be caused to change from one transformation series to another. These elements usually do not reach stability in one step but are required to degrade by a series of transformations to a Z below 83 for stability.

There are now 96 known elements with some 260 stable isotopes and so far some 340 unstable isotopes known. Of the stable isotopes those with even numbers of protons and neutrons predominate over the total of those with odd numbers of protons and even numbers of neutrons, or even numbers of protons and odd numbers of neutrons, in the ratio 152 to 55 to 54. About 85 per cent of the earth substance is made up of atoms of even Z and A and most of them have a value of Z below 20.

171. MASS ENERGY RELATIONS — NUCLEAR STABILITY

Another aspect of nuclear constitution now presents itself. Since the mass of the proton is 1.00813 atomic mass units while that of the

neutron is 1.00893 atomic mass units, it should be possible to calculate the mass of any nucleus, for atomic nuclei are composed of Z protons and $A - Z$ neutrons. When this calculation is attempted it is found that the sum of the masses of neutrons and protons *exceed the observed isotopic atomic weight M* , as given by mass spectrograph, by a small but definite amount B . That is,

$$1.00813 Z + 1.00893 (A - Z) - M = B.$$

Hence when neutrons and protons associate under the powerful exchange forces to give a nucleus there is a *loss of mass*. This *mass loss can only appear as radiation or energy*, as given by the Einstein law

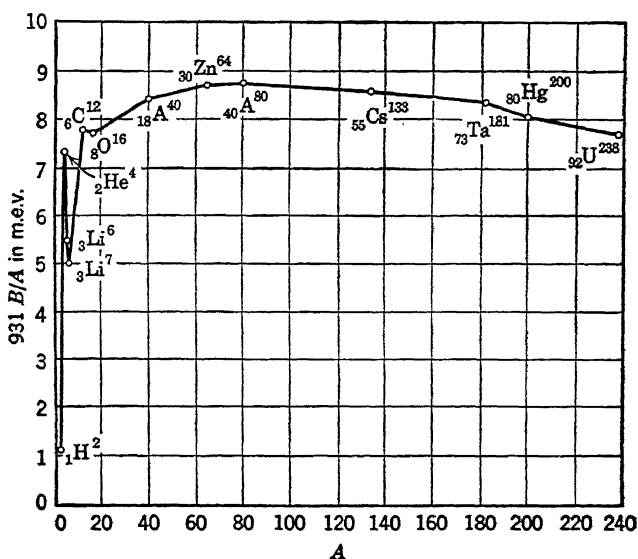


FIG. 187. Plot of binding energy in m.e.v. per nuclear particle in nuclei of the elements plotted against atomic mass number.

$E = mc^2$. Since B is always positive it measures, in *mass terms*, the heat or energy of formation of the nucleus from its constituents. To convert B to energy in terms of m.e.v. it is only necessary to remember that 1 atomic mass unit according to Einstein's law was 931 m.e.v.

Hence when a nucleus is formed from protons and neutrons the energy $931 B$ m.e.v. is given out. Thus $931 B$ measures the *negative potential energy of the nucleus*, i.e., its *stability*. To pull the nucleus apart to its constituent parts would take an impact in excess of $931 B$ m.e.v. of energy.

As an example, the He atom can be used. It is made of 2 protons and 2 neutrons and the value of M observed is 4.0028 atomic mass units. Hence $2(1.00813) + 2(1.00893) - 4.0028 = 0.03132$. Thus

$B = 0.0313$ mass units and $931 B = 27$ m.e.v. of energy. If 4 grams of ${}_2\text{He}^4$ are created from neutrons and protons 190,000 kwh of energy would be obtained. This is a process which, by means of the so-called carbon cycle, occurs in the interior of our sun and many stars under the pressures and temperatures there obtaining. It is the source of the sun's energy. This process cannot occur on the earth's surface because its nuclear particles are too sparse and their energies are too low.

If calculations such as for He are carried out for all elements the isotopic atomic weights M of which are known the *stability chart* can be plotted for the elements. That is, $931 B$ could be plotted against A . Actually, this is not too significant, as B increases with the number of nuclear particles. More significant will be $931 \left(\frac{B}{A}\right)$, i.e., the *binding energy per nuclear particle*. This is plotted against A in Fig. 187. It is seen that for low A the stability oscillates through wide limits, but as an A of about 20 is approached it reaches values of nearly 8 m.e.v. per particle. It then slowly increase to nearly 9 m.e.v. per particle for A above 60 and remains nearly constant up to values of A in the neighborhood of 150. Thereafter, it declines to values near 7.5 m.e.v. with U. The curve is of great interest.

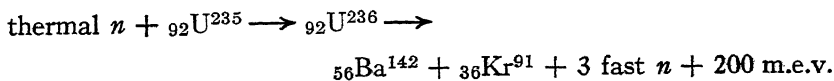
172. NUCLEAR ENERGY SOURCES AND FISSION

Mankind has for ages sought for cheap sources of power. While chemical energy comes to around some few e.v. per reacting atom it is noted that nuclear energies are on the scale of m.e.v. per reacting nuclear particle. Thus ever since the discovery of radioactivity it has been man's desire to utilize this compact source of energy. Fig. 187 shows that nuclear energy can be gained in two ways.

The first way of gaining nuclear energy is by *synthesizing the more stable atoms of A between 60 and 140 from lighter atoms*, i.e., making more stable atoms from less stable lighter ones. The energy gain per nuclear constituent could be very large, but there would be few such particles. This is the source of energy in the sun and the stars. On the earth this source of energy is difficult to achieve. The reason is that the reacting-nuclei must be brought close enough together against coulomb repulsive forces between the Z 's to cause them to react. The energy to do this can probably be gained from the new devices. But at present the energy expended to produce a reaction is greater than the energy gained. This follows since for every nucleus synthesized, of the order of 10^4 to 10^7 projectiles of high energy would be wasted in achieving this because of poor target practice. For example, consider a well-known reaction. It requires 1 m.e.v. protons to disrupt the ${}_3\text{Li}^7$ nuclei to give two ${}_2\text{He}^4$ nuclei, gaining 16 m.e.v. in the process. But the ${}_3\text{Li}^7$ nuclei are hard to hit — they are small. In trying to find an Li nucleus in a lattice of Li metal the proton projectiles will en-

counter the *electrons* of the *extranuclear atom*. The cross section for ionization of these electrons is of the order of 10^{-14} cm² or less. The cross section for nuclear reaction is probably 10^{-24} cm². Thus *the greater proportion of the protons will fritter away their 1 m.e.v. of energy in removing extranuclear electrons of Li metal*. In fact, it takes some 20,000 protons of 1 m.e.v. to create one nuclear explosion in Li, in which 16 m.e.v. are gained. This is not a profitable *gain* of energy. It is, however, the trouble with all nuclear *synthetic* reactions known today. In the sun the temperatures in the interior are so high that the "temperature ionization" has removed all the extranuclear electrons of all the atoms. Nuclear target practice in the sun is good.

The second way of gaining nuclear energy comes by using the other end of the curve of Fig. 187. For if an atom like ${}_{92}\text{U}^{238}$ could be split into two atoms of atomic weight 119 around 1.5 m.e.v. per nuclear particle could be gained. This would yield about 180 m.e.v. per atom. For a long time it was believed impossible to achieve this process because of the very poor target practice with nuclear particles. However, in 1939 Otto Hahn discovered that neutrons fired into U seemed to give radioactive Ba. As a result of the combined work of many theoretical and experimental physicists, following Hahn's announcement, it became clear that the neutrons were causing ${}_{92}\text{U}^{235}$ to split into two parts. The two parts were of unequal atomic mass, but both lay in the region of stability. It further became clear that in the process large amounts of energy were liberated. While only one in perhaps thousands of neutrons of appropriate velocity caused the reaction, there was a saving circumstance. This was that the neutron-proton ratio in ${}_{92}\text{U}^{235}$ is 143 to 92, whereas in the more stable split products the ratio is 86 to 56. That means that there are *excess neutrons* formed in the splitting process. This nuclear splitting process differs from previously observed nuclear reactions. It is an expected consequence of the behavior of the "water-drop" model Bohr evolved for heavy nuclei, which occurs when Z approaches 100. It does not occur below ${}_{90}\text{Th}^{232}$. It was called *fission* by the physicists. A typical fission reaction is that given by



It must be added that ${}_{56}\text{Ba}^{142}$ and ${}_{36}\text{Kr}^{91}$ are highly unstable nuclei that reach stability by a series of successive β decay reactions, with much γ ray emission.

It was at once seen that if a large mass of ${}_{92}\text{U}^{235}$ could be obtained so that all neutrons generated by fission could be retained, they would ultimately slow down and produce more fission. This would then produce an *explosive* chain reaction. For if 1 neutron were added to such a lump it would produce fission and 3 more neutrons. These

in turn could cause 3 atoms to fission and give 9 neutrons and so. In fifty steps 3^{50} atoms would have fissioned. This amounts to the atoms in 235 grams of ${}_{92}\text{U}^{235}$. The energy liberated in this process would be 1.5×10^6 kwh. If liberated in 10^{-6} second this would give a super-explosion.

On the basis of this reasoning, threatened with attack by such explosives made by the Axis powers in World War II, American scientists, engineers, and industrialists in an unprecedented cooperative effort produced the atomic bomb and the *uranium-plutonium fission pile liberating in a controlled reaction enormous amounts of nuclear energy*. The bomb assisted in the defeat of Japan and in ending the war. The problem of utilizing the nuclear energy for peaceful purposes still lies in the future. As far as it is possible to foretell at present, the intense γ and neutron radiation liberated in the fission process limit the applicability of this sort of power to very large and centralized power plants.

173. CHARACTER OF NUCLEAR REACTIONS

It is seen that the study of nuclear process has resulted in some revolutionary industrial, or technical, developments. The world of physics is still, however, very far away from knowing much about nuclear forces or structure. There are already some of the tools for study at hand. We are developing new and more powerful ones. Up to the present nuclei can be bombarded with ${}_1\text{H}^1$, ${}_1\text{H}^2$, ${}_2\text{He}^4$, n , and γ rays in energies ranging from 1 m.e.v. to 40 m.e.v. in some cases. In the resulting interactions we can also get out ${}_1\text{H}^1$, ${}_1\text{H}^2$, ${}_2\text{He}^4$, n and γ rays. This gives some twenty-five different reactions in which, in theory, any given nucleus could be involved. With more powerful accelerators other projectiles of larger Z could be added. The energies up to the present amount to some 40 to 50 m.e.v. α particles or protons or 20 m.e.v. deuterons. Actually, however, all five agents named above do not react with a given nucleus, nor are all the five agents emitted in the transformations of one nucleus. With, however, 260 stable isotopes to work on, the possibilities for reactions to be studied are quite extensive.

In addition, unstable isotopes can become stable by emitting positive or negative electrons, so that the emission of electrons from nuclei as another mode of reaction must be added to the twenty-five possible reactions just mentioned. Again, 1 fast neutron can cause a heavy nucleus to emit 2 or more slower neutrons. Then again, a very few of the elements, ${}_{92}\text{U}^{235}$, ${}_{91}\text{protoactinium}^{231}$, ${}_{92}\text{U}^{234}$, and ${}_{94}\text{plutonium}^{239}$, fission with thermal neutrons, whereas ${}_{92}\text{U}^{238}$ and ${}_{90}\text{Th}^{232}$ fission rarely (small cross sections), with fast neutrons. There are also instances on record where unstable nuclei have become stable by capturing a negative electron from the K shell of electrons about the nucleus instead of

emitting a positron. Thus on occasion electrons can enter nuclei as reacting particles through a process known as *K capture*.

174. OTHER ULTIMATE PARTICLES

Despite all of the sources of information at hand today, we still know virtually nothing of nuclear forces, nor do we know all of the ultimate particles of nature. Familiar to us as a result of what has gone before are the proton, the neutron, and the positive and negative electrons. Besides these other particles are known to exist. One of these has been observed in cosmic rays. This is the mesotron or meson. It can have a positive or negative charge. It has a limited lifetime of some 2×10^{-6} second, particularly if its motion is not fast. It has a mass of some 200 electronic masses. It is produced in some sort of powerful nuclear encounters in the outer atmosphere. It transforms into a positive or negative electron and a burst of γ ray energy when it decays. It is claimed that mesons have been produced in the 100 m.e.v. betatron.

In addition to the mesotron, physicists have a most unpleasant and ghostly sort of a particle to deal with. In all beta-ray changes the beta rays, positive or negative electrons, escape from the nuclei with an energy comprising a *continuous spectrum of values*. Now nuclear energies are very precisely governed and are accurately equal in nuclei of the same Z . The isotopic atomic weights are quite precisely constant as are the energies of emission of α particles, γ rays, etc., in nuclear change. Where α rays, etc., are not emitted with uniform energies, groups of them will have different energies. The differences between energy groups are *always compensated for* by subsequent or preceding γ ray emissions. Thus beta rays appear to escape with all sorts of energies under a certain maximum energy in any reaction, and yet *the nuclei are all left with the same final energy*. Again, when beta rays escape, the atomic recoil does not indicate conservation of nuclear momentum with escaping β rays alone. Thus it appears as if in beta-ray change the basic conservation laws were failing. As a result, W. Pauli, Jr., proposed that when *beta rays escape from nuclei there also escapes a neutrino*. This *neutrino has no charge*. It does have a *mass that is almost entirely relativistic*. It escapes simultaneously with the beta ray, but in the opposite direction. Between them, the neutrino and the beta ray carry off the balance of the total energy of the reaction, and conserve momentum. Since the neutrino does not cause ionization, or ion trails in cloud chambers, it can have no charge. There are many indications that the neutrino, or its ghost-like equivalent, exists. Recent careful studies on the recoil of nuclei with beta-ray emission confirm its existence.

What the future holds in the discovery of new particles no one can guess. It is at this stage of ignorance that the discussion of nuclear constitution and atomic structure must be left.

175. THE ACHIEVEMENT OF HIGH-ENERGY PARTICLES

The problem of the production of high-energy particles can be attacked in two ways. The first is to generate high potentials and let these act on the particles in vacuum. The other is to give the particles a succession of accelerations of relatively low potential. The first method is the most obvious one, but it runs into serious difficulties. These have to do with electrical breakdown. In theory, with simple systems the electrical breakdown depends on the field strength X . Thus if a high potential, V , is to be worked with X can be kept low by increasing the distances d involved. In practice, the breakdown does not work out in this fashion. The reason is that with high potential high stresses can develop at unsuspected places, largely because there are very few perfect insulators and air under high stress is particularly unsatisfactory. Charge therefore creeps over surfaces and builds up local breakdown stresses. In practice, it has been impossible to insulate for total potential differences of more than 2 million volts in air at atmospheric pressure, no matter what the distances practically achievable. At higher pressures, especially if the air is contaminated by halogen containing gases like "Freon," which have high-breakdown field strengths, the limit at present is some 4 million volts. Thus the high-voltage generating devices which developed in the years 1928-1932 never achieved really high-energy particles, nor were they particularly successful in application. They may be listed as follows:

1. Insulated transformers arranged in cascade on ordinary 60-cycle alternating current. Two million volts between two insulated sections were obtained at Stanford. One million volts obtained at California Institute of Technology has been applied across a long tube for generation of x-rays and later fast protons.

2. J. D. Cockcroft and E. T. S. Walton utilized the principle of charging capacities in parallel and discharging them in series in order to build up potential (see page 225). At the Cavendish Laboratory at Cambridge they did this in a continuous fashion, using electron tubes. They generated about 1 million volts. With this they accelerated protons and disrupted ${}^7_3\text{Li}$, getting a seventeenfold energy gain for each disrupted nucleus. This was the first all-manmade nuclear transformation. The so-called surge generator, using the same principle without vacuum-tube switching, has successfully been used by General Electric and Metropolitan Vickers research laboratories to give high potentials simulating lightning discharge breakdown. Several million volts have been obtained in this fashion.

3. Van de Graaff built a very powerful static machine (see page 174). With this up to 4 m.e.v. particles have been obtained in a pressure chamber with Freon gas at Westinghouse Research Laboratories.

The other approach to the high-energy particle problem was under-

taken by E. O. Lawrence and his students, and was later developed by others. The various devices developed will each be presented in detail in sections 176–179.

176. THE LINEAR ACCELERATOR

At an earlier period German scientists had proposed that particles could be accelerated by giving them successive shots of energy. In Fig. 188 an arc source of ions is shown with a slit and initial accelerating field. These ions, for example Hg^+ from an arc A (Fig. 188), with initial energy E_0 enter the highly evacuated chamber C in which there are a series of coaxial sections of tube, $T_1 - T_9$, one after the other with small gaps between them. These tubes are

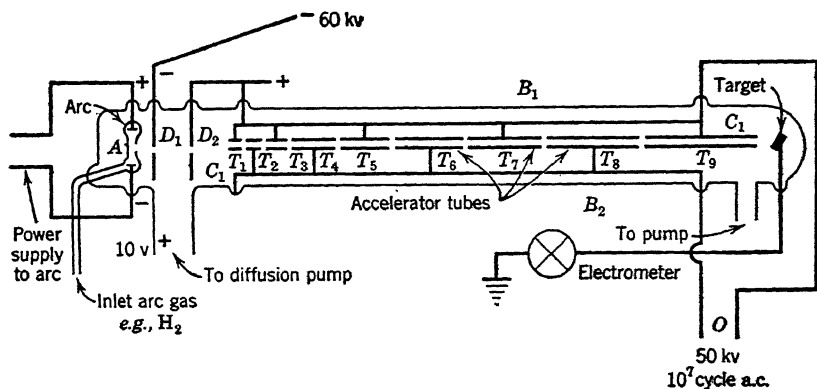


FIG. 188. Pre-radar linear accelerator.

connected alternately to one of two bus bars, B_1 and B_2 , going to two sides of a high-frequency oscillator, O , giving, say, 50 kv and some 10^7 cycles. Consider a group of the ions in the beam from the source that enters the gap between the diaphragm D_2 with energy $E_0 = 60$ kv and the first tube T_1 , as T_1 is at its negative peak and D_2 is positive. They receive a slug of, say, 50 kv of energy in addition to their initial energy of 60 kv at D_2 . If now while they traverse T_1 the potential changes so that they reach the gap between T_1 and T_2 at the time when T_1 is a maximum positive and T_2 negative, then they will get another slug of energy of 50 kv. This can go on indefinitely for the number, n , of tubes in series. In Fig. 188 there are nine gaps and the final energy will be 510 kv. The ion will then emerge with an energy $E = E_0 + 50n$, where n is the number of accelerating gaps.

Now as increasing the energy of material particles speeds them up, $v \propto \sqrt{E}$, the time of oscillation $\frac{T}{2}$ being constant, it is clear that the length l of the tubes must increase. With $v = \frac{2l}{T}$, $v \propto \sqrt{E}$ and with

T constant l increases as \sqrt{E} . Again, frequencies having enough energy to charge up the system were restricted to about 10^7 cycles until World War II. Thus the tubes had to be long, large l , and the energy gain was limited, by relatively few stages of T_1 , T_2 , etc. Accordingly, Coates in Lawrence's laboratory in 1937 finally achieved Hg^+ ions of about 2 m.e.v. in a tube some 20 feet long. Mercury was used since these were the slowest ions, requiring the least length of tube of any ion available. To have obtained 2 m.e.v. protons would have taken a tube fourteen times as long. At radar frequencies of 10^{10} cycles the same tube today would with adequate energy give 17 m.e.v. protons. It is feared, for reasons to be given, that the ion current at such energies would be very weak.

The system of this sort encounters other difficulties, as has been mentioned. In the simple theory above the ideal case of ions at the peak of the cycle only was considered. Now for a few accelerations with finite tube spacing, etc., there can be some latitude in timing for the band of ions that stay *in step* and get through with full acceleration. As more and more steps are added, however, the timing allowance becomes narrower and the current in the high-energy beam falls off rapidly. Stated otherwise, owing to various causes, among them focusing of the beam, a certain fraction of the slug of ions initially in step is lost at the crossing between successive tubes. Thus with many steps the beam will become too weak to use. As indicated, the attenuation is related to the question of magnetic and electrostatic focusing of the beam. In consequence of many studies since 1938 concerning what is called phase stability (i.e., keeping the particles in step) and focusing, in the cyclotron and other accelerators, it has been discovered that in the type of linear accelerator described above not much more can be achieved with any beam intensity, despite increased frequencies, than was achieved by Coates. This ignores the question of the problems encountered when particles begin to achieve relativistic velocities. The relativity correction becomes serious for electrons of the order of 2×10^5 e.v. energy and will become serious for protons of about 9 m.e.v. energy, or α particles of 36 m.e.v. energy.

Since World War II sources of high-frequency power delivered in bursts have been developed for radar. These give electromagnetic waves of from 1 to 10 cm with considerable energy. The frequencies corresponding to these are 3×10^{10} and 3×10^9 cycles per second, respectively. Furthermore, the great experience now gained with cavity resonators which can be made to replace the tubes T_1 , T_2 , etc., of the old linear accelerator brings promise of the ability to coordinate in the same device the conditions for maximum phase stability and maximum focusing. In the old accelerators the conditions for maximum phase stability were diametrically opposed to maximum focusing and with any great increase in energy there would have been a devastating reduction in beam intensity. In consequence, the Radiation

Laboratory at the University of California under the direction of Luiz Alvarez is now building up a linear accelerator using radar oscillators and cavity resonators capable of giving 100 m.e.v. at the outset. The particular value of the linear accelerator lies in the fact that the cost of construction goes up only in direct proportion to the increase in energy achieved. In order to reach the particle energies of 300 to 1000 m.e.v. needed to unlock the doors to a knowledge of the nature of the ultimate particles, it will later be seen that such considerations of cost are essential.

177. THE CYCLOTRON

As early as the year 1928 E. O. Lawrence was attempting to devise suitable high-energy generators based on the principle of repeated acceleration, for he foresaw the difficulties that would be encountered by the high-voltage generators. He also saw that the difficulty with the linear accelerator lay in the length of the path needed since the first high energy linear accelerators were being developed under his direction. In an attempt to solve the difficulty of the long tube it occurred to him that it might be a good idea if the particles could have their paths bent into a spiral since this did not require so much space. To force charged particles into a spiral or circular path requires a magnetic field at right angles to the beam. Lawrence therefore considered the equations for a particle moving in a circular path in a magnetic field and receiving increments of energy every time it crossed from one equivalent of his linear accelerator tube to the next. To his delight he discovered that as the energy of the particle increased, its radius of curvature of path increased, but the *time taken to make the circle was constant*. This meant that it was possible to use an oscillator of fixed period to accelerate the particles in a confined space and add many more increments of energy than was possible in a linear accelerator. The instrument devised and named the *cyclotron* by Lawrence operates as follows:

Consider a pillbox of the order of a meter in diameter and 4 cm deep. It is made of sheet metal and sawed in half across a diameter. The two D-shaped halves shown as D_1 and D_2 of Fig. 189 are insulated and attached to two sides of a high-frequency oscillator giving a potential V_0 of from 20 to 50 kv at 10^7 cycles. These D 's are inserted in a larger evacuated chamber placed between the poles of a large electromagnet. This has pole faces larger than the D 's and gives a uniform magnetic field normal to the D 's of the order of 10,000 oersteds. Near the center and between the D 's is a hot filament emitting electrons. The pressure in the tank is low but not so low as not to leave enough residual gas of the character desired to give some ions by electron bombardment of the gas atoms. The gases used are He, H_2 , and H_2^2 .

At the time of peak potential between the D 's, ions are created by ionization of the gas and the positive ions are accelerated to the D which is negative. Once inside the D they follow a semicircular path of radius ρ (fixed by the field H and their initial velocity), and emerge at the slit between D 's at a certain time later. If in this time the D they are in becomes positive and the opposite one negative, then the ion hops across the slit, getting the full energy due to the potential difference between the D 's. The ion then starts a path with an in-

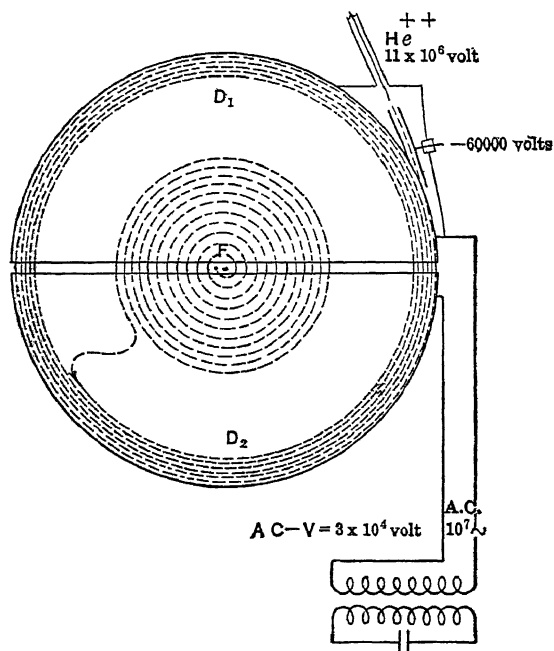


FIG. 189. Schematic diagram of the cyclotron.

creased radius of curvature ρ and arrives at the opening between the D 's in just half a revolution. For what Lawrence had discovered was that once an ion had made its semicircle in a D and emerged just in time for a maximum potential it would continue to arrive at the slits, irrespective of velocity and path radius, ρ , just at the same time in the cycle. That is, once in phase an ion *would remain in phase* and could be accelerated until the radius of its path reached the limit of the magnetic field, or the sides of the pillbox.

This condition may be seen at once from the equations (some of which are by now quite familiar to the student). The centrifugal force in an orbit of radius ρ is $f = \frac{mv^2}{\rho}$. In a magnetic field H this

force just equals the force Hev on the electron by the field. Here v is the velocity of the particle, in this case a positive ion of ${}^1\text{H}^1$, ${}^1\text{H}^2$, or ${}^2\text{He}^4$; m is its mass, and e is its charge. Thus since $\frac{mv^2}{\rho} = f = Hev$ it follows that $v = \frac{He\rho}{m}$. The time of a half-circle is then t and this is related to the velocity v by $v = \frac{\pi\rho}{t}$. Thus

$$\frac{\pi\rho}{t} = \frac{He\rho}{m}, \text{ or } t = \frac{\pi m}{He}.$$

It is seen that t depends on H , e , and m . As long as these are constant, t is constant. Thus an ion once in step remains in step with the high-frequency alternating potential of period T . If m varies because of relativistic effects then the constancy of time t does not hold. Actually, it has been possible to carry heavy ions right up to the relativistic threshold in energies by altering the field H at the edge of the magnet by using soft-iron shims. For a full period of the alternating potential applied to the D 's $T = \frac{2\pi m}{He}$. This is the fundamental equation of the cyclotron.

The gain in energy of the ion is then $E = 2nV_0e$, neglecting the small initial energy of the particle that started it on its expanding orbits. Here V_0 is the peak value of the high-frequency potential applied to the D 's the form of which is $V = V_0 \sin \frac{2\pi t}{T}$. While then

E is given by $E = 2nV_0e$, and while V_0 is limited by the character of the generator and the electrical characteristics of the D circuit it is not V_0 that determines or limits E . Actually, for each E there is a corresponding velocity v and a value of the radius of path curvature ρ . The larger the ρ the larger v and hence E . There are, however, physical limitations, determined primarily by cost, which influence the size of ρ and of H . Since E depends in the first instance on $H\rho$ it is desirable to make these as large as possible.

Now to get a magnet with a 10-inch gap that will give a uniform H of the order of 10,000 oersteds over pole pieces some meter in diameter requires a very massive and expensive layout in the form of a magnetic circuit and copper for the windings. It is thus the value of $H\rho$ obtainable with any given financial budget that determines the limit in energy available. To evaluate this the maximum ρ_0 is calculated

that can be obtained for a given H . Now since $\frac{1}{2}mv^2 = E$, $v = \sqrt{\frac{2E}{m}}$, and $\frac{e}{m} = \frac{v}{H\rho_0}$ it follows that $\frac{e}{m} = \frac{\sqrt{2E/m}}{H\rho_0}$. This at once yields $E =$

$\frac{1}{2} H^2 \rho_0^2 \frac{e^2}{m}$, with E in ergs if e is in e.m.u. Expressed in volts $E_{\text{volts}} = \frac{1}{3.2 \times 10^{-12}} H^2 \rho_0^2 \frac{e^2}{m}$. Since ions of different charge and mass are being dealt with, if the ionic charge is z electrons, i.e., 1.6×10^{20} z e.m.u., and A is the atomic weight in terms of hydrogen, the equations become $E_{\text{volts}} = 4.825 \times 10^{-5} H^2 \rho_0^2 \frac{z^2}{A}$. This makes E_{volts}

for proton and α particle $4.825 \times 10^{-5} H^2 \rho_0^2$ and for the deuteron $2.412 \times 10^{-5} H^2 \rho_0^2$. If H is made about 10^4 , $H^2 \rho_0^2$ becomes, respectively, 2.03×10^{11} and 5.62×10^{11} for a 36- and a 60-inch diameter cyclotron. This would give 9×10^6 and 27×10^6 e.v. protons or α particles. In actual practice, values somewhat higher than those indicated have been obtained by increasing H . The 60-inch cyclotron, with improved fields, at the Crocker Laboratory in Berkeley gave some 40 m.e.v. α particles toward the end of World War II.

From the value of E set by the field and the physical size of the cyclotron the number of turns around the cyclotron made by the ion in getting its energy can be calculated for $E = 2 n V_0 z$. If z is 2, as for α particles, and V_0 is 20 kv, then $n = \frac{27 \times 10^6}{4 \times 2 \times 10^4} = 338$ for the 60-inch cyclotron. To get the frequency needed the equation $T = \frac{2 \pi m}{H e} = \frac{2 \pi z}{9.65 \times 10^7} = 6.5 \times 10^{-8} z$ is used. The frequency for

particles with $H = 10^4$ and $z = 2$ is then $N = \frac{1}{13 \times 10^{-8}} = 7.7 \times 10^6$ cycles. This defines the general operating conditions. Actually there are a number of complicated considerations. There will be less dispersion of ions from the beam, and thus stronger beams, the smaller the number of times the particle has to go around. It is thus desirable to make n as small and V_0 as large as possible. Again the frequency N cannot entirely be chosen at will. Once decided on it is more or less fixed by circuit constants for optimum output. Thus some final adjustment must be made in varying the field H to bring out the beam at a maximum.

To take the particles out of the beam two electrostatic deflecting plates are placed at the side of the D 's tangential to the beam, as shown in Fig. 189.

When the various disturbing factors are considered it seems remarkable that the cyclotron works at all. The factors such as scattering by molecular impacts, straying of the particles in phase, and dissipation of the beam by magnetic and electrical defocusing could cause a complete failure of the device for high energy particles. By chance, however, the design of the D 's and the character of the magnetic field exerted a strong focusing action. Energy loss from col-

lisions in the gas at the rapidly acquired high energies were a minimum. Thus with remarkable engineering skill and sheer hard work and patience the present 60-inch cyclotron was made to give 200 microamperes of deuterons at 16 m.e.v. and 50 microamperes of α particles at 32 m.e.v. In contrast, Ra C' gives 3.7×10^{10} α particles per second per gram of radium in equilibrium with an energy of 8 m.e.v. Thus the cyclotron gives 32 m.e.v. particles in a narrow (3 cm²) directed beam equivalent to all the α particles emitted over all directions from 4 kg of radium C' at 8 m.e.v. of energy. It would take 2000 kg of Ra to give a *directed* beam of similar intensity at one-fourth the energy.

The extension of the cyclotron to 184-inches diameter for achievement of energies of 100 m.e.v. presents a formidable problem, first, because of the relativistic corrections needed, and second, because of defocusing and loss of intensity in the beam with many more revolutions. In 1940 it was planned to alter the shape of the field H near the edges to take care of the 12-per-cent relativistic change in m . It was planned also to make D voltages, V_0 , of the order of 1000 kv to reduce the value of n and lessen the defocusing.

178. THE SYNCHROTRON AND THE FREQUENCY-MODULATED CYCLOTRON

These alterations became unnecessary with the discovery of the principle of the frequency-modulated cyclotron and of the synchrotron developed quite independently and nearly simultaneously during World War II by Veksler in Russia and by E. M. McMillan at the University of California.

The investigations hinged on the question of phase stability for particles in going around the many times required to get the high energies. If the sinusoidal potential wave of the alternating current which accelerates the particle between the D 's is regarded it is noted that in normal operation the particles caught in step with the crest, A , of the wave between the D 's are utilized (see Fig. 190). If now a particle is on the crest and somehow is perturbed, i.e., by change of m due to relativity, by collision, or otherwise, it will drop out of the beam as it will not appear at the gap in time to get its full energy in later transits. If m changes with v , or energy, the ion will progressively drop behind, gaining less energy at each step. For as m increases relativistically, t increases according to $t = \frac{2\pi m}{He}$ and the particle will then be lost from the beam. This will be especially true for high energies and many revolutions n , when m changes relativistically.

If, however, a particle in the relativistic region crosses the D 's as the wave is passing through zero phase at B in Fig. 190, it will always *remain in phase* with B . It will, however, *gain no energy*. For if it

gets ahead in phase to *C* in Fig. 190, it gains energy. Then relativistically m increases and the period of circulation given by $t = \frac{2\pi m}{He}$ is increased. This retards the ion and passes it back in phase to *B*. The same applies for particles which drop *behind* in phase to *D*. Hence particles in phase at the point *B* on the wave remain locked in phase and real phase stability exists.

Now this does not give accelerated particles, as a particle must lie above *B* to gain energy. However, for phase-stabilized particles the following procedure can be applied to give them energy. If H is increased for such particles in such a fashion that the particles stay in the phase stable region, keeping $T = \frac{2\pi m}{He}$ constant, the only way in which the particle can remain locked in phase is to increase m , that is,

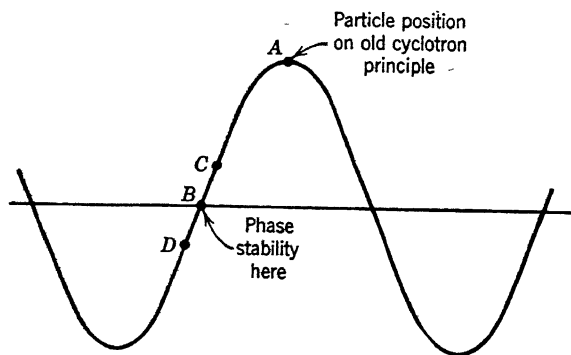


FIG. 190. Positions of particles in the cyclotron and phase stability.

to increase the energy. What this means is this: Increasing H will start to throw the particle above *B* in phase since t is decreased. If it is near *B*, that is, if the change in H was slow and small enough not to change t too much, then the particle gains energy to remain in phase. That is, slowly changing H relative to the 10^{-7} second involved in oscillation will cause the particle to oscillate about *B* in phase in such a fashion as to remain in phase stability but gaining energy. The equations have been set up and solved by McMillan and give rather large latitudes for practical work. The energy gain in increasing H comes from the induction in the particle orbit which is discussed under the betatron. The same result can be accomplished by changing T , keeping H constant.

This principle of phase stability of McMillan and Veksler has several applications in the achievement of high energies. It can be applied directly to electrons and especially protons in the relativistic range. The device is then called the *synchrotron*. It can also be

used with the cyclotron to make it adaptable for smaller relativistic increases for heavier particles. For the large relativistic changes it is best to vary H slowly, say 60 times a second, and use the quarter cycle when H increases. This is the condition of operation for the *synchrotron*. For application to the 184-inch cyclotron where relativistic mass changes of only 12 per cent are expected it suffices to alter T . The device is then called the *synchrocyclotron*, or, better, the *frequency-modulated cyclotron*. It must be observed, however, that both of these devices operate on a principle of phase stability and *not on the principle of the cyclotron*, which is that of constant t .

At McMillan's suggestion the 37-inch cyclotron was frequency-modulated to see whether particles would gain energy. The capacity of the oscillator in that cyclotron was varied by having it a rotor with a large number of radial plates about its circumference. As it rotated the plates were alternately between two sets of fixed stator plates, thus giving a maximum of capacity, or in the open between two sets of stators with minimum capacity. Thus the capacity and hence T was altered some 1000 times a second. This is very slow compared to the 10^7 oscillations per second of the circulating ions in the D 's. With this method, despite a purposely distorted magnetic field, within about six weeks of work protons of 7 m.e.v. were obtained. The currents obtained although about 0.1 of those achieved with the cyclotron were still encouraging.

If applied to electrons the device as indicated has still greater possibilities. It can be used to accelerate electrons by keeping T constant and varying H instead. The orbits approach a constant radius. A constant radius permits use of a magnet with pole pieces in the shape of an annulus, if electrons can be injected at high initial energy. The use of such pole pieces allows the energy of the particles to be increased up to 300 m.e.v. at less cost than would be required for the betatron, which gives 100 m.e.v. electrons, since the total mass of iron needed increases directly with the radius whereas in the betatron it increases as the cube of the radius.

McMillan proposes for electron acceleration a *peak* field of 10,000 oersteds, a final ρ_0 of 100 cm, and an injection energy of electrons at 300 kv, with an initial orbit 78 cm in radius. The actual magnet will be of 1 meter radius but only 22 cm wide. The total flux will be one-fifth that of the 100 m.e.v. betatron for the same energy. The potential, V , on the accelerating electrodes depends on the rate of change of H . For a 60-cycle variation of H , V will be 10,000. This should give about 300 m.e.v. electrons.

Both the *synchrotron* and the *synchrocyclotron* thus will operate in modulated bursts as H or T is altered from 60 to 1000 times a second. The currents will not be as great as those achieved in the cyclotron. However, the energies achieved will be of such magnitude as to make important studies possible.

179. THE BETATRON

The final high-energy device to be discussed is the betatron developed by Kerst at Illinois in 1938. It is now available in commercial production to give appreciable quantities of 20 m.e.v. x-rays. There is one device built by the General Electric Company giving up to 100 m.e.v. electrons and x-rays. The principle utilized differs from the cyclotron and the synchrotron and has definite pedagogical value in that it is a *magnetic induction accelerator*.

Consider a charge e moving with velocity v in an orbit of radius ρ in a magnetic field H with a total flux of ϕ maxwell's, or lines, included in the area of the orbit. This charge e in such an orbit is equivalent to a current in a circular loop of radius ρ . If ϕ is changed, lines of force must cut the *equivalent conductor* and generate an e.m.f. in the wire. Such an e.m.f. will accelerate electrons in a wire and increase the current. If the current is instead an electron constrained to move in a circular orbit of *fixed radius* in vacuum by the magnetic field, nothing is altered. The electron acted on by the e.m.f. will increase its velocity or better its energy. Now if the relativistic range of electron velocities is being considered, v is so near c that actually v will not change much, but E , the electron energy, and hence its relativistic mass, will increase.

The equations must now be considered. Expressing e in electromagnetic units, the velocity of light, c , commonly seen in the equations may be omitted. The condition for the orbit of radius ρ is as before

$$\frac{e}{m} = \frac{v}{H\rho}, \text{ or } mv = eH\rho. \text{ The electrical field acting on the electron in}$$

the orbit to accelerate it when ϕ changes will be $X_\phi = \frac{\text{e.m.f.}}{2\pi\rho} =$

$$\frac{1}{2\pi\rho} \frac{d\phi}{dt}, \text{ for } d\phi \text{ lines cut the single wire circuit in } dt \text{ second. If now}$$

the magnetic field is designed in a special way ρ can be maintained constant. Then as ϕ changes the change will not alter ρ but will merely be expended in increasing the electron energy or mass. Since

$$\rho = \frac{mv}{eH} \text{ and } e \text{ is constant } \rho \text{ will remain constant at } \rho_0 \text{ if } \frac{mv}{H} \text{ is constant.}$$

Thus if $\frac{mv}{H}$ is kept constant the electromagnetic induction increases mv

and hence the electron energy. The orbit is $\rho = \rho_0$ and is constant. The condition for this situation can readily be computed. The force on the electron is $f = eX_\phi$ during the change of flux. Then from

$$\text{Newton's second law it can be written that } eX_\phi = f = \frac{d(mv)}{dt} =$$

$$e \frac{d\phi}{dt} \frac{1}{2\pi\rho}. \text{ This yields at once } mv = e(\phi - \phi_1)/2\pi\rho_0 = eH\rho_0. \text{ Thus}$$

the condition is that $H = (\phi - \phi_1)/2\pi\rho_0^2$.

Therefore the change in flux through the orbit must be twice that which would occur if the magnetic field were uniform in space. This is clear at once since if H is uniform in the orbit $\phi - \phi_1 = \pi \rho_0^2 H$. The solution of this equation indicates, however, that $\phi - \phi_1 = 2 \pi \rho_0^2 H$ if the orbit remains at ρ_0 . This lays down the requirements for a betatron magnet. There must be a strong over-all field H to give a large flux ϕ and the field must be specially shaped, being strong in the center and weaker at the edges. The energy given the electrons is easy to calculate and does not require that ΔE be determined from

$$\Delta E = \int E' i dt = \int e N \frac{d\phi}{dt} dt,$$
 where N is the number of revolutions per second. All that is needed is to note that the final magnetic field H , resulting from ϕ , is acting on an orbit of radius ρ_0 . Thus the final energy must be given by $H\rho_0$. For electrons discussed on page 426 it was shown that the relativistic equation allows one to write $H\rho = \frac{1}{3} \times 10^4 \sqrt{KE(KE + 1.02)}$, with KE in m.e.v. Thus the energy is

$E = 3 \times 10^{-4} H\rho_0 - 0.51$ in m.e.v. In Kerst's first betatron he had $\rho_0 = 7.5$ cm and $H_{\max} = 1200$ oersteds. Placed in the equation this should have given 2.2 m.e.v. electrons, and such were observed.

The practical design and operation of a betatron, or induction accelerator, faced great difficulties. The electrons are injected with an initial high energy from outside of ρ_0 . The injector shoots electrons into the field in which ρ_0 misses the injector. The field H was changed by a 600-cycle-per-second alternating current passed through the windings by induction from a primary wound around the core. That is, the main field H was constant and $\frac{dH}{dt}$ was superimposed by a 600-

cycle alternating current by induction. The 600 cycles was used in order to get a large enough change in flux superposed in this fashion. In the ascending quarter phase, while H went from H_1 to H , in each of these 600 cycles per second the electrons in the orbits ρ_0 were accelerated to $H\rho_0$. Thus the bursts of high-energy electrons occur at the end of each ascending-current quarter phase, 600 times a second. Aside from the specially shaped magnetic field some device had to be arranged to get the electrons out of ρ_0 to strike a target when they had reached maximum energy. This was accomplished by having part of the flux through the center of the orbit pass through disks of easily saturated soft iron. Thus on increasing H , toward the end of the acceleration the field changes in shape. H is increasing, but with saturation ϕ cannot increase as rapidly as H . Thus from the relation it is seen that $\phi = 2 \pi \rho_0^2 H$ requires ρ_0^2 to decrease on saturation. The electrons then shrink in their orbits and strike an inner target, giving x-rays.

It is seen that the betatron requires exceptionally fine magnetic design. Its success despite all the difficulties, which have no place

for presentation here, speaks well for the engineering skill of the designers.

180. CONDUCTION IN GASES

Thus far, the conductivity of metals, ionic solids, and solutions has been dealt with. Very little has been said about the conductivity of gases. This subject is one of vital importance industrially. We live in a gaseous atmosphere and most of our electrical systems are bathed in gases. Thus the insulating properties of gases, and especially their failure, are of paramount importance to industry as a limitation on the potentials that can be used. Furthermore, gaseous conductors have certain properties that make them invaluable for certain purposes. The gas-filled luminous signs, the fluorescent lamps, the mercury-arc rectifiers and converters, arcs, and the gas-filled relay switches that make radar possible, all are applications of conduction through gases.

1. Gases as Insulators. Normally gases are practically perfect insulators. In the gaseous state atoms or molecules are free. The dielectric constant is very near 1 and the thermal energies of impact are low, of the order of 0.03 e.v. at 0° C. Thus there is no way in which a gas can become a conductor unless some agency can give electrons in atoms or molecules the ionizing energy. As seen before, this ionizing energy ranges from 3.5 e.v. for Cs, which is usually not present in gases under normal conditions, to around 15 e.v. for most molecular gases, and up to 24.5 e.v. for He gas.

2. Rendering Gases Conducting. To render a gas conducting it must be subjected to bombardment with electrons usually in excess of 10 e.v. energy, to illumination by ultra-violet light usually of energy more than 10 e.v., to the action of x-rays, or α rays from Ra, or the gas must be subjected to temperatures usually in excess of 1200° C. Such energetic agencies are in general not plentiful in Nature unless mankind intervenes.

The natural radioactivity of the earth and of impurities in metals, together with the few cosmic rays traversing space, do cause some removal of electrons from atoms and thus ionization of the air. This amounts perhaps to 20 electrons per cubic centimeter per second from all causes. Thus usually with loss by recombination there may be present some 1000 ions of each sign per cubic centimeter at sea level at any time. The current due to 20 ions per centimeter per second was too small for measurement and study until improved techniques following the era of x-rays made this possible. However, Coulomb in 1785 suspected that the loss of charge from a sphere in air was faster than could be accounted for by conduction over the silk suspension. It was also known that heated gases and flames conducted electricity. In fact, William Gilbert had noted this in 1600. The specific conductivity of a saltless flame around 2000° C is 5×10^{-6}

mhos/cm, whereas if it has a concentration of 6.28×10^{14} Na atoms per cubic centimeter sprayed into it the specific conductivity is 1.4×10^{-4} mhos/cm. The temperature of the flame with the Na atoms is such that an occasional atomic encounter between fast atoms can ionize the low ionizing-potential metal atoms present.

The fact cannot be escaped, however, that, given a high enough accelerating field to cause ions to multiply, normal air although a splendid insulator has always enough ions or electrons present to start a breakdown.

When x-rays and radium became available for rendering the air conducting rapid progress was made in the understanding of its character. X-rays act by removing either Compton electrons, which are not very copious ionizers since these recoil electrons are of low energy, or by removing *K* or *L* electrons. Since x-rays from almost any anticathode like W, or Mo, have far more energy than the *K* or *L* electrons of the gaseous atoms, the electrons escaping from the atoms can ionize many times. Thus the average W x-ray will on ionizing O or N atoms give some 59,000 e.v. of excess energy to the photoelectron it ejects. As this electron wanders through the gas it tears out electrons from any atoms coming some 10^{-7} cm or less from its path. On the average it will knock out an electron for every 33 volts of energy it has to expend. The difference between 33 volts and the actual ionizing energy of some 16 volts is squandered in exciting atoms to radiation or molecules to vibration. Thus one such x-ray photoelectron will produce some 1200 positive ions and secondary electrons. Some of the secondary electrons get enough energy to form ions of their own.

3. Gaseous Ions. The electrons removed from molecules in air will wander around rapidly for a while but will eventually attach to some O_2 molecule, giving an O_2^- ion. In pure N_2 , H_2 , He, Ne, A, etc., electrons will not attach to form ions. In such gases they will diffuse very rapidly because of their small size and light mass. Thus they reach the walls and electrodes and leak to earth. Positive ions being larger and very massive diffuse much more slowly.

Usually gases are not pure enough to have electrons permanently free. Since electrons make of the order of 10^{11} or more impacts a second with gas molecules at 760-mm pressure and room temperature, it is a very, very pure gas in which they cannot, within a tenth of a second, find one molecule like O_2 , SO_2 , or Cl_2 , and countless organic molecules to which they can attach. Hence under most conditions it will be found that electrons do not exist long in a free state in gases near atmospheric pressure. Thus in general *positive and negative ions* are found in a conducting gas. In electrical discharges and in high electrical fields at low pressures the electrons move too violently to be caught and here positive ions and free electrons must be dealt with.

In very pure gases in 10^{-5} to 10^{-4} of a second after they are formed

it is found that the ions, positive and negative, are just the positive remainders of the ionized atoms or molecules, or the molecules and atoms of the gas that have picked up a negative electron. After longer intervals the molecules have picked up some impurity or other for which the sign of charge has a particular affinity. Thus positive ions have a high affinity for NH_3 , ethyl ether, and HCN , whereas negative ions react strongly with such molecules as SO_2 or alcohols to form complex ions. Thus aged ions may be complexes of one or two molecules added to the originally ionized molecule. In some few instances one or two polar molecules such as NH_3 , HCl , or H_2O will attach to form complex ions. Usually the complex ions are characteristic of the sign of charge and some specially active impurity present. The difference in affinity of positive and negative charges for different impurities causes the positive and negative ions to have somewhat different sizes in different gases. Thus in some gases the positive ion is smaller, in others the negative ion is smaller.

Ions can be studied by their velocities in gases. There are many ways of measuring the velocities. One method starts the ions at one surface and uses an alternating potential, preferably of square waveform, to accelerate them to a collector or pull them back. By increasing the potential until they can just cross the distance d to the collector in one half-cycle, $\frac{T}{2}$, the velocity can be measured. Velocities are

strictly proportional to the field strength X . Thus $v = kX$, where k is a constant called the *mobility*. It differs for positive and negative ions in the same gas because of differences in make-up with the impurities mentioned. Its value ranges from a few tenths of a cm/sec per volt/cm for organic vapors up to about 10 cm per second for H_2 at atmospheric pressure and 22°C . In very pure gases in short time intervals values of 26 cm/sec per volt/cm have been observed in He at atmospheric pressure. The mobility in a given gas for greater ages (in excess of 0.01 second) has little meaning as it depends on unsuspected impurities beyond control.

For formed ions k varies inversely as the gas density, ρ , i.e., $k = K \frac{\rho_0}{\rho}$. If temperature is constant, k varies inversely as the gas pres-

sure, p , i.e., $k = K \frac{760}{p}$, where p is in millimeters of Hg. K is called the *reduced mobility* or the *mobility constant*.

Mobility is somewhat insensitive to changes in temperature if density remains constant. The reason for this temperature stability is that most of the resistance an ion encounters in the gas is caused by the dielectric attractive forces of its charge on gas molecules. Only part of its resistance results from direct physical impacts. Thus while increased temperature decreases mobility by increasing the rate of

collision it decreases the forces causing collision. In this respect there is some similarity between gaseous and solution ions.

4. Ion Recombination. If positive and negative ions are left in a gas by themselves, the charges will neutralize by what is known as *recombination of ions*. The rate of disappearance dn in time dt is given by an equation of the form $dn = -\alpha n^2 dt$. Here n is the number of positive and negative ions per cubic centimeter. Usually $n_+ = n_- = n$ because ions are generated in the gas from neutral gas molecules. The equation can be integrated and leads to the relation, $n =$

$\frac{n_0}{1 + n_0 \alpha t}$. Here n is the number of ions per cubic centimeter after t seconds left out of the n_0 which were present at $t = 0$. While the process appears simple, both conditions of measurement and the changing character of carriers (electrons or ions) in gases leads to some seven different theoretical expressions for α characterizing different processes. For ordinary ions at 0°C and 760 pressure α is of the order of 10^{-6} . For positive ions and free electrons it is of the order of 10^{-10} . In most electrical fields used the loss of ions by recombination is relatively small.

5. The Formation of Negative Ions. The formation of negative ions from electrons and gaseous molecules, or atoms, by attachment is a most complicated process. There are various different mechanisms depending on the character of the molecules and electron energy. In general, attachment is a chance process, being governed by an equation of the form $n = n_0 e^{-hN dx}$, where h is the probability of attachment and N is the number of impacts between electrons and molecules per centimeter that could end in attachment. h may be dependent on electron energy. This chance process of electron attachment makes measurements complicated. All the electrons do not attach at once and because of rapid electron diffusion there is much straggling and spread of electrons and ions in time and space in electrical fields.

6. Electron Velocities in Fields in Gases. The behavior of free electrons in fields in gases, where they do not attach, is complicated. The electrons move very swiftly at random, colliding in the order of 10^{11} times or more, per second, with molecules and being deflected. Superposed on this *random* motion is a directed *drift* velocity in the field direction produced by the force Xe of the imposed field. The electrons further gain energy from the electrical field and lose energy in impacts with molecules. The number of impacts per second is a complicated function of the energy of the electron and the molecular species and varies differently in the various gases in a given energy range. In consequence, the energy of electrons in an electrical field is a complicated and varying quantity with field strength X and pressure p . It is usual in such investigations to study the average velocity and energy of the electrons in terms of $\frac{X}{p}$, where X is the field strength

and p is the pressure. Since the experiences of electrons individually is so varied uniform energies cannot be expected and the individual electrons have their energies distributed about a mean value which is a function of $\frac{X}{p}$. The distribution of energy among the electrons is not even of the orthodox form usually observed in energy distribution and the *form* in many instances even varies with $\frac{X}{p}$.

When $\frac{X}{p}$ reaches such values that electrons can suffer exciting and ionizing impacts the energy functions change again. Methods of measuring the energy distribution of electrons without disturbing the distribution do not exist. In simpler cases the distribution of energies can be computed.

As a result of this situation the electrons move in the field with a drift velocity v which is not proportional to X , or to $\frac{1}{p}$, as is the case for ions. If $K = \frac{v}{X} p$ is computed, as with ions, K decreases as $\frac{X}{p}$ increases. v increases with $\frac{X}{p}$ but more slowly than with $\frac{X}{p}$. Measurements of v over a large range of $\frac{X}{p}$ have been made on type gases so that v is known. It is very much higher than for ions. At 760 mm and 0° C, v in N_2 is of the order of 10^4 cm/sec per volt/cm compared to 2.2 cm/sec per volt/cm for negative ions. Theory for the velocity below excitation energies is in good agreement with experiment.

7. Ionization Currents in Gases. When an ionized gas is placed between two plates in a parallel-plate condenser system and a uniform field of strength X is applied a current flows as long as the ionization of the gas continues. Thus a gas subject to ionization by a beam of x-rays, or a section of a flame, or a gas ionized by application of a high field when placed in an electric field will transmit a current as long as the ionization continues. Unlike aqueous solutions, where large quantities of ions are present and their motion is very slow, *ion currents in gases, for X running some tens or more of volts per centimeter at 760 mm will not obey Ohm's law*, that is, the current i will not be proportional to the applied potential V . At very low values of X or V , i will vary as V . As V increases, i increases more slowly than proportional to V and eventually approaches a *saturation*, or constant, value. The cause for this is not difficult to understand. Gaseous ions move rapidly and most sources of gaseous ions do not furnish great ion densities. Thus x-rays give 10^7 ions per cubic centimeter per second under the very best conditions, in gases. In discharge tubes ion concentrations of 10^8 to 10^9 per cubic centimeter occur. Contrast

this with the 2.7×10^{19} ions per cubic centimeter in a solution of 58 grams of NaCl in 22.41 liters of water with those ions moving at the speed of 0.00035 cm per second in unit field. Thus in gases, except for relatively small losses by recombination and diffusion, as the fields increase ions are dragged out to the electrodes nearly as fast as formed. As the speeds increase with increasing V and X , less and less loss by recombination and diffusion occurs. Finally, in *theory* at least, a limiting saturation current i_0 should be reached in the gas when *all ions are removed as fast as created*, with no loss.

There is, however, another circumstance which limits gaseous ion currents, it was discussed on page 134 in relation to solutions. This is the difference of ionic mobilities. In gases like air, where the values of the mobility are 2.2 cm per second for negative ions and 1.6 cm per second for positive ions in unit field, the effect is not pronounced. Even then, however, there will be an excess of positive ions at the cathode and an excess of negative ions at the anode. These excesses will be different in magnitude because of the differences in mobilities. In any event, they will create *positive space charges* at the cathode and *negative space charges* at the anode. The electrical fields will, therefore, no longer be uniform between the electrical plates. The field X_c at the cathode will be smaller than X_a at the anode, while in mid-gap the field X_0 will be weak over a considerable range and less than X , the uniform applied field. Thus it will happen that $X_a > X_c > X > X_0$. These actions tend to reduce the current flow, as potential increases, relative to the Ohm's law case with uniform potential gradient, or field strength. The effects are very pronounced where there are electrons and positive ions with widely differing mobilities, as in most arcs and glow discharges. The relation between current and potential usually found has the form, $V = Ai + Bi^2$. The Ai is the Ohm's law term and the Bi^2 comes from space charge effects, rapid removal of carriers, etc. If i is small and B is small, then near $i = 0$, V is nearly equal to Ai , so that $\frac{V}{i} = A$, or constant, i.e., Ohm's law is approached.

8. Ionization by Collision. If V is increased with a given ionization between plates as just indicated, the current parabolically approaches what appears to be saturation. When, however, high values of the electrical field X are reached the current again begins to increase. The region where this increase occurs is dependent on pressure and on the nature of the gas used. In general, when electronic phenomena in gases are being dealt with the ratio $\frac{X}{p}$, field strength in volts per centimeter divided by gas pressure in millimeters, is used as the significant variable. Thus in air at $\frac{X}{p} = 20$, in H_2 at $\frac{X}{p} = 10$, and

in pure A and Ne at $\frac{X}{p} = 5$ and 2, respectively, the increase alluded to sets in. The increase in current rises asymptotically into an exponential increase. It is terminated at some value of $\frac{X}{p}$ and at a very high relative value of i by a spark or a sudden electrical breakdown.

The phenomenon was explained by J. S. E. Townsend as follows. If the starting point is a current of i_0 photoelectrons liberated from the cathode, the current i received at the anode is given by $i = i_0 e^{\alpha x}$ after traversing x cm of gas-gap between the plates. The constant α is known as the first Townsend coefficient. It is evaluated by obtaining the slope of the $\log \frac{i}{i_0} - x$ curve. From the logarithmic form of the

equation above it is seen to be given by $\log \frac{i}{i_0} = \alpha x$. The coefficient α

is found to vary with $\frac{X}{p}$, in terms of a relation of the form $\frac{\alpha}{p} = f\left(\frac{X}{p}\right)$.

This function, $\frac{\alpha}{p} = f\left(\frac{X}{p}\right)$, follows a peculiar curve shaped like a

slanting S. It starts to rise asymptotically at the values of $\frac{X}{p}$ given

above for the different gases. Its increase is first exponential, in $\frac{X}{p}$,

then more nearly parabolic. It passes through a linear increase at a

point of inflection and then flattens out parabolically about the $\frac{X}{p}$

axis to a nearly constant value at the top of the S for high values of $\frac{X}{p}$.

The meaning of α is simply explained. As $\frac{X}{p}$ increases, some of the

electrons in the energy distribution gain enough energy in the field, despite losses, to ionize the gas molecules. Then if one electron is started at the cathode it may, after going a distance Δx , create a new electron by ionization. In the next interval Δx , both the original and the new electron, create electrons. Then there will be 4 electrons. These 4 in the next interval Δx will create 4 more electrons, and so on. Thus the electrons will double in number for each Δx covered. This ionization process is called *cumulative ionization*. It leads to the equation

$i = i_0 e^{\alpha x}$ with the distance given above $\Delta x = \frac{1}{\alpha}$, that is, α is the

number of new electron-ion pairs created per centimeter of progress by one electron in the field direction.

$\frac{\alpha}{p}$ has been evaluated for various gases and is an important feature

in all calculations of break down in gases. In *divergent* fields where X varies very rapidly with distance x these quantities α for uniform fields *can no longer be used*. New quantities giving i in terms of i_0 are being obtained for application in such cases.

If now the $\frac{i}{i_0} - x$ curves are studied as $\frac{X}{p}$ increases further, it is found that the curves rise still more steeply than given by $e^{\alpha x}$. This steep increase occurs at larger values of $\frac{X}{p}$ only. It cannot be followed experimentally for a large range of changes in x because a spark occurs very shortly after it begins. At lower values of $\frac{X}{p}$ the $\frac{i}{i_0} - x$ curves terminate abruptly in a spark without the abnormal increase described above.

Townsend explained the rapid increase to higher currents by assuming that a new process began to occur for high $\frac{X}{p}$. At that time he thought that the *positive ion could cause ionization by impact with molecules*, and ascribed the new effect to ionization by positive ions. Later research has shown that positive ions can indeed ionize, but not below an energy of some 70 e.v. Even then, the ionization is a highly selective process, possible only between certain ions and atoms. In gaseous discharges of the Townsend type *positive ions* do not get much more than 2 to 10 e.v. at most and thus cannot ionize. It turned out, however, that several other secondary processes producing the same effect can occur. Two of the most prominent are (1) the liberation of secondary electrons from the cathode by the impact of positive ions, and (2) the liberation of electrons from the cathode by photoelectric action. The photons come from the gas excited by electron impacts accompanying the ionization. Such processes give an equation for $\frac{i}{i_0}$ quite similar to that observed by Townsend at higher $\frac{X}{p}$ and experimentally indistinguishable from it. The equation for positive ion bombardment as a process of liberation reads $i = i_0 \frac{e^{\alpha x}}{1 - \gamma e^{\alpha x}}$. Here γ is the *chance* that one of the $e^{\alpha x}$ positive ions created by a single electron in going a distance x will on arrival at the cathode liberate a new electron. The quantity γ and its equivalent for photoelectric effects have been measured and studied for several gases and values of $\frac{X}{p}$. In general, γ increases with $\frac{X}{p}$ but begins at much higher values of $\frac{X}{p}$ than does α . It begins at about $\frac{X}{p} = 200$ in N_2

with Pt cathode and at about $\frac{X}{p} = 400$ in H_2 with Pt cathode. Photoelectric effects set in at very low values of $\frac{X}{p}$. In N_2 they appear at $\frac{X}{p} = 40$ and in H_2 at $\frac{X}{p} = 5$ with Na-coated cathodes. The phenomenon is, however, very much affected by absorption of photons in the gas. These phenomena are very important in the breakdown mechanism of gases.

9. Spark Breakdown. If the equation for i is regarded in terms of i_0 at low pressures with high $\frac{X}{p}$ using the γ it is noted that in $i = \frac{i_0 e^{\alpha x}}{1 - \gamma e^{\alpha x}}$ the value of i becomes indefinitely great if $1 - \gamma e^{\alpha x}$ approaches 0. This indicates that a *stationary current* i could be maintained without an external source of i_0 photoelectrons from the cathode if $\gamma e^{\alpha x} = 1$. This merely means that if the chance γ that a positive ion created in the gap will liberate a *new* electron from the cathode, multiplied by the $e^{\alpha x}$ electrons created by an *initial* electron, is less than 1, there will be no self-sustained current. And i_0 externally created electrons will be needed to give the steady state current i indicated above. If, however, $\gamma e^{\alpha x} = 1$, then once a current is started it will be self maintaining without any i_0 . If $\gamma e^{\alpha x}$ is greater than 1, the *current will cumulatively and spontaneously grow*. Townsend's steady state equation cannot apply to either of these conditions. The growth when $\gamma e^{\alpha x} > 1$ will build up space charges in the gap. Now it is axiomatic in discharge theory that when potentials, or fields, reach such values that a given current condition increases the ionization and builds up space charges some new and more efficient mechanism giving currents at lower voltage will occur. Thus it is not surprising that when V or X reach values V_s , or X_s , such that $\gamma e^{\alpha x}$ for a gap of length x is greater than unity, space charges will develop and the Townsend current will break down to give a glow discharge at low pressures, or an arc at higher ones. This breakdown is irreversible. It can proceed gradually but if $\gamma e^{\alpha x}$ is even slightly greater than 1 it takes place abruptly, i.e., in 10^{-5} second or less. Such a transition in its broadest meaning defines an electrical spark. In some instances the breakdown is noisy, destructive, and explosive in character because of the great amount of energy liberated in a small volume, at other times it is a sudden current transition indicated by the flip of a meter needle.

Thus an electrical spark is defined as an irreversible transient phenomenon marking the transition from one current mechanism in a gas to another at higher currents. The condition $\gamma e^{\alpha x} = 1$ then sets the threshold value, through the value of $\frac{X}{p}$ and hence of V necessary

to permit such a transition to occur. It is *not* directly connected with the equation $i = i_0 \frac{e^{\alpha x}}{1 - \gamma e^{\alpha x}}$, which applies to a steady state current.

By chance, however, it *coincides* with the condition for an indefinitely great i .

The character of the growth and development of a spark has been a source of study for many years. It appears now that at lower pressures and gap lengths the mechanism is that indicated by Townsend's equations, involving a γ and the criterion given above. The time intervals observed for the development of the spark in such low-pressure breakdown correspond to those calculated on the basis of the movement of the slow positive ions across the gap x and the building up of space charges.

When, however, it was attempted to apply this theory to sparks at atmospheric pressure, in other words, the common spark familiar to all, difficulties were encountered. In the first place, at lower $\frac{X}{p}$ and higher p sparks take place without any γ or its equivalent. Sparks and discharges take place from a highly charged positive point. Again the time for sparks at atmospheric pressure to build up is of the order of 10^{-7} second, smaller by a factor of 100 to 1000 than the time for positive ions to cross a centimeter of gap and build up a space charge as Townsend's theory demands.

The mystery was solved nearly simultaneously by Loeb and Meek at California and by H. Raether in Germany. Loeb had arrived at the streamer theory, which Meek placed on a quantitative basis through studies on the positive point corona where no cathode mechanisms occur. Raether arrived at his conclusions by the study of suppressed discharges, using timed impulses of short duration and studying the results by the cloud tracks produced by adiabatic expansion of the gas saturated with water vapor.

The mechanism is as follows. A photoelectron starts across the gap from the cathode in the high field. When it arrives at the anode x cm away it has created $e^{\alpha x}$ electrons and ions. In the nature of the process

50 per cent of the ions are created in the last ionizing path $\Delta x = \frac{1}{\alpha}$ cm

from the anode. Seventy-five per cent are created in the last $2 \Delta x = \frac{2}{\alpha}$ cm

from the anode. The rapidly moving $e^{\alpha x}$ electrons created are swept into the anode leaving behind half of the slowly moving $e^{\alpha x}$ positive ions in an exceedingly minute volume. At atmospheric pressure this volume is 0.4 mm long and perhaps 0.1 mm or less in radius with 1.2×10^7 ions. The dense positive ion charge produces an additional space charge field at its tip of value X_1 . This adds vectorially to the existing field X in the gap. Photoelectrons created by the excitation accom-

panying the creation of 1.2×10^7 ions is very intense. In a mixed gas like air, or any relatively impure gas, the photons created can ionize some of the *molecules in the gas*. Absorption is rapid for such wave lengths as ionize, so that the photoionization is produced in a relatively confined volume near the tip. As a result around the tip of the space charge photoelectrons are created in a field $X + X_1$. These begin to ionize very effectively and create more ions as they are drawn to the tip. The electrons newly created move up the positive space charge to the anode and the streamer has thus advanced in length into the gas. This process keeps up until the fields $X + X_1$ get too weak to advance the streamer, or until the streamer approaches the cathode.

The streamer constitutes a nearly conducting channel. As it approaches the cathode it seriously distorts the imposed field X . A terrific burst of photoelectrons from the cathode and their progeny bridges the space between cathode and space charge and a heavy potential wave runs up the channel. This causes the intense ionization, illumination, and heat characterizing the spark. In a sense the positive streamer acts like an initial tear in a mechanically stressed medium. As it advances it concentrates the electrical stress in a narrow region leading to easy breakdown.

The time scale of the streamer breakdown is that of the crossing of the first electron and its progeny, viz., about 10^{-7} second for a 1-cm gap in air at normal pressure. The streamer travels by photoelectric action plus electron motion in enhanced fields. Its velocity is of the order of 10^8 cm per second, somewhat faster than the initial electron. The final potential wave that lights up the channel travels at very high speeds, of the order of 10^9 cm per second or more. Thus the spark develops within time intervals of the order of electron travel across the gap, which is consistent with the observed speeds. It requires no cathode mechanism or γ . It occurs only at pressures and under conditions where photoionization can be dense about the space charge. It is independent of photoelectric emission from the cathode. It decreases in time of formation with overvoltage, i.e., potentials above the threshold. It accounts for branched and zigzag or crooked sparks. Most of these characteristics cannot conveniently be accounted for by the previous theory.

As pressures go down and $\frac{X}{p}$ increases it is probable that the photoelectric ionization decreases and actions such as γ at the cathode increase. Thus at some lower pressure, for a given gap length, sparks will pass more easily by Townsend's mechanism than by streamer. The pressure where this occurs is not certain. It probably occurs below 100-mm pressure in air for a 1-cm gap. All very long sparks such as high-tension flashover and lightning operate by streamer mechanism or by modifications of the streamer mechanism.

10. Forms of Gaseous Discharge. In considering forms of discharge it is possible, aside from the transient spark, to differentiate three essential types. These are glow discharges, arcs, and coronas. The latter comprise glows and brushes from pointed or sharply curved conductors. These are all more or less stable self-sustaining current conditions, the economy of which is different from the *dark currents* due to cumulative ionization, or electric field intensification of ion currents. The currents are much larger and the discharges are self maintaining. The negative corona is closely similar in structure to the glow discharge except that it is confined in a much smaller region about a point. The positive corona partakes of a streamer-like mechanism or a modified streamer spread laterally over a surface.

11. The Glow Discharge. The glow discharge occurs in gases in the lower-pressure ranges from 0.1 mm of Hg up to some 100 mm of Hg in the inert gases. In the high-pressure ranges it is at times difficult to distinguish it from the arcs. Acting at lower pressures, glow discharges thus must be confined in tubes and the insulating tube walls play a definite role in the glow economy. Currents in the glow discharge may run from microamperes to amperes. They are more usually in the milliampere range. Glow discharges require high potentials in contrast to arcs. Current densities are low. As potentials increase, the glow may with heating of the cathode gradually go over to the arc. Usually the change from glow to arc is very abrupt, via a spark, the current in the arc jumping up by a large factor while the potential falls.

The glow discharge is characterized by a definite luminous and electrical structure. The cathode will usually be covered by a glow composed of radiation by incoming neutralizing positive ions. The glow may be a small patch on the cathode or it may cover the whole front of the cathode surface, even extending slightly to the rear. When the glow area is small the potential and current density remain nearly constant and the current increases as the resistance in the external circuit decreases and more current can flow. The glow area thus expands with increase in current and current density is constant. This is called the region of *normal cathode fall of potential*. It is characterized by a relatively low and constant value of the cathode fall of potential V_c . As the glow covers the surface further current and current density increase require an increase in potential and an increase in the cathode fall of potential V_c . The cathode fall of potential is characteristic of all discharge conditions. It is a region next the cathode of sharp drop in potential. It comes from a positive ion space charge in the gas, resulting from the high velocity of electrons compared to positive ions. The potential fall V_c ranges from some tens of volts up to 1000 volts or more at lower pressures and in some gases. It is a vital part of the glow-discharge mechanism.

Prominent, next to the cathode at relatively lower pressures, but always present, is the *Crookes dark space*, which is the visual manifestation accompanying V_c . Its width varies from some centimeters at very low pressures to fractions of a millimeter near 1-cm pressure. It is the region over which the cathode fall occurs and is only relatively dark. It is populated by hordes of positive ions moving relatively slowly towards the cathode from the negative glow. At the cathode the electrons liberated by the impact of these positive ions (these positive ions having some hundred or more volts of energy from V_c when they strike) are highly accelerated and shoot through the dark space quickly. The field is so intense and they move so much under its influence that they are little deflected and consequently do little ionizing. This makes the space relatively dark. Next to the Crookes dark space is the luminous *negative glow*. This region is one where the field due to V_c is no longer active. The fast electrons strike atoms or molecules. They are deflected and experience tortuous paths with little field guidance. They thus move at random in the glow, ionizing all the time. Thus there is intense ionization and much luminosity, especially in the more ultra-violet region because of the fast electrons. In the region of the glow away from the cathode the potential might dip down a bit negative to what it is at the gas edge of the positive space charge owing to the accumulation of electrons and loss of positive ions.

Beyond the negative glow there follows a fainter dark space known as the *Faraday dark space*. In this the potential again rises and accelerates the hoard of newly formed electrons into the long luminous *positive column*. It results primarily from the intense negative space charge produced in the negative glow and the conducting region beyond. It is relatively dark in consequence of the fact that the electrons from the glow have not picked up ionizing energy and are being accelerated in it. It is quite prominent at high pressures and is then nearer the cathode.

The *positive column* is the *luminous column* in the neon signs or any glow discharge. The other regions mentioned are usually concealed in such signs by black paint. This column is essentially a gaseous conducting channel. The potential fall along it, i.e., the field, is weak. It is just strong enough to cause the ionization to keep the channel conducting despite losses of ions and electrons. The electrons required are generated in the negative glow and fed into the column by the Faraday dark space. Their number is maintained constant by ionization in the column. The gradient in the positive column is low. The column can be lengthened at will without too much increasing the potential applied. If the potential gradient in the column is X_1 the increase in potential, ΔV , needed for an increase in length by l is $\Delta V = lX_1$.

At the positive electrode there is another short rise of field strength

and potential termed the *anode fall of potential*. It accelerates the electrons from the column in their last paths to give the needed positive ions for maintaining the column by ionization in front of the anode, causing an anode glow.

Thus the glow discharge operates by electrons fed in at the cathode end of the column through the intense ionization in the negative glow of electrons liberated by a γ mechanism by positive ions at the cathode. The positive ions are generated near the anode. In the column there is much excitation of light by the slower electrons. Some recombination takes place in the column. Most of the loss of ions and electrons in the column comes from diffusion to the walls and recombination at the walls. The ionization in the column just makes up this loss. The color of the light emitted depends on the nature of the gas and the energy of the electrons in the column. It varies from the blues of A and Hg through the yellows of Na to the reds of Ne.

Much ultra-violet light is generated in most glow discharges. It cannot escape through the glass walls and is thus lost by conversion to heat. The modern so-called "fluorescent" tubes are merely glow-discharge tubes with A and Hg in them. They are operated so as to produce ultra-violet light. The tube walls are coated with rare-earth oxides. These fluoresce brightly under ultra-violet light. Thus these tubes merely use the wasted ultra-violet light to give bright visible light.

12. The Arc Discharge. The arc, in contrast to the glow discharge, operates at somewhat lower potentials with much larger currents. It can do this since its mechanism for generating its electrons does not require a high cathode fall to liberate them by positive ion bombardment. Instead of this, it utilizes its high current densities to furnish electrons at much lower potentials. The mechanism by which the arcs of high melting-point metals accomplish this is well understood. That of low-melting-point arcs like Na, Hg, Cu, etc., is not well understood in that no quantitative theory is at present applicable. The electrons in the high-melting-point metal arcs are produced by very high current densities that heat the cathode to the 2000 to 4000° C needed for strong thermionic emission (see section 189).

The cathode fall in an arc is only some 10 to 20 volts, but the current density is of the order of 10^4 amperes per square centimeter. Since to carry large currents arcs need many electrons and ions most arcs operate at pressures above 1 cm. Thus the cathode fall of potential in arcs extends over distances of the order of about 0.01 cm or less. Arcs carry currents roughly from tenths of an ampere to hundreds of amperes and current densities up to the order of 10^5 amperes per square centimeter of area. While the potential fall in arcs is in general low, the field strengths in the column of the arc are somewhat higher than in the glow discharge. If, however, the

ratio of field strength to pressure in the column of an arc is taken it is very much *less* than in the glow. Arcs operate on alternating or direct current. The column of an arc is self contained without walls. The higher pressures reduce diffusive losses. Thus ionization in the column requires smaller energy input to make up for losses than in the glow. There is some talk of "temperature ionization" as being the mechanism operating in arcs. *Some* arc columns are hot enough to ionize atoms of the alkali and alkaline earth elements and occasional molecules by collisions between them. However, such arc columns also have hoards of free electrons of which the energy gain from the field gives them average energies at least ten times the energies of molecules at the elevated temperatures. Under those circumstances the expression *Saha equilibrium* "temperature-ionizing" mechanism can hardly be used, since such electrons are efficient ionizers.

As to what happens in Hg and Cu arcs to generate the needed electrons, there is not much doubt, qualitatively. The phenomenon is at present not amenable to calculation. The mass of positive ions flowing into the cathode of such an arc per second, each bringing in 10 to 20 e.v. of energy, is imposing. It is the current in amperes divided by 1.6×10^{-19} electronic charges per second. These relatively slow positive ions probably do not generate many electrons on impact at the cathode. Owing to the volatile nature of the cathode material the impact of charged atoms at an equivalent energy of some 10^6 ° C is *not* going to leave these ions on the surface, *once they are neutralized*. In fact, observation shows that they are bounced back to the gas as neutral atoms and carry with them in addition some one-tenth as many neutral atoms knocked out from the surface. These quantities of incoming and outgoing ions have all been measured in Cu and Hg arcs. Thus there is in the gas immediately adjacent to the cathode in low-boiling-point arcs a blast of neutral atoms of from 5 to 10 or more volts of energy. These will strike the gas molecules in their neighborhood and can produce copious ionization of the Saha type, which is *not* an equilibrium phenomenon. This is probably the origin of the heavy electron currents which cause such arcs to operate.

The quantitative theory of the arc has not been worked out. The high temperatures preclude the study of potentials and currents in the arc column by probes, as can be done for the glow discharge. Furthermore, the regions of interest are too narrow for much study. Empirical study of many arcs in air has revealed that an equation in this form is generally applicable: $V = \alpha + \beta l + \frac{\gamma + \delta l}{i}$. Here α , β , γ , and δ are constants, while l is the gap length, V the applied potential, and i the current. It is noted that the larger i , the lower V . This is merely in keeping with what has been said about the

distinction between arc and glow. The arc can operate at a low potential if the electrons can come from a cathode mechanism that does not depend primarily on V . Since thermionic emission depends on the heating and this depends on the current, it is clear that the greater the i , the greater the economy of the current. The current in an arc is thus controlled by the external resistance rather than by its own mechanism. In fact, it is the conditions of external resistance and circuit constants that determine whether a given spark will end in no current, a glow, or an arc. Arcs are favored by low line resistance and a plentiful power supply. The fact that V varies as $\frac{1}{i}$ gives the so-called negative current characteristic of the arc.

This has industrial applications.

Arc Rectifiers. The fact that a metal like Hg can be used as one electrode and a more refractory metal as the other electrode, and that the economy of the arc is different for different cathodes, indicates that arcs could be used for rectifying a-c currents. The time required to get the low-boiling-point Hg arc going (since it does not involve direct heating), in contrast to that of getting a W electrode heated to thermionic emission, causes such an arc to pass current with Hg as the cathode, but not when the tungsten is the cathode. Thus an arc between W and Hg electrodes with a-c passes current when Hg is negative but not when W is negative. Such rectifiers are used widely in industry today. They pass large currents since there is less space charge limitation of the current than for pure electron filament emitter sources, the positive ions present reducing the electron space charge.

13. Corona Discharges. The operation of corona discharges is not too different from the mechanisms already discussed. If there is a sharp point and a potential is placed on it, the field becomes very high, as shown in section 73. If the point is positive, free electrons generated in the gas near the point, or liberated from negative ions by impacts in the high field, move toward the point. In the last free paths between collisions, the electrons get a very high energy. Thus on striking molecules or atoms, they will be deflected. By virtue of their high energy, instead of going straight to the positive point they may bat around quite a while, creating many ions in a confined region. Then one of two things can occur: They can, just as in a plane gap at near atmospheric pressure, start a positive streamer. This will move outward from the point until the divergent field gets too weak and it stops. On the other hand, it can, by photoionization, spread the ionization around quite a small area over the surface of the point. Both of these occurrences happen in positive-point corona. The electrons produced are all at once picked up, but when the slow-moving positive ions get too dense, the field around the point is weakened. Thus the positive corona and early streamers have a

tendency to choke themselves off by space charge fouling. As fields get higher, however, the streamers can propagate farther into the gap. Inhibitory fouling ceases, and in place of the occasional short streamer and the flickering bright blue film at the tip of the point, longer streamers project into the gap way beyond the region of high field. Such streamers give a characteristic brush-like appearance. In air they are bright electric blue in color. The ions they leave behind them produce space charges, and these show the general purple haze characteristic of a low-voltage discharge in air as they equalize their local fields. Thus the so-called brush discharge is a positive corona with a purple haze pierced by myriads of bright blue streamers.

As the potentials increase still more, the streamers propagate to the cathode. Once they strike this with sufficient intensity a spark ensues, followed by a glow, a power arc, or extinction, depending on circuit constants. In very pure H_2 and N_2 streamers do not readily form, as photoionization is poor. In very pure argon, because of metastable atoms, they form so readily that the first streamer leads to a spark breakdown. Usually the phenomena as first described are observed. Corona streamers have some 10^9 ions per centimeter of length. They travel with a speed of the order of 10^8 cm per second. They carry away enough charge so as to shock excite electrical systems. They are responsible for the noisy corona static which interferes with radio in airplane flight and near power lines.

If a point be negatively charged, the situation is different. Electrons liberated from the negative point by ultra-violet light, or by impact of positive ions in the high field region, move *outward* from the point. In their first few paths they gain a great deal of energy. Thus, quite like electrons in the Crookes dark space of a glow discharge, they begin to collide in the gas and produce ions some short distance out, where the field is lower. There thus appears a region of luminosity of bright blue color in air *detached* from the metal point. This bright patch is essentially a sort of negative glow. The Crookes dark space is against the cathode. The electrons move beyond the negative glow into a sort of Faraday dark space and accelerate in the increased positive field toward the anode resulting from the negative glow ionization. Positive ions from the negative glow are drawn to the cathode and form new electrons on impact. At any rate, plenty of electrons are furnished from the cathode region by photoelectric effect at the cathode, by photoelectric ionization *in the gas*, and by positive ion impact. At low fields and higher pressures usually only the negative glow is observed. If the potential increases, or the pressure is reduced, the whole glow-discharge-tube structure, including a faint purple positive column sometimes looking like a shaving brush, is observed.

Sometimes in gases where electrons can attach to form slow

negative ions the ions formed near the beginning of the positive column or even in the glow itself will choke off the sustained discharge. In consequence, until the field clears off these ions, the discharge cannot continue. Thus an irregular intermittent negative corona may be obtained. If at the cathode a copious source of electrons is provided to ensure that the discharge starts again as soon as the space charge clears, the discharge can take on a beautifully regular relaxation-oscillator-like character. This was first observed by M. D. O'Day. It was studied by G. W. Trichel and finally explained in its details by Loeb, Hudson, Kip, and Bennett. The frequency of the pulses depends primarily on the time of sweeping out of the negative ions. Breakdown from negative points occurs when a positive streamer mechanism can form in the gas and propagate outward.

Both positive and negative point coronas have been observed since time immemorial about pointed conductors during high electrical stress, e.g., in thunder showers or about high-tension lines. They were called St. Elmo's fire. St. Elmo's fire at sea was considered to augur well for a ship's safety. Perhaps if the time rate of advancing potential gradients is not too steep the corona protects by space charge production. Generally, however, coronas can be regarded as danger signs indicating unpleasantly high tension, and a source of radio interference.

14. Vacuum Sparks. Needless to say, the critical study of gaseous conduction is very complicated and, both purely scientifically and industrially, there is yet very much to learn about it. It pays in this respect to keep one's mind open and observe. As a concluding example, this story is apropos. Dr. E. B. Rosa of the Bureau of Standards one day suggested to one of his young men that he investigate the vacuum spark. Perhaps after reading this sketch the reader may say, as did this young man, "But theoretically, there can't be a spark in vacuum." It is true that in vacuum there is no gas to break down and give a spark. Dr. Rosa replied: "I am not interested in what the theory says—sparks *do* occur in the very best vacua." This statement was correct. Theory also cannot be wrong. Thus somehow the spark must create its own gas. Actually, R. A. Millikan had studied such sparks and used them in his vacuum spectrograph. The mechanism is very interesting. The electron theory of metals indicates that if applied electric fields become high enough electrons can actually be "pulled" out of metals. Better stated, a field can be applied so strong that it lowers the potential barrier holding electrons in a metal lattice and some of the electrons will escape. This really requires fields of the order of 10^8 volts per centimeter. However, when on the order of 10^6 volts per centimeter is placed on what is called *smooth surfaces* (10^5 of a centimeter is the finest roughness that can be *seen*), there are little points which have local fields materially higher.

Thus, on highly polished tungsten wires, as potentials were raised, in the best vacuo, the sharpest points began to emit electrons. These, with tens of thousands of volts potential difference acting, shot across to the outer cylindrical anode. They liberated a burst of gas and ionized it. The positive ions formed from the gas then threaded back to the tungsten wire. When they struck the point they fused it, liberated more gas, and left a melted pit at the point of impact. If the metals were not carefully outgassed, enough gas was evolved to give a general breakdown and a power arc. Ultimately W. H. Bennett was able to work out the theory of self-focusing beams deriving from field-emitted electrons and thus complete the theory of the vacuum spark. In fact, it is known today that in industrial installations it is not possible to go much higher than about 4×10^5 volts per centimeter in vacuum between metal surfaces without danger of a "vacuum" spark.

CHAPTER XXVIII

PHOTOELECTRIC AND THERMIONIC EFFECTS

In the rapid adaptations of modern scientific discovery to industrial and practical purposes there are two more or less recently discovered and developed sets of electrical phenomena which are of wide application and of considerable importance. One of these is the so-called photoelectric effect and the other one is the thermionic effect. In a book of the scope of this one the widespread use of these phenomena demands their inclusion. However, in keeping with the spirit of this book the subject will not be dealt with in its practical applications to any great extent, nor is there space to discuss anything but the underlying foundations of the many different applications.

A. THE PHOTOELECTRIC EFFECT

181. EARLY DISCOVERIES

In 1887 Hertz, the discoverer of the electromagnetic waves, noticed that the spark in his receiving set passed less readily (i.e., required a shorter spark gap) if the spark were screened from the spark in his primary sending set causing the oscillations. In a masterly research investigation, which could be chosen as a model of scientific analysis, he determined the nature of the phenomenon. He showed that ultra-violet light from the primary spark caused the spark in the receiving set to pass more easily if it fell on *one* of the electrodes, namely, the cathode. The radiations effective traveled in straight lines, were transmitted, reflected, and refracted by certain substances, while others absorbed them. The wave lengths of light effective were well beyond the visible toward the violet end of the spectrum. He found that an electrical arc was more effective than the spark. The work was carried further at once by Ebert and Hallwachs. The latter studied the action of the light from an arc on electrostatically charged bodies. It was found that the light falling on a negatively charged zinc plate connected to an electroscope caused the electroscope to discharge rapidly, while if the zinc plate was positive there was no discharge noticeable. The effect was shown to be caused by *ultra-violet light*. Thus the *photoelectric effect* (literally light-electric effect) was discovered. The action of the light resides on the metal, the air being inactive, and is very much a function of the condition and nature of the metal on which it falls. A clean plate of an electropositive metal

like aluminum or zinc was more effective than iron, and corrosion destroyed the effect entirely. Hallwachs finally concluded that negatively electrified particles travel away from the plate when it is illuminated, and that this occurs for an uncharged plate but is more effective if the plate is negatively charged. The residual charge on illuminating the uncharged plate was positive, proving the escape of negative

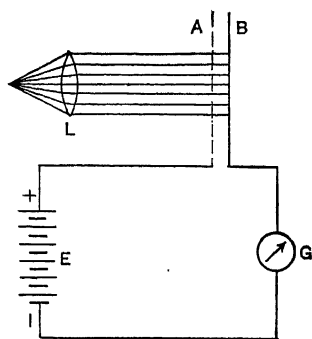


FIG. 191a. Device for the study of the photoelectric effect using a galvanometer.

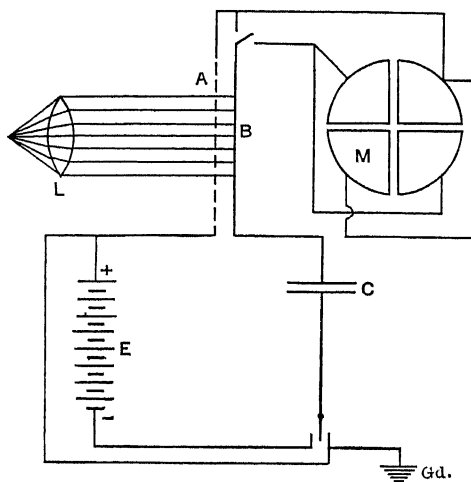


FIG. 191b. Device for the study of the photoelectric effect using a quadrant electrometer.

particles to be the cause. For the study of this phenomenon Stoletoew devised the simple circuits shown in Figs. 191a and 191b. In these the light from an arc or burning magnesium ribbon focused by a quartz lens L falls on a plate B after passing through a gauze A . A and B form two plates of a parallel-plate condenser connected to the battery E through a galvanometer G , Fig. 191a, or an electrometer M , Fig. 191b. The plate B must always be negative. Fleming in 1909 found that if an alloy of Na and K was used on B while A was Pt an e.m.f. between A and B of 0.6 volt could be built up as the result of the flow of negative electricity from B to A , using scheme b. This potential it was early discovered is the *contact difference of potential* (formerly called the Volta difference of potential), an intrinsic property of the metals which was first observed by Volta in his studies on the galvanic cell. Fleming obtained a current of 5.4 microamperes, presumably with the battery, and a galvanometer of resistance of 180 ohms. Righi had previously shown that the new photoelectric effect took place in vacuum with even better results than in the presence of gases. In 1890 Elster and Geitel showed that the effect in vacuum was reduced when a magnetic field parallel to the surfaces was used.

182. LIBERATION OF ELECTRONS BY LIGHT

The discovery of the electron by J. J. Thomson as a result of the investigations of the nature of the bluish streamers from the cathode causing x-rays in a highly evacuated tube led to a speedy identification of the negative electrical particles of Hallwachs with electrons. The experimental proof of this identity was made by Thomson in 1899. From then on the development of the subject owes much to Philipp Lenard in 1900 and thereafter. He checked Thomson's results, using the photoelectrons in vacuum by a slightly different method. The fact that the particles were electrons liberated by light led to the name photoelectrons given them today, and the effect is known as the photoelectric (or light-electric) effect. Lenard next studied the current as influenced by the potential between the illuminated plate or electrode and an opposing electrode in vacuum as a function of the potential difference between the electrodes. He found that at 0 potential difference he got a small but definite current. As he made the illuminated electrode more and more negative relative to the other electrode the current increased rapidly at first and then more slowly, reaching a maximum or "saturation" value at 100 volts or more, thereafter remaining constant. This indicated that as the electrical field increased more and more electrons that had been liberated by the light were drawn to the positive electrode, thus increasing the current, and that in his apparatus at 100 volts practically all electrons liberated were drawn out and contributed to the *saturation* value of the current. Together with other results this effect of the field indicated that the electrons were liberated in all directions and at different velocities. The reason that at low fields the currents were smaller lies in the fact that owing to this directional effect the weak fields let many electrons get back to the illuminated plate, or escape to the walls of the vessel. At higher fields the electrons were drawn more and more nearly directly across from cathode to anode by the field.

Lenard also found that as the illuminated plate was made slightly positive the current was reduced but did not stop until the plate was positive by some 2.1 volts relative to the collecting plate. He correctly interpreted this as due to the fact that *some electrons were liberated at high initial velocities*. That at 2.1 volts no more electrons crossed meant that the fastest electrons which were headed directly for the collecting electrode from the illuminated electrode had only just enough energy to reach the collecting electrode against the potential of 2.1 volts that retarded them. Since the electron carries a charge of e units ($e = 4.80 \times 10^{-10}$ e.s.u.) the work done on the electron in going from one electrode to the other against a field of V volts is Ve ergs of work (V in volts being properly expressed in e.s.u.). To be able to move against a field of V volts an electron of charge e must have a velocity v given by the relation $Ve = \frac{1}{2}mv^2$, where m is the mass

of the electron in grams. Hence v in centimeters per second = $\sqrt{\frac{2Ve}{m}}$. If V is 1 volt = $1/300$ e.s.u., $e = 4.80 \times 10^{-10}$ e.s.u., and $m = 9.04 \times 10^{-28}$ gram, it is seen that an electron able to move up against a potential of 1 volt (or an electron that has been freely accelerated by a potential difference of 1 volt) will have a velocity of 5.94×10^7 cm per second. It is seen that the velocity is then actually given by $\sqrt{V} 5.94 \times 10^7$ cm per second, V being expressed in volts. The velocity of the fastest of the photoelectrons observed by Lenard therefore must have been about 0.86×10^8 cm per second. At lower positive potentials of the emitting plate slower electrons can cross to the collector as well as electrons the directions of which are not normal to the collecting electrode, but of which the components of velocity in that direction are such that they exceed $5.94 \times 10^7 \sqrt{V}$ cm per second. Lenard and others before him also found that the presence of gases reduced the currents under otherwise identical conditions. This is the result of a number of causes, namely:

1. Change in the nature of the metal surfaces from which electrons were emitted as a result of adsorption of gas or chemical reactions.

2. Reduction of energy of electrons by collisions with gas molecules and an angular scattering of electrons by the latter resulting in their ultimate return to the illuminated plate. The electrons are emitted with a high energy of agitation of the order of 100 times thermal energies. Thus, in a gas where elastic impacts occur, they have a high velocity of diffusion compared to that given by the field. Hence their tendency to diffuse at any appreciable gas pressure is such as to drive them back to the electrode from which they came in appreciable numbers despite the action of the field in pulling them away. In very high fields (near sparking) or at lower pressures more and more escape recapture and get to the positive electrodes, as shown by Bradbury.

3. By loss of electrons to molecules in the formation of slowly moving negative ions by electron attachment in certain gases, notably O_2 , SO_2 , and Cl_2 .

It should be noted that the maximum velocities of the electrons observed depended not only on the *nature of the material of the illuminated electrode* but also on some condition inherent on the *combination of the metals* used in both electrodes if they were in any way different. This phenomenon involving both electrodes had been earlier observed and correctly attributed to *contact potentials* between metals; these potentials act to accelerate or retard the photoelectrons and their value must be determined and added to or subtracted from the applied potentials in any quantitative study. The contact potential is the potential difference which acting on unit charge represents the work required to move an electron from the one metal to the other through

their surfaces against the surface electrical fields existing in the metals. These potentials are related to the values of C , μ , and χ , discussed later on page 517.

In the discussion of all that has gone before it has been assumed that the intensity and the nature of the light source were kept constant. It is not surprising that Lenard found that the saturation current varied with the light intensity, and that, in fact, the current was directly proportional to the light intensity. This law has been amply verified since and has been found to hold for light intensities which in total vary in the ratio of 1 to 10^5 , although in any one experiment the study has not been made for a single range of intensities of more than a hundredfold. It is, however, unquestionable that in the range studied quite accurate proportionality exists. It is not surprising, therefore, that in the great technical applications of today the proportionality between the electrical current in the photoelectric effect and the light intensity should give a valuable means for converting varying light intensities to proportionately varying electrical intensities. It is, in fact, on this basis that some of the modern "talkies" as well as the new television and many other devices operate.

183. THE EINSTEIN PHOTOELECTRIC LAW

Although it was early known that the frequency of the light played an important role in the photo effect the exact nature of the process was little understood. It was known that the effect was produced by ultra-violet light. The earlier workers Stoletow and Elster and Geitel found further that there was a limit on the red side of the spectrum below which ultra-violet light ceased to be active. It was further found that the wave length of the ultra-violet light which marked this threshold varied with the metal used. In other words, there appeared to be a *long wave-length* (low-frequency) *limit* to ultra-violet activity which varied with different metals used. It was also found that this threshold depended somewhat on the gas and the condition of the surface. Another early observation was the discovery of the effect known as photoelectric fatigue. It was observed particularly in the presence of gases, or residual gases, that the photoelectric effect varied with the time of illumination and that with prolonged illumination the quantity of current decreased. The fatigue is today little understood, and has been much reduced by the elimination of reactive gases and features causing the formation of surface films.

In 1905 a new era dawned in the investigation of the photoelectric effect. This was in part due to some observations of Lenard which indicated that the maximum velocity of the photoelectrons, as measured by the potential against which they could reach the collecting electrode, depended on the frequency of the ultra-violet light. This fact stimulated Einstein to discuss the effect on the basis of the then

rather newly developed quantum theory. As a result of his attempt to find an accurate equation for black-body radiation Max Planck in 1900 had arrived at the conclusion that a satisfactory equation could be obtained only by assuming that radiant energy (light) was emitted or absorbed in units or quanta. He had found that the size of this unit or quantum varied with the frequency ν of the radiation and was given by a whole multiple of the quantity $h\nu$, where h was a new universal constant of nature known as the unit of quantum action. As shown on page 33, ν is a frequency and has the dimensions of the reciprocal of time or $\frac{1}{T}$, while $h\nu$ has the dimensions of energy. Hence h has the

dimensions of an energy times time, or ML^2T^{-1} . It is also of importance to note that mvr , the *moment of momentum*, has the dimensions of ML^2T^{-1} so that h has at the same time the dimensions and perhaps the properties of a moment of momentum, which is called action, as well as energy multiplied by time. Hence h is the unit of action. This peculiar dualistic property of the quantity h was destined to play an enormously important role in the hands of Bohr when he came to unraveling the spectra of the elements in relation to the new Rutherford theory of atomic structure. Basing his views on Lenard's observations concerning the frequency of the incident light and velocity of photoelectrons, and on a rather extreme interpretation of the notion of the role of the quantum in light, Einstein derived a mathematical equation expressing a relation between the frequency of the light and the electron velocity in photoelectric emission. This equation may be written

$$\frac{1}{2}mv^2 = eV = h\nu - P.$$

In this equation m is the mass of the electron, v its velocity, e the charge, V the potential necessary to stop the electron (i.e., the volt equivalent of its velocity), and P the energy necessary to remove an electron from the metal with 0 velocity. That is, P represents the energy just to set an electron free from the interior and bring it through the surface of the metal. By bringing the frequency ν down to lower and lower values it is seen that $v = 0$ at $h\nu_0 = P$, where ν_0 is the frequency for which electrons emerge with no velocity. This frequency, it turns out, is the *limiting frequency* just spoken of, below which no more electrons can emerge, that is, ν_0 is the *red threshold* of the photoelectric effect. Einstein's equation then can also be written as $\frac{1}{2}mv^2 = h\nu - h\nu_0$. The quantum theoretical viewpoint on which Einstein derived the equation was an extreme one, and he assumed not only absorption or emission in quanta as had Planck, but also that the light was actually quantized in space. Later developments led him to conclude that his initial assumptions were untenable and hence that the deduction was not valid. With a peculiar irony of fate, however, the next researches of Ladenburg, Richardson and Compton, and

Hughes indicated that the energy of the photoelectrons was proportional to the frequency of the incident light, although the proof was not extensive enough to be conclusive. Finally in 1914-1915 Millikan and his pupils developed a technique of making a careful test of the law and *accurately verified it*. One of the great drawbacks in investigation had up to this time lain in the difficulty of getting *clean outgassed surfaces*. Using high vacuums made possible by improvements in vacuum technique, and a new method of *shaving* the surfaces of his photoelectric metal in vacuo, thereby getting fresh surfaces (the soft alkali and alkaline-earth metals were largely used), Millikan was able to make a real test of the law. These experiments showed that the equation held very accurately. The source of error due to the contact potential was eliminated by proper control methods (i.e., independent evaluation of the contact potentials), so that the true energy of the photoelectrons could be measured, the values of ν_0 for the different metals were determined, and finally a new and for that period very accurate value of h was obtained. Within a year or two the work of William Duane and D. L. Webster established the law accurately for the photoelectrons from x-rays. The final and most accurate verification in the latter field came from the work of Maurice de Broglie and of Ellis between 1921 and 1925. In fact, the equation is now so well established that it furnishes a method of obtaining the wave length and frequency of electromagnetic waves too short to be analyzed by crystal structure methods, such as, for instance, the γ rays of radium.

184. THE INTENSITY OF PHOTOELECTRIC EMISSION

With the general principles of the photoelectric emission established, perhaps a few words might be said concerning some of its more detailed aspects. First mention might be made of the proportionality of photoelectrical current intensity and the light intensity. This law of proportionality holds up to current densities of the order of 3×10^{-7} ampere per square centimeter. Further increase in intensity could cause apparent deviations unless care is taken to increase the field between source and collector, for with high current densities *space charges due to accumulations of large numbers of electrons can increase the saturation voltage necessary*, as will be seen later in connection with thermionics. Hence the necessity of the use of *saturation currents in all measurements* must be emphasized. For very small currents proportionality still holds down to 10^{-9} erg per square centimeter incident light intensity of blue light, or 200 quanta per square centimeter. Below this the currents must be amplified by ionization by collision, and emission is so slow as to be sporadic. The *average* emission even down to 10^{-11} erg per square centimeter per second incident light intensity (2 quanta per square centimeter per second) still is proportional to the intensity if measured so as to get an average

value. The first electrons are liberated at the instant of illumination even over time intervals of 10^{-9} second. This fact caused some difficulty in the understanding of the action of light, for with the minute energies used it was impossible to understand how one electron in a metal surface could instantly (10^{-9} second) gain a whole quantum unless light went in particles. Wave mechanics has, however, removed this difficulty entirely.

When the study of the law of emission as a function of intensity is carried out for the use of light incident at angles differing from normal, care must be taken to remember that it is the *absorbed* light which causes emission. Hence as the proportion of absorbed to reflected light varies with the angle of incidence and reflection, a variation of the photoelectric intensity as a function of the angle of incidence for a constant light source should be expected. The work of Elster and Geitel and of Pohl showed that the change of emission with angle was quite accurately proportional to the *absorbed* light intensity for different angles.

It being understood that below a certain frequency ν_0 no light is able to call forth photoelectric currents, it is of interest to see how the

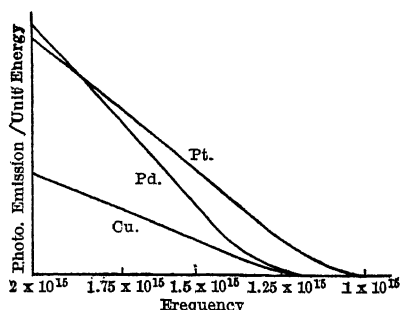


FIG. 192. Photoelectric current per unit incident light energy as a function of frequency of incident light in cycles per second.

current changes with frequency above the threshold. It is best in this case to plot current per unit light energy *absorbed* at various frequencies of incident light. The type of curve obtained for different metals in such measurements is shown in Fig. 192. The curves begin at the long wave-length limit, are concave upward, increasing more rapidly with ν than proportional to ν , and approach a linear relation. It might be expected from classic reasoning that the curves should flatten out at the higher frequencies and show a portion concave to the origin.

This variation has not been observed thus far experimentally *except with the alkali metals*, as will be seen later. It is also to be expected on the newer concepts of the electrical behavior of metals. It is probable, however, that the point of inflection in most measurements lies well above frequencies that can be controlled in the laboratory in such experiments. Various attempts have been made at deducing proper equations for this phenomenon, but until the recent work of Pauli and Sommerfeld on the theory of the nature of the electronic state in metals no success had been achieved. More recently on the basis of the Sommerfeld theory of the state of the electrons in metals Wentzel has made what appears to be a promising approach. The nature of the electron

theory of metals is well beyond the scope of this text and hence cannot be included, although a word might be said concerning the nature of the new theory.

185. THE SOMMERFELD-PAULI CONCEPT OF THE FREE ELECTRONS IN METALS

As a result of the high conductivity of metals and the optical properties of metals as stated in Chapter XII the electrons in metals were assumed to be free in the crystal lattice of the metal in considerable numbers and to share in the thermal energy of the metal. In order that a *free* electron inside a metal and possessing 0 energy can escape from the metal a certain energy, of value C , must be given to it, that is, C is the *work* required to pull an electron out of a metal surface into free space against the attractive forces. It can be called the *true work function*. This work C is dependent on the nature of the metal (i.e., its atomic species and crystalline state). In general, it represents a rather high energy value of the order of magnitude of that required to move an electron against a field of 10 to 20 volts. Were the electrons at *rest* in the metal and also *free*, then the photoelectric threshold would be represented by the relation that $C = h\nu_0$. Now, in general, electrons have long been assumed to be free in metals as a result of optical studies on the reflecting power of metals, as well as from considerations dealing with the peculiar constancy of the ratios of electrical and heat conductivities in metals known as the Wiedemann-Franz law. The estimated number of free electrons was of the order of 1 or more free electrons per atom present (i.e., in silver about 6×10^{22} electrons per cubic centimeter).

If such electrons are *free* it was believed that they should be moving about among the atoms of the metal crystal lattice with an average energy of motion given by $\frac{1}{2}mv^2 = \frac{3}{2}kT$ (where T is the absolute temperature and k is the gas constant R divided by the number of molecules in a gram molecule), due to interchange of heat energies with the atoms of the lattice. At 0°C this energy of motion amounts on the average to about 0.0356 e.v. of energy. Hence the work to remove an electron at 0°C would not be C , but should on the average be $C - 0.0356$ volt. However, if the 6×10^{22} electrons in a cubic centimeter of silver, for example, have this freedom of energy exchange with the atoms, they might be expected to take on a large amount of heat as the temperature of the body is increased, for each electron acts like an atom in energy exchange and there are 6×10^{22} electrons per cubic centimeter. If the electrons acted according to classic theory this would increase the *atomic heats* of all metals by 50 per cent above the value *observed* by Dulong and Petit (atomic heats of pure crystalline metals = 6 calories). The fact that metals have the atomic heats given by Dulong and Petit's law thus indicates that *somehow the free*

electrons do not behave in a strictly classic manner. It is at this point that Sommerfeld improved the situation by applying the *nonclassic*

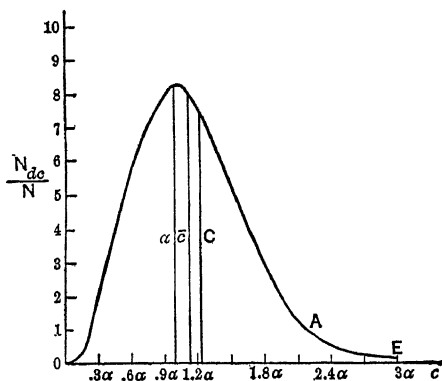


FIG. 193. Maxwell's law for the distribution of velocities among molecules. Ordinates are $\frac{N_{dc}}{N}$ in per cent, abscissas c for $\frac{dc}{\alpha} = 0.1$.

quantum statistics of Fermi and Dirac, together with the famous *exclusion* principle of Pauli, to electrons in metals.

Although the detailed reasoning by which the law of distribution was derived cannot here be entered into the nature of the reasoning may be indicated. Normally, particles in an atmosphere (i.e., atoms in a gas, or electrons in an electron atmosphere) which are in thermal equilibrium have their velocities distributed around a mean value represented by the famous Maxwellian distribution law. This

says that out of a total of N particles in the gas the N_{dc} particles having a velocity between c and $c + dc$ are given by the law that

$$N_{dc} = \frac{4}{\alpha^3 \sqrt{\pi}} N c^2 e^{-\frac{c^2}{\alpha^2}} dc.$$

This law is pictured in Fig. 193, where $\frac{N_{dc}}{N}$ is plotted against c for values of $\frac{dc}{\alpha}$ equal to 0.1. In the figure α is the most probable velocity

and is represented by the peak of the curve. This velocity α is determined by the absolute temperature T of the gas, and is related to T by the equation

$$\frac{m}{2} \alpha^2 = kT = \frac{R_A}{N_A} T.$$

Here m is the mass of the particle (atom or electron) and k is the gas constant R_A per mole divided by the number of atoms in a mole.

It is for some purposes more convenient to divide the velocity c into components u , v , and w along the X , Y , and Z axes, and to express the chance that a particle has simultaneously velocity components between u and $u + du$, v and $v + dv$, and w and $w + dw$ as

$$f(u, v, w) du dv dw = \frac{du dv dw}{\alpha^3 (\pi)^{3/2}} e^{-\frac{m(u^2+v^2+w^2)}{2kT}} = \frac{du dv dw}{\alpha^3 (\pi)^{3/2}} e^{-\frac{u^2+v^2+w^2}{\alpha^2}}$$

This form of the function is shown plotted against $c = \sqrt{u^2 + v^2 + w^2}$ in Fig. 194.

This law holds very well for gases, particularly as gases do not exist as such at very low temperatures. Under the conditions which might obtain at temperatures near absolute zero in gases and which do obtain for electrons in metals at room temperatures this law cannot be realized. The reason for this lies in the fact that even the translation energies of a gas or an electron assemblage can be *quantized* by using the principle of action. The conditions in the gas can be accurately pictured in terms of the momenta mu , mv , and mw along the three axes and by the positions x , y , z of the separate particles. Pauli's exclusion principle (see page 120) says that no more than 2 electrons of opposite spin can simultaneously occupy a cell dmu , dmv , dmw , dx , dy , dz , less than h^3 in moment of momentum space. Thus the space designated by mu , mv , mw , x , y , and z is cut up into cells of h^3 in size. Only two electrons of opposite spin may have values of mu , mv , mw , xy , z ending in any given cell. If now the *concentration of atoms or electrons is high*, space is so limited per atom or electron that there would be at the *low temperatures more than 2 electrons for each state, if the ordinary Maxwellian distribution* is assumed as just given. For under such conditions all the electrons or atoms are crowded into the narrow limits under the bell-shaped curve. But since there can be only 2 electrons in each separate state the Fermi-Dirac statistics give a new distribution law which must vary in form with the density of the electron gas, and with the temperature in a fashion to be described below. The new law may be written as follows:

$$f(u, v, w) du dv dw = \frac{2 m^3}{h^3} \frac{du dv dw}{e^{\left[\frac{m}{2} (u^2 + v^2 + w^2) - \mu \right] / kT} + 1}.$$

In this equation all symbols are as before except that Planck's h appears here as a result of quantization, and the 2 refers to the fact that 2 electrons may occupy each cell. The quantity $\mu = \left(\frac{3n}{\pi} \right)^{2/3} \frac{h^2}{8m}$ to a first approximation is the important new factor which appears. In this term n represents the number of electrons per cubic centimeter of the metal. For values of $\frac{m}{2} (u^2 + v^2 + w^2)$ (the kinetic energy of an electron) less than the critical constant μ , it is seen that for relatively low values of kT (i.e., lower temperatures T) the exponent of e has a negative sign and is large. Hence the exponential term vanishes in comparison to 1 and there is a distribution law of the form

$$f(u, v, w) du dv dw = \frac{2 m^3}{h^3} du dv dw$$

which is constant. At the value of a velocity

$$v_0 = \sqrt{\frac{2\mu}{m}} = \sqrt{u^2 + v^2 + w^2}$$

corresponding to $\mu = \frac{m}{2} (u^2 + v^2 + w^2)$, for small values of kT , the exponential term becomes 0 and the exponent changes sign rapidly, increasing as v rises above v_0 . For higher values of v the 1 in the equation may be neglected, and $f(u, v, w) du dv dw$ drops sharply in an

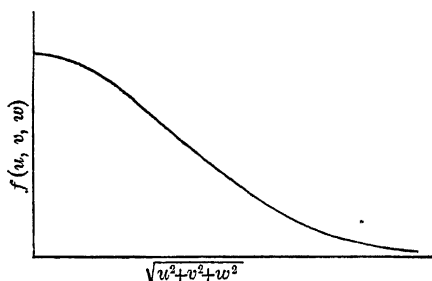


FIG. 194. Chance that a molecule has simultaneously the velocity components u , v , and w plotted as ordinates against the combined velocity $\sqrt{u^2 + v^2 + w^2}$ as abscissas.

exponential manner to 0, remaining at 0, following, however, a curve of the Maxwellian form shown in Fig. 194. If T is very nearly 0 the new distribution curve is that shown in Fig. 195 by the full rectangular lines, the drop occurring at the velocity v_0 above. The higher the value of μ (i.e., the greater the electron density), the higher the value of T for which a given rate of fall occurs, and the higher lies the critical

velocity v_0 , or the critical energy $\frac{m}{2} v_0^2$. The interpretation of this

curve now becomes obvious, for it is to be remembered that at very low temperatures (for T near absolute 0) there were so many low energy states among the electrons that they exceeded the number allowed according to the arrangement of states given by the Fermi-Dirac statistics. These surplus electrons exist in a free state in the Fermi distribution which gives them far more energy than the temperature of the metal permits, because they cannot occupy lower energy states and hence must go to the higher ones that are vacant. Accordingly, in a metal at low temperatures there are a great many electrons occupying positions of high energy content which are unable to share their energy with the atoms of the metal. The electrons in this peculiar state are called *degenerate*. The reason for the degeneracy is that an electron in the rectangular distribution can only leave its position for another one if there is a vacant cell or state and if it has energy to reach that state. Below v_0 there are no vacant states. Just above it there are. At absolute 0 there is no thermal energy to carry it beyond v_0 to a vacant state. Above absolute 0 there is energy. The electrons just below v_0 get carried over by lattice heat impacts just beyond v_0 . This accounts for the tail AE (Fig. 195). Such electrons are not degenerate. At absolute zero all the electrons are degenerate,

and the distribution of energy among the degenerate electrons is that given by the rectangle of Fig. 195. In this condition the number of electrons of velocity components du, dv, dw is constant up to the critical combined velocity v_0 ,

$$v_0 = \sqrt{(u^2 + v^2 + w^2)_0},$$

and then 0 thereafter. Only those electrons that are at the maximum energy μ of the Fermi distribution and next the unoccupied levels can alter their energy and share in the regular thermal energy exchanges. Hence the nondegenerate electrons have energies of value

$$\mu + \frac{m}{2} (u^2 + v^2 + w^2).$$

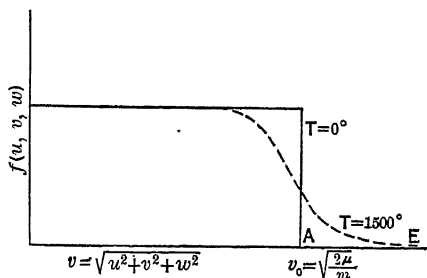


FIG. 195. Distribution of energy among free electrons in metals on the basis of the Fermi-Dirac statistics as applied by Sommerfeld.

The few electrons with the Maxwellian energy superposed on μ at the lower temperatures are the ones which cause electrical and heat conductivity, and thermionic and thermoelectric phenomena. It is these which leave the upper portion of the rectangular Fermi distribution at 0° absolute and go to the asymptotic Maxwellian foot as T increases. In silver at 0° C these amount to about one-sixtieth of the free electrons in the Fermi distribution. Hence these contribute only a small amount to the molecular heat above the Dulong and Petit law.

As T is increased the exponential function for velocities less than v_0 begins to be appreciable, and the exponential term with negative exponent adds to the 1 and makes the curve fall before v_0 is reached. For values of the velocity $\sqrt{u^2 + v^2 + w^2}$ greater than v_0 the exponential increases more slowly the greater the kT , and hence the fall is more pronouncedly asymptotic, and for a metal with 1 electron per atom at 1500° absolute the end of the distribution law curve takes on the appearance shown by the curved dotted line of Fig. 195. At a $\sqrt{u^2 + v^2 + w^2}$ with an energy greater than μ for large kT the exponential term is greater than the 1 and the distribution law approaches a law of the form

$$f(u, v, w) du dv dw = \frac{2 m^3}{h^3} e^{\frac{-\frac{m}{2}(u^2+v^2+w^2)}{kT}} du dv dw,$$

which is closely similar to the form of the Maxwellian law as given in Fig. 194. In other words, the new law at high temperatures even for metals takes on the Maxwellian form.

The situation can thus be summarized by saying that at ordinary temperatures in a metal the small electron mass and the high electron density lead to a distribution in which most of the electrons are free but degenerate, having a high energy of which the limiting value is μ . As temperatures increase, more and more electrons become free to engage in energy exchanges and the distribution function becomes modified in form, the higher velocity limit conforming more and more to the Maxwellian distribution as these energies are superposed on the energy μ .

186. THE INFLUENCE OF THE SOMMERFELD-PAULI THEORY ON PHOTOELECTRIC EMISSION

It is now of interest to inquire into the magnitudes of these quantities and to see how this new limiting energy will alter the photoelectric effect. For a metal like silver with 1 electron per atom n takes the value of 6×10^{22} electrons per cubic centimeter. With this value of n a value of μ can be calculated which gives v_0 as about 5.5 equivalent e.v. of energy. Hence whereas at room temperature the average thermal energy of electrons is about 0.0356 e.v. there are present in the *metal* an appreciable number of electrons with an energy of 5.5 electron volts. Now though these electrons cannot partake of thermal equilibrium relations, they are *free*, and can respond to the vibrations of ultra-violet light. Hence if ultra-violet light of frequency ν_0 strikes a metal with an absolute work function C , but with degenerate electrons of maximum energy not 0.0356 volt but equal to μ volts, the electrons will be emitted with zero energy if $h\nu_0 = \chi = C - \mu$. Hence χ gives the actual *observed value of the work to pull an average* electron out of the metal. If the problem of the current as a function of the frequency ν is handled from the standpoint of these suppositions using wave mechanics, Wentzel shows that for $h\nu$ greater than C (i.e., all electrons can be removed by the light), the current i is proportional to $\nu^{-\frac{1}{2}}$. For values of $h\nu$ between C and χ the current i is proportional to

$$\nu^{-\frac{1}{2}} \left[1 - \left(\frac{C/h - \nu}{\mu/h} \right)^{\frac{1}{2}} \right],$$

while at a value of $h\nu$ below χ (i.e., below ν_0) there is no current. In the region between C and χ at low temperatures it is the degenerate electrons that give rise to i . Hence since most measurements are made in this region it is not surprising that the *photoelectric current is practically independent of temperature*, for most electrons are degenerate up to temperatures of 1000° C or more. At temperatures where the electrons cease being degenerate there should be a definite effect of temperature. The effect is masked, however, by the thermionic effect in this region.

From the second equation of Wentzel, which in general covers the usual conditions, it is seen that the current reaches a maximum value and then decreases. It now happens that this maximum lies in the far ultra violet for all except one group of metals, that is, for the alkali group. It is possible to calculate μ for data taken from the optical constants. χ is observed experimentally so that both C and μ are available. From these data one can calculate the equivalent wave lengths λ in angstrom units for the constants C/h , μ/h , and ν_0 for two metals. They are

	$\lambda_{(C/h)}$	$\lambda_{(\mu/h)}$	λ_0
Na.....	2490	3930	6800
Cu.....	1110	1760	2660

From this table it is seen that for Na the limiting wave length λ_0 is in the red of the visible spectrum, and the frequency corresponding to the energy C , $\lambda_{(C/h)}$, lies well into the ultra violet, but in a region easily achieved. For copper the threshold wave-length λ_0 is already in the ultra violet, not far removed from the strong ultra-violet mercury line 2537, while C lies in the experimentally difficultly accessible *extreme ultra violet*.

Sub-stance	Long-wave-length threshold in Å units	μ expressed in equivalent wave length in Å u. by $3 \times 10^{10} h/\mu$	μ in equivalent volts	χ observed in volts		Approximate value of C in equivalent volts
				From photoeffect	From thermionic effect	
Li	5200-5160	2630	4.7			
C	2565-2615			4.72-4.81	4.3	
Na	5830	3930	3.14	1.80-2.12	1.8	5.0
K	6120-7200	6050	2.04	1.2 -2.02	0.46-1.55	4.0
Cu	2665-2750	1760	7.01	4.07-4.63	3.85-4.00	11.5
Ag	3210-3390	2250	5.5	3.75-4.05	2.60	9.5
Rb	10000	6980	1.77	1.2	1.45	2.6
Cs	10000	8150	1.52	1.2	0.7 -1.36	2.2
Zn	3020-4010			3.08-4.10	3.02	
Au	2600-2730			4.33-4.75		
Fe	2870-3150			3.92-4.36	4.04	
Hg	2600-3040			4.05-4.75		
Pt	1850-3020			3.63-6.5	5.0 -6.0	
W	best 2840 2300-2735			4.52-5.36	4.31-4.53	

It is not strange then that the early work with the alkali metals showed a distinct maximum for the photoelectric emission. Pohl and Pringsheim further showed that this selective photoelectric effect, as it was called, depended on the direction of vibration of the light. The effect is a maximum when the electrical vector of polarized light

is parallel to the plane of incidence (i.e., when the light vibrations have marked components perpendicular to the surface). The metals show peaks of activity at approximately the following wave lengths: Rb at 5000 angstrom units; K at 4500; Na at 3400; and Li at 2800. The theory of Wentzel predicts just such an action with a dependence on the electrical vector as indicated, though at present in approximate form only. An idea of the magnitudes of the quantities λ_0 , μ expressed in equivalent volts of energy and as an equivalent wave length by the relation $3 \times 10^{10} \hbar/\mu$, χ in equivalent volts observed from the photo effect and the thermionic effect, and finally $C = \mu + \chi$, is given in the accompanying table. It is to be noted among other things that the quantity χ comes both out of photoelectric and thermionic effects, a result to be expected from this theory.

187. FURTHER FACTS CONCERNING PHOTOELECTRIC EMISSION

The distribution of the velocities of photoelectrons liberated by different wave lengths from different metals is of the same general form for all metals and frequencies, and differs only in the magnitude of the currents found, according to Ramsauer. The form of the curve is a sort of bell-shaped curve when energy of the emitted electrons is plotted against the number of electrons emitted. The peak or most probable energy in these curves is between one-third and one-half the maximum emergent energy. Such a curve of velocities of *emitted* electrons might indicate the form of the energy distribution of the degenerate electrons in the metal, but is, however, involved with the question as to the initial direction of emergence, and the nature of the surface layers. The results of Ramsauer do not appear to be of very general application for substances other than certain pure metals, and surface films play a very disturbing role.

The photoelectric effect occurs not only for metals but for a large number of nonmetallic substances. The light does not only have to liberate the electrons of these nonmetallic substances from the solid or liquid surface, but is also required first to set free electrons in the substances, for nonmetallic substances are characterized by the absence of free electrons in any quantities. In general, very short wave lengths are required for these substances, and the long wave-length thresholds required for substances like NaCl, KCl, CuCl, etc., are seen to lie in the region of 1800–2000 angstrom units. The action of light on AgI, on S and on anthracene indicates internal liberation of electrons in the visible region with external emission only in the ultra violet. In all cases the currents are small, as relatively little light is absorbed near the surface (in comparison to metals), so that many electrons cannot get out. The vapors of the metals are also photoelectric, and it has been shown by Williamson, Mohler, Lawrence, and others that ultra-violet light liberates electrons in vapors of the alkali metals and Hg. In

gases like N_2 , O_2 , and H_2 the effect should also be observed, but the long wave-length limit lay so far in the ultra violet as to preclude early detection. The effects are also small owing to the difficulty of getting strong sources of short wave lengths with existing high absorbtion.

Finally it might be stated that in recent years the action of a high external electrical field on the photoelectric effect has yielded the fact that the threshold frequency is shifted to longer wave lengths if a powerful field is allowed to act to pull the electrons out, as shown by Lawrence and Linford. This is in agreement with the theory of Schottky as to the nature of the electrical fields at the surface of metals, and with Wentzel's theory of the metallic surface forces.

The photoelectric effect is used wherever it is desired to convert changes of light intensities to electrical or mechanical effects. With certain restrictions it can be used for *measuring light intensities*, provided the *selective actions of some wave lengths* are corrected for. It can be used for producing electrical effects proportional to varying light intensities. It has been used for recording speech in talking moving pictures. Here the varying light intensities produced by the reception of sound waves on an electrical receiving system sensitive to sound vibrations are photographed on the same films recording a moving picture. This ensures synchronism between speech and action. After developing the film the varying intensities of the light record on the film are projected onto a photoelectric cell by means of a beam of light passed through the film. The electrical response of the cell is reproduced as sound in a loud-speaker through electrical amplifying devices. Photocells are also used in the transmission of pictures of objects by radio in television. The light intensities at various points on the object being viewed are thrown in rapid succession onto a photocell, which in turn modifies the intensities of signals emitted by a radio sending device. At the receiving end the signals are amplified up, synchronized with the sending device, and used to produce light-intensity variations by changes of electron current impinging on a fluorescent screen, such as is used in oscillographs. The intensity modulated electron beam is caused to scan over the fluorescent screen by static deflectors in synchronism with the scanning of the televised image. More recently in the Farnsworth device the picture is projected onto the cathode photoelectric surface and the picture in terms of intensity of electron emission is converted to a cathode-ray beam picture in vacuum. This picture is moved up and down and horizontally over the surface of the anode (in which there is a small hole) using magnetic deflecting coils properly synchronized outside the tube. The electrons entering the hole are caught on an electrode and amplified, producing a variable signal on the grid of the first tube of the transmitting system. This gives signals the intensity of which varies with the photoelectric intensity of the picture on the cathode. Zworykin has invented still another way of using the photoelectric effect in

television, which is also very successful. Both these devices utilize the electron multiplier tubes first developed by Farnsworth for solution of the television problem; see page 523. The photoelectric effect is used widely in the control of any phenomena, electrical or mechanical, by light signals.

The advantages of use of the photoelectric effect in light-intensity measurements are:

1. The close proportionality between light intensity and current.
2. The very rapid time response of signals with no apparent inertial effects.

3. The extreme sensitivity of the effect. (The effect with amplification is possibly more sensitive than the eye for light detection. The eye can detect light intensities of the order of one quantum of blue-green light per square millimeter per second, an effect easily detectable with the photocell.)

The disadvantages of the photoelectric effect in light-intensity measurements are:*

1. All photocells are *selective* in their action. Certain wave lengths are far more effective than others. Hence great care in the adaptation of such cells and the interpretation of results obtained therewith is required.

2. The unsatisfactory degree of constancy of emission. The photoelectric emission varies with time and requires constant verification.

3. The unsuitability of the phenomenon for measurements of wave lengths much greater than 6000 angstrom units.

The commercial photoelectric cells have either a central anode or a central cathode. Cells with a central anode are most frequently used. The cell consists of a glass or quartz (for ultra-violet light) bulb coated on the inside with a sensitive conducting layer of a photoelectric metal. This is usually one of the alkali metals, very often in the form of a hydride (i.e., combined with hydrogen). This coating is removed from a given area of the cell to make a window for the admission of the light. The central anode usually in ring form is insulated from the layer of metal, and both anode and metal coating or cathode are connected to outside contact points by sealed-in leads. The cells are usually highly evacuated, except for certain cells which have a residual gas for amplifying the currents by ionization by collision. The usual cells employ enough potential between cathode and anode to give a saturation current, i.e., about 100 or so volts. The gas-filled cells use a much higher potential (just below sparking) to cause the electrons emitted by the metal to multiply the current by

* The development of commercial phototubes has in some measure overcome these inherent difficulties. Phototubes sensitive in the infra red have successfully been used by all nations in World War II. Limits of sensitivity are not known but cannot go far below 12,000 Å.

ionization by collision. The alkali hydride cells are most sensitive in the visible but are not very constant. The hydride is formed by a glow discharge between cathode and anode in the presence of H_2 at a pressure of a millimeter or so of mercury. The gas is later pumped out. Constant cells of very high efficiency are given by Zn-Cd cells if ultra-violet light can be used. Straight alkali metal cells are also used for the visible end of the spectrum and, if well made, are fairly constant. Today sensitive surfaces are also formed on an oxidized silver plate coated with Cs and very slightly oxidized again. These have a very high efficiency.

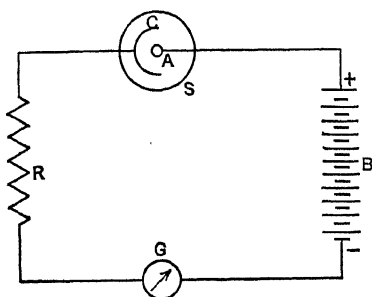


FIG. 196. Simple circuit for the use of a photoelectric cell.

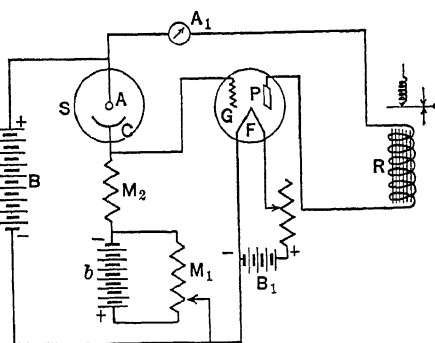


FIG. 197. Amplifying circuit used to work a relay by means of a photoelectric cell.

To measure light intensities the cell can be used as indicated in Fig. 196. S is the cell, C the cathode, A the anode. B is a battery of 90 to 200 volts, G is the galvanometer, and R is a resistance of some 5000 ohms.

In Fig. 197 an amplifying circuit and photocell are shown for working a relay R in response to light signals. B is a battery of 90–135 volts, M_1 is a grid bias variable resistance of 10^4 ohms, with a battery b for giving a bias to the grid of the three-electrode tube. The cell is S , the cathode C being connected to the grid G of a three-electrode amplifier tube. M_2 is a resistance of 1 to 10 megohms between the bias and the cathode C . The anode A is connected through a milliammeter A_1 through the relay coil R to the plate P of the three-electrode tube, the positive pole of the main battery B also going to A . B_1 is the filament-lighting battery of the filament F of the three-electrode tube which is attached to the negative pole of the battery B , and the positive pole of the bias b through the variable resistance M_1 . On illumination the positive charge on C due to loss of electrons overcomes the negative bias on b which is set to interrupt the electron current from the filament F to the plate G . The instant the bias is neutralized the current flows in the three-electrode tube and operates the relay.

Recently a new and powerful photoelectric cell has come into commercial use. This utilizes the effect of light in allowing a current to pass to a metal from the contact of a *semiconductor* like CuO to the

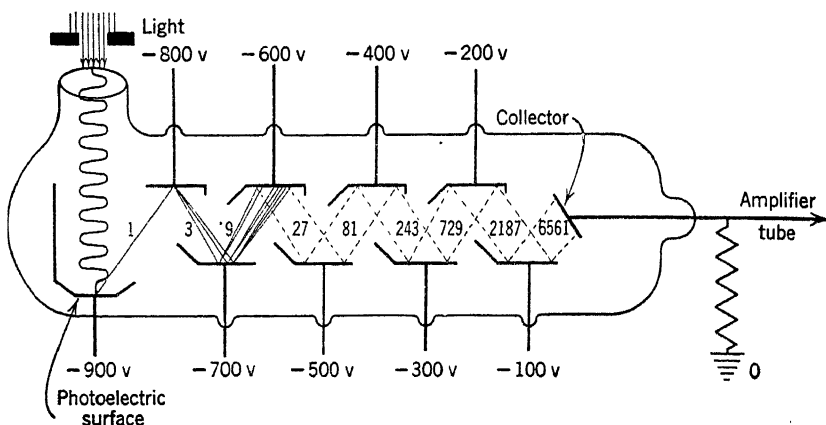


FIG. 198a. Eight-stage phototube multiplier.

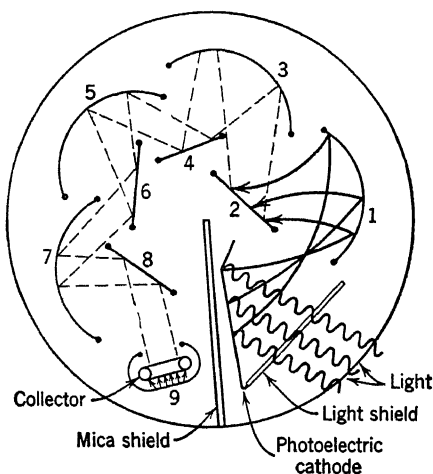


FIG. 198b. RCA type-931 phototube multiplier.

conductor Cu in close contact with it. One such commercial cell goes under the name of "photronic" cell. These cells have a very high efficiency and are quite stable. The theory of the action of semiconductors is little understood at present and is the subject of much study. The commercial advantages of these cells are such that they are widely used.

It had long been known that electrons of sufficient energy on strik-

ing metal surfaces could knock further electrons out of the surfaces. The yield increases with electron energy up to a point where the electrons penetrate too deeply into the surface to permit secondary electrons of low energy to escape. It then decreases with further increase in energy. It depends on the nature of the metal, or the surface, and on its condition. In general, the yield obtained was low, ranging between 1 to 2 electrons from the surface per incident primary. This still applies to most metals. However, certain oxides of metals like Ba, some special metal surfaces, and, above all, composite surfaces of Ag with AgO and Cs give rather startling gains of up to 9 electrons per incident primary. The very sensitive surfaces are not very stable, but stable surfaces giving 4 to 6 electrons per primary have been achieved. This makes their use highly profitable for amplifying weak photoelectric or electronic signals such as obtained in television. If there are five stages of successive amplification with a gain of sixfold per stage the amplification will be 5^6 or 16,625 times. Various devices have been perfected to utilize this principle, which are shown schematically in Figs. 198a and 198b, and which are self explanatory. The surfaces used are of the high-gain type. They are indispensable to television, where weak photoelectric currents require amplification before they can operate the tubes.

B. THE THERMIONIC EFFECT

188. THE DISCOVERY OF THE THERMIONIC EFFECT

We can now turn to the second of the two phenomena to be discussed, namely, the thermionic effect. The early experiments on static electricity of Du Fay (1737), Watson, Priestley, and others indicated that the air in the neighborhood of hot bodies has the power of conducting electricity. These early observations were not pushed any further and it was not until Becquerel in 1853 observed that air near objects at white heat was unable to insulate, even for a few volts potential difference, that any real advance in the study of this field was made. This fact was extended by Blondlot to include as low a potential difference as 0.001 volt. He also found that the current was not proportional to the potential difference for small potentials, a very important fact as we shall later learn. It was Elster and Geitel who made the first systematic investigation of the phenomenon in 1880. It will be recalled that these same men were also pioneers in a study of the photoelectric effect. They heated wires by an electrical current and measured the current conducted to a neighboring electrode as a function of the potential between hot wire and electrode. It was observed that a current was established in the absence of any potential difference. This current built up the potential between the electrode to a certain value and then ceased, except in sufficient measure to

maintain the potential built up. The effects obtained varied widely. With Pt in air the potential was positive at low temperatures (dull red heat), increased to a maximum at red heat, and fell to near zero at white heat. In vacuum the potential at higher temperatures not only went to 0 but became progressively more and more negative. Thus the wires appeared to give off positive electricity at low temperatures and negative electricity at high temperatures. The effects varied with the metals used. Cu gave mostly positive charges, carbon exclusively negative charges.

Branly, approaching the problem with a different technique, confirmed the results of Elster and Geitel. Edison in his early work on the carbon-filament lamp found that the filament became positive while an independent electrode in the lamp bulb became negative when the filament was sufficiently hot. This was analogous to the results of Elster and Geitel and was proved so by Preece and by Fleming in 1885 to 1896. J. J. Thomson's work leading to the discovery of the electron and the investigations on positive and negative gaseous ions led to the opinion that the very hot filaments gave rise to negative ions, especially in the presence of gases. It was then believed that the hot metal acting on the gas ionized the gas, and McClelland, a pupil of Thomson's, in 1899 showed that in fact the air drawn from the neighborhood of the hot filament did contain negative ions. This was proved by a measurement of the velocity of the ions, these ions being somewhat slower than the normal ions in air. This was caused by a faulty technique, for now the ions formed in dry air by a hot filament are known to have the same velocity as normal negative ions. McClelland found that the current from a hot wire increased with the potential applied, finally reaching a *constant saturation* value. J. J.

Thomson, as might be expected, measured the ratio $\frac{e}{m}$ of charge to

mass for the ions from a hot carbon filament in 1899, and showed the *negative carriers to be electrons*, or the same as the constituents of cathode rays. Hence the negative currents from hot carbon, and from the hotter metals like platinum, were electrons liberated in vacuum and thus not a product of gaseous disintegration produced by the hot wire as previously assumed. The ions observed by McClelland were simply electrons that had attached to air molecules or vaporized platinum particles. *The electrons are emitted directly from the incandescent bodies at high temperatures in vacuo.* By 1906 J. J. Thomson had

shown by magnetic measurements, similar to those used in $\frac{e}{m}$ de-

terminations, that the positive electricity emitted by Pt at lower temperatures was on particles of a molecular mass (either ions of Pt or lighter ions attributed to the gas present), and that some heavier ions were also present as observed by McClelland. These are now at-

tributed to large positively charged solid particles of platinum torn off, or liberated, from the hot wire as atoms that condense to solid particles.

189. THE LAWS OF THERMIONIC EMISSION

By 1901 the pioneer work on thermionic emission which has made possible most of the modern advances in the field of thermionics, and which netted its author the Nobel prize, was begun by O. W. Richardson. This work was an experimental investigation of the variation of the electron emission from hot bodies as a function of temperature. The problem was a difficult one from a technical point of view, par-

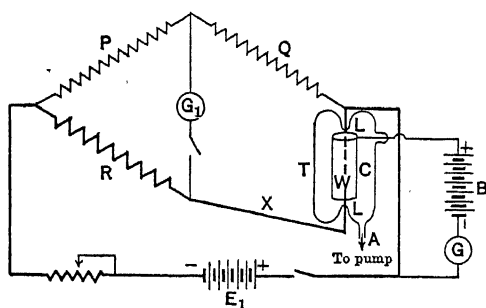


FIG. 199. Arrangement for a study of thermionic emission.

ticularly at its inception. The body to be studied is if possible used in the form of a small wire *W* supported in the axis of a cylindrical tube by heavy leads *L* as shown in Fig. 199. The wire is surrounded by a cylinder *C* of length greater than the wire and coaxial with the glass tube. The cylinder and heavy leads are passed out through the glass tube by glass to metal seals, and a side tube *A* is provided for exhausting the air. Owing to intense heating by the filament, all occluded gases must be baked out and removed by heating the glass to near its softening point, heating the metal parts to incandescence by an induction furnace, and bombarding the metal parts by a heavy discharge of electrons. These operations are carried out while the tube is connected to a powerful high-vacuum pump. Such a tube is then constant in further investigations and should yield reliable results. Modern tubes are sealed off from the pump on exhausting and are often kept gas-free by distilling a small amount of reactive metal like an alkali or Mg into the tube to act as a "getter" for gases given off. The temperature of the incandescent filament is controlled and kept constant by measuring the filament resistance, changing the heating current as the temperature changes. The filament is made one arm of a Wheatstone bridge (*X*) of Fig. 199, the other resistance in

that arm being R and having such a nature that it can carry the heating current without marked change in temperature. The other arm of the bridge has the ordinary ratio resistances P and Q set to give a balance with the hot wire at the proper temperature. The potential E_1 operating the bridge is the same one that gives the heating current. Any change in temperature of the filament changes the resistance and throws the bridge out of balance. The filament temperature can be determined by an optical pyrometer, or by a thermocouple. Cali-

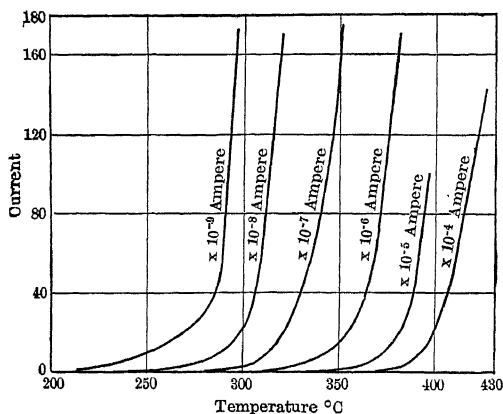


FIG. 200. Richardson's curves for the thermionic emission as a function of filament temperature.

brating the bridge to read temperature may also be accomplished by observing the melting points of specks of fusible substances placed on the filament, the bridge current being noted as each speck melts, so that the current can be plotted to indicate temperature.

In this fashion, by connecting the tube as indicated in Fig. 199, a study of emission as a function of temperature can be made. The battery B furnishes the potential driving the current through a galvanometer or electrometer G . The potential of B must be such as to give the *saturation current* from the filament, as it had been early observed by Blondlot that the current varied with potential for low values of the potential across the tube. It will be found that the potential to give this saturation increases with the temperature of the filament and hence as the current increases. A characteristic set of curves found by Richardson for Na from 217° to 427° C are shown in Fig. 200.* It is seen that the saturation electron current from a hot filament rises exponentially with the temperature of the filament, following a law deduced by H. A. Wilson and Richardson theoretically

* The author wishes to acknowledge the courtesy of the editors of the *Proceedings* of the Royal Society of London, of Messrs. Longmans, Green & Co., and of Professor O. W. Richardson for permission to reproduce these curves.

as early as 1901, and later further developed by Richardson. The equation was developed on the basis of the electron theory of metallic conduction which evolved in the work of Drude, Riecke, and Lorentz, following J. J. Thomson's discovery of the electron. As was stated in the early part of this chapter, the electrons were believed to be free in metals in considerable numbers and to be in thermal equilibrium with the atoms of the substance at a temperature corresponding to that of the body. If it is assumed that the energy of the electrons is distributed according to Maxwell's law, then as the temperature rises the velocities and energies of the electrons increase as described before. It will be noticed on observing the curve of Fig. 194, showing Maxwell's distribution for electrons emerging from a surface, that there are always present in the distribution some electrons of very high kinetic energy. If now the energy of motion of those electrons which are moving toward the surface of the metal, and near it, exceeds the potential energy of the force field holding the electrons inside the metal, the electrons will, just as do atoms inside a liquid or a solid, escape through the surface (i.e., evaporate). Hence on heating a metal with free electrons to a high enough temperature so that the energy of a portion of the faster electrons (i.e., those in the tail AE of the distribution curve, Fig. 193) exceeds the so-called retarding potential at the surface, these electrons will escape, i.e., the hot metal will emit electrons. In close analogy with the kinetics and thermodynamics of vapor molecules escaping from a liquid surface (i.e., evaporating) Richardson and Wilson developed the equations for emission. There are two equations resulting from somewhat different assumptions as to conditions in the metal. χ may be used to designate the energy required to remove an electron from the metal, i.e., the so-called *work function* mentioned before. In the event that the number of free electrons n per cubic centimeter is independent of the absolute temperature T , and that χ is independent of T , the equation becomes

$$i = n\epsilon \sqrt{\frac{k}{2\pi m}} T^{1/2} e^{-\chi/kT} = A_1 T^{1/2} e^{-\chi/kT}.$$

Here ϵ is the charge of the electron, m is its mass, and k is the Boltzmann gas constant per atom $\frac{R_a}{N_a}$.

If χ is independent of T , while n is proportional to $T^{3/2}$ the equation takes the form

$$i = A_2 T^2 e^{-\chi/kT}.$$

In more recent years Dushman has deduced the theory more rigorously and obtained the equation

$$i = \frac{2\pi k^2 m \epsilon}{h^3} T^2 e^{-\chi/kT}.$$

Here e is the charge on the electron while e is the base of the natural system of logarithms.

The new statistics of Fermi and Dirac as recently applied to the problem of electron emission by Sommerfeld and Fowler gives the current i as

$$i = \frac{4 \pi m e k^2}{h^3} D T^2 e^{-\frac{C-\mu}{kT}},$$

where D is the reflecting factor of the surface for electrons and $C - \mu = \chi$. This differs from Dushman's theoretical equation only by the factor 2 brought in by the electron spin (2 electrons for each energy state), and by the uncertain value of the coefficient D for internal reflection of electrons by the surface which, however, may cancel the 2 by taking on the value $\frac{1}{2}$. The quantity D enters the problem here as a result of the application of the newer wave mechanics and cannot be further considered. This equation, as the notation indicates, takes account of the high energy μ of the degenerate electrons.

It is seen then that, in general, except for the first equation the law of thermionic emission may be written

$$i = A T^2 e^{-\chi/kT} = A T^2 e^{-b/T}.$$

It may be recalled that χ is *not the real work function* but the *apparent* one, C being the real work function, and χ being of importance because the upper energy μ of the degenerate electrons reduces the work to get out of the metal: χ is given by $\chi = C - \mu$, and indicates the energy of thermal agitation beyond μ which is required by electrons in order to escape. In most work the constant χ/k is written as b or b_2 . On the basis of Dushman's derivation the constant A has been evaluated as 60.2 amperes per square centimeter per degree squared, a value agreeing within the limits of certainty with the constant of the Sommerfeld theory. As seen before, $\chi = C - \mu$, and hence b is a characteristic of the metal used and depends on the work to pull electrons out of the metal. C is the absolute work function, and μ is the limiting energy of the fastest electrons in the Fermi distribution.

The equation has been tested numerous times, one of the latest and best tests having been in the work of Davisson and Germer, and later of Germer. For a test of the equation over large ranges of i owing to the exponential term it is simpler to plot $\log i$ against $\frac{b}{T}$. If the

equation is correct the resulting curve should be a straight line of which the slope at once gives $\log AT^2$. Such a curve, taken from Germer's paper in the *Physical Review*, Vol. 25, 805, 1925, is shown in Fig. 201. The way in which the observed points lie on a straight line indicates that the exponential law is accurately fulfilled even for a range of values of i from 1 to about 10^{10} .

The value of A obtained from the data for the element W is in good agreement with Dushman's value of A . It was early pointed out by Richardson, however, that in attempting to distinguish between the T^2 and the $T^{1/2}$ in the first two equations deduced, the rapid variation of i with $e^{-b/T}$ masked the effect of T^2 outside the exponential to such an extent that no decision between the two equations could be arrived at. The same condition holds for the value of A . This lies in the fact that between 1000° absolute and 2500° absolute i for W varies by a ratio of 10^{15} while $T^{1/2}$ varies by 1.58 and T^2 by only 6.25. What really emerges from theory and experiment is clearly that an equation of the form $i = AT^x e^{-b/T}$ fits the facts to a high degree of precision, with a good degree of probability that $x = 2$, and that the value of A observed by Davisson and Germer and computed by Dushman is of the right order of magnitude. The values of b observed are fairly accurate and consistent for similar surfaces and further agree within experimental limits with the values of $\chi/k = \frac{C - \mu}{k}$ determined by the photoelectric effect.

Hence it seems fairly certain that the Dushman theory and the newer Sommerfeld theory are correct in general outline, though the more minute details are uncertain.

The relations established, however, have important implications, for they show that certain theoretical assumptions are confirmed. It was early shown theoretically by Richardson that if a Maxwellian distribution of velocities exist for electrons inside the metal they emerge through the surface with the Maxwellian distribution maintained, even though in escaping from the surface they have done work in escaping. This same condition holds for an evaporating liquid, both the molecules of the liquid and the escaping vapor having the Maxwellian distribution of velocities, the average energy of the vapor in *equilibrium* being that of the liquid. Now the test of this expected behavior on the part of the electrons from a hot metal lies in the proof of the exponential law as predicted by the equations above. This is seen to have been verified to a high degree of precision, for the plot of $\log i$ against $\frac{b}{T}$ is accurately a straight line over a tremendous range. However, in the light of the newer Sommerfeld theory the electron

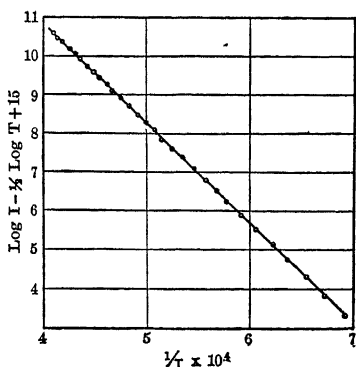


FIG. 201. Germer's test of Richardson's equation for thermionic emission.

energy distribution in the metal is not Maxwellian, but is given by the Fermi-Dirac statistics. It happens that for high values of T the number of degenerate electrons decreases and the square curve of Fig. 195 goes over for higher temperatures to a distribution of which the highest energy electrons more and more resemble in their velocity distribution the Maxwellian distribution, as in the dotted portion of Fig. 195. In the limit of very high temperatures the Sommerfeld equation yields the usual Maxwellian form. What then occurs is that the faster electrons (those corresponding to the asymptotic dotted portion of Fig. 195) are the ones to escape most often, and since the average energy is μ and the total work of escape is C , the electrons escaping have an energy in excess of $C - \mu = \chi$ in the measure of the asymptotic portion of the curve of Fig. 195, which is closely Maxwellian. It is then in conformity with the new Sommerfeld theory that the thermionic emission yields a Maxwellian distribution for the relatively few electrons emitted, while inside the metal the large proportion of the electrons have the Fermi-Dirac distribution.

The values of the constants of the equation for thermionic emission of a few metals are given below.

$$i = AT^2 e^{-b/T} = AT^2 e^{-\chi/kT}$$

$$A = 1.80 \times 10^{11} \text{ e.s.u. per cm}^2 \text{ deg}^2 = 60.2 \text{ amp per cm}^2 \text{ deg}^2.$$

$$k = 1.371 \times 10^{-16} \text{ erg per degree}$$

$$\chi_0 \text{ in volts } (\chi \text{ at } 0^\circ \text{ absolute}) = 300 \frac{kb_0}{e} = 8.61 \times 10^{-5} b_0 \text{ volt.}$$

It is thus seen that by varying the temperature the saturation current can be varied at will and the approximate emission to be obtained from any filament at a given temperature if b is known can be predicted.

RECENT THERMIONIC CONSTANTS FOR DIFFERENT METALS

Element	A	b_0	χ_0 in volts
Platinum.....	1.7×10^{11} *	72,500	6.27
Molybdenum.....	60.2	50,900	4.38
Tantalum.....	60.2	47,200	4.07
Tungsten.....	60.2	52,400	4.52
Thorium.....	60.2	38,900	3.35
Calcium.....	0.12	35,000	3.02
Nickel.....	26.8	32,100	2.71
Carbon.....	5.93	45,700	3.93

* Recent work indicates that the high value of A for Pt results from minute traces of impurity which can be removed only by outgassing near the melting point over thousands of hours.

190. THE CURRENT-POTENTIAL CHARACTERISTIC OF THERMIONIC EMISSION

It is next essential that the student obtain a knowledge of how the current varies with the field applied, for in applications of the thermionic emission the field applied plays a most important role. An experimental arrangement such as shown in Fig. 202 in which the current from the filament F , or an equipotential hot cathode to a plate P , is measured by a milliammeter A or galvanometer as a function of the voltage serves for such a study. The curves obtain

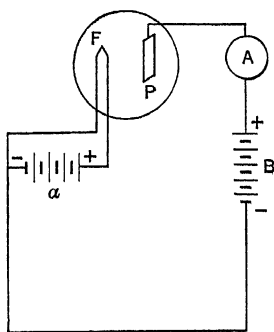


FIG. 202. Measurement of the current-potential characteristic of thermionic emission.

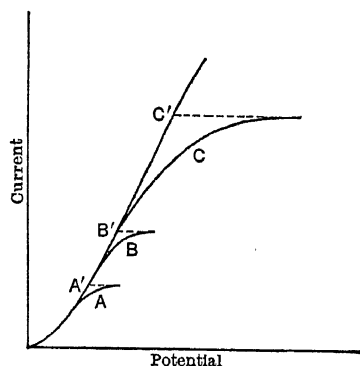


FIG. 203. Current-potential characteristics of thermionic emission at three temperatures increasing from A to C .

for different temperatures T_1 , T_2 , and T_3 of the cathode source are shown in Fig. 203. It is seen that the curve starts at 0, rises along what might appear to be a parabolic curve, varying as some power of the potential greater than unity, and then reaches a saturation value, or at least flattens out and approaches a saturation value gradually. This form of the curve was early recognized to be conditioned by the fact that for heavy electron emissions with a weak field between anode and cathode the electrons emitted formed a cloud in front of the cathode which gave what is known as a *negative space charge* (i.e., a negative charge in space due to a volume distribution of negative electrons). Thus while the field draws electrons away from the cathode the accumulated negative charge of the electrons in the space between anode and cathode exerts a force driving the electrons back to the cathode and limits the current. When the applied field becomes so high that the electrons are drawn across to the anode with such speed that there is little electron accumulation between the plates (small space charge), then the maximum current possible at the temperature is acquired and the saturation current is reached. In 1906, J. J. Thomson computed the limitation

of space charge on the current in vacuum assuming the electrons all to emerge with the same velocity. In 1911, C. D. Child gave the solution assuming the initial velocity of emission to be zero (an assumption which is not seriously in error, for the electrons emerge with velocities equivalent to a fraction of a volt only). More rigorous equations on the basis of the real distribution of velocities are the work of Langmuir and Schottky in 1913 and 1915. The derivation of Child does not give the gradual transition of the curve as saturation is approached, but this region is not of fundamental importance and for the present purposes Child's derivation and equation will suffice.

Let it be assumed there are two plates, the cathode being at 0 potential, the anode at $+E$ volts, and the distance between anode and cathode being d . The plane of the cathode passes through the origin and the distance d is measured parallel to the x -axis. Now one of the important contributions of the mathematical physicist Poisson lay in setting up an equation relating potential and the volume density of the electrical charges at a given point; see page 392. The equation is an expression of the continuity of flux of lines of electrical force, and the contribution made by the accumulation of charges per unit volume at the point in question. As will be remembered, the number of lines of force per square centimeter normal to the flux expresses the electrical field strength X . The rate of change of this field produced by a flux of lines of force due to ρ charges per cubic centimeter in an infinitesimal element of volume at x centimeters from the cathode, due to the accumulation of electrons between the cathode and anode, is $4\pi\rho$, as there are 4π lines of force per unit charge (see page 182) and there are ρ charges per cubic centimeter at x . Hence $\frac{dX}{dx} = 4\pi\rho$ (see

page 392). But in Chapter XIV it was shown that $X dx = dV$, where dV is the change in potential over a distance dx , hence $\frac{dX}{dx} = \frac{d^2V}{dx^2}$.

Poisson's equation for the electron accumulation at a point between the plates can then be written as $\frac{d^2V}{dx^2} = 4\pi\rho$. Here it is seen that

ρ must vary with x , for near the cathode the electrons will be densest, and at the anode there will be no electrons. At saturation, of course, the density will be more nearly uniform (electrons being rapidly drawn away from the cathode), but for lower potentials the electron density near the cathode will be great, falling off towards the anode.

The field strength at any point x will not be $\frac{E}{d}$ but will vary in a more complicated manner as shown in Fig. 204, where curve A is the gradient $\frac{E}{d}$, while curve B shows the distortion produced by the space charge.

The curve *B* exists for 0 velocity of electron emission. If there is a finite velocity of emission the curve at first falls below the 0 axis and then rises above, as shown by the curve *C* of Fig. 204. At any point *x* the electron has fallen through a potential *V*, which is the potential difference between 0 and the potential *V* at *x*. The electron has therefore obtained a kinetic energy $\frac{1}{2}mv^2 = Ve$. Finally the current density *i* (current per square centimeter normal to the line of flow) is given by $i = \rho v$. Here *v* is the electron velocity, *m* is the mass of the electron, and *e* is its charge.

Substituting

$$\rho = \frac{i}{v} \text{ and } v = \sqrt{\frac{2Ve}{m}}, \text{ whence } \rho = \frac{i}{\sqrt{\frac{2Ve}{m}}},$$

for ρ in Poisson's equation we have

$$\frac{d^2V}{dx^2} = 4\pi i \sqrt{\frac{m}{2Ve}}.$$

Multiply both sides by $\frac{2dV}{dx}$ and integrate between the limits $V = V_0$

at $x = 0$, and $V = V$ at $x = d$, $\left(\frac{dV}{dx}\right) = \left(\frac{dV}{dx}\right)_0$ at $x = 0$, and $\frac{dV}{dx} = \frac{dV}{dx}$ at $x = d$.

$$\left(\frac{dV}{dx}\right)^2 - \left(\frac{dV}{dx}\right)_0^2 = 8\pi i \sqrt{\frac{2m}{e}} (V^{3/2} - V_0^{3/2}).$$

Assume for convenience that the cathode is grounded so that $V_0 = 0$.

$\left(\frac{dV}{dx}\right)_0$ is also 0 at the cathode where the electrons have no initial velocity, for if $v = 0$ at $x = 0$ there is no motion, hence the field *X*

is 0 at $x = 0$ but $X = \frac{dV}{dx}$, thus $\left(\frac{dV}{dx}\right)_0 = 0$. This is seen in the curve

B of Fig. 204, where the curve is asymptotic to the *x*-axis (i.e., *V* has 0 rate of change with the distance *x* at this point). Accordingly we can write

$$\left(\frac{dV}{dx}\right)^2 = 8\pi i \sqrt{\frac{2mV}{e}}.$$

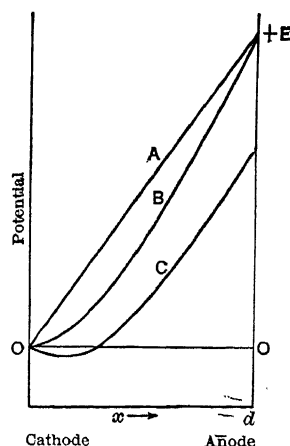


FIG. 204. Potential distribution between cathode and anode with thermionic emission from the cathode: *A*, normal potential drop, with no space charge; *B*, drop neglecting initial electron velocities; *C*, actual drop.

Integrating this with $V = 0$ at $x = 0$ and $V = E$ at $x = d$ (i.e., at the anode) we get

$$i = \frac{1}{9\pi} \sqrt{\frac{2e}{m}} \frac{E^{3/2}}{d^2}.$$

Since π , e , and m are constants it follows that the space charge limited current (i.e., below saturation) is given by

$$i = 2.33 \times 10^{-6} \frac{E^{3/2}}{d^2} \text{ ampere per square centimeter,}$$

or if the area of the cathode is A we can write

$$i_A = 2.33 \times 10^{-6} \frac{AE^{3/2}}{d^2} \text{ ampere.}$$

It is seen then that the current increases as the three-halves power of the potential driving the current, and inversely as the square of the distance between anode and cathode. The curve follows this law until the saturation current (A' , B' , and C') of Fig. 203 is reached, after which it is constant and is given by the Richardson equation depending on the cathode temperature. The presence of the velocity distribution rounds off the otherwise sharp break of the $E^{3/2}$ curve at the saturation voltage so that saturation is much more gradually approached and the curves A , B , C of Fig. 203 take the rounded form of the experimental curves. The initial velocities also serve to prolong the rise of the asymptotic foot of the curves near the axis, giving a slower initial rise. There is still a minor correction to be used where the filament has a drop in potential due to the iR drop of the heating current. The rounded curves illustrated in Fig. 203, which contain all these factors as they represent experimental curves, are called the *characteristic curves of the two electrode tube*, the name given the *thermionic vacuum tube* with anode and hot cathode.

191. THE THERMIONIC RECTIFIER

It is at once seen that an evacuated tube with a hot cathode having a potential difference placed between cathode and anode will react in such a way as to have its current varied as the potential difference varies. If an alternating current is impressed across such a tube with an electron emission from the cathode it will be seen that as long as the cathode is positive and even a little beyond this value no current will flow from anode to cathode, as the electrons are held at the cathode by a combination of space charge forces and retarding field. As soon as the cathode becomes distinctly negative the current of electrons flows to the anode. If the alternating e.m.f. rises well above the saturation value for the current a current results that rises.

rapidly from 0 from the beginning of the negative phase on the cathode, reaches saturation, and continues until the impressed e.m.f. drops down to zero, decreasing along the characteristic curve as the e.m.f. falls below saturation value. Hence we see that such a tube can *rectify* (i.e., give a direct current) for one-half the wave of an alternating e.m.f. This property of the two-electrode vacuum tube is

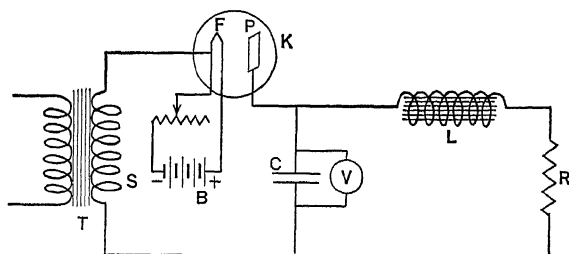


FIG. 205a. Single-valve rectifier.

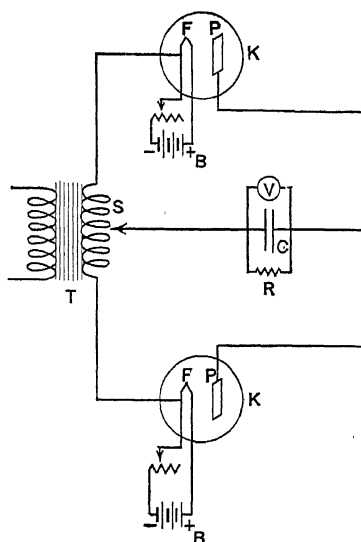


FIG. 205b. Double-valve rectifier.

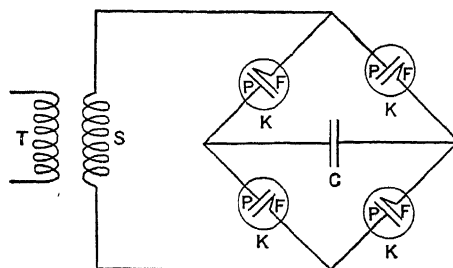


FIG. 205c. Four-valve rectifier.

used very widely today in obtaining direct currents of high potential, from alternating potentials. The two-electrode tube is called the *thermionic valve* or *rectifier*, and a most common form sold in this country is called a *kenotron*. To rectify both halves of the wave two valves acting in inverse senses can be used. If power is being drawn from the tube it is seen that for each valve the current is flowing half the time when there is no supply through the rectifier.

Hence the potential falls during the time of the positive phase of the cathode. It is thus customary to attempt to reduce the irregularities in potential to a minimum and to smooth out the effect of the half-cycle surges by accumulating the charge on capacities of large value. The higher the frequency of the alternating potential the less serious will be the irregularities or ripples of the rectified potential, and hence it is deemed best to use at least a frequency of 500 cycles in the alternating potential to be rectified where smoothness of the rectified potential is required. Greater regularity in high-potential d-c gener-

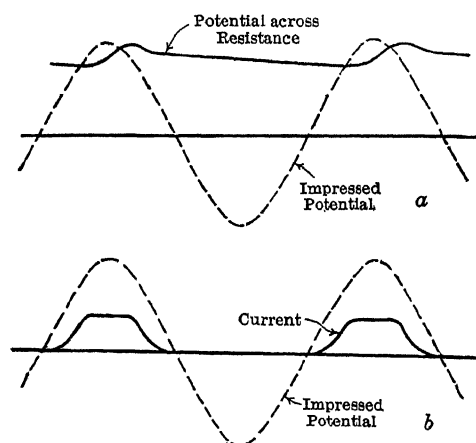


FIG. 206. Impressed potentials and rectified potentials and currents for a single-valve rectifier.

ation for x-ray work using valves is obtained by inserting inductances of high value in the supply lines to "choke" out the ripples due to time variations in current. These are called choke coils. A single valve rectifier as used in rectification is shown with connections in Fig. 205a; Fig. 205b shows a double valve rectifying both halves of the alternating potential, and Fig. 205c shows a four-valve arrangement giving even more complete rectification. The figures are self explanatory, *F* being the filaments, *P* the plates, *K* the rectifiers, *S* the secondaries of the transformers, *T* supplying the power to be rectified, *B* the batteries heating the filaments, and *V* the voltmeters measuring the rectified potential whereas *L* are choke coils, *C* are capacities, and *R* are resistance loads. In Fig. 206a the impressed and rectified potentials of the scheme of Fig. 205a are shown, the dotted lines being the impressed and the full lines the resultant rectified potential on *K* and *C* with a small load on *C* through the high resistance *R*. In Fig. 206b the currents flowing into *C* are shown compared to the impressed potential shown in the dotted curves. Figures 207a and

207*b* show similar curves for the potentials on *K* and *C* for currents in the scheme of Fig. 205*b*.

These rectifiers are made for all sorts of uses from that of giving plate voltage in radio sets (B battery eliminators) to the rectifiers for thousands of volts and many amperes (power rectifiers) used in powerful radio transmitting stations, or for high potentials (10^5 volts) and currents of the order of milliamperes in x-ray work. To obtain more power and to avoid space-charge limitations at lower potentials

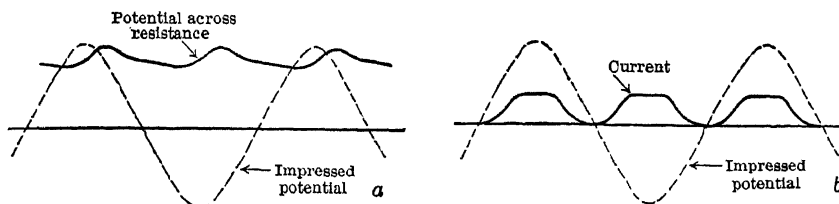


FIG. 207. Impressed potentials and rectified potentials and currents for a double-valve rectifier.

gas-filled thermionic rectifiers are now used. The gases are argon or mercury vapor at low pressures. These give positive ions on bombardment by electrons. The positive ions moving more slowly in the gas help to reduce electron space charge by neutralizing it. In addition the electrons produced by ionization also help. With these tubes amperes of current can be rectified at low voltages in place of the milliamperes observed with space-charge limitation.

192. THE THREE-ELECTRODE TUBE

In about 1908 a new feature was added to the thermionic rectifier valve which made it the basis of all modern developments in radio communications. It was devised as a radio detector replacing the troublesome crystal, electrolytic rectifier, and the coherer by Lee De Forest. Since De Forest's original device was imperfect (it contained gas) and since the inventor little understood the theory of the device it is much to his credit that he made the great step forward which at once and for many years to come solved the radio communication problem. The detector was later developed as an oscillator for generating undamped electrical oscillations. De Forest called the tube the *audion detector*. The device used by De Forest had already in 1902 been used by Lenard to control photoelectric currents, and was devised in a different form by O. von Baeyer in 1908 for the control of thermionic currents. De Forest it appears, however, was the first one to exploit the device in radio detection and obtained the patent on it.

What De Forest did was to introduce a third electrode in the form

of an open grid of coarse mesh between the filament and plate of an ordinary thermionic rectifier. Hence such tubes are often called *three-electrode* tubes. The principle of the operation of the three-electrode tube is about as follows. It was seen that in the rectifier the current through the tube was determined by the effect of the accumulated space charge due to the electrons between the anode and the cathode. This was controlled to some extent by the potential between filament and plate or anode. By placing a grid of which the potential can be controlled from the outside into the space between anode and hot cathode the power of influencing the current through the tube by an outside agent is obtained. Were the third electrode a solid plate it would be possible to have nearly complete electrostatic screening of the anode from the cathode, and were the cathode and grid connected then the whole drop of potential would take place in the space between the third electrode and the anode so that no electrons could get through. If now the third plate is a screen of fine wires it will act to screen the cathode from the anode to a large extent. If the potential of the open grid and cathode are equal then it will be found that the anode will not be completely screened but that a few lines of force will go through the grid from anode to cathode. Hence a few electrons can wander through the grid as a result of the attenuated field, and a much reduced electron current over that in the absence of the grid from cathode to anode results. The larger the meshes of the grid the less the screening action of the grid and the greater the "transparency" of the grid for electrons. If the grid is given a potential which is not that of the cathode, but is that which the plate that it occupies would normally take on for a given current, the current to the anode will flow practically as if the grid were not there, except for a small absorption of electrons by the grid due to the area of its wires. If the grid is made more positive than the position which it occupies would normally take on, it is clear that this field between grid and cathode will *stimulate* the flow of current from the cathode by neutralizing some of the space charge or altering the space charge distribution. Hence a grid positive to the cathode by an amount greater than that of the space in the absence of the grid will increase the current to the anode, beyond that which would exist in the absence of the grid. If the grid is more negative than the place which it occupies would normally become, then it will act to repel electrons back to the cathode, thus causing an increase of the space charge at the cathode, so that the current is decreased. It is accordingly seen that owing to its strategic position between anode and cathode a grid can even by a slight negative or positive change in its potential produce a considerable change in the current through the tube, even though the meshes in the grid are very wide. The fundamental equation for the action of such a grid was worked out by H. J. Van der Bijl in 1913, and was developed from a study of the

device shown in Fig. 208. The tube FGP having a glowing filament F , a grid G , and a plate P is connected through an ammeter A to a battery in the plate-filament circuit of potential E_p . Between F and G there is a battery E_g putting a potential between F and G , making G negative to F . Omitting the contact potential between F , G , and P , we can now consider the action of the grid. Owing to the field between P and F at some point (say the plane where the grid is located) there is produced a potential which can be set as the same as that of a plate of potential $\frac{E_p}{\mu}$ placed at the position of the grid, where μ is a number by which E_p must be divided to give the potential produced at this point by E_p . This represents the field existing in

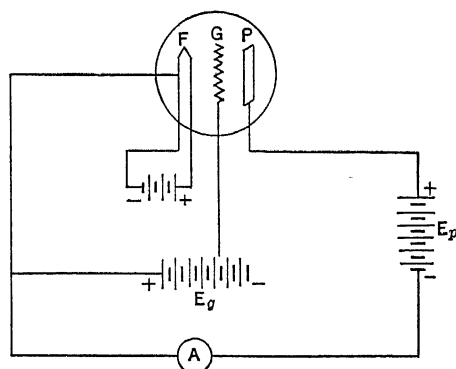


FIG. 208. Arrangement for a study of the characteristic curves of a three-electrode tube.

the absence of the grid due to the potential between P and F . This field as stated is positive, and acts to draw electrons from F , through the point where G is located, towards P . Now some electrons reach G , but with open meshes most of them go through the grid towards P . Thus the current can be stimulated by replacing the field due to P by a potential of value $\frac{E_p}{\mu}$ on the grid, nearly to the extent that the current would be stimulated by E_p alone in the absence of the grid. If the maximum velocity of emission of electrons from the cathode in equivalent volts is V and a contact potential K exists between F and P , and if $K + V = \epsilon$, it can be seen that if a potential $-E_s$ be placed on the grid equal and opposite to that which the plane of the grid would have owing to E_p , the field of E_p will be annihilated and no electron can leave the filament and reach P . Hence the negative stopping potential E_s of the grid (i.e., the negative potential applied to the grid which will stop a current due to a potential E_p on P)

is given by

$$E_s = \frac{E_p}{\mu} + \epsilon.$$

Hence it is seen that the action of the grid on the electrons from the filament is dependent on E_g the grid potential, $\frac{E_p}{\mu}$, and ϵ . That is, the effective potential driving electrons from F to P in a three-electrode tube is $\frac{E_p}{\mu} + E_g + \epsilon$. If $E_g = -\left(\frac{E_p}{\mu} + \epsilon\right)$ it is seen that the net result is 0 and no current will flow. If E_g is positive the current is stimulated, whereas if it is negative and less than $\frac{E_p}{\mu} + \epsilon$ the current is reduced. In general, then, Van der Bijl's equation for a current i in the tube can be written as

$$i = f\left(\frac{E_p}{\mu} + E_g + \epsilon\right),$$

where $f()$ means some function of the expression in parentheses and of course also of the filament area, temperature, etc. The exact form of the function is uncertain, but it is of an approximate form

$$i = \alpha \left(\frac{E_p}{\mu} + E_g + \epsilon\right)^2$$

and varies with the geometrical form of the tube. The factor μ was found by Van der Bijl to be a constant for a given tube and depends on the diameter of the wires and the meshes and the location of the grid between F and P . Among other things μ expresses the maximum amplification obtainable for a given tube and is called the amplifying factor or better the *voltage amplification constant*. This equation is the fundamental equation of the thermionic amplifier.

Now the property which the third electrode imparts to the tube is that it endows the rectifier with a very sensitive sort of a trigger (the grid), which can stop (E_g negative and equal to or greater than $E_s = \frac{E_p}{\mu} + \epsilon$) or stimulate the flow of current through the tube. In this way a small potential E_g relative to the plate potential E_p can cause a current to flow or cease flowing. Hence very feeble oscillations on G (small E_g) can be made to cause a current of much larger magnitude to flow from P to F in synchronism with themselves. By the successive building up of the currents by impressing the oscillations in the plate-filament circuit of a first tube on the grid and filament of a second tube and so on for several stages, the weakest oscillations can be amplified up to values such as to be easily converted to sound or so as to cause marked electrical effects.

In this triggering action it is clear that the vital quantity is the change in current I_p caused by a small change in E_g relative to E_p . This leads to the definition of a very important quantity in circuit work, namely, the *voltage amplification constant*, μ . On page 540, it was shown that the control of the current depended on the value of E_g relative to $\frac{E_p}{\mu} + \epsilon$. This equation relates E_g and E_p but contains

the troublesome constant ϵ . If, however, I_p , the current, is kept constant for the same tube and from curves like those in Fig. 210 the change in value of E_g at constant current for a change in value of E_p is taken an evaluation of μ can be reached without ϵ . Hence the voltage amplification constant is defined as $\mu = \left(\frac{E_p - E_{p'}}{E_{g'} - E_g} \right)_{I_{\text{const.}}}$ since

E_g is negative. Thus $\mu = \frac{\Delta E_p}{\Delta E_g} = \frac{e_p}{e_g}$, for constant current, I_p . The

action of the space charge about the filament in reducing current flow is the equivalent of a resistance r_p called the *plate resistance*. Ohm's law can be applied to the tube and the external circuit resistance

r , viz., $i_p = \frac{\mu e_g}{r + r_p} = \frac{e_p}{r + r_p}$. Note that this really differs from

Ohm's law since i_p and e_p are the *changes* in current ΔI_p and ΔE_p , *not* the full currents and potentials. Thus r_p is the internal resistance of the tube. Since tubes operate with alternating current, r_p is called the "plate impedance" of the tube. If $r = 0$ then $r_p = \frac{e_p}{i_p}$.

The reason for the use of ΔE_p and Δi_p to define r_p is clear. The three-electrode tube does not have a linear characteristic. The plate current I_p as a function of E_p is the sort of tilted-S-shaped curve of Fig. 210. $\frac{E_p}{I_p}$ gives a value of R_p that has no real significance considering tube

control. On the other hand, at any point on the curve $\frac{\Delta E_p}{\Delta I_p} = \frac{e_p}{i_p} = r_p$

has a definite slope which is characteristic of the operation of the tube *at that point*. In working with tubes it is important at times to know how the current I_p changes as E_g is altered, for the purpose of the grid is to change I_p by small changes in E_g . To this end a quantity called the *mutual conductance* g_m , sometimes also called the *transconductance*

S_m , is used. It is defined by the relation $g_m = \frac{i_p}{e_g} = \frac{\mu e_g}{r_p} \bigg/ e_g = \frac{\mu}{r_p}$.

The quantity g_m also varies with plate voltage E_p over the range of the characteristic current, since it depends on r_p . For commercial triodes operating in their normal range r_p runs from about 800 to 150,000 ohms. The quantity g_m varies for commercial tubes under normal conditions from 200 to 5000 micromhos. The value 1000 micromhos means that I_p changes by 1 milliamperes when $\Delta E_g = 1$ volt.

A simple circuit for a study of this action is shown in Fig. 209. The oscillations entering at O are impressed between the filament F and a grid G . The filament F and the plate P are connected through a resistance R and hot wire milliammeter A to a battery E_p . a is the filament battery rendering the filament incandescent, and E_g is a bias battery for varying the potential of G negatively to F in order to let the oscillations impressed on the grid work over a particular part of the range of action of the grid on the current i between P and F . With a negative E_g on G equal to or greater than E_s there will be no

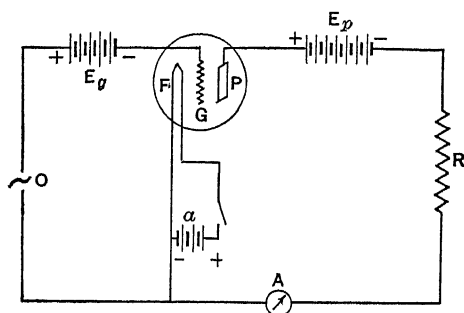


FIG. 209. Circuit illustrating the amplifying action of a three-electrode tube.

current through R . Under other conditions a current flows from P to F (electrons from F to P), and with oscillations of sufficient amplitude on the grid an intermittent current flows through R , giving an intermittent iR drop in R in synchronism with the oscillations in O . This iR drop can be impressed between filament and grid of a second tube, or else R can be replaced by a coil coupled inductively* to a coil in a second circuit. If the oscillator be omitted the current i to the anode or plate can be studied as a function of E_p and of E_g . This gives a set of curves for different values of the two independent variables. Such a set of curves are reproduced in Figs. 210 and 211, which result from the measurements of Van der Bijl.† In Fig. 210 the current is plotted against E_p for different values of E_g , and in Fig. 211 the current is plotted against E_g for different values of E_p . In Fig. 211 it is seen that the value of E_g required to reduce the current to 0 is the more negative the greater the E_p . The curves are the *characteristic curves* of the *three-electrode tube*.

If it is desired to rectify the incoming oscillations it is seen that, when these oscillations pass through the 0 phase, the current should

* The amplified oscillations in the branch $FARE_pP$ can be impressed between the grid and filament of a second tube by means of either the resistance method or else by inductive coupling, the conditions of operation determining the method used.

† These curves were taken from H. J. Van der Bijl's *The Thermionic Vacuum Tube*, and are produced through the courtesy of the McGraw-Hill Book Co.

just be 0 (i.e., the bias voltage E_g should have the value shown in Fig. 211 for the foot of the curve corresponding to the E_p used). Then on the negative half of the wave no current will pass. During the whole positive phase the current will flow, starting at zero and rising along the characteristic curve up to the maximum current drawn by the peak positive potential impressed on G by the oscillations, and then falling off to 0 along the curve, the current curves showing much the same form as in the two-electrode rectifier, Fig. 206b. If, on the other hand, it is merely desired to *amplify* the oscillations positive and negative, *without distortion*, the *straight portion of the characteristic*

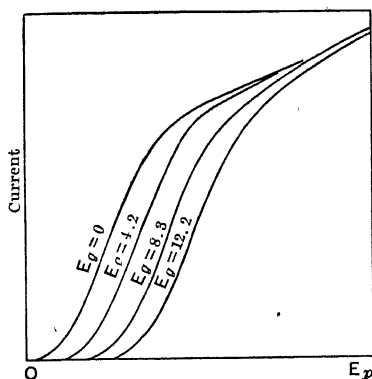


FIG. 210. Characteristic curves for a three-electrode tube, current plotted against *plate voltage* for different grid potentials.

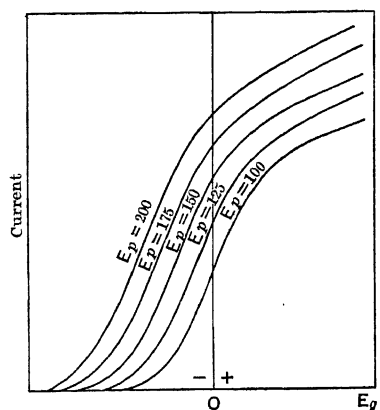


FIG. 211. Characteristic curves for a three-electrode tube, current plotted against *grid potential* for various plate voltages.

alone must be used. For such amplification it is assumed that the *voltage amplitude* of the incoming waves is *small* compared to the *range along the x-axis of the curves of Fig. 211* over which the slope of the curves is constant. Take, for example, an E_p of 200 in Fig. 211. The curve is nearly linear between an E_g of 23 volts and 8 volts. The oscillations must then have a total amplitude of less than 15 volts. E_g must accordingly be set so that the 0 value of the impressed e.m.f. lies at about 15 volts. Then positive and negative oscillations will cause changes in current which are proportional to the potential variations. These variations in current can be made to give potential variations of the order of μ times the incoming amplitudes, and an amplifier is the result.

In general, radio signals consist of a train of sine-wave-form high-frequency oscillations the intensity of which varies in a fashion dependent on the vibrations impressed on the sending oscillator by the voice through a microphone or by a telegraph key. These oscillations

attenuating rapidly with distance, are transmitted as a wave train through space and reach an antenna or collecting aerial wire of large absorbing power tuned to resonance with the frequency of the incoming oscillations. These are communicated by means of a radio transformer (having no iron) to an amplifying tube or series of such tubes. The amplified oscillations are thus intensified to a point where they can be rectified, and are converted by a rectifying three-electrode tube to unidirectional surges of varying intensity and duration. If the transmitted signal is a sound signal the integrated unidirectional surges have frequencies so low (10,000 cycles or less) that they permit of amplification by iron-core transformers, hence they are once more placed on an audio-frequency three-electrode-tube amplifier capable of a high energy output. This in turn acts on the coils of a sound generator or loud-speaking microphone. The myriads of devices used to amplify and reproduce such signals make it impossible in a book of this scope to describe any particular circuits or class of circuits, so that only the principles are stated. Furthermore, the types of tubes and the methods of achieving various goals are changing so rapidly that it hardly pays to describe circuits that are out of date before a book is off the press.

In the 1930's advantages were found in using tubes with more than one grid. A tube with two grids is called a tetrode. It has one grid which acts as the usual grid in the triode and is named the *control grid*. The second grid, known as the *screen grid*, shields the control grid from changes on the plate without affecting the static potentials on the plate. It surrounds the plate fairly completely. It connects into the plate battery so as to place it at an appropriate potential above the filament. These tubes have high values of μ , but in addition have a very high r_p in the order of megohms. Tubes having five electrodes with three grids are known as pentodes. They have the *control grid* and the *screen grid*, as with tetrodes. In addition, they have a third grid called the *suppressor grid*. It is placed between the screen grid and the plate. It is connected to the cathode filament. It has a peculiar function which comes from using fairly high potentials on the plates. The electrons striking the plate knock out secondary electrons if they have high enough energies. These secondary electrons from the cathode make the characteristic curve of the tetrode have an undesirable dip in its course. The suppressor grid repels the secondary electrons from the plate back to the plate. This, then, gives the advantages of the tetrode without the disadvantages. The values of μ for such tubes range from 80 to 1500, with r_p running from 2×10^4 to 2×10^6 ohms and g_m ranging from 400 to 6000 micromhos.

193. THE THERMIONIC OSCILLATOR

Not long after the development of the three-electrode tube as an amplifier and rectifier its use for an even more important purpose was

discovered. As was seen in Chapter XXV a capacity connected to an inductance which naturally has some resistance will, if the electrical state of the circuit be disturbed, lead to an electrical oscillation. The period T of the latter is given approximately by the equation $T = 2\pi\sqrt{LC}$, where L is the self-induction and C is the capacity. Owing to the resistance of the circuit such oscillations are highly damped and are useless for any but telegraphic communication (i.e., they cannot be modulated by the voice). The damping also reduces the range of distances over which signals may be sent. Various attempts were made to produce sustained (i.e., nearly undamped) electrical oscillations of high frequency. Among these the successful attempts led to the design of generators of 100,000 cycles or more of relatively small energy output, and of high-frequency oscillating arcs such as the Poulsen and Chaffee arcs. The latter were relatively successful and gave large power outputs with sustained oscillations, which were, however, incapable of being used for anything but telegraphy as the intensity of the oscillations could not be controlled or modulated by sound waves, etc. But the three-electrode tube solved the problem of furnishing sustained oscillations which are capable of modulation and led at once to the development of radiotelephony and television, as well as to increasing the range of radio communication in general.

The exact manner in which the oscillating power of these tubes was discovered is somewhat obscure. No one name is associated with this discovery in the literature, and two facts probably served to make it difficult to trace its origin. One of these is that this phase was developed for commercial uses and hence the secrecy surrounding patentable inventions was a contributing cause to the slow dissemination of information. Secondly much of the development occurred during World War I, 1913 to 1919, under cooperative and needless to say secret investigations which were divulged only after 1919 in a completed form. The fact that the output energy of a three-electrode-tube amplifier is greater than the input energy, and that hence a part of the output energy might be fed back to sustain the oscillations of the input portion of the system, was doubtless the idea that must have struck many workers and led to the development of the three-electrode-tube oscillator. The period of the oscillation of such a device will depend, as do the periods in any oscillating circuits, primarily on the values of self-induction and capacity in the circuit, and hence these act as the "timing clocks" in such oscillators. The operation of one or two types of oscillators will be indicated here in order to show the nature of this use of the three-electrode tube, though no attempt will be made to discuss the quantitative theory.

An excellent example of a simple oscillating circuit is the Hartley circuit pictured in Fig. 212. A capacity C is connected to two inductances L_1 and L_2 in series. The circuit L_1L_2C , in which C may be variable, having self-induction capacity and resistance (of the coils),

will oscillate electrically if disturbed as given by the equations of section 143 (i.e., this is the "time clock" of the circuit). A battery B is placed in series with the plate and F through L_2 . As the current from B starts to flow through L_2 when the filament F is heated, the

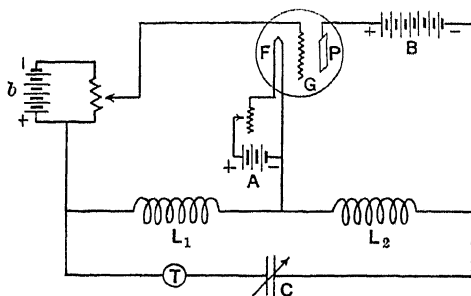


FIG. 212. The simple Hartley oscillating circuit.

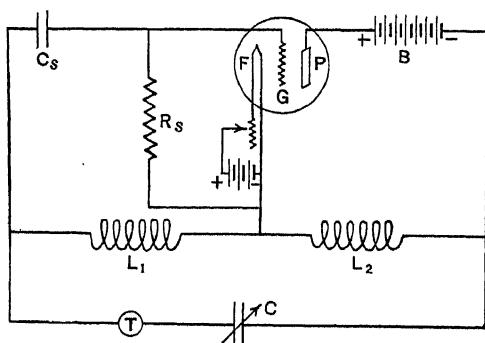


FIG. 213. The simple Hartley oscillating circuit using a grid-leak instead of a bias battery.

potential drop across L_2 starts a current charging C through L_1 . The system CL_1L_2 then oscillates with a period T approximately given by $T = 2\pi\sqrt{C(L_1 + L_2)}$ if the resistance of L_1 and L_2 is small. The oscillation sets up an alternating e.m.f. between F and G which is aided by the bias battery b placing G on the sensitive part of its characteristic curve. The value of L_1 relative to L_2 is so chosen that the e.m.f. between F and G is in the proper phase relation with the oscillation in the circuit CL_1L_2 to give a sustained oscillation. Thus as the potential between F and G causes the current through the tube from P to F to vary, this same current *reinforces* the oscillations in the condenser system CL_1L_2 . In this manner continuous undamped oscillations occur in CL_1L_2 of any desired frequency depending on L_1 , L_2 , and C , which can be transferred to any desired system by

coupling coils or other devices. The test for the presence of oscillations and adjustment of the circuit is facilitated by the use of a thermogalvanometer T in the circuit CL_1L_2 . The above circuit is sometimes modified by replacing the bias battery b by a capacity-grid leak system for charging the grid to a proper potential. This arrangement is shown in Fig. 213. The circuit is the same except that b is replaced by the condenser C_s and the resistance R_s . The resistance R_s is of rather high value (order of a megohm) and C_s and R_s are so chosen that the accumulations of electrons on G due to the electron current from F to P keep G at the proper potential with respect to F . C_s

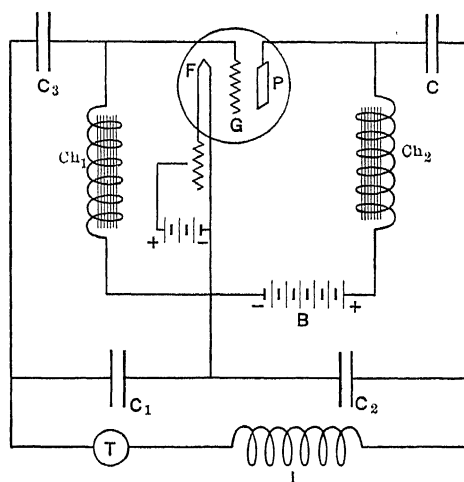


FIG. 214. The simple Colpitts circuit.

may vary from 0.0005 microfarad to 0.002 microfarad or more. Colpitts used a circuit in which the *capacity is divided*, to play the role of interrupter and oscillator instead of the divided inductances as used above in the Hartley circuit. A type of Colpitts circuit is shown in Fig. 214. In this circuit the oscillating circuit, or "timing clock," is the circuit C_1C_2L . The capacity is $\frac{C_1C_2}{C_1 + C_2}$ and the period T is approximately

$$T = 2\pi \sqrt{L \left(\frac{C_1C_2}{C_1 + C_2} \right)}.$$

The driving battery B drives the electron current through F to P except as the oscillations on G modify it. The current runs through an iron-core choke coil Ch_2 which acts to damp the radio-frequency oscillations through the battery. Ch_1 , an iron-core high-impedance choke coil, replaces the grid resistance used in the Hartley circuit,

while C_3 fulfills the same function as in the latter circuit. C is a very high capacity which establishes a high-frequency connection between B and P , and the condenser system C_1C_2L . The oscillations are impressed on the grid by being picked off across C_1 which is in series with C_2 .

A third type of oscillation generator is shown in Fig. 215. The oscillating circuit is CL_2 , T being the thermogalvanometer. The oscillations in CL_2 are placed between G and F through the bias battery b . The driving battery B operates to drive electrons from F to P .

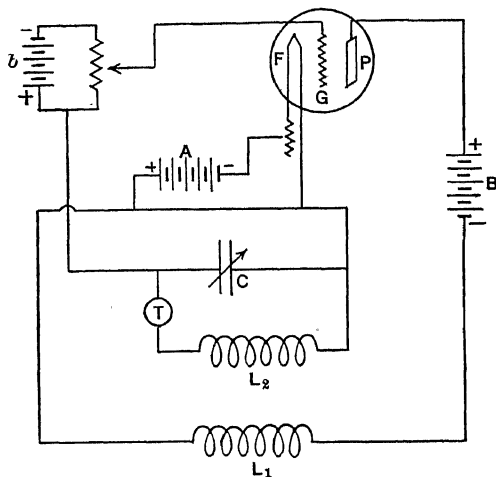


FIG. 215. Tuned grid circuit oscillator of the Hartley type.

through an inductance L_1 closely coupled to L_2 ; the period is influenced by CL_2 and the mutual inductance M between L_1 and L_2 . As soon as the plate current begins to flow through L_1 an oscillation is induced in L_2C which maintains such a phase relation to the current from B that sustained oscillations are generated in L_1 which may be picked up by a circuit tuned to L_1 as desired. The energy of oscillation in CL_2 comes from the inductive coupling with L_1 and from the battery B .

The variations of these circuits and the manipulation of the oscillations so obtained by amplification, rectification, etc., are indefinitely great. There is thus no place in this book for a further discussion of the application of the thermionic emission to the modern developments of applied science, sufficient theoretical consideration having been given to lay a foundation for further study.

194. PIEZOELECTRIC AND MAGNETOSTRICTIVE OSCILLATORS

Perhaps a word or two might be said about two more sets of electrical phenomena which are now widely used in the work on oscil-

lating systems. One of these makes use of the so-called piezoelectric (pressure-electrical) effect in quartz first studied by Pierre Curie. If a quartz crystal is cut so that two of its faces are perpendicular to one of the electric axes, and if the faces are rendered conducting by placing metal plates thereon, or by sputtering with metal, it will be observed that whenever the crystal is subjected to mechanical stress (i.e., either compressed or elongated along its electrical or mechanical axes) an electrical displacement takes place in the crystal, one electrode becoming positive and the other negative. The charges on the plates on compression are reversed on tension, and hence by *compressing and expanding the crystal an alternating potential difference* is set up on the plates which can be amplified to any desired magnitude by thermionic amplifiers. Now such a quartz crystal will have a *mechanical* frequency of vibration dependent on the thickness of the crystal and its elastic constants. A similar behavior is manifested by crystals of Rochelle salts. Hence it is seen, by making the crystal vibrate mechanically, that electrical oscillations of great constancy can be obtained. Thus the result is a new sort of "timing clock" (the mechanical vibrations of crystals), which can be used in place of the usual electrical oscillators. The value of these crystals, however, lies in a further property. If properly timed electrical charges are placed on the crystal faces by means of the plates, mechanical vibrations will be introduced on the quartz crystal. Hence these crystals when in tune with an electrical oscillation can be made to vibrate quite violently. The frequencies of these vibrations are, of course, well below the customary radio frequencies, but lie well above the frequencies of audible sounds (i.e., between ten thousand and some hundreds of thousands of cycles per second). They are therefore called *supersonic vibrations*. The energies of the supersonic waves obtained are prodigious, as the mechanical efficiency of the crystals is high and the electrical energy input is almost entirely converted to sound. Furthermore, these supersonic waves of short wave-length are not diffracted by ordinary objects like ships, but cast sharp shadows and travel in nearly straight lines. If an oscillator and a parabolic reflector send out a beam of such waves these will be reflected back by any solid bodies (hulls of ships in water, or the ocean bottom). The echo or reflected wave can then be picked up by a tuned crystal amplifier converted to d-c surges of the modulated frequency and heard on a telephone. From the velocity of sound and time elapsed between sending and receiving a signal the distance of the reflecting body can be measured. These supersonic transmitters and receivers served as the most successful of the devices for detecting submarines in World Wars I and II. Analogous devices at audible frequencies known as *fathometers* are installed in ships for determining automatically the depth of water. The development of supersonic devices is largely based on the work of Paul Langevin, who developed the first submarine detectors. The mechanical effects produced by the con-

centrated energy in supersonic waves are such as to cause intense local heating, to break up emulsions, and to cause chemical reactions. There is no doubt a great future in the application of these crystals to many problems. It appears for instance, that the constant frequency of these crystals has revolutionized the problems of timing and timekeeping. This phenomenon is today used in such problems as those of maintaining the constancy of astronomical clocks.

The use of a similar mechanical effect of *electromagnetic* nature is also being developed today. A cylindrical bar of iron of a certain length and diameter has its own mechanical periods of oscillation. It can vibrate in tune to its natural frequency or multiples thereof. G. W. Pierce at Harvard has made use of the magnetostrictive effects of magnetizing currents in a solenoid about such a bar, to set the bar into oscillations in its natural period. These mechanical vibrations of a bar produced by the magnetostrictive effects of currents of proper frequency can be transmitted to a sounding board of adequate design. Hence by properly tuning a bar to resonance with a radio signal of appropriate frequency the modulations impressed on this radio frequency by a microphone adequately amplified can be impressed on the amplitude of vibration of the bar. In this way the sounding board can be made to reproduce the audible notes impressed on the microphone with very great precision. What the future of such magnetostrictive devices in practical application will be remains for the future to show.

Piezoelectric properties are now finding another use. Up to the present the faithful conversion of mechanical sound energy to electrical energy for amplification and transmission purposes employed microphones using the change in resistance of loosely packed carbon particles with sound-wave-produced pressures, the changes in capacity or the production of induced currents by movement of diaphragms induced by sound, etc. The carbon type produce "frying," owing to poor contacts. The other types are expensive and many of them have low-frequency resonance periods of their own which distort. The piezoelectrical crystal can pick up compressional sound waves and convert them into electrical impulses in a strictly linear fashion. The natural frequencies of such crystals are far from the audible region. Good contacts can be used. Hence today the use of microphones employing artificially grown Rochelle salt crystals of reasonable price is becoming widespread.

195. TUBE COUNTERS

In the early investigations in radioactivity, it became clear that the energies possessed by single α particles of radium were such that it was considered possible to count the individual particles in terms of the effects produced by these energies. Accordingly in the years

1903 to 1912 three devices were developed which achieved this end. C. T. R. Wilson showed that the trail of ions left behind by an α particle or a β ray could be made visible by adiabatically expanding air saturated with water vapor by just the proper amount. At room temperature and about atmospheric pressure an adiabatic expansion between 1.25 and 1.31 times the volume causes a selective condensation about droplets of vapor that have acquired a charge by picking up an ion. Thus the ion trails become visible in strong illumination as fine lines of fog particles. These can be photographed and counted. Possibly nuclear study owes as much to this method as to any other single discovery, and Wilson very fittingly was awarded the Nobel prize. At about the same time Crookes observed the brilliant flashes of light made by impact of single α particles on phosphorescent zinc sulfide and diamond. Regener and Rutherford's pupils, Geiger and Marsden, developed the method of scintillation counting. With this not only was the charge on the α particle (counting the number of α particles per second and measuring their charge in e.s.u.) as well as the number of atoms in a gram-molecule evaluated, but the proof of the nuclear atom rested on the scintillation-counting method. At the same time Geiger attempted electrically to multiply the 10,000 to 100,000 ions produced by an α particle by means of ionization by collision in a high electrical field and thus to get for each particle, a current which he could measure or record electrically.

Among other partially successful counters was the point counter. A negatively charged point raised to a potential high enough to cause a corona discharge was exposed to the beam of α particles. When the ions from an α particle entered the sensitive region of the point, the ions started a discharge from the point which was a continuous corona. For certain points, however, the heavy bombardment of the needle by $+$ ions destroyed its further emitting power for electrons (presumably by destroying its adsorbed gas layer) and the effective discharge terminated abruptly, after, however, having passed enough current to affect an amplifier and to work a counter. The period of recovery of such a point was some 10^{-3} second, and such counters were used in some of the later work on disintegrations. The counters were capricious on account of the peculiar type of active surface and were thus uncertain and unsatisfactory.

Finally in 1928 Geiger and Mueller developed a counter which is now the indispensable adjunct of all radiation laboratories and is being carried over to engineering investigations. They went back to a positive corona wire in a coaxial negative cylinder and used a high resistance in series with the wire to prevent complete breakdown. In principle it is quite simple. As shown in Fig. 216, it consists of a small cylindrical tube T of copper 1 cm in diameter, 1 to 5 cm long, that is slightly oxidized. Other substances may be used, including alkali metals, though these are too photoelectrically active in most

instances and must be used in the dark. This tube is sealed into a glass tube with a coaxial tungsten wire W of about 1 mil diameter. This wire must be smooth and should be flashed to remove rough spots. The wire is positive and operates in the neighborhood of 1000 (500–2000) volts, depending on the gas filling of the tube and the pressure. Ne and He may be used at near atmospheric pressure. Usually H_2 , O_2 , or N_2 are used at around 10 to 60 mm pressure. Purity is not essential. V is a very constant d-c source of potential,

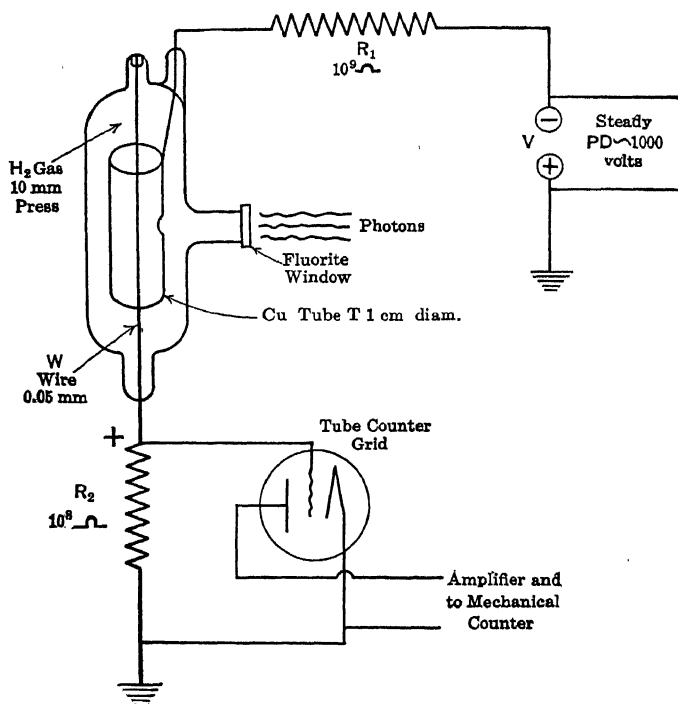


FIG. 216. Wiring diagram of a Geiger counter.

as stated, of about 1000 volts. Its negative side goes to the cylinder through a resistance R_1 which ranges from 10^7 to 10^{10} ohms and determines the action of the counter (Geiger high resistance). The positive pole of the d-c source is grounded; the positive tungsten wire W is grounded through an amplifier resistance R_2 of 0.5×10^6 to 10^7 ohms. The filament and grid leads to a vacuum amplifier tube or a thyratron gas-filled amplifier tube are taken from the two ends of R_2 . Thus if a current flows from the ground through R_2 as the result of a discharge through the system WT , the potential iR_2 changes the bias on the amplifier and a current will flow, working a counter. Now the positive wire breakdown corona due to the

potential difference between W and T has a definite potential V_c at which a *self-maintaining* discharge current of milliamperes starts. Below the starting range for the corona there is a small region of potential from V_c to V_s where the tube WT will transmit enhanced currents as long as electrons are furnished by T from outside or internal influence. This region from V_c to V_s is very limited, being only a few volts below the complete breakdown value V_c . The current flowing under these conditions is small and of the order of 1 micro-ampere. Below the potential V_s for the current induced by ions the only current observed is an intensified current due to photoelectrons from the tube which cannot be measured. If now a sensitive tube has the P.D. between W and T at some point above but near V_c , a single electron *liberated* by light, γ rays, β rays, or α particles from T will proceed across the tube to W . There in the intense field of W it will ionize intensively and a current will start to flow. This ionization is accompanied by light emission, and this liberates more electrons from T . Thus the current will grow in the tube and charge will flow up R_2 . Since the area of T is large the light emission in the space WT will cause a high degree of emission from T , and so it would be suspected that the tube would go into corona. The tube will, in fact, become more and more conducting. As it does this, current begins to flow in R_1 . Accordingly as the current in WT increases, the iR drop of potential in R_1 increases and lowers the potential across WT below the potential V applied. Thus V rapidly falls below the value V_c and V_s . At the same time a space charge in WT builds up that still further lowers potential drop at W . As soon as the current builds up in the tube WT , the discharge ceases to operate and as the light from the discharge dies out (in some 10^{-6} second except for He and Ne, etc.) the current flow ceases. The tube then recovers and is ready for the next count. The rate of recovery depends on the capacity of the wire and the character of W and T . It cannot be reduced below 10^{-5} second. Thus ionizing radiations and probably even single photoelectrons can be counted at the rate of 10^5 per second or less. With the improvement in vacuum tube techniques the self, or resistance, quenching of the initiated corona discharge can be dispensed with by the use of quenching circuits using appropriate tube combinations. This has notably extended the frequency range in counting. Counts can automatically be recorded by the use of scaling circuits. These detectors for cosmic rays, γ rays, and light quanta are proving of very great importance.

196. RADAR AND ULTRA-HIGH-FREQUENCY ELECTRIC-WAVE GENERATION

The subject of radar, which is the application of ultra-high-frequency radio waves to various uses such as detection, ranging, signaling, etc., constitutes a science all its own. Technical advances

have been so great during World War II that it is impossible to deal with this subject, even in an elementary fashion, in a textbook of the scope of this one. In the electromagnetic spectrum of radiations a group of waves are seen extending from the short wave-length radio region, perhaps from more than 1 meter wave length (3×10^8 cycles, or 300 megacycles) to 1 centimeter wave length (3×10^{10} cycles, or 30,000 megacycles), the dimensions of which begin to approach the physical dimensions of common objects. Thus they will interact with common objects to be reflected, refracted, and diffracted. They therefore are waves which it is possible to treat more or less as light waves are treated. For it is possible to reflect them and to focus them into beams, and thus to concentrate energy along given paths. This allows the directions of echo-producing objects to be determined and makes it possible to determine distances of reflecting objects from the elapsed times of passage of these waves in transit to and from the reflector. Furthermore, they are waves which are interfered with only by material bodies of considerable physical size. Thus they can penetrate rain, fog, or clouds, and can detect objects such as planes or ships. Using scanning procedures such as in television the shorter waves can be used to throw pictures of reflecting objects onto the radar screen.

Waves of this character were present in Hertz's early oscillations. They were studied further by Nichols and Tear and others at the end of the nineteenth century. With techniques then available, it was impossible to get single frequencies in great enough intensity to make practical use of these waves. Two things were lacking: oscillators of adequate high frequency and power, and proper means of detection and guidance. Since such waves are not at all unlike light waves in their behavior, optical considerations can be applied, with the difference that the wave lengths involved are in the centimeter range instead of the 10^{-5} cm range.

What emerged, when in the early 1930's physicists and engineers turned their attention to the microwave regions, was the use of *cavity resonators* of appropriate frequencies and of *wave guides*. These devices, well known in acoustics and in optics, served to amplify and conduct the oscillations produced. Their development and perfection went hand in hand with the various devices for generating high-frequency oscillations. Of these, essentially two types were developed successfully during the war, the *magnetron* and the *Klystron*. Finally, detection was simplified through an increased knowledge of semiconductors and the improvement of rectifying crystals, as well as the great improvement of high-sweep-speed oscilloscopes.

The magnetron was first developed and described by A. W. Hull of the General Electric Company in 1921. It was the subject of some later study, but no one realized its possibilities until the demands of radar secretly developed by the armed services of several countries

for aircraft detection in the early 1930's made its exploitation imperative. While the theory of the original simple magnetron oscillator of Hull is pretty well developed, the theory of the later successful and powerful magnetrons has not been worked out. They are very effective empirically developed devices the form of which grew out of experience in many commercial developments. The Klystron was proposed and initially developed in 1937 to 1938 by R. H. and S. F. Varian. The successful perfection of this device owes much to the development of the cavity resonators and allied studies by W. W. Hansen and D. L. Webster.

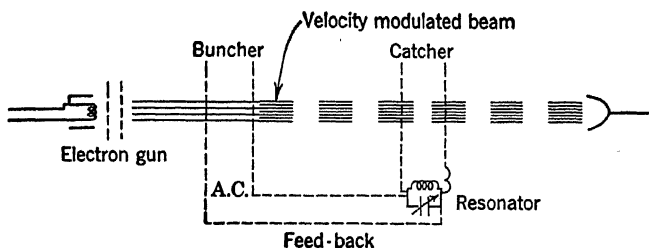


FIG. 217a. Principle of the Klystron.

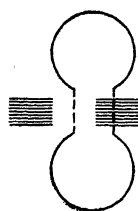


FIG. 217b. Cavity resonator excited by bunched waves.

The principle of the Klystron is as follows: A beam of electrons of uniform velocity emitted from a filament and electron gun passes between two grids shown under the title *buncher* in Fig. 217a. These grids are connected to a high-frequency electrostatic field, which actually could come from a feed-back line from the oscillator. Suppose one group of electrons reach the grids during the retarding phase of the field. These electrons will be slowed down. The field is, however, decreasing in retarding value and passes through 0 while more electrons in the beam are continuing through. The electrons entering at this point are unchanged in velocity. As the field goes positive the later electrons are speeded up. Thus what was originally a homogeneous uniform stream of electrons is, after passing through the grids of the buncher, a group with the first electrons retarded, the later ones accelerated. Thus with the frequency of the buncher the electrons are broken up into groups bunched in space. This is known as a *velocity modulated beam*. This bunched beam then crosses two more grids in the *catcher*, which can be connected to what is essentially a *tuned circuit*. This circuit is tuned to the frequency with which the electron bunches pass. It is thus thrown into oscillation. The beam then proceeds to the anode. Now in ultra-short-frequency waves, T the period is small, and it is difficult to get regular electrical circuits with values of L and C small enough to give 1 cm waves.

It was the discovery that a cavity in a metal will resonate very effectively and with very little energy loss to electrical oscillations of high frequency that made the Klystron and the magnetron possible. The theory of the cavity resonators and wave guides, etc., belongs more to the optical and acoustical sections of physics and cannot well be explained until these principles are mastered in subsequent courses. In any case, the fact that such doughnut-shaped cavities as that shown in cross section in Fig. 217b will resonate when shock-excited by a bunched electron beam going through the grids makes the Klystron

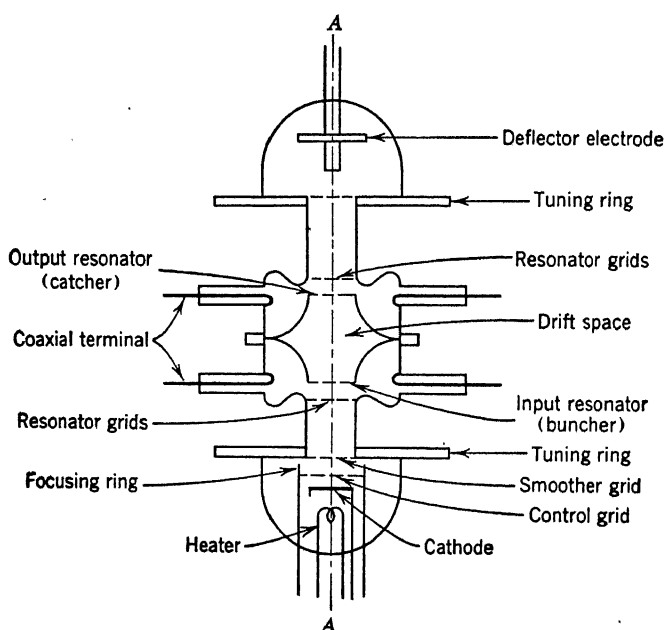


FIG. 217c. Schematic diagram of a Klystron.

possible. The combination of the mechanisms of Figs. 217a and 217b makes the realization of Fig. 217c clear. The oscillation for the buncher is obtained by feed-back from the catcher. The energy comes from the energy in the electron beam which is reduced on passing through the catcher in exciting oscillations by induction. It enters the collector with decreased energy.

The magnetron depends on the effect of a magnetic field on the motion of electrons coming from a filament in a concentric cylindrical condenser. These electrons are constrained to circular or cycloidal orbits under the combined action of the crossed electric and magnetic fields. These orbits are executed with the high velocities of electrons in vacuum. The character of the orbits is such that they approach

more or less closely and in a regular way to sections of the cylinder. In consequence, they induce mirror images in the cylinder and oscillating currents in the cavity walls. With proper tuning, these can produce oscillations of great amplitude.

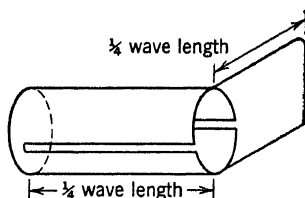


FIG. 218a. Plate.

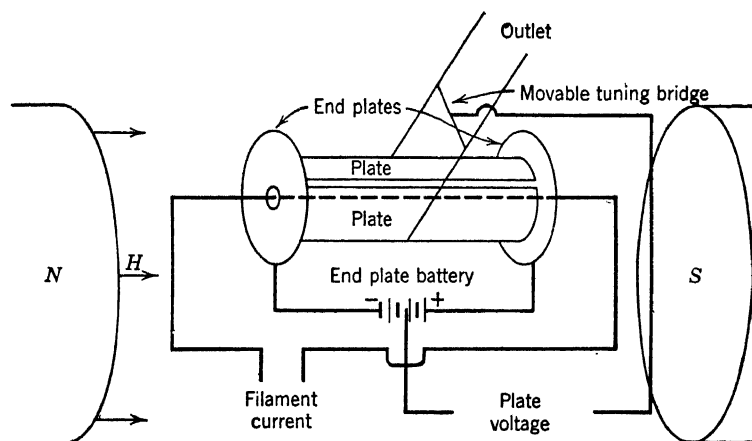


FIG. 218b. Early split plate magnetron, vacuum jacket omitted.

The split anode magnetron which was an early variant of Hull's original device has the appearance shown in Figs. 218a and 218b. The structure of the anode is shown in *a* with the wires leading to the tuned *wire* system, since cavity resonators did not exist in that day. In Fig. 218b is shown the system without a vacuum jacket placed between the poles of a magnet. Figure 219a shows the current to the outer plate for a solid cylindrical anode with the radial electric field from filament to the anode and a magnetic field of varying strength H parallel to the filament axis. As long as the paths shown for a constant radial electric field as in Fig. 219b are not bent by H so that they fail to reach the cylinder, the current i is constant. At a field H_c , the *cutoff field*, the paths of the electrons cease to reach the cylinder. For larger fields H the paths are short circles or cycloids. In Figs. 220a and 220b it is seen what happens to the paths with H fixed

beyond cutoff when the two halves are at uniform potential and when they are at a different potential due to oscillations. To prevent electrons from accumulating in the space when they fail to hit the plates, there

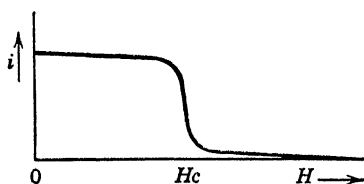


FIG. 219a. Current magnetic field relations in a concentric cylindrical magnetron. H_c = cutoff field.

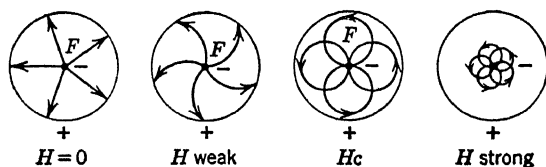


FIG. 219b. Electron orbits in a concentric cylindrical magnetron. Note the orbits at H_c —the cutoff field.

is a small difference of potential applied by the end plate battery to the end plates of Fig. 218b to cause the electron paths to spiral out axially.

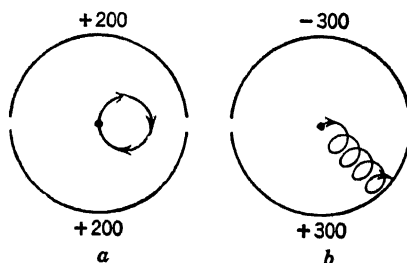
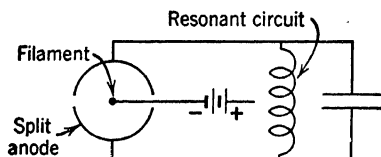


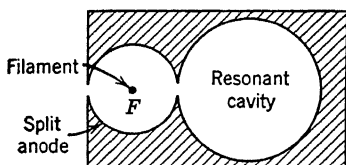
FIG. 220. Split anode magnetron. Electron orbits above H_c for plate halves equal potential and different potentials.

Figures 221a and 221b show in principle how the system behaves electrically. A slight disturbance making one plate different in potential from the others, e.g., an outside stimulus to the wire system, and the electrons beyond cutoff at H_c leave the paths of Fig. 220a and go to the plate as in Fig. 220b. This at once feeds energy to the tuned resonant circuit of Fig. 221a. This drives the plates from one polarity to the other in tune with its own oscillations. In actual practice the self-induction and capacity of Fig. 221a are replaced by

a cavity resonator as in Fig. 221*b*. The leads and the wave guides for taking out the energy are not shown. They are peculiar to the art of radar. A cross-sectional view of a modern magnetron oscillator as used in radar is shown in Fig. 221*c*. The hot-oxide-coated



a. Electrical circuit idealized.



b. Electrical circuit with resonant cavity.

FIG. 221. Split anode magnetron

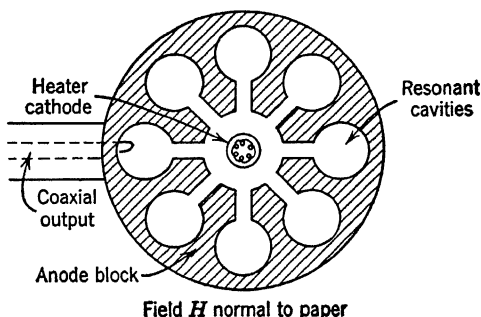


FIG. 221*c*. Schematic diagram of a high-output magnetron.

cathode, indirectly heated, is used instead of a filament. The resonant cavities are cross-connected in alternate pairs in order to keep in step. The magnetic field is normal to the plane of the paper. The electrons which do not reach the walls strike the cathode surface with much energy and cause added secondary emission. With such tubes remarkably large power output can be given with very high frequencies for short periods of time. The radar generators are pulsed with high voltage at regular intervals in pulses lasting from fractions of a microsecond up. Thus some 3×10^4 waves of 1 cm length are emitted in a radar pulse. It is the pulsing which permits such large energy outputs without wrecking the filaments.

PROBLEMS BASED ON CHAPTERS III AND IV

1. Given two *very* long magnetized steel ribbons placed vertically in line so that the *N* pole of the upper one is 1 cm from the *S* pole of the lower one which is adjacent to it. The pole strength of the *N* pole is 100 units, that of the lower one is 98 units. The lower magnet can just be lifted against gravity by the upper magnet. Taking g as 980 cm per sec², what is the mass of the lower magnet? If it is 0.5 mm thick and 5 mm wide and the density is 8, how long is it? *Ans.* $f = 10$ grams; $l = 50$ cm.

2. A magnet bar some 30 cm long is suspended at its center so that it lies horizontally and is suspended by a fine vertical wire such that a force of 10 grams causes a deflection of 10° in angle. This constitutes a Coulomb balance. An *identical* magnet with vertical axis, when placed so that at equilibrium its *N* pole is 1.41 cm from the *N* pole of the suspended magnet, causes the suspended magnet to be at rest with an angle of 5.1° with its rest position. Neglecting the effect of opposite poles and the earth's field, calculate the pole strength m of these magnet bars, given the acceleration of gravity as 980 cm per sec². *Ans.* $m = 100.3$.

3. Calculate the following quantities:

(a) The field H 10 cm from an isolated south pole of strength 300 units in *magnitude* and *sense*. *Ans.* $H = 3$.

(b) The torque on a magnet of length 20 cm and pole strength 200 in the earth's field H of horizontal intensity 0.25 oersted (1) when it is *perpendicular* to this field, and (2) when it is *parallel* to the earth's field but with its north-seeking pole south. *Ans.* (1) $G = 1000$; (2) $G = 0$.

4. Two magnets *A* and *B* are placed with their axes at right angles and the poles initially directed as indicated in Fig. P1. The moment of magnet *A*, M_A , is known to be 200 units. That of *B*, M_B , as well as the earth's field H parallel to the axis of *B*, is not known. *A* and *B* are short magnets compared to the distance of their centers from the small compass needle *C*, which is 10 cm in each case. The compass needle *C* takes an angle $\theta = 45^\circ$ with the axis of *B*. When the magnet *B* is turned through 180° the compass needle *C* points along the axis of *A*, i.e., $\theta = 90^\circ$. Calculate H' , the field due to *B*, and H , the field of the earth, and thus M_B , given $M_A = 200$. *Ans.* $H = 0.2$ oersted; $H' = 0.2$ oersted; $M_B = 100$.

5. At a point *C* in the earth's field, a small compass needle had a period T of 2 seconds. A bar magnet *B* of moment $M = 150$ was placed with its center 10 cm magnetically north of this point, its north-seeking pole being nearest the small compass needle and its axis passing through the center of the needle. The compass needle reversed its direction and its period changed to 3 seconds. Calculate the earth's field, given that the field r centimeters from the center of a bar magnet of moment M along its axis is $H_M = 2M/r^3$. *Ans.* $H = 0.208$ oersted.

6. In an otherwise field-free space two short magnets are placed as shown in Fig. P2, the one *B* having its axis along the normal to the center of the other called *A*. The moments of *A* and *B* are the same and are 200 units, and their centers are 22.6

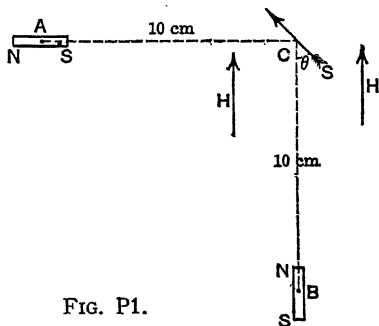


FIG. P1.

cm apart. At some point on the line of centers AB a small compass needle C makes an angle of 45° with AB , the north pole pointing in the direction indicated in Fig. P2.

(a) Locate the point C , given that the field due to B is $H_B = 2 M/r_B^3$, the field due to A is $H_A = M/r_A^3$.

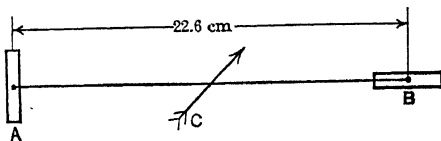


FIG. P2.

(b) Calculate the magnitude of the resultant force at C and the period of oscillation of C if its moment of inertia is 5.65 absolute units and its magnetic moment is 20.

(c) Identify the north and south magnetic poles of A and B .

To aid numerical computation you are given $\sqrt[3]{2} = 1.26$. Do not carry work to more than three significant figures.

Ans. (a) C is 10 cm from A and 12.6 cm from B ;

(b) $F = 0.282$ dyne, $T = 6.29$ second.

7. An isolated north pole of strength 100 units is at a point O . Calculate the force in dynes on a S magnet pole of unit strength placed 0.05, 0.1, 0.2, 0.4, 0.8, 1.0, 2.0, 5.0, 10, and 20 cm distant. Plot this force, which represents the magnetic field about O , on coordinate paper as a function of distance. Assuming the point O to be placed at the center of the sheet of plotting paper in the plane of the paper, calculate the force on a pole of 45 units pole strength at 1, 2, and 5 cm distance from O , and designate by lines the loci of points where the force on this pole is the same as the values deduced. These represent lines of equal force and also equal field strength of the values given. What is the field direction at any point on these lines?

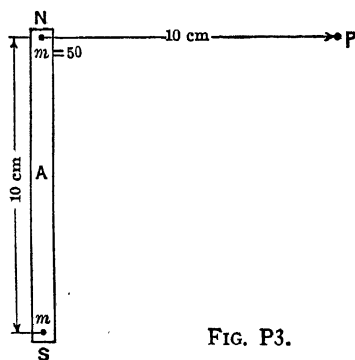


FIG. P3.

8. In the diagram of Fig. P3, A is a magnet the length of which is 10 cm. Find the magnitude and direction of the resultant field at a point P , 10 cm distant from the N pole of A on a line at right angles to A , if the pole strength m is 50 units. This problem is to be solved numerically. *Ans.* 0.369 dyne, 61° clockwise from NP .

9. Given the magnet NS of length 10 cm, and $m = 200$, placed with N upward on the paper. Calculate the field in oersteds and its direction at the point O , to the right of NS and 6 cm from N and 8 cm from S . This may be solved graphically or numerically. *Ans.* $H = 6.37$ oersted; direction = 30° clockwise from NO .

10. In the lecture demonstration experiment a small bar magnet 6 cm long weighing 14 grams was shown to be supported with its axis 2 cm from an identical bar magnet concealed in the wooden box below it as shown in Fig. P4. The N pole of the suspended magnet is just over the N pole of the magnet in the box. These mag-

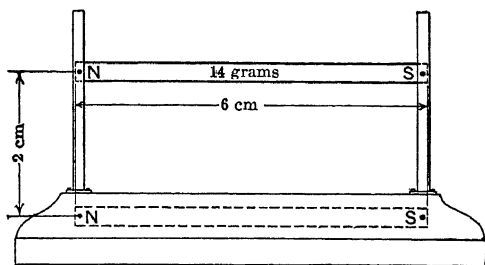


FIG. P4.

nets are made of an alloy developed by the Western Electric Company and represent examples of as powerful a magnet as can be obtained for small permanent magnets of this type. Calculate the pole strength of these magnets from the data given above, taking into account attractive forces as well as repulsive forces, and assuming both magnets to be of equal strength.

Ans. 168 unit poles.

11. Solve problem 10, using a length of 5 cm, a mass of 20 grams, and a distance between axes of 2 cm. Ans. 203.3 unit poles.

12. Given a steel knitting needle K floated in a vertical position by means of a cork C on water as shown in Fig. P5. It is 20 cm long, and has a pole strength of 50 units. Its N pole is 5 cm from the N end of a bar magnet A , 15 cm long, having a pole strength of 100 units, lying horizontally with its axis parallel to the earth's field H of strength 0.20 gauss and having its N end pointing to the N geographic pole. Calculate the resultant force parallel to the surface of the water for two situations: (a) neglecting the action of both S poles but not of H on the S poles; (b) taking into account the S poles. What is the percentage error due to a neglect of the S poles?

Ans. N poles 187.5 dynes; N and S poles 182.0, 2.9%.

13. Calculate the field, i.e., force on unit pole, in magnitude and direction relative to the magnet axis at a point P indicated in Fig. P6. The magnet is 10 cm long between poles and its pole strength m is 200 units, the poles being as designated in the figure.

Ans. $H_R = 31$ oersted $2^\circ 50'$ clockwise from NP ; NP makes angle $54^\circ 10'$ with axis.

14. Solve problem 13 with the length of NP equal to 5 cm, the vertical height 3 cm, the base of the triangle 4 cm, and the pole strength 200 units.

Ans. $H_R = 7.15$ oersteds, $\phi = 139^\circ$; this makes H_R 5° clockwise from NP . NP makes an angle of 36.9° with the magnet axis SN .

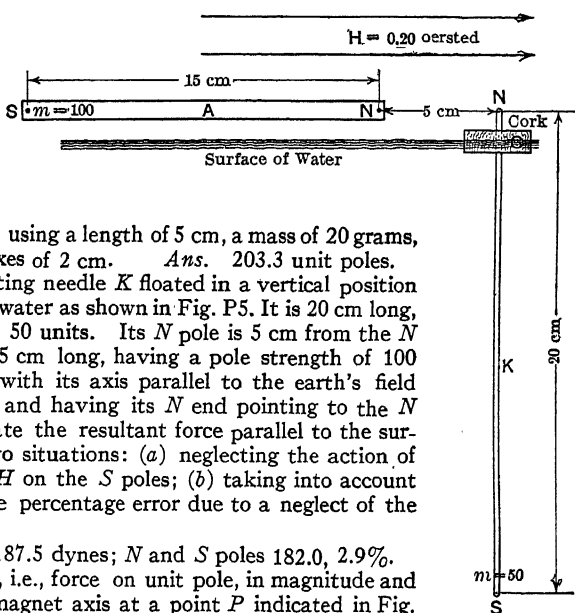


FIG. P5

15. Calculate the field strength at a point P on the diagram of Fig. P6 of problem 13 in the presence of the earth's field of $H = 0.1$ gauss parallel to SN and in the direction S to N . In this case the distances 1.5, 2.0, and 2.5 cm of problem 13 are replaced by 3, 4, and 5 cm. The pole strength is 100 units, and the magnet is 10 cm long.

Ans. $\alpha = 37^\circ$; $F_1 = 4$ oersteds; $F_2 = 0.54$ oersted; $\theta = 36^\circ$; $R' = 3.58$ oersteds
 Angle between east-west line and $R' = 31^\circ 50'$.
 Resultant with earth's field = 3.63 oersteds.
 Direction of resultant = $33^\circ 10'$ from east.

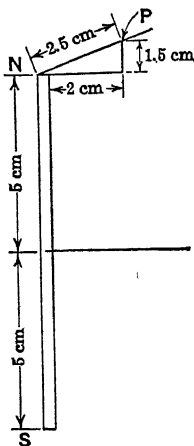


FIG. P6.

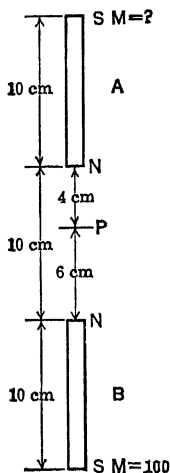


FIG. P7.

16. The two magnets A and B of Fig. P7 were placed with their axes in line and their north poles opposing and 10 cm apart. Each magnet was 10 cm long. Magnet B had a known pole strength of 100 units. Magnet A was of unknown pole strength. Plotting the fields by means of iron filings in the absence of any outside field showed a neutral point, i.e., zero force, to occur at P , 4 cm distant from the north pole of A and 6 cm distant from the north pole of B . Calculate: (1) the strength of A , taking into account the effect of the north poles only; (2) the strength of A , including the south poles as well. The calculation should be made to slide rule accuracy only (three significant figures). (3) Calculate the error made by neglecting the south poles. *Ans.* (1) North poles only 44.4 units; (2) N and S poles 41.6 units;

(3) 6.7% error.

17. Given two bar magnets A and B of length 10 cm with axes parallel and separated by 10 cm. The N pole of A is opposite the S pole of B . Calculate the field at a point P midway between the two magnets on a line joining their centers when (a) A has $m = 200$ and B has $m = 100$, and (b) A has $m = 200$ and B has $m = 200$.

Ans. (a) Field along AB towards $A = 2.83$ oersteds; (b) zero field.

18. Calculate the resultant field in magnitude and direction at a point P on the axis of the magnet A , Fig. P8, of 40 units pole strength and 5 cm length, 20 cm from its center at its north end, and 20 cm from the center of a magnet B which lies on the axis of A but is at right angles to this axis and has a length of 5 cm and a pole strength of 80 units, its north end being upward. This answer need not be given more accurately than within 5 per cent. *Ans.* 0.0707 oersted, 45° clockwise from axis of A .

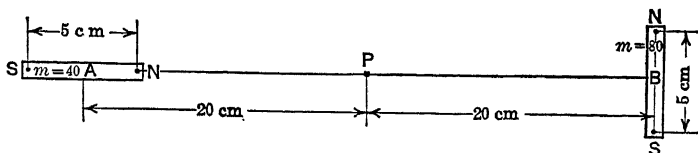


FIG. P8.

19. Draw Fig. P9 to some convenient scale on a sheet of paper, preferably graph paper, and use a scale to measure lengths to avoid lengthier calculations. A north pole is at A with strength 1000 units. The south pole of the magnet of which A is the north pole is at B. CD is a line parallel to AB 10 cm below AB. The point marked 0 is directly below A and the point marked 20 is directly below B. These distances are in centimeters. Measuring the distances from A to the points

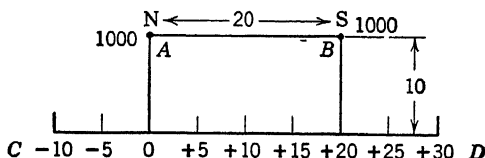


FIG. P9.

-10, -5, 0, 5, etc., the fields due to the N pole at A can be calculated at each point. Note the direction of the field. Then compute the *vertical component* of the field due to A at each place, i.e., the component parallel to AO. Do the same for B with each point indicated, being careful to note the direction of the field. Note that numerical values for B will be the same as for the symmetrical points for A so that calculation is simplified. Then proceed to calculate the *resultant* vertical component of the field at each of the points from -10 to +30. Plot these on the diagram in red or any other distinctive color and fair a curve through the ends. This wavy curve represents the *vertical component of the field of a bar magnet AB along CD at a distance one-half the length of the magnet*. If now it is considered that (except, of course, for a change in scale for all magnitudes) this diagram would represent the magnetic field due to the longitudinal component of a *ship's magnetism at a depth equal to half the ship's length*, it assumes more interest. The average ship, as a result of construction, acts like a permanent magnet with either a N or a S pole in the bow depending how it was headed in building. In water at the depth indicated the vertical component of the field has the form just calculated. It is what would be recorded on a moving tape if just the longitudinal magnetism is picked up by a vertical fluxmeter or a magnetometer coil as the ship moved over it. Knowing this variation in field, the German enemy in 1939-1945 arranged a magnetic dip needle about a horizontal pivot which could just be held in a horizontal position in the earth's field by a delicate hairspring. When a ship passed over the needle, located in a mine case on the bottom, the vertical force, if great enough, either tipped the magnet up or down, thus closing a contact and detonating the notorious magnetic mine below the ship. To counteract such mines coils were wound around the ships with currents to neutralize the fields due to vertical magnets in the ship running from funnels to keel and the longitudinal magnetism was removed by deperring—a process akin to the demagnetization of a watch. To check on a ship's safety it was run over the fluxmeter coils mentioned and the vertical component of the magnetism was recorded on a tape. The ship's signature observed when the vertical magnet was compensated by degaussing coils was just of the form calculated.

Ans. Resulting curve is like a sine curve with a peak of -10 oersted under N pole, 0 at 10 cm, and 10 oersteds under S pole at +20 cm.

20. Calculate the value of the vertical component of the field for the case shown in Fig. P10. This gives a facsimile of the signature of a ship produced by its vertical component of magnetism. On undegaussed ships this component is very prominent. Under normal conditions it masks the effect of the horizontal component of problem 19. When a ship is properly degaussed by compensating coils this signature is largely replaced by that caused by the permanent longitudinal magnetism studied in connection with problem 19. Solve as directed in problem 19.

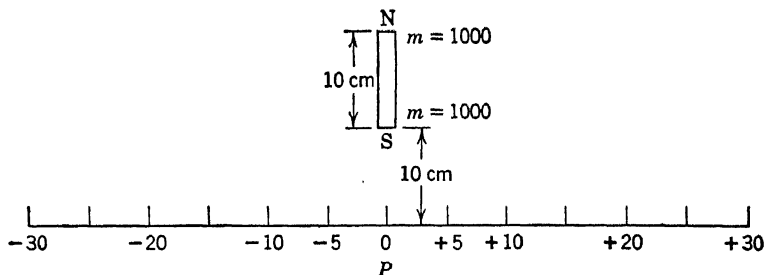


FIG. P10.

Ans. The curve has a blunt peak of $+7.5$ oersteds at 0 cm. At about 25 cm the vertical component is zero and thereafter becomes negative, ultimately approaching zero asymptotically at $\pm \infty$. The negative dip in the curves comes from the fact that whereas the repelling north pole is more distant than the south pole the distances from N and S rapidly equalize while the vertical component favors the N pole.

21. A bar of copper is suspended in a stirrup by a fine steel wire and its position accurately set perpendicular to the horizontal component of earth's field H . The bar is then removed and replaced by a powerful bar magnet of magnetic moment $M = 4 \times 10^4$ units. This magnet sets itself so that it makes an angle of 30° with its initial position and 60° with the earth's field. The radius of the wire is 0.030 cm and its coefficient of rigidity $\eta = 5 \times 10^{11}$. The length of the wire is 38.4 cm. Calculate the horizontal component H of the earth's magnetic field to an accuracy not exceeding three significant figures. Remember that the angles used in the formulas are in radians.

Ans. 0.252 oersted.

22. A bar of copper is suspended by a fine steel wire by means of a stirrup in a position perpendicular with the earth's field H the horizontal component of which is at this place 0.15 oersted. When the copper bar is removed and a magnet of moment $M = 6.78 \times 10^4$ units is placed in the stirrup the magnet takes on a position in the field such that the wire and stirrup are twisted through φ degrees with the perpendicular to the field (the original rest position), and with $90 - \varphi$ degrees or θ° with the field. If the radius of the wire is 0.03 cm, its coefficient of rigidity is 4×10^{11} , and its length is 30.7 cm, calculate the angle φ which the magnet makes with the perpendicular to the earth's field. Do not carry computations farther than three significant figures. (Note when the equation is properly set up an expression of the form,

$\frac{\cos \varphi}{\varphi} = A$, results in which A has a numerical value. To solve such an equation

plot the values of $\frac{\cos \varphi}{\varphi}$ as ordinates for various angles φ from 0 to π radians, against φ as abscissas. At the point A on the plot read off the value of φ , which is the desired result, and convert to degrees.)

Ans. $30^\circ 36'$.

23. Referring to problem 21, calculate through what angle the torsion head of the suspension would have to be twisted to hold the bar magnet at an angle of 80°

with the field, using the torsional constant given in problem 21 and the value of the earth's field there calculated of 0.252 oersted. (To solve this problem remember that there is at 80° with the field a torque on the magnet; this torque must be equal to the torque on the suspension produced by twisting the torsion head.) *Ans.* $4^\circ 45'$.

24. A bar of copper is suspended on a fine steel wire by means of a stirrup in a position perpendicular to the horizontal component of the earth's magnetic field, which at this place is 0.20 oersted. When the copper bar is replaced by a bar magnet of moment $M = 4 \times 10^4$, the magnet takes on a position in the field such that the wire and stirrup are twisted through ϕ° with the perpendicular to the earth's field, and with $90 - \phi^\circ$ or θ° with the field. If the radius of the wire is 0.04 cm, its coefficient of rigidity is 3×10^{11} , and its length is 40 cm, calculate the angle ϕ which the magnet makes with the perpendicular to the earth's field. (Note that when the equation is properly set up an expression of the form $\frac{\cos \phi}{\phi} = A$ results, A having a

numerical value. To solve this equation plot $\frac{\cos \phi}{\phi}$ as ordinates against ϕ expressed in radians from 0 to π radians. At the point A on the plot read off the value of ϕ desired, and convert to degrees. *Ans.* $14^\circ 40'$.

25. A fine phosphor-bronze suspension 31.4 cm long had a stirrup at its lower end. In this stirrup was mounted a copper bar and the bar adjusted so that it was directly perpendicular to the earth's field. It was then removed and a bar magnet 10 cm long and of pole strength 200 units placed on the stirrup. The stirrup was observed to deflect through an angle θ . Given the radius of the wire $r = 0.02$ cm, the coefficient of rigidity $\eta = 4 \times 10^{11}$, and the horizontal component of the earth's field H as 0.20 oersted, calculate the angle θ . (Hint: the force on a magnet in the earth's field at a given angle must equal the torque produced by twisting the suspension through 90° less the angle. This gives an equation in which some number is equal to the ratio of $\theta/\cos \theta$. To find the value of θ plot a curve for $\theta/\cos \theta$ as a function of θ in the neighborhood of this value and from the constant evaluate θ . Note also that θ must be in radians.) *Ans.* $7^\circ 05'$.

26. A bar magnet of unknown moment was pivoted in the earth's field and its period was found to be 2π seconds. When loaded with a ring of moment of inertia 2.50 g-cm^2 , its period was 3π seconds. Calculate the moment of the bar magnet if the horizontal component of the earth's field was 0.25. *Ans.* $M = 800$.

27. The horizontal intensity of the earth's magnetic field is 0.20 oersted. If the angle of dip is 70° , what is the total intensity? How would you represent such a field graphically? The declination or variation of the compass in Berkeley is $18^\circ 30'$ east of true north. If the compass has an error of $2^\circ 30'$ east, what compass course must you steer to sail due east, or 90° true? A ship's course is 155 by magnetic compass, the variation is 18° E , and the error is 3° W ; what is the true course of the ship? Justify by diagram the general statement that in converting a magnetic bearing to a bearing in the true geographical direction an easterly error (deviation or variation) is additive. Having justified this rule, the memory of it in the form, "when *correcting*, easterly errors are additive," makes calculations such as the above simple.

Ans. $H = 0.585$ oersted; Compass course = $N 69^\circ \text{ E}, 170^\circ \text{ True}$.

28. A bar magnet of unknown moment M was placed with its axis horizontal, and perpendicular to the horizontal component of the earth's field H , which was also unknown. A small compass needle 17.1 cm from the center of the bar magnet was observed to be deflected 45° with the earth's field. The magnet was then mounted on a frictionless pivot in the earth's field and its period was observed to be 2π seconds. The magnet was then loaded with a ring of moment of inertia 125 g-cm^2 and the period was observed to be 3π seconds. The approximate length of the magnet was 5 cm. Calculate M , H , and the pole strength m of the magnet. Work to three significant figures only. *Ans.* $M = 500$; $m = 100$; $H = 0.2$.

29. A bar magnet of unknown moment M was placed with its axis horizontal, and perpendicular to the horizontal component of the earth's field H , which was also

unknown. A small compass needle 17.1 cm from the center of the bar magnet was observed to be deflected 45° with the earth's field. The magnet was then mounted on a frictionless pivot in the earth's field and its period was observed to be 3π seconds. The magnet was then loaded with a ring of moment of inertia 100 g-cm^2 , and the period was observed to be 4π seconds. The approximate length of the magnet was 5 cm. Calculate M , H , and the pole strength m of the magnet. Work to three significant figures only. *Ans.* $M = 378$; $H = 0.151$ oersted; $m = 75.6$ units.

30. Given a bar magnet of moment $M = 500$ units and of length 5 cm placed parallel to the horizontal component of the earth's field, the intensity of which is 0.25 oersted, with the north magnetic pole of the magnet toward the earth's south magnetic pole. Locate quantitatively the position of any neutral points, i.e., points where the resultant field is zero, about the magnet using the approximate formulas.

Ans. 12.6 cm on a line drawn perpendicular to axis of magnet at center.

31. A bar magnet of moment $M = 200$ was placed with its center north of and 10 cm distant from a small compass needle of moment M' and moment of inertia I . The south pole of the bar magnet, whose axis was parallel to the horizontal intensity of the earth's field, was nearest the small compass needle, whose period was observed to be π seconds. When the bar magnet was reversed with its north pole nearest the compass needle the compass needle reversed its direction and its period was observed to be $\pi\sqrt{3}$ seconds. Calculate the intensity of the earth's field H at the small compass needle. Using the same data, calculate the value of the field if the compass needle had not reversed its direction.

Ans. $H = 0.2$ oersted; $H = 0.80$ oersted.

32. A bar magnet of unknown moment M was placed with its axis horizontal, and perpendicular to the horizontal component of the earth's field H , which was also unknown. A small compass needle 31.1 cm from the center of the bar magnet along its axis was observed to be deflected 45° with the earth's field. The magnet was then mounted on a frictionless pivot in the earth's field and its period was observed to be π seconds. The magnet was then loaded with a ring of moment of inertia 450 g-cm^2 , and the period was observed to be 2π seconds. The approximate length of the magnet was 10 cm. Calculate M , H , and the pole strength m of the magnet. Work to three significant figures only.

Ans. $M = 3000$; $H = 0.2$; $m = 300$.

33. A bar magnet 10 cm long of pole strength 100 was placed with its axis parallel to the earth's field. In one case its north magnetic pole N was north, in the other case it was south relative to the earth's field H of strength 0.2 oersted. Calculate the position of the neutral points relative to the center C of the magnet in each case, assuming the magnet isolated. Where the north pole was south calculate the position of the neutral point, neglecting the effect of the south pole, using approximate equations. How does this differ from the accurate calculation?

Ans. $n = 17.1 \text{ cm E and W of center of magnet}$; 21.5 cm N and S of center; 27.4 cm S of center.

34. A bar magnet of moment M was placed with its center north of and 10 cm distant from a small compass needle of moment M' and moment of inertia I . The north pole of the bar magnet, the axis of which was parallel to the horizontal intensity of the earth's field, was nearest the small compass needle, with a period that was observed to be $\pi\sqrt{3}$ seconds. When the bar magnet was reversed with its south pole nearest the compass needle the needle maintained its direction and its period was observed to be π seconds. Given H , the intensity of the earth's field, as 0.2 oersted, calculate M , the moment of the bar magnet. Use approximate formula for field at a point on the axis of a magnet which involves only M . *Ans.* $M = 50$.

35. Two identical bar magnets of moments $M = 400$ each are placed as indicated in Fig. P11 with axes parallel and polarities opposite in a uniform field of horizontal intensity 0.20 oersted. Locate quantitatively the position of the neutral points (i.e., points of zero intensity) using approximate formulas for the field perpendicular to the axis of a bar magnet at its center.

Ans. \perp to center of A , left side, 12.62 without B , 12.45 with B . Another 9.18 cm to the right of A , neglecting A , north and south of B on axis, 15.9 cm.

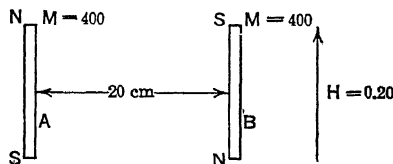


FIG. P11.

36. A bar magnet of moment M was placed with its center north of and 10 cm distant from a small compass needle of moment M' and moment of inertia I . The north pole of the bar magnet, the axis of which was parallel to the horizontal intensity of the earth's field, was nearest the small compass needle with a period that was observed to be $\pi\sqrt{4}$ seconds. When the bar magnet was reversed with its south pole nearest the compass needle, the needle reversed its direction and its period was observed to be $\pi\sqrt{3}$ seconds. Given H , the intensity of the earth's field, as 0.2 oersted, calculate M , the moment of the bar magnet. *Ans.* $M = 700$.

37. Assume that a magnetic field instead of being uniform is expressed by $H = H_0 + x \frac{dH}{dx}$ (i.e., increasing as x increases from a point where it is H_0). Here x is a

distance along the direction in which H changes, and $\frac{dH}{dx}$ is the rate of change of H with x . Hence in x centimeters the change in the field will be $x \frac{dH}{dx}$, assuming the

value of $\frac{dH}{dx}$ constant along x . Prove that a small magnet of moment $M = ml$ making an angle θ with x will experience a force, toward or away from the region of most intense field, of magnitude $M \cos \theta \frac{dH}{dx}$. Whether the force is attractive or repulsive depends on whether the S or the N pole of the magnet causing the field is in the strongest field.

38. The electron is nearly a point charge and has a magnetic moment $M = 0.92 \times 10^{-20}$ absolute unit. The axis of the electronic magnet is parallel to a field in which $\frac{dH}{dx} = 10,000$ oersteds per centimeter, its N pole being in the strongest part of the field. The electron is carried by an atom of H of mass 1.65×10^{-24} gram which is moving with the velocity of thermal agitation, and which is 2.5×10^5 cm per second along the face of the N pole and perpendicular to the magnetic field. How far and in what sense will the atom be deflected if it moves in the region where $\frac{dH}{dx} = 10,000$ oersteds for a distance $r = 20$ cm? The field is strongest near the N pole of the magnet. Recall that a body accelerated under a uniform force moves $S = \frac{1}{2} at^2$ cm in t seconds. Here $a = f/m$, where f is the force of the field on the atom. t can be found from the velocity v and the distance r . These are actually the conditions under which the famous Stern-Gerlach experiments were done. *Ans.* 1.78 mm.

PROBLEMS BASED ON CHAPTERS V AND VI

1. Given two long straight parallel wires, such as might feed a motor as indicated in Fig. P12, separated by 2 cm. The current flows up in one and down in the other.

If the current carried by them is the same and equal to 2 amperes calculate the resultant field H in oersteds at a point P , 2 cm from one and 4 cm from the other, and indicate the direction of the resultant field.

Ans. 0.10 oersted normal and into plane of paper.

2. A pair of very long wires run vertically upward parallel to the y -axis. They are 6 cm apart measured along the x -axis. Wire A on the left has a current of 10 amperes upward, and wire B has a current of 5 amperes upward. The wires are 2 mm in diameter, and the fields inside of the wires are not to be considered. The space between wires is otherwise field-free. Calculate and plot the fields about wires A and B in the plane of the figure from 10 cm to the left of A to 10 cm to the right of B . Fields into the paper should be plotted upward, fields out of the paper should be plotted downward from the horizontal x -axis. The wire A should be along the y -axis. From the algebraic sum of these two curves plot the resultant field within the limits given.

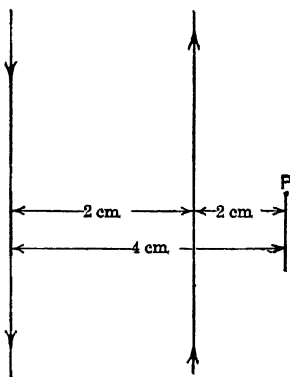


FIG. P12.

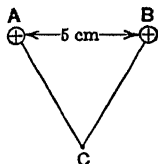


FIG. P13.

3. Given two long parallel wires 5 cm apart, with currents flowing outward from the plane of the paper as indicated by the circled crosses A and B , Fig. P13. Find the resultant field H at the point C placed at the corner of an equilateral triangle of sides 5 cm with A and B , if the current is 5 amperes in each wire.

Ans. 0.35 oersted parallel to AB .

4. A compass needle of moment M and moment of inertia I is found to have a period of π seconds in the earth's field H . When a downward current in a vertical wire due west of the needle and 10 cm distant of strength 10 amperes is turned on, the needle still points north but its period is increased to $\pi\sqrt{5}$ seconds. Calculate the strength of the earth's field.

Ans. 0.25 oersted.

5. A current flowed in a long straight vertical conductor. At a point 10 cm magnetically west of the conductor a small compass needle of which the period in the earth's field $H = 0.20$ oersted had been 10 seconds was observed to point in the opposite direction and the period was observed to be 17.3 seconds when a current flowed in the wire. Calculate the current in the wire in amperes, and state whether it was up or down.

Ans. 13.5 amp up.

6. A current flowed in a long straight vertical conductor. At a point 10 cm magnetically east of the conductor a small compass needle of which the period in the earth's field $H = 0.20$ oersted had been 10 seconds was observed to point in the same direction and the period was observed to be 14.12 seconds when a current flowed in the wire. Calculate the current in the wire in amperes, and state whether it was up or down.

Ans. 5 amp down.

7. Two vertical wires A and B in a field-free space have currents in them. When the current in A and B is upward a compass needle of moment of inertia I and magnetic moment M has a period of 2π seconds, at a point C on the line between A and B , 5 cm from A and 10 cm from B . When the current in A is reversed the compass needle reverses and has a period of π seconds. What is the current in B , given the current in A as 10 amperes?

Ans. 12.0 amp.

8. Two vertical wires A and B , A being to the left of B , perpendicular to the plane of the paper, are in an otherwise field-free space 20 cm apart. There is an upward current in A of 10 amperes. Midway between the two, i.e., 10 cm from A and B , a small compass needle C points downward in the plane of the paper. Its period of oscillation is 2 seconds. When the current in A is reversed the compass needle still points downward but its period is 1 second. Calculate the magnitude and sense of the current in B .

Ans. 16.7 amp up.

9. In the earth's field of horizontal intensity 0.20 oersted a suspended magnet had a period of 1 second. The same magnet placed in the center of a plane circular coil having a radius 15.7 cm and 75 turns, so placed that its field is opposed to the earth's field, does not reverse and has a period of 2 seconds when a current flows through the coil. Calculate the current in the coil.

Ans. $i = 0.05$ amp.

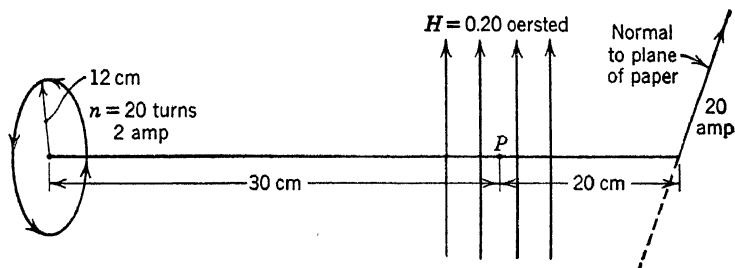


FIG. P14.

10. In a laboratory experiment a student was investigating the field due to a circular coil of twenty turns, radius 12 cm, at a point P , 30 cm from its center along its axis. The current in the coil was 2 amperes and in the direction of the arrows in Fig. P14. He measured the field by observing the deflection of a compass needle in the field of the coil and the horizontal component of the earth's field, which was 0.20' oersted and at right angles to the axis of the coil. Unknown to the student there was in the wall 20 cm distant a vertical power-supply-line wire carrying a current of 20 amperes upward. Calculate the angle θ of the compass needle with the earth's field and the resultant field, (a) when the power line was running (indicate direction in diagram), (b) when the power was off.

Ans. (a) 0.108 oersted, due east; (b) 0.227 oersted, $28^\circ 28'$ E of N.

11. In a laboratory experiment a student was investigating the field due to a circular coil of forty turns, radius 20 cm, at a point P , 40 cm from its center along its axis. The current in the coil was 3 amperes and in the direction of the arrows in Fig. P14. He measured the field by observing the deflection of a compass needle in the field of the coil and the horizontal component of the earth's field which was 0.25 gauss and at right angles to the axis of the coil. Unknown to the student there was in the wall 25 cm distant a vertical power-supply-line wire carrying a current of 20 amperes upward. Calculate the angle θ of the compass needle with the earth's field and the resultant field, (a) when the power line was running (indicate direction in diagram), (b) when the power was off.

Ans. (a) $77^\circ 55'$ east, $H_r = 0.43$ oersted.
(b) $55^\circ 30'$ east, $H_r = 0.34$ oersted.

12. A student attempted to measure the earth's field as follows: A plane circular coil radius 10 cm of twenty-five turns was placed with its axis perpendicular to the horizontal component of the earth's magnetic field. At a point on the axis 20 cm from the plane of the coil and east of it a small compass needle was placed. It pointed to the magnetic north when there was no current in the coil and was deflected 45° when 1.43 amperes flowed through the coil. When the experiment was repeated on the second day another student reported that the deflection he had obtained was $26^\circ 35'$ with the earth's field. The instructor investigated and learned that on the second day a vertical 10-ampere power line had been in use, which building plans indicated to be located east on the axis of the coil and in the wall beyond the compass. (a) What was the true value of the earth's field at the point? (b) How far away was the wire, and in what direction did the current flow?

Ans. (a) 0.202 oersted; (b) 10 cm down.

13. A student attempted to measure the earth's field as follows: A plane circular coil radius 20 cm of fifteen turns was placed with its axis perpendicular to the horizontal component of the earth's magnetic field. At a point on the axis 20 cm from the plane of the coil and east of it a small compass needle was placed. It pointed to the magnetic north when there was no current in the coil and was deflected 45° when 1.43 amperes flowed through the coil. When the experiment was repeated on the second day another student reported that the deflection he obtained was $26^\circ 35'$ with the earth's field. The instructor investigated and learned that on the second day a vertical 15-ampere power line had been in use, which building plans indicated to be located just on the axis of the coil and in the wall beyond the compass. (a) What was the true value of the earth's field at the point? (b) How far away was the wire and in what direction did the current flow?

Ans. (a) $H = 0.24$ oersted; (b) 12.51 cm down.

14. Two plane coils of 100 turns each, of radius 20 cm, are placed parallel to each other and on the same axis 20 cm apart with currents of 1 ampere flowing in the same sense about each coil. Calculate and plot the resultant field along the axis of the coils as a function of the distance between them. These are known as Helmholtz coils. Of what use can they be?

15. The most powerful factor on a ship giving the vertical magnetic field below the keel that actuates magnetic mines is the vertical component of magnetism running from the funnels to the keel. This is produced by the vertical steel in the central portion of the ship. It acts as if a vertical magnet were placed in the center of the ship. Part of its moment is induced and temporary and varies with magnetic latitude. The greater portion of it is, however, semipermanent. To compensate this component of ship's magnetism a horizontal coil is run around the inside skin of the ship with a current sent through it to give a field opposing that of the equivalent magnet acting. On a small scale model it is desired to know how well the field of the

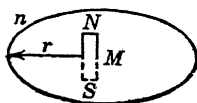


FIG. P15.

coil along its axis can compensate the field for the bar magnet along the same axis. A scale model will be used for this calculation. It can be multiplied by numerical factors to give the proper full scale sizes as needed.

The bar magnet M is short, less than 1 cm, and its axis is vertical as shown in Fig. P15. It has a moment $M = 111$ units with S pole down. The horizontal plane of the coil bisects the vertical magnet. It has a radius r of 5 cm and 100 turns.

The field along the axis of the magnet can be taken as $H_m = \frac{2M}{r^3}$. Calculate the

field H_m at a point 5.0 cm below the plane of the coil. Calculate the current i in amperes needed to make H_c , the field of the coil, equal in magnitude to H_m at 5 cm below the coil. What is the direction of the current needed to compensate H_m at 5 cm below the coil? Then calculate H_c and H_m at points 3, 10, and 20 cm below the plane of the coil. What conclusions can be drawn concerning the use of the coils in ships for compensating the vertical component of a ship's magnetism? Note also

that in designing coils the important factor is not i but ni so that one can either make n small and i large, saving critical copper, or make n large and i small. By varying i the field H_c can be adjusted to compensate H_m at any depth and any magnetic latitude. In actual practice the peculiar magnetic structure of a ship is such that coils around their inner shell compensate the vertical field far better than is the case in such a calculation. Thus fortunately for the safety of ships the effective use of magnetic mines limited the depth below the ship's keel at which mines could profitably be used to a rather narrow range of values. This made it possible in World War II to "degauss" ships successfully against magnetic mines and thus combined with effective magnetic minesweeping to save the extremely critical shipping. Note also that degaussing of a ship so changes its magnetism that the magnetic compasses used aboard have to be corrected when degaussing currents are on. This was accomplished by bleeding off some of the current from the degaussing coils by shunts and undegaussing the immediate region about the compass.

Ans. H magnet at 3, 10, and 20 cm = 8.2, 0.22, 0.0278 oersteds,
 H coil at 3, 10, and 20 cm = 3.16, 0.447, and 0.068 oersteds.

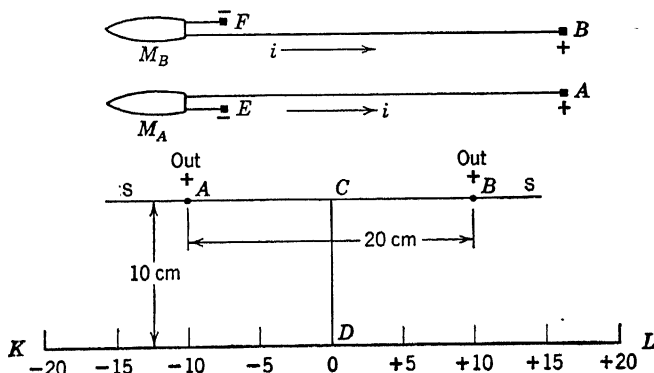


FIG. P16.

16. In World War II areas where the presence of magnetic mines were suspected were swept by various devices. For instance, ships towed submerged skids having permanent magnets attached that gave strong local fields. A more successful method used the device indicated in the upper part of Fig. P16. Here M_A and M_B are two minesweepers towing long insulated floating conductors, i.e., *sweep tails*, on the surface. They had powerful d-c generators or batteries that passed heavy currents through the sweep tails A and B . These had electrodes at the ends and in Fig. P16 they are positive electrodes. The current return through the sea water goes to electrodes at E and F close to the sweeper. The concentrated current in the cables A and B gives the fields needed. In the lower part of Fig. P16 (drawn to reduced scale) the cables A and B are shown in cross section lying on $S-S$, the water surface. The current flows out at A and B , and is 100 amperes. AB is 20 cm long. Calculate the vertical component of the sweep-tail field along a line KL 10 cm below $S-S$ and parallel to it. The field most interesting is that between abscissas ± 10 . Calculate the vertical components of the field at abscissas, 0, ± 1 , ± 2 , ± 5 , ± 8 , ± 10 , ± 15 , ± 20 and plot. By proper spacing of AB relative to depth CD quite an extensive area can be swept.

Ans. H_D 2 oersteds up, $H_{\pm 10}$ 0.4 oersted up.

17. The distance of an electron from the nucleus, i.e., the radius of the innermost Bohr orbit, in the H atom is 0.53×10^{-8} cm. The velocity of the electron in the orbit is 2.18×10^8 cm per second. The charge on the electron is 1.6×10^{-20} absolute e.m.u. Calculate the magnetic field in oersted at the center of the orbit

Ans. 4.16×10^{-3} oersted.

18. A beam of cathode rays having an energy of 10,000 electron-volts (i.e., having electrons that have fallen freely through 10,000 volts) has individual electrons of charge 1.6×10^{-20} absolute .m.u. with velocities of 5.9×10^9 cm per second. The beam is 1 mm in diameter, and in 1-cm length of the beam there are 10^8 electrons. Calculate the magnetic field at the edge of the beam. If 1 electron were moving by itself and its velocity were that given above, what would be the magnetic field at 1 atomic radius of 10^{-8} cm and at its own surface which could roughly be set at 2×10^{-13} cm? *Ans.* 0.378 oersted; 1.89×10^{-2} oersted; 945 oersteds.

19. A lightning discharge struck a copper lightning rod of cross-sectional area 0.10 cm^2 and fused a piece 100 cm long. The density of copper is 8.5 grams per cubic centimeter, its specific resistance is 2×10^{-6} ohms \times cm, its melting point is 1100°C , its latent heat of fusion is 42 calories per gram, and its specific heat is 0.12 calorie per gram. The rod was fused and the heat of the discharge went to heating it to its melting point and fused it. The discharge lasted 1×10^{-4} second. The outside temperature was 0°C . With these data calculate:

- The weight of copper melted.
- The heat liberated in the process.
- The resistance of the lightning rod.
- The average current in amperes.
- The potential across the rod if the current was constant for the 1×10^{-4} sec.
- The power liberated in watts and the quantity of electricity in coulombs.
- The magnetic field in oersted 1 meter from the rod.
- If the flash which struck the rod was 1000 meters long in the air and the P.D. was 2.5×10^8 volts, the following data are required: (1) the energy in joules liberated in the air; (2) the *apparent* resistance of the air path of the spark as a whole and the resistance per meter.

Ans. (a) 85 grams; (b) 1.476×10^4 cal; (c) 2×10^{-3} ohm; (d) 5.55×10^6 amp; (e) 1110 volts; (f) 6.18×10^8 watts, 55.5 coulombs; (g) 1110 oersteds; (h) (1) 1.39×10^{10} joules; (2) 0.45 ohm per meter.

20. A lightning discharge struck a copper lightning rod 1 cubic centimeter, in cross-sectional area and fused a section 50 cm long. The density of copper is 8.5 grams per cubic centimeter, its specific resistance is 2×10^{-6} ohm \times cm, its melting point is 1100°C , its latent heat of fusion is 42 calories per gram, and its specific heat is 0.12 calorie per gram. The rod was fused, and the heat of the discharge went to heating it to its melting point and fused it. The discharge lasted 10^{-3} second. The outside temperature was 0°C . With these data calculate:

- The weight of copper melted.
- The heat liberated in the process.
- The resistance of the lightning rod.
- The average current in amperes.
- The potential across the wire if the current was constant for the 10^{-3} sec.
- The power liberated in watts and the quantity of electricity in coulombs.
- The magnetic field in gauss 5 meters from the rod.
- If the flash which struck the rod was 500 meters long in the air and the P.D. was $(0.5) \times 10^8$ volts, the following data are required: (1) the power in watts liberated in the air; (2) the apparent resistance of the air path of the spark as a whole and the resistance per meter.

Ans. (a) 425 grams; (b) 7.395×10^4 cal; (c) 10^{-4} ohm; (d) 17.55×10^6 amp; (e) 175.5 volts; (f) 3.09×10^8 watts, 1755 coulombs; (g) 7.02×10^3 oersteds; (h) (1) 87.75×10^{14} watts; (2) $R = 28.5$ ohms, $\rho = 0.057$ ohm/m.

21. A current passed through a tangent galvanometer, and through a copper voltmeter connected in series. In the voltmeter, metallic copper was deposited at the cathode. The galvanometer had twenty-five turns, a radius of 10 cm, the horizontal component of the earth's field H was 0.314 gauss, and a deflection of 45° was observed. This same current in 1 hour deposited 0.2365 gram of copper. (a) How

many grams of copper does 1 coulomb deposit? (b) How many coulombs deposit 31.65 grams of copper? *Ans.* (a) 3.29×10^{-4} gram; (b) 96,200 coulombs.

22. A current passed through a tangent galvanometer, and through a silver voltameter connected in series. In the voltameter, metallic silver was deposited at the cathode. The galvanometer had forty turns, a radius of 15.0 cm, the horizontal component of the earth's field H was 0.25 gauss, and a deflection of 35° was observed. This same current in 2 hours deposited 0.8455 gram of silver. (a) How many grams of silver does 1 coulomb deposit? (b) How many coulombs deposit 107.9 grams of silver? *Ans.* (a) 1.123×10^{-3} gram; (b) 96,000 coulombs.

23. A calorimeter having a water equivalent of 240 grams had a coil of resistance R in it. A current of 1 ampere flowed through the coil for 1 min and 40 seconds, raising the temperature 10° C. (a) What was the potential across the coil in volts? (b) What was the resistance, assuming its change with temperature negligible? (c) What was the power consumption in watts?

Ans. (a) 100.32 volts; (b) 100.32 ohms; (c) 100.32 watts.

24. In testing magnetos for airplane engines during World War I all the characteristics of the sparks were required to be known. To test the heat per spark and thus the average ignition current during the spark, a standard spark gap was placed inside a heat-insulated copper block calorimeter and the rate of heating of the block was determined as the magneto operated. The heating curve with the magneto attached giving 360 sparks per minute was compared to calibration curves taken with a heating coil placed in the block, the current through which, and potential across which, could be measured. With a given magneto the curve coincided with that owing to a current of 0.5 ampere at 120 volts. The average voltage across the spark gap was known from oscillograph measurements to be about 200 volts (it started at 6000 volts and dropped to 100 volts during the discharge so as to give an average of 200 volts). The time of the discharge was known by oscillograph to be 0.001 second. (a) Calculate the watts delivered to the copper calorimeter by the heating coil and therefore by the 360 sparks per minute. (b) Calculate the average rate of temperature rise of the copper if its equivalent mass was 1000 grams (specific heat 0.093) before it began to lose much heat by radiation (i.e., on the straight part of its heating curve). (c) Calculate the power delivered in watts per spark from the magneto. (d) Then calculate the average current from this magneto in amperes during the discharge of 0.001 second.

Ans. (a) 60 watts; (b) 0.155° C per second; (c) 10,000 watts; (d) 50 amp.

25. In the demonstration experiment verifying Joule's law of heating, the two beakers with resistances R_1 and R_2 which are in parallel, each carrying half of the current, have 240 cm³ of water, while the one beaker, with all the current flowing through it, has the resistance R_3 immersed in 480 cm³ of water. See Fig. 35. Assume that $R_1 = R_2 = R_3$ is constant and is 10 ohms, and that the potential from the switchboard placed across the whole system is $V = 150$ volts. Neglecting heat loss during heating, starting from 20° C, how long will it take the water to boil at 100° C in the beaker with R_1 and that with R_3 ? *Ans.* For R_3 , 200 sec; for R_1 , 400 sec.

26. Consider two copper wires, one 0.05 mm in radius, the other 1 mm in radius, both 10 cm long. The melting point of copper is 1100° C, the density of copper is about 9, the specific heat is about 0.12 calorie per gram, and the specific resistance can be taken as constant and 2×10^{-6} ohm per cubic centimeter in this experiment. They are immersed in a liquid of which the average heat-conducting power from the surface of those copper wires is on the average 4×10^{-2} calorie per square centimeter of surface per second per degree difference between the liquid at 0° and the temperature of the copper wire. When 10 volts are maintained across both wires they will ultimately melt. Calculate the time of melting t_m in each case, and tell which one will melt first, starting at 0° C, the large wire or the small wire. To facilitate solution you are given that $\frac{1}{1-a}$ when a is nearly 1 is closely $1+a$, and that $\log_e (1+a) = a - \frac{1}{2}a^2$.

Ans. Large wire = 0.0094 sec; small wire = 0.0106 sec.

27. An electric flatiron is wound for a 110-volt circuit and takes 6.6 amperes of current under operating conditions. What is its power consumption? How many calories of heat are developed per hour? What does it cost to operate the iron at the rate of 4 cents per kilowatt hour?

Ans. $w = 726$ watts; $H = 6.27 \times 10^5$ cal per hour; cost = 2.91 cents per hour.

28. The force on an electron in a wire due to the potential driving the current is 1.59×10^{-14} dyne. (a) How many ergs of work were done in moving the electron 1 meter down the wire if the potential fall down the wire was uniform? (b) If the charge (quantity of electricity) on the electron is 1.59×10^{-20} e.m.u., what is the potential across the meter of wire in electromagnetic units and in volts? (c) If 6.29×10^{19} electrons flowed in a second, what was the current in absolute electromagnetic units and in amperes? (d) What was the resistance in ohms per centimeter of wire? (e) What was the quantity of electricity in coulombs that flowed in 1 second? (f) What was the power expended in the 1 meter of wire in watts?

Ans. (a) 1.59×10^{-12} erg; (b) 10^8 e.m.u., 1 volt; (c) 1 e.m.u., 10 amp; (d) 0.001 ohm per centimeter; (e) 10 coulombs; (f) 10 watts.

29. One of the electrons in a current down a wire experiences a force of 1.113×10^{-13} dyne due to the uniform potential gradient down the wire which is causing the current. (a) How many ergs of work were done when the electron moved down 10 meters of wire? (b) If the charge (quantity of electricity) on 1 electron is 1.59×10^{-20} e.m.u., what is the potential across the 10 meters in absolute units and volts? (c) If 4.4×10^{19} electrons flowed in 1 second, what was the current in electromagnetic units and amperes? (d) What was the resistance of the wire in ohms? (e) What was the quantity that flowed in 1 sec in coulombs? (f) What was the power expended in watts?

Ans. (a) 1.113×10^{-10} erg; (b) 7.0×10^9 e.m.u., 70 volts; (c) 7.00×10^{-1} e.m.u., 7.00 amp; (d) 0.010 ohm per cm; (e) 7.00 coulombs; (f) 490 watts.

30. In the cyclotron currents of He^{++} atoms (doubly charged He atoms) of 10^{-4} ampere are achieved. The He^{++} atoms have traversed a total potential of 6×10^8 volts. Each atom carries a charge of 3.2×10^{-20} e.m.u. The work done on a particle has been converted to kinetic energy $\frac{1}{2}mv^2$. The mass of the α particle is 6.60×10^{-24} gram. Calculate: (a) The number of He^{++} atoms passing a point per second. (b) The energy of an He^{++} atom in ergs. (c) Its velocity in centimeters per second. (d) The power expended by the whole He^{++} atom beam if stopped in a block of lead. (e) The He^{++} atoms penetrate 10^{-3} mm of copper before stopping. If the beam has an area of 5 cm², how long will it take to melt the surface of the copper screen put into the beam, neglecting heat loss; given the specific heat of copper as 0.12 calories per gram, its density as 9 grams per cubic centimeter, and melting point 1100° C? Given the heat of fusion of copper = 42 calories per gram.

Ans. (a) 3.12×10^{14} per sec; (b) $w = 192 \times 10^{-8}$ erg; (c) $v = 2.41 \times 10^9$ cm per second; (d) 600 watts; (e) 5.43×10^{-3} second.

31. The earth has a radius of roughly 4000 km. Assume that it is uniformly charged with electricity and rotates about its axis once in 24 hours. What charge density (a) in coulombs, and (b) in electrons uniformly distributed over its surface would be required to give a field along its axis of 0.5 oersted? (c) What would be the sign of the charge to give the observed polarity? (d) What would be the total charge required? To solve this in an approximate fashion consider the currents needed in a coil of 1 turn equal to the average distance r of the charge from the axis of the earth rotating once in 24 hours.

Ans. (a) 8.8×10^{-16} e.m.u. per cm²; (b) 5.54×10^4 electrons per cm²; (c) negative; (d) 1090 e.m.u.

32. An electrical train on a level track travelling at 50 km per hour requires 7.5 kw-hr per train kilometer. What is the equivalent resisting force in kilograms?

Ans. 2760 kg.*

* Problems 32, 1, 2, 6 and 7 were taken from *Lessons and Problems in Electricity* by Norman C. Page, by the courtesy of the author and The Macmillan Company.

33. An x-ray tube delivering 10 milliamperes of current at 100,000 volts liberates the kinetic energy of its electrons in a very large measure as heat in the target (anticathode). The x-ray energy is a relatively small fraction of the total electron energy set free. The target has a radiating surface of 5 cm^2 . It loses heat largely by radiation according to the equation $H = A\sigma(T_1^4 - T_2^4)t$ ergs in time t . Given $T_2 = 300^\circ$ absolute, $\sigma = 5.7 \times 10^{-5}$ erg per square centimeter per degree⁴ per second. Calculate the equilibrium temperature T_1 in degrees absolute of the target at equilibrium under the radiation loss alone.
Ans. 2432° C.

PROBLEMS BASED ON CHAPTER VII

1. A d-c power station delivers 440 kw at 2200 volts station output to a factory, the resistance of the transmission line being 0.20 ohm. (a) What is the power loss in the line in watts? (b) What is the P.D. across the line at the factory? (c) If the P.D. at the power station were 550 volts for the same power with the same resistance, what would the power loss have been? *Ans.* (a) 8 kw; (b) 2160 volts; (c) 128 kw.*

2. From Niagara Falls to New York City is about 480 km. A power line has a copper conduit 1 cm^2 in cross section. At 0° C the specific resistance of copper is 1.64×10^{-6} ohm cm. (a) What is the resistance of the conduit? (b) If it carries 400 amperes, what is the potential needed? (c) What is the power loss in watts to heating? (d) Had it been possible to send the same power over the line at 10 times the voltage, what would the power loss have been?

Ans. (a) 78.8 ohms; (b) 31,520 volts; (c) 1.26×10^4 kw; (d) 126 kw.*

3. A wire of 4 ohms resistance is drawn out until its length is doubled. If the specific resistance remained constant, calculate its resistance. If drawing had doubled its specific resistance, what would its resistance have been?

Ans. 16 ohms; 32 ohms.

4. The specific resistance of copper is about 1.6×10^{-6} ohm \times cm. Calculate the resistance of 1 cm^3 of copper wire when drawn out into a wire 0.178 mm in radius and when rolled out into a flat sheet 0.1 mm thick taken perpendicular to the face of the sheet. What is the ratio of the two resistances which can be obtained from the same mass of copper in the forms above?

Ans. 1.62 ohms; 1.62×10^{-10} ohm; ratio 1 : 10^{10} .

5. Two electrodes of area 10 cm^2 and 4 cm apart are immersed in a dilute solution of salt. The resistance measured is 0.01 ohm. What is the specific resistance of the solution? What is the specific conductivity in mhos per square centimeter per centimeter, and the conductivity in mhos?

Ans. $\rho_s = 0.025$; $\sigma_s = 40$; $\sigma = 100$.

6. A copper wire of 5.50 ohms resistance at 20° C is plunged into liquid air at -197° C . The temperature coefficient of copper is 3.8×10^{-3} in this range. What is the resistance of the wire at liquid air temperature? *Ans.* $R_{197} = 1.283$ ohms.*

7. An iron wire of resistance 5 ohms is in series with a carbon rod of resistance 90 ohms both at 0° C . Carbon has a temperature coefficient $\alpha = -3 \times 10^{-4}$; that for iron is $\alpha = +5.4 \times 10^{-3}$. What will be the resistance at 100° C ?

Ans. 95 ohms.*

8. The average temperature coefficient for tungsten wire is 5.1×10^{-3} above 20° C . An ordinary light bulb has a "cold resistance" at 20° C of 9.7 ohms. When burning normally it has a resistance of 121 ohms. What is its temperature in degrees C? If it consumes 100 watts, what current does it draw? What is the potential across it? *Ans.* 2270° C ; current 0.91 amp; potential 110 volts.

9. Manganin wire used in making standard resistance box coils has a temperature coefficient $\alpha = 1.5 \times 10^{-6}$. Its specific resistance is 4.11×10^{-6} ohm \times cm. If a resistance box made up of this material and the dial settings calibrated at 0° C had been read to give the resistance when readings were taken at room temperature of 22° C , what percentage error would have resulted? *Ans.* 0.033%.

10. Copper has a specific resistance of 1.8×10^{-6} ohm \times cm. A telegraph line of 150 km length is to be run from one town to the next. The length of the wire will thus be 300 km of copper wire of diameter 0.356 cm (about No. 8 wire). It requires 0.319 ampere to work the relay at the receiving end. (a) How many Daniell cells of e.m.f. = 1.15 volts each will be required to operate the line? (b) If the temperature ranges from -15°C in winter to $+42^\circ\text{C}$ in summer, and if the temperature coefficient of resistance of copper is 0.004 ohm, per ohm per degree C, how many cells will be required to care for this change, and at what time of the year must more cells be used?

Ans. (a) 151 cells; (b) 35 cells, summer.

11. Determine the resistance between the points *A* and *B* in the circuit shown in Fig. P17. The numerals denote the resistance of each conductor in ohms.

Ans. 2.34 ohms.

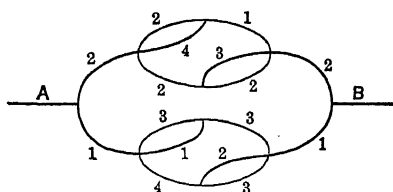


FIG. P17.

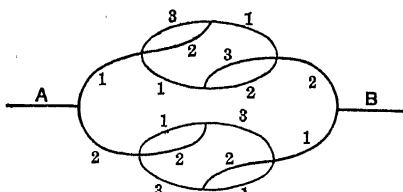


FIG. P18.

12. Determine the resistance between the points *A* and *B* in the circuit shown in Fig. P18. The numerals denote the resistance of each conductor in ohms.

Ans. Upper branch 4.1 ohms; lower level 4.83 ohms; total 2.22 ohms.

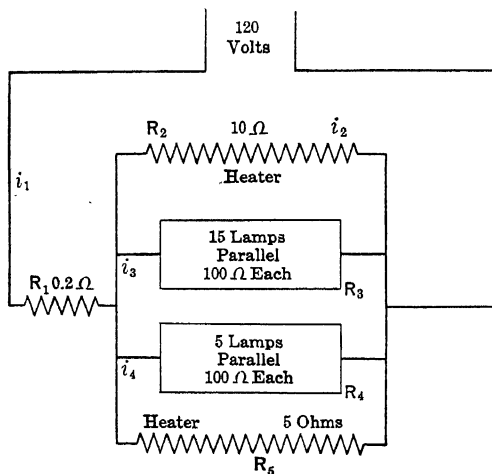


FIG. P19.

13. The circuit depicted in Fig. P19 is typical of the circuits used in wiring houses. In order to choose the wiring to be used it is essential to know the currents in each branch. (a) Calculate the currents i_1 , i_2 , i_3 , i_4 , and i_5 as well as the potential across R_1 , R_2 , R_3 , R_4 , R_5 when all the lights are burning. (b) Calculate the potentials across the resistances R_1 and R_2 , R_3 , R_4 , R_5 which are in parallel when the current in R_5 one of the heaters is 0, that is, with the circuit R_5 cut out. This

change in potential across the lamps shows why it is that lights dim when electric heaters are put on. *Ans.* (a) $i_1 = 54.5$ amp, $i_2 = 10.9$ amp, $i_3 = 16.35$ amp, $i_4 = 5.45$ amp, $i_5 = 21.8$ amp, $v_1 = 10.9$ volts, $v_2 = 109$ volts; (b) $v_1 = 6.8$ volts, $v_2 = 113.2$ volts.

14. The circuit depicted in the diagram of Fig. P20 is typical of the circuits used in wiring houses. In order to choose the wiring to be used it is essential to know the currents in each branch. (a) Calculate the currents i_1 , i_2 , i_3 , i_4 , and i_5 as well as the

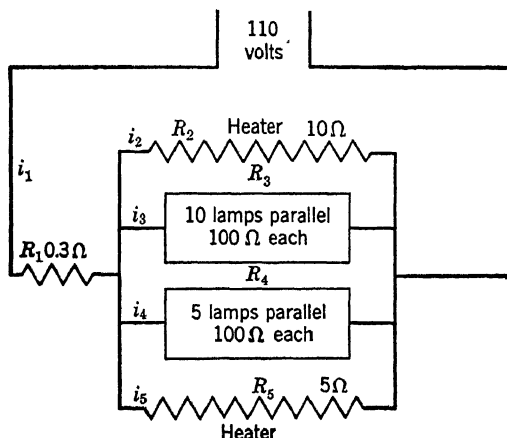


FIG. P20.

potential across R_1 , R_2 , R_3 , R_4 , R_5 when all the lights are burning. (b) Calculate the potentials across the resistances R_1 , and R_2 , R_3 , R_4 , R_5 which are in parallel when the currents in R_5 one of the heaters is 0, that is, with the circuit R_5 cut out. This change in potential across the lamps shows why it is that lights dim when electric heaters are put on. *Ans.* (a) $i_1 = 43.6$ amp, $i_2 = 9.69$ amp, $i_3 = 9.69$ amp, $i_4 = 4.89$ amp, $i_5 = 19.4$ amp, $v_1 = 13$ volts, $v_2 = 96.9$ volts; (b) $v_1 = 7.7$ volts, $v_2 = 102.3$ volts.

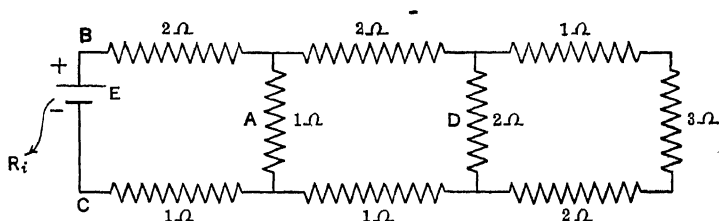


FIG. P21.

15. Given the circuit of Fig. P21 with the values of the resistances marked on the diagram. You are also given the e.m.f., E , of the cell as 10 volts and its internal resistance R_i as 1 ohm. Calculate:

(a) The resistance of the external circuit with resistance A in place. (Hint: Start at the right-hand end.)

(b) The resistance of the external circuit with $A = 1$ ohm removed.

(c) The current through the circuit in (a) and (b) above.

(d) The P.D. across the points BC when A was present and absent.

- (e) The current through D when A is in and when it is out.

Ans. (a) 3.82 ohms; (b) 7.5 ohms; (c) 2.075 amp, 1.176 amp;
(d) 7.925 volts, 8.824 volts; (e) 0.238, 0.881.*

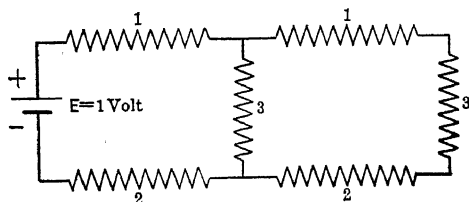


FIG. P22.

16. Given the resistances and a battery shown in Fig. P22, calculate the whole resistance of the circuit attached to E , assuming negligible internal resistance. What is the current in the whole circuit and in each of the 3-ohm resistors?

Ans. $\frac{1}{15}$, $\frac{2}{15}$, $\frac{3}{15}$, amp.*

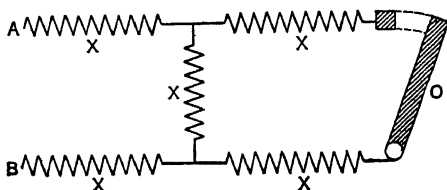


FIG. P23.

17. Given the circuit of Fig. P23. The five equal resistances have a value of x ohms. The switch O has a resistance of 0 ohms. What is the resistance of the circuit for a potential attached to A and B when O is open and when O is closed?

Ans. O open, $3x$; O closed, $2\frac{2}{3}x$.*

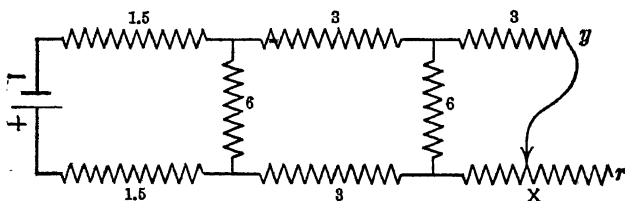


FIG. P24.

18. A cell is connected to a circuit shown in Fig. P24, the battery giving an e.m.f. of 14 volts with negligible internal resistance. The variable resistance r can be altered from 0 across xy to ∞ . What is the range of potential across xy which can be obtained?

Ans. 0 to 4 volts when r goes from 0 to ∞ .

19. On charging two storage batteries in series with an e.m.f. of 12.24 volts at 8 amperes current, what must the applied potential be if the internal resistance of each cell is 0.05 ohm?

Ans. $V = 11.44$ volts.

20. A storage battery of e.m.f. 2.2 volts, $R_i = 0.05$ ohm, is opposed to a dry

* Problems 15, 16 and 17 were taken from *Lessons and Problems in Electricity*, by Newell C. Page, by the courtesy of the author and The Macmillan Company.

cell of e.m.f. = 1.5 volts and $R_i = 0.1$ ohm, the + terminals and - terminals being connected together. What is the resultant current in sense and direction?

Ans. 0.466 amp with + terminal of the storage cell positive.*

21. Two cells can be placed either in opposition to each other or aiding each other. If in the two cases $E_1 = 2.2$ volts, $R_{i1} = 0.05$ ohm, and $E_2 = 1.5$ volts, $R_{i2} = 0.10$ ohm, and if 0.20 ohm is in series with the cells in both cases, what is the ratio of the currents i_0 and i_a with cells opposed and aiding?

Ans. $\frac{i_0}{i_a} = 0.189$.

22. A long wire is cut into ten pieces which are equally long. These are twisted together to make a cable. If the length of the separate wires was such as to give them a resistance of 10 ohms each, what was the initial resistance of the wire and what is the ratio of the resistance of the wire to the resistance of the cable?

Ans. $R_{\text{wire}} = 100$ ohms; $\frac{R_{\text{wire}}}{R_{\text{cable}}} = 100$.*

23. Grid leaks used in radio are made up of metal-coated glass and range in value from 1×10^6 to 5×10^6 ohms. They are mounted in clips on Bakelite, which is *not* a good static insulator and is especially poor when coated with dust and moisture. A 2×10^6 -ohm grid leak is to be used. What must be the insulation resistance used with this leak such that in use it is not 2 per cent less than its rated value?

Ans. $R = 9.8 \times 10^7$ ohms.

24. Given a battery of e.m.f. E and internal resistance R_i . Under what conditions will the potential V across the external resistance R_e be a maximum when R_e can be varied over a wide range? Under what conditions will the current i be a maximum?

Ans. $i = \text{max at } R_e = 0$; $V = \text{max at } R_e = \infty$.

25. A Daniell cell gives a potential reading of 1.05 volts when a voltmeter of 10,000 ohms resistance is placed across a 10-ohm resistance in series with the cell. When the 10-ohm resistance is replaced by one of 5 ohms, the voltmeter reads 0.95 volt. Calculate the e.m.f. and internal resistance of the Daniell cell.

Ans. $R_i = 1.18$ ohms; $E = 1.17$ volts.

26. A power main gives a potential reading of 110 volts when a voltmeter of 10,000-ohm resistance is placed across a 20-ohm resistance in series with the main. When the 20-ohm resistance is replaced by one of 10 ohms the voltmeter reads 100 volts. Calculate the e.m.f. and internal and line resistance of the circuit.

Ans. $R_i = 2.22$ ohms; e.m.f. = 122.2.

27. A dry cell gives a potential as measured by a very high-resistance voltmeter of 1.45 volts when shorted by 10 ohms, and 1.25 volts when shorted by 2 ohms. What are the e.m.f. and the internal resistance? What is the power consumed *inside* the cell and in a 0.0255-ohm resistance when the cell is shorted by a 0.0255-ohm resistance if there is no polarization? As the cell is well insulated thermally, about how many calories would be created in 40 minutes under these conditions? If the mass of the cell is 300 grams and its specific heat is about 0.5 calorie per gram, what rise in temperature would occur? Can you see why drawing such a current ruins a dry cell?

Ans. $R_i = 0.417$ ohm; e.m.f. = 1.51 volt; 46.5° C.

28. The chemical reactions in a cell generate electrical energy at the over-all rate of 1 watt per ampere. A current of 10 amperes is being drawn. As a result of this current heat is generated in the cell at 1 watt. Calculate the e.m.f., the potential across the external circuit, and the internal resistance of the cell.

Ans. 1 volt, 0.9 volt, 0.01 ohm.*

29. A dry cell has an e.m.f. of 1.5 volts and an internal resistance of 0.05 ohm. What current and what potential will it give across resistances of 100, 10, 1, and 0.1 ohms, respectively?

Ans. $R = 100$, $i = 0.01499$, $V = 1.499$;
 $R = 10$, $i = 0.1492$, $V = 1.492$;
 $R = 1$, $i = 1.429$, $V = 1.429$;
 $R = 0.1$, $i = 10.00$, $V = 1.000$.

* Problems 20, 22 and 28 were taken from *Lessons and Problems in Electricity*, by Newell C. Page, by the courtesy of the author and The Macmillan Company.

30. A "power pack" used to rectify an alternating current for radio sets gives 1000 volts across a 100,000-ohm "bleeder" resistance. Of this 100,000 ohms a section of 20,000 ohms at the ground end is tapped off to furnish 200 volts as a bias voltage for part of a complex circuit. To check the voltage a good 200-volt d-c voltmeter of resistance 20,000 ohms was placed across the 20,000-ohm section by a careless student. He was amazed to find that the voltmeter did *not* read 200 volts, although he had checked his resistances carefully. What potential did he read, neglecting the internal resistance of the power pack relative to the change in the line?

Ans. 111.1 volts.

31. A circuit has an e.m.f. E and an internal resistance R_i . It has an external resistance R_e . The heating H in R_e is given by Joule's law in terms of the current i and t . The current i is governed by E and R_e and R_i . Set up the equation for the heating, and express R_e as a multiple a times R_i , i.e., $R_e = aR_i$. By differentiating to obtain $d\left(\frac{H}{t}\right)/da$, and setting this equal to 0, we find the value of R_e which

makes $\frac{H}{t}$, the rate of heating, a maximum. Show that the maximum rate of heating

is given when $a = 1$ and $\left(\frac{H}{t}\right)_{\max.} = \frac{E^2}{4R_i}$. What is the maximum heat evolution which can be obtained from a dry cell for which $R_i = 0.05$ ohm and e.m.f. = 1.5 volts? Prove that, when the cell is giving its maximum value of $\frac{H}{t}$ or power output,

its efficiency is 50 per cent. *Ans.* $\left(\frac{H}{t}\right)_{\max.} = 1.125$ watts per second.

32. Use is made of "ballast" resistors for reducing fluctuations of current with potential. These consist of fine iron wires in a gaseous, thermally conducting atmosphere of H_2 gas. The current is run through these at a value where the i^2R gain from the wire is just compensated by cooling by the gas. Now for many substances, e.g., metals, resistance increases with temperature. Hence if the voltage V increases across the line the resistor heats up, R increases, and the current decreases to its value before the increase in V . Given a wire in equilibrium with cooling $A\theta$ just compensated by heating due to the current $0.24 Vi$, where A is the wire surface area, E its heat loss per square centimeter per second per degree temperature above the gas, θ the temperature difference between wire and gas, V the potential, i the current, and t the time. It is also given that since $i = \frac{V}{R}$, $\frac{V}{R_0(1+a\theta)} = i$, where R_0 is the resistance at 0°C and a is the temperature coefficient of resistance. For metals it is positive; for carbon it is negative. Prove that the wire at equilibrium will act as a "ballast" tube if a is positive, and establish the condition for this action. (Hint:

Eliminate θ from the equation and evaluate $\frac{di}{dV}$. The condition that $\frac{di}{dV} = 0$ is the condition for the ballast action.) Given the following data: $E = 4 \times 10^{-6}$ calories per square centimeter per degree per second; radius of wire = 0.2 mm; length of wire = 50 cm; specific resistance of pure iron = 15×10^{-6} ; a for pure iron = 60×10^{-4} . Calculate the current i in amperes at which the wire will act as a "ballast" and stabilize a fluctuating current.

Ans. 0.54 amp.

PROBLEMS BASED ON CHAPTER VIII

Note that some of these problems do not require solution by the Kirchhoff's law equations, but are intended to be so solved. They may also be solved more rapidly by the method of the equivalent Y or by Maxwell's method. It is suggested that the

formal Kirchhoff procedure be used first and that the shorter methods be applied only to the more difficult problems.

Note these problems are quite general and can be modified in any way to suit appropriate practical arrangements. The *numerical values* can be substituted at will by the instructor from year to year.

1. Given the values of the resistances and potentials of Fig. P25, calculate the value of the current i_g in R_g .

$$\begin{aligned} E &= 2 \text{ volts.} \\ R_E &= 1 \text{ ohm.} \\ R_g &= 20 \text{ ohms.} \\ R_P &= 3 \text{ ohms.} \\ R_Q &= 4 \text{ ohms.} \\ R_X &= 3 \text{ ohms.} \\ R_R &= 3 \text{ ohms.} \end{aligned}$$

Ans. $i_g = \frac{9}{1277}$ amp, current upward.

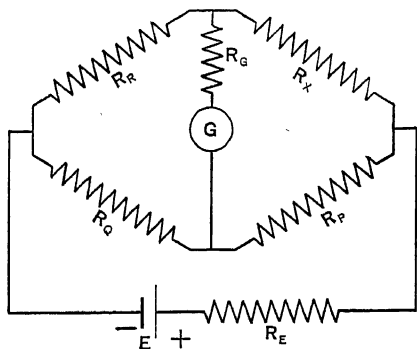


FIG. P25.

2. Given the values of the resistances and potentials of Fig. P26, calculate the value of the current i_g in R_g .

$$\begin{aligned} E &= 2 \text{ volts.} \\ R_E &= 1 \text{ ohm.} \\ R_g &= 40 \text{ ohms.} \\ R_P &= 2 \text{ ohms.} \\ R_Q &= 3 \text{ ohms.} \\ R_X &= 3 \text{ ohms.} \\ R_R &= 4 \text{ ohms.} \end{aligned}$$

Ans. $i_g = \frac{9}{2017}$ amp, current upward

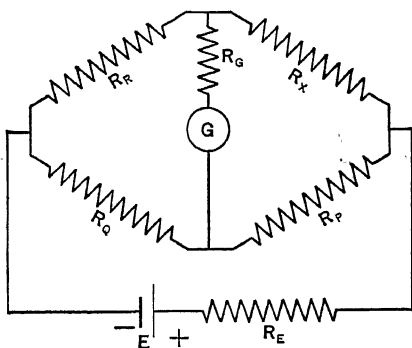


FIG. P26.

3. Given the values of the resistances and potentials of Fig. P27 as seen below, calculate the value of the current i_g in R_g .

$$\begin{aligned} E &= 1 \text{ volt.} \\ R_E &= 1 \text{ ohm.} \\ R_g &= 10 \text{ ohms.} \\ R_P &= 3 \text{ ohms.} \\ R_Q &= 4 \text{ ohms.} \\ R_X &= 2 \text{ ohms.} \\ R_R &= 3 \text{ ohms.} \end{aligned}$$

Ans. $i_g = \frac{1}{807}$ amp, current upward.

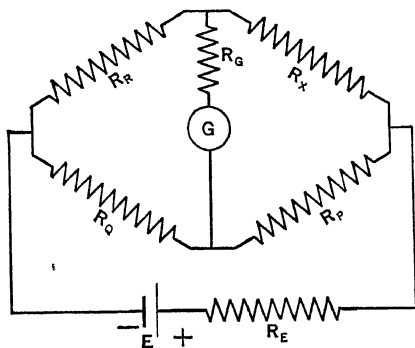


FIG. P27.

4. Given a Wheatstone bridge network which is in "balance" ($i_g = 0$), prove that the battery and galvanometer may be interchanged without destroying the balance.

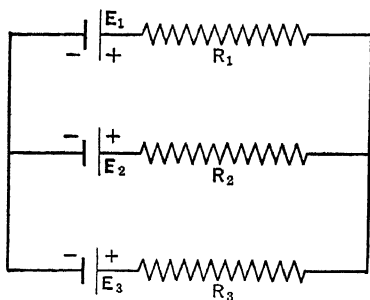


FIG. P28.

5. Given the circuit of Fig. P28, calculate the currents i_1 , i_2 , and i_3 .

$$E_1 = 3 \text{ volts.} \quad R_1 = 3 \text{ ohms.}$$

$$E_2 = 2 \text{ volts.} \quad R_2 = 2 \text{ ohms.}$$

$$E_3 = 1 \text{ volt.} \quad R_3 = 1 \text{ ohm.}$$

$$\text{Ans. } i_1 = \frac{5}{11}; i_2 = \frac{2}{11}; i_3 = \frac{7}{11}.$$

6. Given the circuit of Fig. P29 with the following data, calculate i_1 , i_2 , and i_3 in R_1 , R_2 , and R_3 .

$$E_1 = 3 \text{ volts.} \quad R_1 = 1 \text{ ohm.}$$

$$E_2 = 2 \text{ volts.} \quad R_2 = 2 \text{ ohms.}$$

$$E_3 = 1 \text{ volt.} \quad R_3 = 3 \text{ ohms.}$$

$$\text{Ans. } i_1 = 1; i_2 = 0; i_3 = 1.$$

7. Given the circuit of Fig. P29 with the following data, calculate i_1 , i_2 , and i_3 in R_1 , R_2 , and R_3 .

$$E_1 = 3 \text{ volts.} \quad R_1 = 2 \text{ ohms.}$$

$$E_2 = 2 \text{ volts.} \quad R_2 = 1 \text{ ohm.}$$

$$E_3 = 2 \text{ volts.} \quad R_3 = 2 \text{ ohms.}$$

$$\text{Ans. } i_1 = \frac{7}{8} \text{ amp; } i_2 = \frac{6}{8} \text{ amp; } i_3 = \frac{13}{8} \text{ amp.}$$

8. Given the circuit of Fig. P30 with the following data, calculate i_1 , i_2 , and i_3 in R_1 , R_2 , and R_3 .

$$E_1 = 1 \text{ volt.} \quad R_1 = 1 \text{ ohm.}$$

$$E_2 = 3 \text{ volts.} \quad R'_1 = 1 \text{ ohm.}$$

$$E_3 = 2 \text{ volts.} \quad R_2 = 2 \text{ ohms.}$$

$$R'_2 = 1 \text{ ohm.}$$

$$R_3 = 1 \text{ ohm.}$$

$$\text{Ans. } i_1 = \frac{7}{11} \text{ amp; } i_2 = \frac{12}{11} \text{ amp; } i_3 = \frac{18}{11} \text{ amp.}$$

9. Given the circuit of Fig. P30 with the following data, calculate i_1 , i_2 , and i_3 in R_1 , R_2 , and R_3 .

$$E_1 = 3 \text{ volts.} \quad R_1 = 1 \text{ ohm.}$$

$$E_2 = 1 \text{ volt.} \quad R'_1 = 1 \text{ ohm.}$$

$$E_3 = 2 \text{ volts.} \quad R_2 = 2 \text{ ohms.}$$

$$R'_2 = 1 \text{ ohm.}$$

$$R_3 = 1 \text{ ohm.}$$

$$\text{Ans. } i_1 = \frac{17}{11} \text{ amp; } i_2 = \frac{4}{11} \text{ amp; } i_3 = \frac{21}{11} \text{ amp.}$$

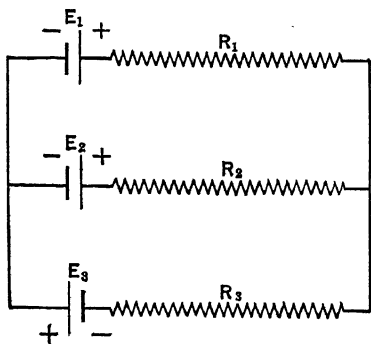


FIG. P29.

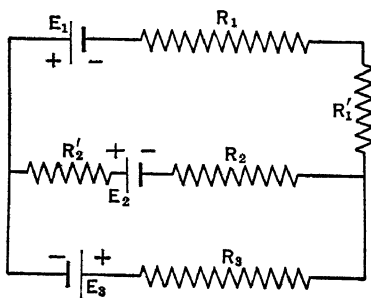


FIG. P30.

10. In the circuit of Fig. P31 the polarities of E_1 and E_2 are fixed. R_1 , R_2 , and R_3 may be varied to suit the conditions to be outlined. E_3 is fixed in value but may be connected either opposing E_1 and E_2 , switch A closed, B open, or aiding E_1 and E_2 switch B closed and switch A open. Set up Kirchhoff's laws for this circuit for the two cases A closed or B closed. From these laws deduce the algebraic relations between the values of E_1 , E_2 , and E_3 and R_1 , R_2 , and R_3 required to give the following conditions:

(A) E_3 opposed to E_1 and E_2 .

(a) $i_3 = 0$, (b) $i_2 = 0$, (c) $i_1 = i_2 = 0$.

(B) E_3 aiding E_1 and E_2 .

(a) $i_1 = 0$, (b) $i_2 = 0$, (c) $i_1 = i_2$, (d) $i_3 = 0$.

Note: The answer is to be given in the form such as appears in one of the cases $\frac{E_1 - E_2}{E_3 - E_2} = \frac{R_1}{R_2}$. Thus if E_1 , E_2 , and E_3 are given these special cases can be predicted in the choice of R_1 and R_2 and avoided. Note that in one case the condition imposed is physically impossible. Which one is it?

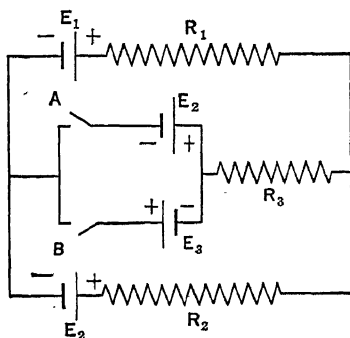


FIG. P31.

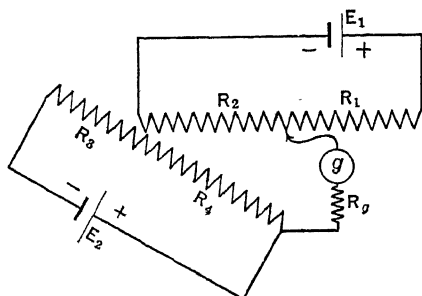


FIG. P32.

11. In the circuit of Fig. P32 solve for the currents i_1 , i_2 , i_3 , i_4 , and i_5 , in R_1 , R_2 , R_3 , R_4 , and R_5 .

$E_1 = 5$ volts.

$E_2 = 2$ volts.

$R_2 = 2$ ohms.

$R_1 = 2$ ohms.

$R_4 = 2$ ohms.

$R_3 = 3$ ohms.

$R_5 = 10$ ohms.

Ans. $i_1 = \frac{161}{132}$ amp; $i_2 = \frac{144}{122}$ amp; $i_3 = \frac{42}{122}$ amp; $i_4 = \frac{59}{122}$ amp;
 $i_5 = \frac{17}{122}$ amp, current downward.

12. In the circuit of Fig. P32 solve for the currents i_1 , i_2 , i_3 , i_4 , and i_5 , in R_1 , R_2 , R_3 , R_4 , and R_5 .

$E_1 = 3$ volts.

$E_2 = 2$ volts.

$R_2 = 1$ ohm.

$R_1 = 4$ ohms.

$R_4 = 3$ ohms.

$R_3 = 2$ ohms.

$R_5 = 10$ ohms.

Ans. $i_1 = 0.63$ amp; $i_2 = 0.48$ amp; $i_3 = 0.49$ amp;
 $i_4 = 0.34$ amp; $i_5 = 0.15$ amp.

13. In the circuit of Fig. P32 solve for the currents i_1 , i_2 , i_3 , i_4 , and i_o , in R_1 , R_2 , R_3 , R_4 , and R_o .

$$E_1 = 4 \text{ volts.}$$

$$R_4 = 2 \text{ ohms.}$$

$$E_2 = 3 \text{ volts.}$$

$$R_3 = 3 \text{ ohms.}$$

$$R_2 = 1 \text{ ohm.}$$

$$R_o = 10 \text{ ohms.}$$

$$R_1 = 2 \text{ ohms.}$$

Ans. $i_1 = \frac{238}{178} = 1.338 \text{ amp; } i_2 = 1.324; i_3 = 0.595; i_4 = 0.606; i_o = 0.011.$

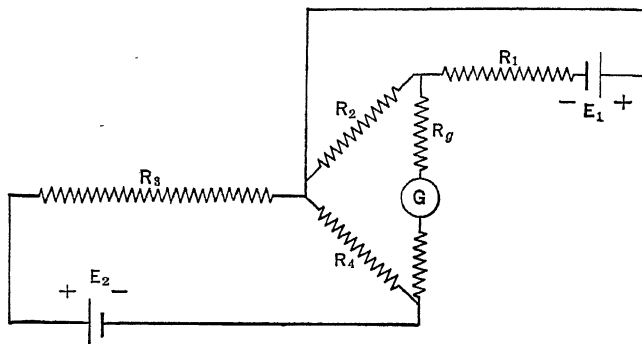


FIG. P33.

14. Given the circuit of Fig. P33, determine the condition that the current i_o through R_o shall be zero.

Ans. $i_o = 0$ when $E_1[R_2R_3 + R_2R_4] = -E_2[R_2R_4 + R_1R_4]$.

15. Given the circuit of Fig. P34, calculate the resistance through the network.

$$E = 1 \text{ volt.}$$

$$R_2 = 2 \text{ ohms.}$$

$$R_4 = 1 \text{ ohm.}$$

$$R_6 = 2 \text{ ohms.}$$

$$R_1 = 1 \text{ ohm.}$$

$$R_3 = 3 \text{ ohms.}$$

$$R_5 = 2 \text{ ohms.}$$

$$R_7 = 3 \text{ ohms.}$$

Ans. $i = 0.555 \text{ amp; } R = 1.8 \text{ ohms.}$

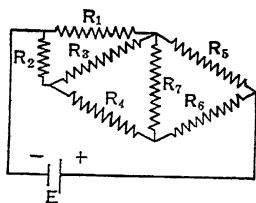


FIG. P34.

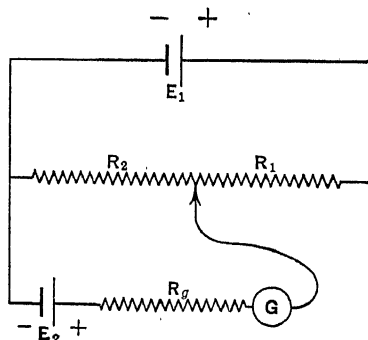


FIG. P35.

16. Given the circuit of Fig. P35, determine the condition that the current i_o shall be zero. Also given a fixed value for E_1 , E_2 , and R_o , what is the condition when not in balance that a change in R_1 will produce a maximum change in i_o ? That is, what is the condition of maximum sensitivity of the device in terms of R_1 , R_2 and the combined resistance $R_1 + R_2 = R$?

Ans. $i_o = 0$ when $R_1 = \frac{R_2(E_1 - E_2)}{E_2}$ or $R = \frac{E_1 R_2}{E_2}$.

17. Given the circuit shown in Fig. P36 with the constants given, calculate the currents in the different resistances.

$$E_1 = 3 \text{ volts.}$$

$$R_4 = 2 \text{ ohms.}$$

$$E_2 = 2 \text{ volts.}$$

$$R_5 = 1 \text{ ohm.}$$

$$E_3 = 1 \text{ volt.}$$

$$R_6 = 2 \text{ ohms.}$$

$$R_1 = 1 \text{ ohm.}$$

$$R_7 = 3 \text{ ohms.}$$

$$R_2 = 2 \text{ ohms.}$$

$$R_8 = 2 \text{ ohms.}$$

$$R_3 = 3 \text{ ohms.}$$

$$R_9 = 1 \text{ ohm.}$$

$$\text{Ans. } i_1 = \frac{1}{2} \text{ amp; } i_2 = \frac{1}{2} \text{ amp; } i_3 = \frac{1}{2} \text{ amp; } i_4 = 0 \text{ amp; } i_5 = 0 \text{ amp; } i_6 = \frac{1}{2} \text{ amp; } i_7 = \frac{1}{2} \text{ amp; } i_8 = \frac{1}{2} \text{ amp; } i_9 = \frac{1}{2} \text{ amp.}^*$$

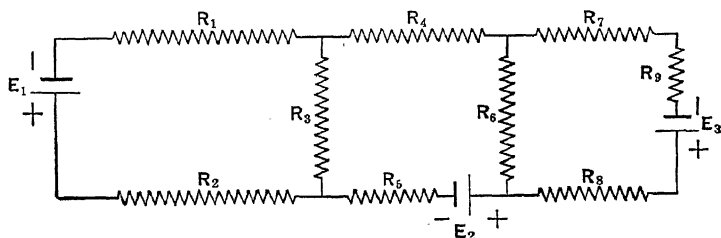


FIG. P36.

18. From the equation for the current in the galvanometer branch i_g in a Wheatstone bridge not in balance, calculate the relation between resistances R_P , R_Q , R_R , and R_X to give a maximum sensitivity to the bridge, i.e., for making changes in i_g a maximum for a given change in R_R . What are the effects of increasing E and decreasing R_g ? What practical limitations could one expect to be placed on the latter modes of increasing the sensitivity of the system?

$$\text{Ans. Maximum sensitivity if } R_R = R_X \text{ and } R_P = R_Q.$$

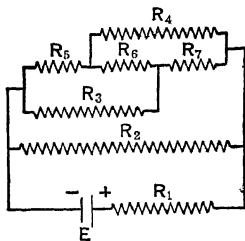


FIG. P37.

19. Given the circuit of Fig. P37, solve for the currents in the various branches.

$$E = 1 \text{ volt.}$$

$$R_4 = 1 \text{ ohm.}$$

$$R_1 = 1 \text{ ohm.}$$

$$R_5 = 1 \text{ ohm.}$$

$$R_2 = 1 \text{ ohm.}$$

$$R_6 = 1 \text{ ohm.}$$

$$R_3 = 1 \text{ ohm.}$$

$$R_7 = 1 \text{ ohm.}$$

$$\text{Ans. } i_1 = \frac{2}{3} \text{ amp; } i_2 = \frac{1}{3} \text{ amp; } i_3 = \frac{1}{3} \text{ amp; } i_4 = \frac{1}{3} \text{ amp; } i_5 = \frac{1}{3} \text{ amp; } i_6 = 0 \text{ amp; } i_7 = \frac{1}{3} \text{ amp.}$$

20. Solve problem 19 for the following values of the resistances, using the method of Maxwell or of the equivalent Y.

$$E = 1 \text{ volt.}$$

$$R_4 = 2 \text{ ohms.}$$

$$R_1 = 1 \text{ ohm.}$$

$$R_5 = 2 \text{ ohms.}$$

$$R_2 = 2 \text{ ohms.}$$

$$R_6 = 3 \text{ ohms.}$$

$$R_3 = 1 \text{ ohm.}$$

$$R_7 = 2 \text{ ohms.}$$

$$\text{Ans. } i_1 = 0.522 \text{ amp; } i_2 = 0.239 \text{ amp; } i_3 = 0.189 \text{ amp; } i_4 = 0.142 \text{ amp; } i_5 = 0.124 \text{ amp; } i_6 = 0.0177 \text{ amp; } i_7 = 0.170 \text{ amp.}$$

* Problem 17 was taken from *Lessons and Problems in Electricity*, by Newell C. Page, by the courtesy of the author and The Macmillan Company.

PROBLEMS BASED ON CHAPTERS IX, X, AND XI

1. A current of 10^{-6} ampere gives a deflection of 15 cm on a galvanometer in which the scale is 50 cm from the mirror. What are the figure of merit and the sensitivity in megohms? *Ans.* $k = 3.3 \times 10^{-9}$ amp/mm; $s = 303$ megohms.

2. Given a d'Arsonval galvanometer made according to the following specifications: It has a coil of 1000 turns, length 10 cm, width 0.6 cm. The field H is 600 oersteds. It has a phosphor-bronze suspension of modulus of rigidity $\eta = 6 \times 10^{11}$ dynes per cm^2 , whose radius is 3×10^{-3} cm and whose length is 2π cm. The moment of inertia of the coil is 200 g-cm². Calculate: (a) the galvanometer constant; (b) the figure of merit; (c) its sensitivity in megohms; (d) its period of oscillation. Can you see why a much greater sensitivity might become impractical in a galvanometer like this? Could this galvanometer be used to measure the current from a thermocouple giving an e.m.f. of 1.5×10^{-5} volt for 1° temperature difference if its resistance is 100 ohms?

Ans. (a) $K' = 3.38 \times 10^{-8}$; $K = 3.38 \times 10^{-5}$; (b) $k = 1.69 \times 10^{-8}$; (c) $S = 59.3$ megohms; (d) $T = 25.5$ sec.

3. Given a d'Arsonval galvanometer made according to the following specifications: A coil of 2000 turns, length 5 cm, width 0.8 cm. The field is 1200 oersteds. It has a phosphor-bronze suspension of modulus of rigidity $\eta = 6 \times 10^{11}$ dynes per square centimeter, the radius of which is 1×10^{-3} cm and the length of which is 2π cm. The moment of inertia of the coil is 100 g-cm². Calculate: (a) the galvanometer constant; (b) the figure of merit; (c) its sensitivity in megohms; (d) its period of oscillation. Can you see why a much greater sensitivity might become impractical in a galvanometer like this one? Could this galvanometer be used to measure the current from a thermocouple giving an e.m.f. of 1.5×10^{-5} volt for 1° temperature difference if its resistance is 200 ohms?

Ans. $K' = 1.56 \times 10^{-8}$ abs unit per radian; $K = 1.56 \times 10^{-7}$ amp per radian; (b) $k = 7.8 \times 10^{-11}$ amp per millimeter; (c) $S = 12,810$ megohms; (d) $T = 162$ sec.

4. Given a d'Arsonval galvanometer made according to the following specifications: A coil of 500 turns, length 10 cm, width 0.8 cm. The field H is 400 oersteds. It has a phosphor-bronze suspension of modulus of rigidity $\eta = 6 \times 10^{11}$ dynes per square centimeter, the radius of which is 3×10^{-3} cm and the length of which is 2π cm. The moment of inertia of the coil is 100 g-cm². Calculate: (a) the galvanometer constant; (b) the figure of merit; (c) its sensitivity in megohms; (d) its period of oscillation. Can you see why a much greater sensitivity might become impractical in a galvanometer like this one? Could this galvanometer be used to measure the current from a thermocouple giving an e.m.f. of 1.5×10^{-5} volt for 1° temperature difference if its resistance is 60 ohms?

Ans. (a) $K' = 7.6 \times 10^{-8}$; $K = 7.6 \times 10^{-5}$; (b) $k = 3.8 \times 10^{-8}$; (c) $S = 26.3$ megohms; (d) $T = 18$ seconds; 6.6 mm deflection per degree.

5. When 5×10^{-3} volt is applied to a galvanometer, the deflection is 10 cm on a scale 50 cm distant. When 150 ohms are placed in series with the galvanometer, the deflection is 7 cm. Find the resistance and the sensitivity of the galvanometer.

Ans. 350 ohms, 7.1×10^{-8} amp.

6. An ammeter has a range of 0 to 1 ampere. It is needed to measure currents from 15 to 25 amperes. Its resistance as measured was 1 ohm. (a) How would it be connected in a circuit to read these currents? (b) What resistance if any would be used? (c) How would the true current through the circuit be deduced from them?

Ans. (b) 0.0417 ohm; (c) multiply reading by 25.

7. An ammeter has a range of 0 to 1 milliampere. It is needed to measure currents from 0 to 200 milliamperes. Its resistance as measured was 0.001 ohm. (a) How would it be connected in a circuit to read these currents? (b) What resistances if any would be used? (c) How would the true current through the circuit be deduced from them?

Ans. (b) $R_s = 5.02 \times 10^{-6}$ ohm; (c) $i = 200 i_A$.

8. An ammeter has a range of 0 to 5 amperes. It is needed to measure currents from 0 to 20 amperes. Its resistance as measured was 0.05 ohm. (a) How would it be connected in a circuit to read these currents? (b) What resistances if any would be used? (c) How would the true current through the circuit be deduced from them? (d) Given the specific resistance of pure copper = 2×10^{-8} ohm, calculate a copper wire resistor to use.

Ans. (b) $R_s = 0.0167$ ohm; (c) it is four times the current read; (d) one with $\frac{\text{length}}{\text{area}} = 8350$.

9. It is required to measure potentials from 500 small B batteries, giving from 500 to 1000 volts. The voltmeter available has a range of 0 to 100 volts, and its resistance is 30,000 ohms. Calculate the series resistance required to make this voltmeter read the potential used. What current will this draw from the source? If the internal resistance of this source was 50 ohms, how much error would be made in calling the potential the e.m.f.?

Ans. $R_s = 270,000$ ohms; $i = 0.0033$ amp; error = 0.017% at 1000 volts.

10. High potentials are conveniently read by using a high-resistance tower of accurately calibrated resistors in series with a microammeter or a galvanometer. These resistors must not be overheated. The resistance units in a set of resistors consist of 200,000-ohm units of 10 watts energy dissipation each. They should not be operated steadily above 5 watts. Calculate how many of these will be needed on a 20,000-volt line, and give the range of the meter to be used in series with them to read the current to 1 per cent, giving a full-scale deflection for the maximum safe current. *Ans.* 20 resistors, current 5×10^{-3} amp; must give a minimum unit scale reading for 5×10^{-6} amp.

11. In measurements in the Physical Laboratory of the University of California Dr. F. G. Sanders studied the increase in an electron current due to the action of high fields in causing the electrons to speed up, ionizing gas molecules by impact, and creating new electrons as they move through the gas. Such currents increase exponentially with the distance d of the gas traversed for a given constant value of the field strength X . Thus keeping X constant and varying the distance d over which the electrons ionize cumulatively, the current increases from that of i_0 for 0 plate distance to a value i given by $i = i_0 e^{\alpha d}$, where $\alpha = 0.5$ per centimeter and d is varied from 1 cm to 15 cm. The current i_0 is 1.649×10^{-10} ampere, and a galvanometer of resistance 100 ohms and figure of merit $k = 10^{-11}$ ampere per millimeter displacement at 1 meter is used. Calculate the values of the shunt to be used if he can measure a maximum deflection on his scale of only 16.5 cm (at 1 meter) without error due to the cosine of the angle of deflection and if the minimum allowable deflection is 1 cm, in order that he can measure the currents from $d = 1$ cm to $d = 15$ cm without changing his galvanometer. (Hint: Find the values of i from $d = 1$ to $d = 15$, and then find how many times deflections from 1 cm to 16.5 cm must be used to cover this range of currents; from this the shunts can be calculated.)

Ans. There are an indefinite number of values correct between the limits.

12. Ten liters of pure H_2 gas are required at $27^\circ C$ and 760 mm pressure for an experiment. The H_2 is to be generated electrically and must be generated in a period of less than 5 hours. What current will be required if 96,500 coulombs deposit 1 gram of H_2 gas?

13. Five Daniell cells are connected in series with a water voltameter. They give a current of 0.1 ampere for 1 hour. How much zinc dissolves? How much copper is deposited? How much H_2 and O_2 are liberated?

Ans. 0.609 gram of Zn, 0.593 gram of Cu, 0.0298 gram of O_2 , and 0.00375 gram of H_2 .

14. J. J. Thomson measured the deflection of a stream of electrons in a magnetic and electric field and from the measurements found the ratio of the charge e (the quantity of electricity) on an electron to the mass m to be given by $e/m = 1.81 \times 10^7$ absolute electromagnetic units of quantity per gram mass of electrons. Now the

mass of an electron is $1/1860$ that of the H atom or 8.8×10^{-28} gram. Calculate from these data the quantity of electricity in absolute electromagnetic units carried by the electron. What is the value in coulombs? If each chlorine ion carried as its negative charge 1 electron, and there were 6.06×10^{23} atoms of chlorine in a gram atom, how much electricity would a gram atom of chlorine ions carry? How does this compare with the quantity actually carried by a gram atom of chlorine ions and what does it lead one to conclude about the nature of the charge carried by the chlorine ion?

Ans. $e = 1.593 \times 10^{-20}$ e.m.u.; $e = 1.593 \times 10^{-19}$ coulomb; charge on Cl^- = 96,600 coulombs = 1 faraday; $\therefore \text{Cl}^-$ carries 1 electron.

15. Given that a gram atom of oxygen has in it 6.06×10^{23} atoms. It takes 193,000 coulombs to liberate this gram atom of divalent oxygen atoms. (a) What charge does the divalent oxygen atom carry in coulombs, in e.m.u.? (b) The mass of the hydrogen atom is 1.662×10^{-24} gram, and that of an electron is $1/1860$ that of the H atom. Calculate the ratio of charge to mass, e/m , in electromagnetic units per gram carried by the electrons assuming that their charge is one-half that on the oxygen ion. Compare this with the value of $e/m = 1.81 \times 10^7$ absolute e.m.u. per gram observed by J. J. Thomson for cathode rays, and draw your conclusions.

Ans. $\text{O}^{--} = 3.186 \times 10^{-19}$ coulomb = 3.186×10^{-20} e.m.u.; $e/m = 1.78 \times 10^7$ e.m.u. per gram.

16. A soda-glass light globe was weighed and had a mass of 235.4432 grams. It was then lighted and its tip was immersed in molten NaNO_3 , current being passed through it for 5 minutes. It was then washed and carefully weighed again, its weight being 235.4662 grams. Given the atomic weight of Na as 23.0, valence as 1, and the Faraday constant as 96,500 coulombs per gram-atom equivalent, what current passed through the bulb? If the charge on an electron is 1.59×10^{-20} absolute e.m.u., how many electrons flowed from the filament per second?

Ans. $i = 0.3215$ amp; 2.021×10^{18} electrons per second.

17. An iron beam, part of the foundation of a large building, takes part of the return current from a trolley line. The current is spread over an area of the beam amounting to 20 cm by 20 cm. The current is 0.20 ampere, the ground being negative to the beam. If the beam is 0.5 cm thick, how long will it take to corrode the beam away, provided the corrosion is uniform over this area and assuming that in the moist ground away from air the iron goes into solution as Fe^{++} , or ferrous iron?

Ans. 314 days.

18. A water pipe takes part of the return current from a trolley line. The current is spread over an area of the pipe amounting to 200 cm by 20 cm. The current is 0.05 ampere, the ground being negative to the pipe. If the pipe is 0.5 cm thick, how long will it take to corrode the pipe away, provided the corrosion is uniform over this area and assuming that in the moist ground away from air the iron goes into solution as Fe^{++} , or ferrous iron?

Ans. 34.5 years.

19. A water pipe takes part of the return current from a trolley line. The current is spread over an area of the pipe amounting to 100 cm by 20 cm. The current is 0.02 ampere, the ground being negative to the pipe. If the pipe is 0.4 cm thick, how long will it take to corrode the pipe away, provided the corrosion is uniform over this area and assuming that in the moist ground away from air the iron goes into solution as Fe^{++} , or ferrous iron?

Ans. 31.1 years.

20. A piece of apparatus must be plated with pure gold to resist the action of acids. The deposit must be 0.1 mm thick and the chamber is a cylinder of 5.62 cm radius and 20 cm high. The inside of the cylinder and the base must be plated. The gold is trivalent, and the current density must not exceed 0.005 ampere per square centimeter of gold surface. How many hours will it take to plate the chamber?

Ans. 15.5 hr.

21. Given a dilute solution of NaCl with the Na^+ and Cl^- completely dissociated. Na^+ and Cl^- ions have nearly the same mobility of 4.5×10^{-4} cm per second per volt per centimeter at 0°C each. The solution is 10^{-2} molar (i.e., there are 6×10^{23}

Na^+ ions dissolved in 10^2 liters or 10^5 cm^3 of water). Each ion carries 1.6×10^{-19} coulomb. The electrodes have 10 cm^2 of area and are 2 cm apart. Ten volts are placed across the cell. What is the current? What is the resistance and the specific resistance? What is the conductivity in mhos? If the coefficient of viscosity of water at this temperature is $0.01 \text{ c.g.s. units}$, calculate the effective radius a of the Na^+ ions. Compare this with the value of the radius of the L orbit of the Na^+ , which is $1.3 \times 10^{-8} \text{ cm}$.

Note a caution in solving this problem. Whereas the current around the circuit is completed when one Na^+ and one Cl^- arrive at their respective electrodes both ions move independently in the field. To solve for a both mobility and charge must be converted into e.m.u.

Ans. $i = 4.5 \times 10^{-2} \text{ amp}$; $R = 222.5 \text{ ohms}$, $R_s = 1112.5 \text{ ohms} \times \text{cm}$;
 $\sigma = 4.5 \times 10^{-3} \text{ mho}$; $a = 1.89 \times 10^{-8} \text{ cm}$.

22. Benzene, C_6H_6 , becomes conducting when exposed to x-rays owing to the liberation of electrons by the x-rays. Under the conditions used only 10^6 electrons per cubic centimeter were produced. With electrodes of 100 cm^2 area 1 cm apart and 100 volts across the plates, a current of 1.6×10^{-12} ampere was observed. What is the mobility of the electron in benzene?

Ans. $1 \times 10^{-3} \text{ cm per sec per volt per cm}$.

23. A 1-cm cube of rock salt at 400° absolute has 6×10^{11} Na^+ ions per cubic centimeter. In a unit electrical field ($X = 1$ volt per centimeter) these ions move with a velocity of $10^{-6} \text{ cm per second}$. The charge on an Na^+ ion is 1.6×10^{-19} coulomb. What is the specific resistance of NaCl at this temperature? At 727°C or 1000° absolute the value of the mobility of this ion is $10^{-4} \text{ cm per second}$. In addition the number of Na^+ ions free increases according to the law $N = N_0 e^{-\frac{7660}{T}}$, where T is the absolute temperature. What are the resistance and conductivity of NaCl at 1000° absolute? If 1000 volts were effectively placed across the crystal at this temperature for 5 hours, how much Na would have been deposited? (Note such quantitative tests have been made in Joffé's laboratory on rock salt and Faraday's law has been shown to hold. To work this the cube is placed between two NaCl -coated electrodes to prevent loss.)

Ans. $R_{S400^\circ} = 1.04 \times 10^{13} \text{ ohms}$; $R_{S1000^\circ} = 1.25 \times 10^6 \text{ ohms}$;
 $\sigma_{S1000^\circ} = 8.08 \times 10^{-7}$; $3.43 \times 10^{-3} \text{ gram}$.

24. The Daniell cell, composed of Zn which replaces the Cu of CuSO_4 in solution giving ZnSO_4 , has an e.m.f. of 1.05 volts. The heat of formation of ZnSO_4 is $250,000$ calories per gram-molecule. Given the atomic weight of Cu as 63.7 , its valence as 2 , and J as 4.18×10^7 ergs, calculate the difference in heat of formation of ZnSO_4 and CuSO_4 first per gram and then per gram-molecule and hence the heat of formation of CuSO_4 if entropy can be neglected.

Ans. $201,500 \text{ cal per mole of CuSO}_4$; $1260 \text{ cal per gram CuSO}_4$.

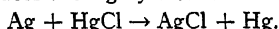
25. In a Daniell cell ZnSO_4 is formed and CuSO_4 is decomposed, depositing Cu on the anode. The heat of formation of ZnSO_4 is $248,000$ calories per gram-molecule. That of CuSO_4 is $197,500$ calories per gram-molecule. From these data calculate the heat set free per gram-molecule of the reaction of the cell, and then per gram Zn consumed. If Zn has an atomic weight 65.4 and a valence of 2 , calculate the e.m.f. of the Daniell cell in volts, given $J = 4.18 \times 10^7$ ergs.

Ans. $50,500 \text{ cal per mole}$; heat per gram $\text{Zn} = 772 \text{ cal}$; e.m.f. = 1.093 volts.

26. In a cell in which Pb reacts with HgCl to give $1 \text{ PbCl}_2 + 2 \text{ Hg}$, the heat of reaction is $22,730$ calories per mole. At about 298° absolute $\frac{dE}{dT} = -0.000145$ volt per degree. Calculate the e.m.f. of the cell.

Ans. 0.5357 volt.

27. In an Ag-Hg cell using a saturated solution of HgCl as an electrolyte the reaction is one in which Ag replaces the Hg by means of a reaction



Both ions are monovalent. The heat of reaction is 1280 calories per mole which is

set free and is *apparently* available for giving current at 298° absolute or 15° C. Calculate the theoretical e.m.f. of this cell, assuming no loss to entropy. The e.m.f. observed is 0.0455 volts. The difference between observation and calculation represents a loss to entropy. From this data determine the entropy loss and hence $\frac{dE_v}{dT}$.

$$\text{Ans. } E_v = 0.0555 \text{ volt; } \frac{dE_v}{dT} = 3.37 \times 10^{-5} \text{ volts per degree } K.$$

28. A small lead storage cell is charged with a current of 1 ampere. The cell is such that 25 grams of water if taken from it will lower the level of the acid in the cell 1 cm. The cells were left to charge overnight at the above-mentioned current. At 5 A.M. the cells were fully charged and the current was used in decomposing the water. How far had the level fallen by 10 A.M., when the current was shut off?

$$\text{Ans. } 0.067 \text{ cm.}$$

29. Given 32 cells of e.m.f. of 1.5 volts each and internal resistance of 0.1 ohm. Find the current through the system when the external resistance is 20 ohms, when it is 0.01 ohm, and find the most efficient arrangement and current for an external resistance of 0.2 ohm.

$$\text{Ans. } i_{P20} = 0.075; i_{S20} = 2.065; i_{P0.01} = 115; i_{S0.01} = 14.93; \text{ maximum efficiency: four cells in parallel; } i_{\max} = 30 \text{ amp.}$$

30. In the old make-and-break ignition on marine engines the maximum instantaneous current drawn out in an arc was needed. Dry cells of an e.m.f. of 1.5 volts each were used. The external resistance of the circuit was 0.2 ohm; the *internal* resistance of the cells was 0.1 ohm each. A dozen and a half cells were used. Calculate the most efficient arrangement of cells and the maximum current drawn.

$$\text{Ans. Three cells in parallel; } 22.5 \text{ amps.}$$

31. When in 1827 Ohm studied what is now called resistance, he had only the very imperfect cells of his time. The currents they gave varied with time (i.e., decreased as time went on after closing the circuit), so that he could get comparable results only when he took instantaneous deflections of his "multiplier." He next resorted to thermocouples and found that the magnetic effect (current i) was for

one couple given by $i = \frac{a}{b+x}$, where x was the length of the wire the resistance

of which he was studying. When m couples were used in series, he found $i = \frac{ma}{mb+x}$.

What action of his cells caused him to give them up? Why did his thermocouples not give him $i = \frac{V}{R}$ directly?

32. A group of seventy-two cells of e.m.f. 1.05 volts and internal resistance 1.2 ohms is to be used in a circuit with variable external resistance R_e . (a) If $R_e = 0$, what arrangement will be used to give the greatest current and what will the current be? (b) If $R_e = 5.3$ ohms, what arrangement could best be used and what is the maximum current? $R_e = 2.4$ ohms is the minimum value a circuit can have. (c) It is desired to get a current of 3 amperes in this circuit with the seventy-two cells. Can this be done?

$$\text{Ans. (a) } i = 63 \text{ amp (cells in parallel); (b) } i = 1.77 \text{ (four cells in parallel); (c) It can not.}$$

33. A group of n cells all alike are connected to a wire the resistance R_e of which is equal to the internal resistance R_i of one cell. Show that i is the same no matter how the cells are arranged.

PROBLEMS BASED ON CHAPTER XII

1. In the thermoscope shown in Fig. P38 the current through the system is 15 amperes. The resistance of the Cu-Bi element in the small bulb of 100 cm³ capacity is 1.0×10^{-3} ohm in each cell. The Peltier coefficient $\pi = 3.0 \times 10^{-2}$ for each

element, and the current flowed for 50 seconds. During this period a quantity of heat was developed in each of the two junctions. This heat must be calculated for each junction. Since the heat capacity of the 40 grams of metal rod (Bi + Cu) heated, together with the small heat capacity of the gas in each bulb, was 2 calories for a degree rise in temperature, calculate the temperature rise observed in each bulb. Given the density of the liquid in each column equal to 1 and the outside

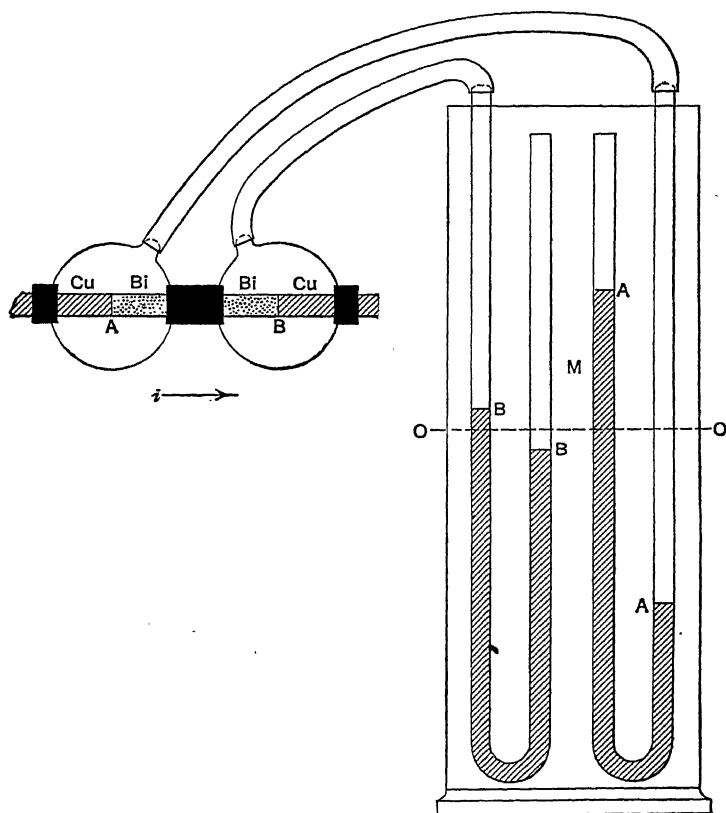


FIG. P38.

pressure as 76 cm of mercury, assuming the air in the bulb at the temperature of the metal, and the room temperature as 20°C . The density of mercury is taken as 13.6. Then calculate the differences of level in the arms of the manometers.

Note: At one junction there is a Peltier heating, whereas at the other there is a Peltier cooling as well as a Joule heating at both junctions. The effects add at one junction and subtract at the other.

Ans. 4.05°C ; -1.35°C ; 14.2 cm; -4.74 cm .

2. In the thermoscope shown in Fig. P38 the current through the system is 20 amperes. The resistance of the Cu-Bi element in the small bulb of 80 cm^3 capacity is $8.0 \times 10^{-4}\text{ ohm}$ in each cell. The Peltier coefficient $\pi = 2 \times 10^{-2}$ for each element, and the current flowed for 120 seconds. During this period a quantity of heat was developed in each of the two junctions. This heat must be calculated for each junction. Since the heat capacity of the 45 grams of metal rod (Bi + Cu) heated,

together with the small heat capacity of the gas in each bulb, was 2.65 calories for a degree rise in temperature, calculate the temperature rise observed in each bulb. Then calculate the differences of level in the arms of the manometer of each bulb. Given the density of the liquid in each column equal to 1, and the outside pressure as 76 cm of mercury, assuming the air in the bulb at the temperature of the metal, and the room temperature as 20°C . The density of mercury is taken as 13.6.

Note: At one junction there is a Peltier heating while at the other there is a Peltier cooling as well as a Joule heating at both junctions. The effects add at one junction and subtract at the other.

Ans. +20.7 cal; -2.3 cal; 27.6 cm; -3.06 cm.

3. In the thermoscope shown in Fig. P38 the current through the system is 17 amperes. The resistance of the Cu-Bi element in the small bulb of 60 cm^3 capacity is 5.0×10^{-4} ohm in each cell. The Peltier coefficient $\pi = 10^{-2}$ for each element, and the current flowed for 150 seconds. During this period a quantity of heat was developed in each of the two junctions. This heat must be calculated for each junction. Since the heat capacity of the 45 grams of metal rod (Bi + Cu) heated, together with the small heat capacity of the gas in each bulb, was 2.65 calories for a degree rise in temperature, calculate the temperature rise observed in each bulb. Then calculate the differences of level in the arms of the manometer of each bulb. Given the density of the liquid in each column equal to 1, and the outside pressure as 76 cm of mercury, assuming the air in the bulb at the temperature of the metal, and the room temperature as 20°C . The density of mercury is taken as 13.6.

Note: At one junction there is a Peltier heating, while at the other there is a Peltier cooling as well as a Joule heating at both junctions. The effects add at one junction and subtract at the other.

Ans. +4.28 $^{\circ}\text{C}$; -0.34 $^{\circ}\text{C}$; 15.15 cm; -1.2 cm.

4. Given the thermoelectric power for iron as $(17 - 0.05t) \times 10^{-6}$ volt, and for a Cu alloy as $(-7.00 + 0.020t) \times 10^{-6}$ volt, calculate the thermoelectromotive force of the couple when one junction is at 0°C and the other junction is at $50^{\circ}, 100^{\circ}, 150^{\circ}, 200^{\circ}, 250^{\circ}, 300^{\circ}, 350^{\circ}, 400^{\circ}$, and 500°C . What is the temperature of the neutral point? The calculation is to be done graphically and accurate to not more than three significant figures, accompanied by plot.

5. Given the thermoelectric power for iron as $(17 - 0.05t) \times 10^{-6}$ volt, and for aluminum as $(-0.80 + 0.004t) \times 10^{-6}$ volt, calculate the thermoelectromotive force of an Al-Fe couple when one junction is at 0°C and the other junction is at $50^{\circ}, 100^{\circ}, 150^{\circ}, 200^{\circ}, 250^{\circ}, 300^{\circ}, 350^{\circ}, 400^{\circ}$, and 500°C . What is the temperature of the neutral point? The calculation may be done graphically or using the calculus and accurate to *not more than three* significant figures, accompanied by plot.

Ans. Neutral point 330°C .

6. An Au-Fe thermocouple has the following values of the powers for Au and Fe $(+2.8 + 0.01t) \times 10^{-6}$ and $(+17.5 - 0.048t) \times 10^{-6}$. Calculate and plot the curve for the e.m.f. of this couple with one junction at 0° and the other at t° , between 0° and 700°C . What is the temperature of the neutral point?

Ans. Neutral point 253.5°C .

7. In the thermocouple of Au and Fe referred to in problem 6, calculate the value of the Peltier coefficient at 0°C and at 500°C .

Ans. $\pi_0 = 4.02 \times 10^{-3}$; $\pi_{500} = -11.07 \times 10^{-3}$.

8. In a demonstration experiment a thermoelectric current in a copper rod produced a magnetic field sufficiently strong to lift a 5-kg weight. From the data given the magnitude of this current can be calculated. The area of the Cu and Ni rod was 1 cm^2 . Its length was 40 cm. The average specific resistance of the copper and Ni in the circuit was 3×10^{-6} ohm \times cm. The temperature of the hot junction was 650°C ; that of the cold junction, 50°C . The thermoelectromotive power of Ni is $(-24 - 0.05t) \times 10^{-6}$ volt, and that of Cu is $(+1 + 0.01t) \times 10^{-6}$ volt.

Ans. 215 amp.

9. In a demonstration experiment a thermoelectric current in a copper rod pro-

duced a magnetic field sufficiently strong to lift a 4-kg weight by its magnetic field. From the data given the magnitude of this current can be calculated. The area of the Cu and Ni rod was 0.8 cm^2 . Its length was 50 cm. The average specific resistance of the copper and Ni in the circuit was $3.5 \times 10^{-6} \text{ ohm} \times \text{cm}$. The temperature of the hot junction was 500°C ; that of the cold junction, 0°C . The thermoelectric power of Ni is $(-24 - 0.05 t) \times 10^{-6} \text{ volt}$, and that of Cu is $(+1 + 0.01 t) \times 10^{-6} \text{ volt}$.
Ans. $2.00 \times 10^{-2} \text{ volt}$, 91.5 amp.

10. Copper-constantan (an alloy of Cu 60 per cent and Ni 40 per cent used for thermocouple work) thermocouples are to be used for measuring the freezing point of solutions of salts running to about -4°C . They are to be accurate to 0.01°C . A galvanometer giving a figure of merit of $k = 5 \times 10^{-9}$ is to be used with a scale of 50 cm at 1 meter. The galvanometer resistance is 500 ohms. How many couples in series (give nearest whole number only) are needed to give this accuracy if the thermoelectric power of constantan is $(-38.105 - 0.0888t) \times 10^{-6}$ and of copper is $(+2.760 + 0.0122 t) \times 10^{-6}$.

Ans. Six or seven couples to give 1-mm deflection.

11. In measuring the heat from the stars the astronomers use vacuum thermocouples made of Bi and Cu placed in series with the most sensitive galvanometers. Using a couple of twenty elements in series and a galvanometer of $k = 10^{-12}$ ampere per millimeter at 1 meter, assuming 1 mm is the best deflection that can be had, calculate the minimum rise in temperature that can be noted at 0°C if P for Bi = -97×10^{-6} and P for Cu, $+1.52 \times 10^{-6}$, resistance 200 ohms.

Ans. $\Delta T = 1.015 \times 10^{-6}^\circ \text{C}$ for 1-mm deflection.

PROBLEMS BASED ON CHAPTERS XIII AND XIV

1. A charge of $+6 \text{ e.s.u.}$ is placed at a point A . A body charged with $+2 \text{ e.s.u.}$ is brought to within 6 cm of A . (a) What is the force acting in magnitude and direction? Had the charge at A been -6 e.s.u. , what would have been the force? (b) What would have been the force with the charge at A as $+6 \text{ e.s.u.}$ when the charge brought to 6 cm was $+1 \text{ e.s.u.}$ The magnitude and direction of this force, i.e., the force on unit charge, is called the electrical field strength at that point and is designated by X . If X is known the force for any charge q can at once be obtained from qX . (c) What is the force in the case above when $q = 1, 2, 5, 100 \text{ e.s.u.}$?

Ans. (a) 0.333; (b) 0.161; (c) 0.161, 0.333, 0.834, 16.1 dynes.

2. What is the force in magnitude and direction on a unit $+$ charge (i.e., field strength) at a point C 3 cm from a point A where the charge is $+9 \text{ e.s.u.}$ and 4 cm from a point B where the charge is -16 e.s.u. when A and B are 5 cm apart?

Ans. 1.414 dynes, 45° to CA and CB .

3. What would be the force in kilograms between two $+$ charges of 1 coulomb separated by 1 meter? Can you see why we practically reach a limit to the amount of charge we can heap on a body?

Ans. 9.2×10^8 kilograms.

4. Find the locus of points about an isolated $+$ charge of 4 units where the electrical field intensity is 0.25 dyne per unit charge. What is the field direction?

5. Two identical conducting pith balls of mass 0.51 gram each are suspended by threads 25 cm long. On being charged and brought in contact they take positions with their centers 10 cm apart. What is the charge on each, assuming $g = 980 \text{ cm per second squared}$?

Ans. 101 e.s.u.

6. Two identical conducting pith balls of mass 1.02 grams each are suspended by threads 50 cm long. On being charged and brought in contact they take positions with their centers 5 cm apart. What is the charge on each assuming $g = 980 \text{ cm per second squared}$?

Ans. 35.5 e.s.u.

7. Two identical conducting pith balls of mass 1.53 grams each are suspended by threads 30 cm long. On being charged and brought in contact they take positions with their centers 10 cm apart. What is the charge on each assuming $g = 980 \text{ cm per second squared}$?

Ans. 159 e.s.u.

8. Two identical conducting pith balls of mass 0.51 gram each are suspended from a common point by threads 40 cm long. On being charged and brought in contact they take positions with their centers 20 cm apart. What is the charge on each?

Ans. 227.6 e.s.u.

9. If the pith balls in problem 8 had been immersed in toluene of dielectric constant 2.5 what would the distance between them have been? Assume $g = 980$ cm per second squared and carry out computations to two figures only. *Suggestions:* the unknown is $r = \frac{1}{2}$ the distance between balls. Two equations between force and r can be set up from which a cubic equation in r results. Solve this equation by successive approximations or graphically.

Ans. 14 cm.

10. A, B, C are the vertices of an equilateral triangle, the radius of the circumscribing circle being 10 cm. A charge of +10 e.s.u. is placed at A , a charge of +10 e.s.u. at B , and a charge of -5 e.s.u. at C . Find the force in magnitude and direction on a charge of +2 e.s.u. placed at the center O of the circumscribing circle.

Ans. 0.3 dyne from O to C .

11. A, B , and C are the vertices of an equilateral triangle, the radius of the circumscribing circle being 10 cm. A charge of +10 e.s.u. is placed at A , a charge of -10 e.s.u. at B , and a charge of +5 e.s.u. at C . Find the force in magnitude and direction on a charge of +3 e.s.u. placed at the center of the circumscribing circle.

Ans. $f = 0.541$ dyne; Force makes angle of $43^\circ 55'$ with side BC , directed toward B .

12. A small ring has a radius of 3 cm. It is charged uniformly over its surface with 50 e.s.u. Find the resultant field at a point A , 4 cm from the plane of the ring on its axis.

Ans. 1.6 e.s.u.

13. Find the field at a point on the axis of a ring of radius a charged with $+q$ units, which is d centimeters from the plane of the ring. At what point will this field be a maximum?

Ans. $X = \frac{qd}{(a^2 + d^2)^{3/2}}$, $d_m = \frac{a}{\sqrt{2}}$.

14. An electron has a charge $e = 4.8 \times 10^{-10}$ e.s.u. It is fired with a velocity of 3×10^9 cm per second (equivalent to an electron that fell through about 250 volts potential difference) horizontally and at right angles to a uniform electrical field of 0.1 e.s.u. (30 volts per centimeter) for a distance of 10 cm. How much will it be deflected toward the lower + plate if its mass is 9×10^{-28} gram? *Ans.* 0.296 cm.

15. Compute the force between an electron in a hydrogen atom and the nucleus if they are point charges of -4.8×10^{-10} e.s.u. and $+4.8 \times 10^{-10}$ e.s.u. and separated by 1×10^{-8} cm. In the K ring of electrons of uranium having a charge of $+92 \times 4.8 \times 10^{-10}$ e.s.u. the K electron having a charge of -4.8×10^{-10} e.s.u. is as close as 5×10^{-12} cm from the nucleus. Calculate the force in this case. Then realizing that the force acts on a body which can be considered as having a radius 3.5×10^{-13} cm calculate the pressure which the electron would exert in these two fields if it could press on some body.

Ans. $F_H = 2.30 \times 10^{-3}$ dyne; Press = 5.97×10^{21} dyne per square centimeter;

$F_U = 8.46 \times 10^3$ dynes; Press = 2.30×10^{28} dyne per square centimeter.

16. An electron in a circular orbit in an H atom is restricted by quantum theory to such orbits that the de Broglie wave length of an electron λ goes into the orbit length a whole number of times, i.e., such that $n\lambda = 2\pi r$, where n is an integer 1, 2, 3, etc., and r is the orbit radius. λ is given by the de Broglie relation that $\lambda = \frac{h}{mv}$, where v is the electron velocity, m is the electron mass, and h is the Planck constant 6.55×10^{-27} erg \times second. The electron is attracted to the nucleus with Coulomb's law $f = \frac{e^2}{r^2}$ where e is the charge on the electron, and r the orbit radius. Centrifugal

force due to the orbital velocity is $\frac{mv^2}{r}$ and just compensates the coulomb force in a stable orbit. From the first two equations the quantum relation between v and r

for a stable orbit can be found. From the second two equations the dynamical condition for any orbit is fixed. By combining quantum stability and dynamical stability equations the radii r and velocities v for stable orbits can be solved for. Given $e = 4.8 \times 10^{-10}$ e.s.u., $m = 9 \times 10^{-28}$ gram, calculate r and v for the case where $n = 1$ de Broglie wave length in the simple H atom. This fixes the "size" of the normal H atom. How much larger will r be if $n = 2$, and $n = 3$?

Ans. $v = 2.215 \times 10^8$ cm per second; $r = 0.522 \times 10^{-8}$ cm; $r_2 = 4r_1$; $r_3 = 9r_1$.

17. When a swift α particle, a doubly positively charged helium atom of mass 6.64×10^{-24} gram, is shot against a massive nucleus of an atom of gold, which can be considered as fixed, it approaches to within 2×10^{-12} cm before its velocity is annihilated and it reverses its direction and separates. The charge on the gold nucleus is $+79 \times 4.77 \times 10^{-10}$ e.s.u. and that on the α particle is $+2 \times 4.77 \times 10^{-10}$ e.s.u. Since at the point where the α particle starts to retrace its path the potential energy equals the kinetic energy which it initially had, with the above data it is possible to calculate the potential energy of the α particle at the point of closest approach and hence know the kinetic energy of the particle. Determine the kinetic energy of the α particle and calculate its velocity in centimeters per second.

Ans. 1.8×10^{-6} erg, 2.33×10^9 cm per second.

18. The distance between the centers of ions of Na^+ and Cl^- in rock salt at room temperature is 2.81×10^{-8} cm. The charge on the Na^+ and Cl^- ions is equal and 4.77×10^{-10} e.s.u. What is the force between the ions in dynes in the crystal and when separated by 10^{-6} cm as they would be in a molar solution? If the solvent is water of dielectric constant 81, assuming the water surrounded an Na^+ ion at a corner of the crystal so that other Cl^- ions, except the nearest one, can be neglected and assuming the water active between them, what are the potential energies of the ions at these distances in water and in the absence of water? Compare these energies with the energy of heat motion of a single water molecule at 20°C which is 6.02×10^{-14} erg and see whether you can explain electrolytic dissociation. For the molecules have an energy of motion spread all about the value above, and some 20 per cent of the impacts exceed an energy of 1.204×10^{-13} erg.

Ans. Force -2.88×10^{-4} in crystal; 2.27×10^{-7} in air; 3.56×10^{-6} in water.

Potential energy -8.1×10^{-12} erg; 2.27×10^{-13} erg; 1.00×10^{-13} erg.

19. Two large circular plates of 1-meter radius are separated by 1-cm distance. They are charged to 27,000 volts, i.e., just below the sparking potential for air. Calculate the force in kilograms between the plates.

Ans. 10.33 kg.

20. Given a charge of -10 units at A . What work in ergs will be required to move a charge of $+5$ units from a distance of 1 cm to 10 cm from A ? What is the potential difference in absolute e.s.u. and in volts?

Ans. $w = 45$ ergs; P.D. $= -9$ e.s.u. or -2700 volts.

21. What is the potential at a point 5 cm from a charge of 5 e.s.u. in e.s.u. and in volts?

Ans. $P = 1$ e.s.u. $= 300$ volts.

22. Plot a curve of the variation of the field and potential with distance from a charge of $+100$ e.s.u.

23. What is the potential at the center of an equilateral triangle with charges $+10$, -5 , $+5$ at the corners A , B , C , respectively, the corners lying on a circle of radius 5 cm?

Ans. $P = +2$ e.s.u.

24. (a) What is the field strength X at a distance r centimeter from an infinitely long wire charged to $+q$ units per centimeter? From the value of X calculate the potential difference between $r = a_1$, and $r = a_2$.

(b) The potential difference between two points r_a and r_b distant r_a and r_b centimeters, respectively, from an infinitely long wire charged with σ charges per centimeter can also be directly calculated from the integral of $\frac{\sigma dl}{x}$, for a point charge σdl at a distance x . The difficulty appears to arise when the limits $+$ and $-\infty$ are inserted. To solve, set up the integral and integrate between the finite limits $+L$ and $-L$. Then evaluate the potential difference between the points distant r_a and

r_b . Rationalize the expression and take the limit as L approaches infinity. This should give the same result as that using field strength, $X = 2\sigma/r$.

$$\text{Ans. P.D.} = 2\sigma \log \frac{r_a}{r_b}.$$

25. Calculate the energy in ergs which an α particle of charge $+9.6 \times 10^{-10}$ e.s.u. acquires in falling through a potential difference of 1 volt, 1000 volts, 200,000 volts, and 6×10^6 volts (the latter potentials have recently been achieved in the cyclotron). What energy per second would a current of 1×10^{-5} ampere of the latter carry? *Ans.* 3.2×10^{-12} erg; 3.2×10^{-9} erg; 6.4×10^{-7} erg; 19.2×10^{-8} erg; 60 joules per second.

26. Given the mass of the electron as 8.99×10^{-28} gram and assuming that mass does not appreciably change with these velocities, calculate the velocities of the electrons in centimeters per second at 1, 10, 1000, 200,000, and 10^6 volts. The charge on the electron is 4.8×10^{-10} e.s.u. Note that the velocities obtained by 10^6 -volt electrons exceed 3×10^{10} cm per second. This is impossible according to relativity, and we see that the increase in m should have been included. *Ans.* 5.96×10^7 , 1.885×10^8 , 1.885×10^9 , 2.67×10^{10} , 5.96×10^{10} , cm per sec.

27. What is the potential at the center of a wire ring of radius r with $+q$ units uniformly distributed over the surface of the ring? *Ans.* $P = \frac{q}{r}.$ *

28. What is the potential at any distance d from the plane of a uniformly charged ring of radius r at a point on its axis? The charge on the ring is $+q$. From this expression deduce the value of the field strength X .

$$\text{Ans. } P = \frac{q}{\sqrt{d^2 + r^2}}, X = \frac{qd}{(r^2 + d^2)^{3/2}}.$$

29. During the period of development of high-tension devices for use in nuclear disintegrations, W. F. G. Swann worked on a device by which steel spheres of 0.7 cm radius, started at zero potential, were charged by induction in vacuum and under gravity came against a condenser plate which marked the high-voltage receiver. The density of steel is 7.6. They fell 10 cm before striking the plate. What was the highest potential that he could have obtained if each sphere had 10 e.s.u. of charge? *Ans.* 3,210,000 volts.

30. The spherical electrode of Van de Graaff's lightning generator has a radius of about 2 meters. A charge of 4×10^6 e.s.u. of quantity is placed on it; calculate:

(a) The potential in e.s.u. and volts.

(b) The charge density on the sphere.

(c) The field strength at the surface of the sphere in e.s.u. and in volts per centimeter.

(d) If it touched a sphere of 4-meter radius, what is the potential on each and the charge on each on separation?

Ans. (a) 2×10^4 e.s.u., 6×10^6 volts; (b) 7.95 e.s.u. per square centimeter; (c) 100 e.s.u. per centimeter, 30,000 volts per centimeter; (d) $q_{200} = 1.33 \times 10^6$ e.s.u., $q_{400} = 2.66 \times 10^6$ e.s.u.; $P = 6.67 \times 10^3$ e.s.u.

31. The earth is a conducting sphere of radius about 5000 km. It is charged with 1×10^{18} e.s.u. of quantity; calculate:

(a) The potential in e.s.u. and in volts.

(b) The charge density on the sphere.

(c) The field strength at the surface of the sphere in e.s.u. and volts per centimeter.

(d) If it touched the moon, an uncharged conducting sphere of 1000-km radius, what is the potential of each and the charge on each on separation?

Ans. (a) 2×10^9 e.s.u., 6×10^{11} volts; (b) 0.318 e.s.u. per square centimeter; (c) 4 e.s.u. per centimeter, 1200 volts per centimeter; (d) $P = 1.667 \times 10^9$ e.s.u.; $q_{\text{moon}} = 1.667 \times 10^{17}$ e.s.u.; $q_{\text{earth}} = 8.33 \times 10^{17}$ e.s.u.

* Problems 27 and 28 were taken from *Lessons and Problems in Electricity*, by Newell C. Page, by the courtesy of the author and The Macmillan Company.

32. An electric charge of $+400$ units is placed on a conducting sphere of radius 2 cm. The same charge is later placed on an isolated body having the same capacity, but of such a shape that one of the ends has a radius of curvature of 1 cm. The potential remains constant. Find the following quantities:

- The potential of the sphere in e.s.u., in volts, and in absolute e.m.u.
- The charge density on the sphere.
- The charge density on the curved end of the second body.
- The number of lines of force per square centimeter emerging at the surface of the sphere.
- The number of lines of force per square centimeter emerging at the surface of the pointed body.
- The field strength close to the surface of the sphere, in absolute units per centimeter and in volts per centimeter.
- The field strength close to the surface of the pointed body.
- If air can stand a field strength of $30,000$ volts per centimeter before it breaks down, will the air break down in either case?
- What would be the force on a unit positive charge 1 cm distant from the surface of the 2 -cm sphere?

Ans. (a) 200 e.s.u., $60,000$ volts, 6×10^{10} e.m.u.; (b) 7.96 e.s.u. per cm^2 ; (c) 15.9 e.s.u. per cm^2 ; (d) 100 lines per cm^2 ; (e) 200 lines per cm^2 ; (f) $30,000$ volts per cm, 3×10^{12} e.m.u. per cm; (g) $60,000$ volts per cm, 6×10^{12} e.m.u. per cm; (h) Yes, breaks down at small sphere; (i) 44.44 dynes.

33. An electric charge of $+600$ units is placed on a conducting sphere of radius 3 cm. The same charge is later placed on an isolated body having the same capacity, but of such a shape that one of the ends has a radius of curvature of 1 cm. The potential remains constant. Find the following quantities:

- The potential of the sphere in e.s.u., in volts, and in absolute e.m.u.
- The charge density on the sphere.
- The charge density on the curved end of the second body.
- The number of lines of force per square centimeter emerging at the surface of the sphere.
- The number of lines of force per square centimeter emerging at the surface of the pointed body.
- The field strength close to the surface of the sphere, in absolute units per centimeter and in volts per centimeter.
- The field strength close to the surface of the pointed body.
- If air can stand a field strength of $30,000$ volts per centimeter before it breaks down, will the air break down in either case?
- What would be the force on a unit positive charge 1 cm distant from the surface of the 3 -cm sphere?

Ans. (a) 200 e.s.u., $60,000$ volts; (b) 5.31 e.s.u. per cm^2 ; (c) 15.93 e.s.u. per cm^2 ; (d) 66.7 lines per cm^2 ; (e) 200 lines per cm^2 ; (f) 66 e.s.u. per cm, $20,000$ volts per cm; (g) $60,000$ volts per cm; (h) Yes, in the case of pointed body; (i) 37.5 dynes.

34. A capacity of 1 microfarad is charged to $10,000$ volts. It is suddenly discharged by shorting through a fine copper wire. If all the energy went to heating the wire and raising the temperature of the gases in the volume of the wire, how many calories were adiabatically liberated? Assume that in the process 10^{-1} cm^3 of gas was heated by the explosion of the wire. Its specific heat is 0.17 , and the density of air is 1.3×10^{-3} . What temperature could one expect from this process? Actually, the specific heat increases with temperature and the energy is dissipated over a larger volume so that these temperatures are not reached.

Ans. 11.6 cal.; $\Delta t = 5.24 \times 10^6$ C.

35. F. G. Dunnington and E. O. Lawrence at the University of California discovered that in a spark discharge the spark path 1 cm long and of radius 4×10^{-5} cm had about one-third of the 2.7×10^{19} gas molecules per cubic centimeter ionized.

Assuming that the electrons in the spark recombined with the positive ions by falling into orbits of radius 0.75×10^{-8} cm from infinity (an assumption justified as the force falls rapidly with distance), calculate the energy in ergs liberated in the recombination in the spark, given the electronic charge as 4.77×10^{-10} e.s.u. Given the mechanical equivalent of heat as 4.18×10^7 ergs per calorie and the heat required to raise 1 cm³ of air 1° as 2.23×10^{-4} calories, calculate the temperature which would be, given the gas if the recombination took place adiabatically. This is only one of the many factors leading to the noise (adiabatic expansion of air) and light (from recombination) of the spark.

Ans. Energy liberated = 1.37 ergs, temperature = 29,200° C. Actually, we cannot speak of temperature in a transient phenomenon such as a spark. The energy density liberated in some 10^{-6} second, however, is about correct.

36. An absolute electrometer has plates 10 cm in diameter separated by 2 mm. It is desired to measure potentials of 3000 volts as accurately as possible. If weights of 0.001 gram are all that it is convenient to use in the instrument, and taking $g = 980$ cm per second squared, how accurately can the potentials of this amount be measured?

Ans. ± 0.160 e.s.u.

37. An absolute electrometer has plates of 20-cm diameter separated by 0.5 mm. It is placed across a d-c source of potential, and 100 grams are needed to balance the applied field. If 1 gram of weight = 980 cm per second squared, calculate the P.D. in e.s.u. A standard voltmeter gave the potential across these mains as 1300 volts. Calculate the value of the ratio of the e.s.u. of potential to the e.m.u. of potential.

Ans. P.D. = 4.4 e.s.u.; 2.96×10^{10} .

38. Given a gold-leaf electroscope with a leaf of mass 0.002 gram. It has a capacity of 6 cm and a plate distance of 5 cm. Given $g = 980$, what potential difference is required to give a deflection of 45°?

Ans. 383 volts.

39. In a study of currents in gases, ions are created by radioactive radiations and by cosmic rays. The electroscope used in measurement has an electrical capacity of 10 cm. Over a period of 1 hour the ions of both signs produced by the radiations in a volume of 1000 cm³ cause a fall of 1.2 volts in the potential of the gold-leaf electroscope. Calculate the ion current in e.s.u. and in amperes. Could a galvanometer be used in this work? As ions of both signs take part in the transport of current and each ion carries 4.77×10^{-10} e.s.u. of charge, calculate how many ion pairs were found per second. How many ion pairs are generated per cubic centimeter per second?

Ans. 3.7×10^{-15} amp; 23.3 pairs per cubic centimeter per second.

40. In a study of currents in gases ions are created by radioactive radiations and by cosmic rays. The electroscope used in measurement has an electrical capacity of 2 cm. Over a period of 1 hour the ions of both signs produced by the radiations in a volume of 500 cm³ cause a fall of 3 volts in the potential of the gold-leaf electroscope. Calculate the ion current in e.s.u. and in amperes. Could a galvanometer be used in this work? As ions of both signs take part in the transport of current and each ion carries 4.77×10^{-10} e.s.u. of charge, calculate how many ion pairs were found per second. How many ion pairs are generated per cubic centimeter per second?

Ans. $i = 1.66 \times 10^{-5}$ e.s.u.; 5.55×10^{-15} amp; ion pairs per second = 3.45×10^4 ; ion pairs per cubic centimeter, 6.9. A galvanometer cannot be used for this work.

PROBLEMS BASED ON CHAPTER XV

1. A conducting sphere 10 cm in radius has 4,000 e.s.u. of charge uniformly distributed over its surface. What is the potential of the sphere in e.s.u. and in volts? How much work was required to charge this sphere? What is its capacity?

Ans. $P = 400$ e.s.u., 120,000 volts; work = 8×10^6 ergs; capacity = 10 cm.

2. An infinitely long wire has a charge of $+q$ e.s.u. per centimeter length. It is

immersed in a medium of dielectric constant D . What are the direction and magnitude of the field X at a point a centimeter from the wire?

$$\text{Ans. } X = \frac{2q}{aD} \text{ normal to wire.}$$

3. A small conducting sphere of radius 1 cm was charged to +6000 volts and immersed in nitrobenzol, $D = 40$. A small sphere with an unknown charge having a mass of 2 grams was found to be in equilibrium with the first sphere when rotating freely about the first sphere with a speed of 1 cm per second, in an orbit of 5-cm radius. Given the law of centripetal force as $f = mv^2/r$, calculate the unknown charge in magnitude, and tell what its sign is.

$$\text{Ans. } q = -20 \text{ e.s.u.}$$

4. Given the radius of the earth as 6370 km, and that of the moon as 1700 km. Assume that, by the action of the ultraviolet light of the sun on the surface of the moon which is unprotected by an atmosphere, the number of electrons that can leave the moon is sufficiently great to give it a potential of +300 volts. The charge on the electron is 4.8×10^{-10} e.s.u. of quantity. With these data calculate:

(a) The capacity of the earth and moon in centimeters, in absolute e.m.u. in farads and microfarads, assuming the earth and moon each isolated in space.

(b) Assuming the moon's surface conducting calculate how much electricity would escape to raise it to 300 volts potential in e.s.u., coulombs, and in absolute e.m.u.

(c) If the moon were to lose the above charge how much energy would be liberated?

$$\begin{aligned} \text{Ans. (a) } C_{\text{earth}} &= 6.37 \times 10^8 \text{ cm.} & (b) \quad Q &= 17 \times 10^7 \text{ e.s.u.} \\ &= 7.08 \times 10^{-13} \text{ e.m.u.} & &= 5.67 \times 10^{-3} \text{ e.m.u.} \\ &= 7.08 \times 10^{-4} \text{ farad.} & &= 5.67 \times 10^{-2} \text{ coulomb.} \\ &= 708 \text{ mf.} & (c) \text{ Energy} &= 8.51 \text{ joules.} \\ C_{\text{moon}} &= 17 \times 10^7 \text{ cm.} \\ &= 1.89 \times 10^{-13} \text{ e.m.u.} \\ &= 1.89 \times 10^{-4} \text{ farad.} \\ &= 189 \text{ mf.} \end{aligned}$$

5. The earth's radius is 6370 km, and that of the sun is 697,500 km. The temperature of the sun is 6000°C , which corresponds to the energy which an electron would get by falling through a potential of 0.34 volt. Thus electrons on the sun due to their heat motions can leave until the sun acquires a positive potential of this value. Given the charge of an electron as 4.8×10^{-10} e.s.u. Calculate:

(a) The capacity of the earth and sun in centimeters, farads, and absolute e.m.u. if they are considered completely isolated in space.

(b) Calculate the quantity of electricity the sun would lose in this way in absolute e.s.u., in coulombs, and the number of electrons which would leave the sun due to their heat motions.

$$\begin{aligned} \text{Ans. (a) } C_{\text{earth}} &= 6.37 \times 10^8 \text{ cm.} & (b) \quad Q &= 7.9 \times 10^7 \text{ e.s.u.} \\ &= 7.08 \times 10^{-13} \text{ e.m.u.} & &= 2.635 \times 10^{-3} \text{ e.m.u.} \\ &= 7.08 \times 10^{-4} \text{ farad.} & &= 2.635 \times 10^{-2} \text{ coulomb.} \\ &= 708 \text{ mf.} & \text{Number of electrons} &= 1.645 \times 10^{17}. \\ C_{\text{sun}} &= 6.975 \times 10^{10} \text{ cm.} \\ &= 7.76 \times 10^{-11} \text{ e.m.u.} \\ &= 7.76 \times 10^{-2} \text{ farad.} \end{aligned}$$

6. An electrically charged cloud 1 km^2 in extent approached to within 100 meters from the earth's surface when it discharged to a copper lightning conductor of resistance 3×10^{-2} ohm melting 400 grams of copper in the 10^{-4} second of the current flow and discharge. The latent heat of fusion of copper is 43 calories per gram, and the copper was raised from 0°C to its melting point 1100°C with a specific heat 0.08. Calculate:

(a) The capacity of the cloud-earth system in centimeters and in farads from the dimensions given, assuming the cloud and earth to form a parallel plate condenser.

(b) The current which flowed for 10^{-4} second and the quantity of electricity transferred.

(c) The potential between the cloud and the earth and from this the sparking potential of a lightning discharge in volts per centimeter.

Ans. (a) $C = 79,700$ cm.

(c) $V = 1.49 \times 10^8$ volts.

$C = 8.84 \times 10^{-8}$ farad.

Sparking potential =

(b) $i = 1.32 \times 10^8$ amperes.

$1.49 \times 10^4 \frac{\text{volts}}{\text{cm}}$.

$Q = 13.23$ coulombs.

7. A condenser such as is used in telephone circuits having a capacity of 1 microfarad was charged to 100 volts. It was then connected to a condenser of 2-microfarad capacity which was at 0 potential. Calculate the energy first in the charged condenser and then in the system of the two condensers. How much loss of energy was there, and how much were the connections raised in temperature, if they weighed 2 grams and had a specific heat 0.08 calorie?

Ans. $E_{\text{before comb.}} = 0.5 \times 10^{-2}$ joule.

$E_{\text{loss}} = 0.333 \times 10^{-2}$ joule.

$E_{\text{after comb.}} = 0.167 \times 10^{-2}$ joule.

$T_{\text{rise}} = 0.00497^\circ \text{C}$.

8. A 1-microfarad condenser is charged to 20,000 volts. It is connected in parallel with a 4-microfarad condenser. What is the resultant potential? What is the energy of charge before and after connecting them? What became of the difference?

Ans. $P = 4000$ volts.

Energy before connection = 200 joules.

Energy after connection = 40 joules.

9. Ten parallel-plate glass condensers of area $1000 \pi \text{ cm}^2$ having $D = 4$, with 1 mm between plates, are charged in parallel with 4×10^8 e.s.u. of quantity. By a switching arrangement they were connected in series in the charged condition. Calculate the potential generated in this way. What was the energy of the charged combinations in series and in parallel? Calling $J = 4 \times 10^7$ ergs per calorie, how many calories were stored on charging?

Ans. $P_{\text{series}} = 400$ e.s.u.

$E_{\text{series}} = 8 \times 10^7$ ergs.

$E_{\text{parallel}} = 8 \times 10^7$ ergs.

$E_{\text{stored}} = 2$ calories.

10. Given a parallel-plate air condenser of area $300 \pi \text{ cm}^2$, plate distance 3 mm. A quantity 2500 e.s.u. of charge is placed on it. A minute charged metal sphere of mass 333/980 grams stays suspended in the field. Calculate:

(a) The capacity of the condenser.

(b) The potential in e.s.u. and volts.

(c) The field strength between the plates in e.s.u. per centimeter and volts per centimeter.

(d) From the force on the sphere in dynes and the field strength, calculate the charge on the sphere in e.s.u.

(Hint: Consider the absolute electrometer.)

Ans. (a) 250 cm.

(c) 33.3 e.s.u. per cm., 10,000 volts per cm.

(b) 10 e.s.u., 3000 volts.

(d) 10 e.s.u.

11. In a large parallel-plate condenser with plates 1 mm apart filled with air a thin flat foil of Al of area $4 \pi \text{ cm}^2$ lying on the lower plate at its center is observed to be just lifted against gravity. It weighs 0.204 gram; $g = 980$ cm per second squared. The capacity of the condenser is 2000 cm. The area of its plates is 4π (200) cm, and the plate distance is 0.1 cm.

(a) What is the potential difference between the plates in volts?

(b) What is the quantity of electricity in e.s.u. and coulombs?

(c) What is the charge density on the plate surface?

(d) What is the field strength in e.s.u. and in volts per centimeter?

(Hint: Consider the absolute electrometer.)

Ans. (a) 600 volts.

(c) 1.59 e.s.u. per cm^2 .

(b) 4000 e.s.u., 1.33×10^{-6} coulomb.

(d) 20 e.s.u., 6000 volts per cm.

12. In a large parallel-plate condenser with plates 2 mm apart filled with air a thin flat foil of Al of area $4 \pi \text{ cm}^2$ lying on the lower plate at its center is observed to be just lifted against gravity. It weighs 0.816 gram. What is the potential differ-

ence between the plates in volts? If the capacity of the condenser is 2000 cm, what is the quantity of electricity in e.s.u. and coulombs?

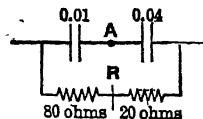
Ans. P.D. = 2400 volts. $Q = 16,000$ e.s.u. $Q = 5.33 \times 10^{-6}$ coulomb.

13. The electrical field produced between an electrified thunder cloud of area $8\pi \times 10^8$ square meters 30 meters above the ground and the ground was so strong that it just lifted a piece of Al foil of density 2.65 and 0.128 cm thick and 1 cm² against gravity. Calculate first the force in grams and dynes exerted on the Al. Then calculate the potential difference between cloud and earth (in absolute e.m.u. and in volts) that are necessary to lift the Al foil. Then calculate the field strength existing between earth and cloud in volts per centimeter. Would this cause a lightning discharge if the potential gradient or field strength for sparking is 1×10^4 volts per centimeter? Calculate the total electrostatic force between earth and cloud in kilograms. This illustrates very simply why it is that one's hair literally stands on end in powerful electrical fields long before a spark passes.

Ans. Force = 333 dynes. Field strength = 27,400 volts per cm.

P.D. = 8.23×10^7 volts. Electrostatic force = 8,530 kg.

14. Two condensers of 0.01- and 0.04-microfarad capacity, respectively, are in series. They are placed in parallel with a 100-ohm resistance R across 100 volts as in Fig. P39.



(a) At what point P in the wire will the P.D. be the same as that of the contact point of the two condensers?

(b) What is the P.D. across each?

(c) What charge have they?

(d) If A is connected to the middle of the 100-ohm resistance by a wire, the P.D. across the whole resistance R being 100 volts, what will now be the P.D. across each condenser?

(e) What will be the charge on each condenser?

Ans. (a) One-fifth the length of R from the (c) 8×10^{-7} coulomb.

point of attachment of the 0.04-mf (d) 50 volts each.

condenser. (e) 0.01 has 5×10^{-7} coulomb.

(b) 80 volts and 20 volts. 0.04 has 2×10^{-6} coulomb.

15. A parallel-plate condenser of area 6280 cm² with a dielectric of value $D = 8$ between the plates 0.04 cm thick is charged to 300 volts. It is then discharged through a ballistic galvanometer of figure of merit $k = 6 \times 10^{-8}$, the period of which is 10π sec. The deflection observed corrected for damping was 111.1 mm. Calculate the quantity of electricity Q in e.s.u. and in coulombs and absolute e.m.u. that was stored in the condenser. From these calculate the ratio of e.s.u. to e.m.u. of quantity. Take $\pi = 3.14$.

Ans. $Q = 10^5$ e.s.u. = 3.333×10^{-6} coulomb.

= 3.333×10^{-6} e.m.u.

$\therefore 3.000 \times 10^{10}$ e.s.u. = 1 e.m.u.

16. (a) A condenser of capacity 12,000 cm was charged to 300 volts. It was discharged through a ballistic galvanometer of $k = 2 \times 10^{-8}$ and $T = 5\pi$ second. What was the deflection in millimeters, and θ in radians?

(b) An electroscope has a capacity of 15 cm. The gold leaf fell through one division on the scale in 1000 seconds, representing a change of 0.3 volt. Calculate the quantity of electricity that ran off in 1000 seconds and thus the current in e.s.u. and in amperes. If the charge on an electron is 5×10^{-10} e.s.u., how many electrons flowed per second?

Ans. (a) 80 mm or 0.04 rad.

(b) $i = 1.5 \times 10^{-5}$ e.s.u. or 5×10^{-15} amp
or 31,200 electrons per second.

17. Given a 100,000-volt power line. The station has a 500-volt static voltmeter of capacity 2000 cm. It has also a standard plate condenser that can take 5000 volts of which the capacity is 20,000 cm. Calculate the dimensions of a parallel-plate condenser, of which the plates to stand the potential must be at least 4 cm apart, such that the voltmeter can be used, and what the multiplying factor is.

Accompany the answer by a diagram showing how the voltmeter will be fixed across the line.

Ans. Area electrodes $582 \times 10^4 \text{ cm}^2$; multiplying factor 20.

18. Given a 100,000-volt power line. The station has a 10,000-volt static voltmeter of capacity 1000 cm. It has also a standard plate condenser which can stand 10,000 volts of which the capacity is 4000 cm. Calculate the dimensions of a parallel-plate condenser the plates of which must be 4 cm apart to stand the potential such that the voltmeter can be used. What is the multiplying factor? Indicate in a diagram how the voltmeters will be fixed across the line.

Ans. Multiplying factor 10; plates: square, $l = 168 \text{ cm}$; round, $r = 94.2 \text{ cm}$.

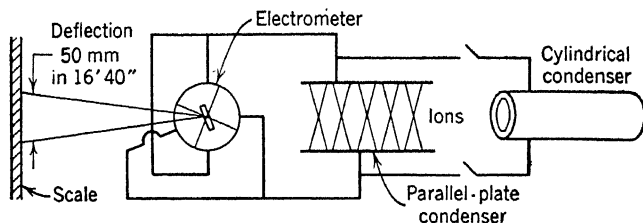


FIG. P40.

19. A quadrant electrometer giving a deflection of 20,000 mm per volt, when connected to the collecting plate of a parallel-plate condenser of area $40 \pi \text{ cm}^2$ and 2 cm high in which the air was uniformly ionized by cosmic rays, was found to give a deflection of 50 mm in $16' 40''$ owing to the ion current received in that time. The electrometer and the air condenser had their capacity determined by charging up the system to a deflection of 100 mm. The arrangement was then connected in parallel with a standard cylindrical air condenser of length 10 cm, inner radius 1 cm, outer radius 1.2 cm as seen in Fig. P40, and the deflection was reduced to 50 mm. Calculate:

- The potential rise in volts and e.s.u. produced by the ion current in the gas in $16' 40''$.
- The capacity of the parallel-plate air condenser in centimeters.
- The capacity of the standard cylindrical condenser in centimeters.
- The relative potential change on connecting the condensers in parallel.
- The capacity of the electrometer and parallel-plate condenser computed from this change.
- The capacity of the electrometer in centimeters.
- The quantity of electricity in e.s.u. carried in $16' 40''$ by the ions charging the electrometer and parallel-plate condenser to the voltage observed.
- The current through the gas in e.s.u. per second and in amperes.
- The number of ion pairs created per second by the cosmic rays per cubic centimeter of gas.

Ans. (a) 2.5×10^{-3} volts, 8.33×10^{-6} e.s.u.; (b) 5 cm; (c) 27.5 cm;

(d) $\frac{V_2}{V_1} = \frac{1}{2}$; (e) 27.5 cm; (f) 22.5 cm; (g) 2.29×10^{-4} e.s.u.;

(h) 2.29×10^{-7} e.s.u. per second, 0.763×10^{-16} amp; (i) 1.91 per cubic centimeter per second.

20. Given that a parallel-plate air condenser with a space d centimeters between the plates has a capacity given by

$$C = \frac{A}{4\pi \left[d - x \frac{D-1}{D} \right]}$$

if its plate area is A when x centimeters of a dielectric of constant D is placed be-

tween the plates. Such a condenser has $A = 560 \pi \text{ cm}^2$, $d = 2 \text{ cm}$. It is connected to an electroscope of capacity 100 cm, and the system is charged to 600 volts. When $x = 0.75 \text{ cm}$ of glass $D = 5$ is placed in the condenser, the charge remaining constant, calculate:

- The capacity of the air condenser.
- The capacity of electroscope and air condenser.
- The quantity Q in e.s.u. placed on the combined system.
- The capacity of the air condenser with glass inserted.
- The combined capacity of (d) with the electroscope.
- The potential of the system on insertion of the glass in e.s.u. and in volts.
- Given the calibration scale of the electroscope as follows, calculate the angle of the leaf before and after insertion and the change in deflection of the leaf.

Volts	420	450	480	510	530	560	600	620
Angle	30	35	40	44	48	51	54	56

Ans. (a) 70 cm; (b) 170 cm; (c) 340 e.s.u.; (d) 100 cm; (e) 200 cm; (f) 1.7 e.s.u.; 510 volts; (g) 10° .

21. In lightning surge generators the following principle for multiplying potentials is used. A potential V is placed on the two identical condensers of capacity C in parallel (see Fig. 102) by closing the switches S_1 and S_2 . After charging the condensers, switches S_1 and S_2 are opened and switch S_3 is closed, putting two oppositely charged plates of the capacities in contact and throwing the capacities in series. What will be the potential V' relative to V for the case above? If now ten such condensers are charged in parallel to 50,000 volts, each having a capacity of 0.002 microfarad, calculate:

- The quantity on each of the condensers when charged in parallel and when connected in series.
- The resultant potential V .
- The energy when all condensers are charged in parallel.
- The energy when all condensers are connected in series.
- What happens in switching from parallel to series to limit the increase in potential to an increase of tenfold instead of a hundredfold as C changes by 100.

Ans. (a) $Q_{\text{parallel}} = 10^{-3}$ coulomb, $Q_{\text{series}} = 10^{-4}$ coulomb; (b) 5×10^5 volts; (c) 25 joules; (d) 25 joules.

22. In the experiment showing that a battery gives a static charge, using the multiplier, the area of the plates was $25 \pi \text{ cm}^2$ and the distance between the two surfaces of metal was 0.05 cm, the dielectric constant of the shellac insulation being 4. The capacity of the lower plate and gold leaf alone is 50 cm, and the potential of the four cells was 6 volts. If the leaf deflects 4° for 40 volts, 5.5° for 50 volts, 7° for 60 volts, and 9° for 70 volts, what was the deflection observed by the class? *Ans.* 7° .

23. What is the capacity of the following combination of capacities in series: $C_1 = 0.01$, $C_2 = 0.02$, $C_3 = 0.05$ microfarad? What are the potentials across each condenser if 100 volts are placed across them?

Ans. $C = 0.00588 \text{ mf}$; $V_1 = 58.8$, $V_2 = 29.4$, $V_3 = 11.8$.

24. An electroscope is connected to an air condenser of capacity calculable from the data below. It is charged to a quantity Q units which remains constant. When a glass plate is inserted between the plates the leaves collapse. Given the data below.

Area of plates $A = 400 \pi \text{ cm}^2$; distance between plates $d = 1 \text{ cm}$.

Glass of dielectric constant $D = 6$; thickness of glass $x = 0.6 \text{ cm}$.

Capacity of electroscope = 100 cm, charged to 900 volts.

$$\text{Capacity of a condenser with dielectric in it is } C = \frac{A}{4\pi} \left(\frac{1}{d - x \frac{D-1}{D}} \right).$$

Volts	500	550	600	650	700	750	800	850	900	950
Angle	42	46	50	54	57	60	63	66	68	70

Calculate

- (a) The capacity of the air condenser.
- (b) The combined capacity of the electroscope and condenser.
- (c) The quantity Q in e.s.u. placed on the combined system.
- (d) The capacity of the air condenser alone with glass inserted.
- (e) The combined capacity of (d) with glass inserted and the electroscope.
- (f) The potential of the combined system on insertion of the glass in e.s.u. and in volts.

(g) Given the calibration scale of the voltmeter above, the angle through which the leaf fell on insertion.

Ans. (a) 100 cm; (b) 200 cm; (c) 600 e.s.u.; (d) 200 cm; (e) 300 cm; (f) 2 e.s.u., 600 volts; (g) 18° .

25. An electrically charged cloud 1 sq km in extent approached to within 100 meters from the earth's surface when it discharged to a copper lightning conductor of resistance 3×10^{-2} ohm and melted 400 grams of copper in the 10^{-4} second of the current flow and discharge. The latent heat of fusion of copper is 43 calories per gram, and the copper was raised from 0° C to its melting point 1100° C with a specific heat 0.08. Calculate:

(a) The capacity of the cloud-earth system in centimeters and in farads from the dimensions given, assuming cloud and earth to form a parallel-plate condenser.

(b) The current which flowed for 10^{-4} second and the quantity of electricity transferred.

(c) The potential between the cloud and the earth and from this the sparking potential of a lightning discharge in volts per centimeter.

Ans. (a) 8.8×10^{-4} farad; (b) 2.7×10^5 amp; (c) 30,550 volts per centimeter.

26. An electrically charged cloud 0.972π sq km in extent approached to within 100 meters from the earth's surface when it discharged to a copper lightning conductor of resistance 3.0×10^{-2} ohm and melted 400 grams of copper in the 10^{-4} second of the current flow and discharge. The latent heat of fusion of copper is 43 calories per gram, and the copper was raised from 0° C to its melting point 1100° C with a specific heat 0.08. Calculate:

(a) The capacity of the cloud-earth system in centimeters and in farads from the dimensions given, assuming cloud and earth to form a parallel-plate condenser.

(b) The current which flowed for 10^{-4} second and the quantity of electricity transferred.

(c) The potential between the cloud and the earth, and from this the sparking potential of a lightning discharge in volts per centimeter. This is materially lower than the normal sparking potential in air in sphere gaps of 30,000 volts per centimeter. Hence we must assume a different mechanism for long discharges in air, based on a principle of self-propagating leader strokes or bolts, with materially higher fields at their tips. Such strokes traveling at velocities of 10^7 to 10^8 cm per second have been observed by Schonland.

Ans. (a) 0.27 mf; (b) 2.7×10^5 amp; (c) 10^4 volts per centimeter.

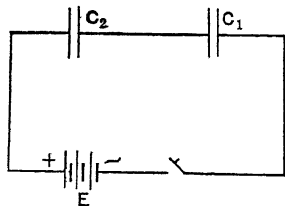


FIG. P41.

27. Given the electrical system illustrated in Fig. P41. C_1 is a parallel-plate condenser having circular plates 4 cm in radius and 1 cm apart used in measuring the velocity of gaseous ions. C_2 is a cylindrical air condenser of capacity 300 cm, and E is a source of potential of 100 volts. Calculate: (a) the total capacity of the system; (b) the potential across C_2 and C_1 , for it is essential that the potential across C_1 be accurately known.

Ans. (a) $C = 3.95$ cm; (b) $V_{C_2} = 1.317$ volts, $V_{C_1} = 98.683$ volts.

28. A 1-microfarad and a 10-microfarad condenser are in series across a 550-volt

power line. Calculate: (a) the charge on each; (b) the potential difference across each; (c) the energy of the charge on each; (d) the energy available from the two in series on discharge.

Ans. (a) $q = 5 \times 10^{-4}$ coulomb; (b) $V_1 = 500$ volts, $V_{10} = 50$ volts; (c) $E_1 = 0.125$ joule, $E_{10} = 0.0125$ joule; (d) $E = 0.1375$ joule.

29. A condenser for high-tension work consisted of 100 glass plates of dielectric constant $D = 2\pi$, of thickness 8 mm, separating 101 metal plates 60 by 120 cm in area, the alternate plates being connected together. This condenser was charged up, and on being discharged through a length of 40 cm of No. 35 copper wire (0.14-mm diameter) weighing 0.0411 gram heated it from 0 to 1100°C and melted it. It takes 131 calories to do this for a gram of copper. Calculate: (a) the capacity of the condenser; (b) the energy of the discharge; (c) the potential to which the condenser was charged in volts. (d) The resistance of the wire was 0.30 ohm; what was the maximum possible current that could have flowed?

Ans. (a) 5×10^{-7} farad; (b) 2.26×10^8 ergs; (c) 9,510 volts; (d) 31,700 amp.

Actually the wire heats and r increases.

30. A condenser for high-tension work consisted of 200 glass plates of dielectric constant $D = 2\pi$, of thickness 6 mm, separating 201 metal plates 100 by 100 cm in area, the alternate plates being connected together. This condenser was charged up, and on being discharged through a length of 100 cm of No. 35 copper wire (0.14-mm diameter) weighing 0.1028 gram heated it from 0 to 1100°C and melted it. It takes 131 calories to do this for a gram of copper. Calculate: (a) the capacity of the condenser; (b) the energy of the discharge; (c) the potential to which the condenser was charged in volts. (d) The resistance of the wire was 0.10 ohm; what was the maximum possible current that could have flowed?

Ans. (a) 1.85×10^{-6} farad; (b) 56.5 joules; (c) 7824 volts; (d) 7.82×10^4 amp.

31. A condenser of capacity 1.00×10^{-3} microfarad is charged to 20,000 volts and discharged 1000 times a second by means of a commutator and spark gap. What is the average power dissipation in watts?

Ans. 200 watts.

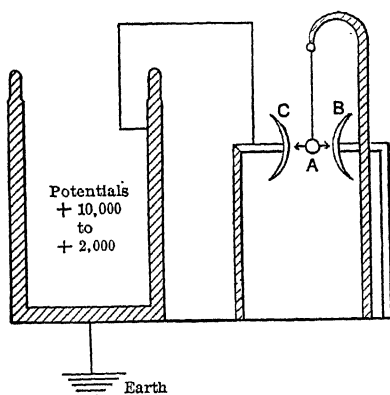


FIG. P42.

32. Given the first electric bell dating from 1750 illustrated in Fig. P42. In this the small insulated sphere A of radius 0.5 cm after being touched to the inner terminal bell-shaped plate C of the condenser of capacity 0.001 microfarad, which has a positive potential of 20,000 volts relative to the bell B , and the outer layer of the condenser which is earthed, is repelled until it strikes the bell B and is discharged. It is then attracted to C until it makes contact and repeats the process. The bell will keep ringing for a long time. If the forces are insufficient for the operation of

the bell when the potential falls to 5000 volts, calculate the number of times the bell rings. (Note: When the sphere A strikes C the capacity of the system is changed and the potential is altered.)

Ans. 2508 round trips; 5016 rings.

33. Given the first electric bell dating from 1750 illustrated in Fig. P42. In this the small insulated sphere A of radius 1 cm after being touched to the inner terminal bell-shaped plate C of the condenser of capacity 0.001 microfarad, which has a positive potential of 10,000 volts relative to the bell B , and the outer layer of the condenser which is earthed, is repelled until it strikes the bell B and is discharged. It is then attracted to C until it makes contact and repeats the process. The bell will keep ringing for a long time. If the forces are insufficient for the operation of the bell when the potential falls to 2000 volts, calculate the number of times the bell rings. (Note: When the sphere A strikes C the capacity of the system is changed and the potential is altered.)

Ans. 1450 trips.

34. A cylindrical condenser consists of a central section (aside from guard ring ends) 20 cm long, the radius of the outer cylinder being 2.100 cm and the inner cylinder 2.000 cm. The amber spacers are 2.5 mm long and 1 mm wide, and there are four of these all told. The dielectric constant for this material is 5.000. Calculate the capacity in centimeters.

Ans. 204.8 cm.

35. An analysis of the behavior of NH_3 molecules has shown that it has a permanent dipole moment $\mu = 1.56 \times 10^{-18}$. Its polarizability is $\gamma' = 5.45 \frac{3}{4\pi N_A}$ cm³. Given $k = 1.37 \times 10^{-16}$, $N_A = 6 \times 10^{23}$, $N = 2.7 \times 10^{19}$, calculate the dielectric constant of NH_3 gas at constant density at 300° absolute and at 900° absolute. What are the ratios of γ' to γ for NH_3 at these two temperatures?

Ans. $D_{300} = 1.00745$; $D_{900} = 1.00297$; $\frac{\gamma'}{\gamma_{300}} = 0.11$; $\frac{\gamma'}{\gamma_{900}} = 0.2485$.

36. Ammonia vapor at 300° absolute has $D = 1.00708$ and $D = 1.00600$ at 500° absolute. The number of molecules per cubic centimeter at constant density at these temperatures is 2.69×10^{19} . Given the Boltzmann constant $k = 1.38 \times 10^{-16}$, calculate the displacement polarizability γ' and the dipole moment μ for the NH_3 molecule.

Ans. $\gamma' = 1.28 \times 10^{-23}$, $\mu = 1 \times 10^{-18}$.

PROBLEMS BASED ON CHAPTERS XVI AND XVII

1. For certain e/m experiments a solenoid 50 cm in diameter and 400 cm long was used. It was covered with a single layer of cotton-covered No. 18 copper wire having seven turns per centimeter and carried a current of 2 amperes. What was the field inside the coil, and what was the total flux?

Ans. 17.61 oersteds; 34,600 lines.

2. The two identical wires A and B each 1 meter long were suspended as shown in Fig. P43 with their ends dipping in two troughs of mercury TT . They were 1.0 cm apart. A current of 15 amperes flowed through the two wires when they were in parallel (key k closed, switch S at P), and 14 amperes flowed through each when they were connected in series (switch S at Q , key k open). Taking $g = 980$ cm per second squared, calculate: (a) the force in grams between the conductors, and the direction of motion when in parallel; (b) the force in grams between the conductors, and the direction of motion when in series.

Ans. 0.115 g wt together; 0.4 g wt apart.

3. The two vertical wires A and B each 1 meter long were suspended as shown in Fig. P43 with their ends dipping into two troughs of mercury TT . They were 0.2 cm apart. A current of 15 amperes flowed through the two wires when they were in parallel (key k closed, switch S at P), and 12 amperes flowed through each when they were connected in series (switch S at Q , key k open). Taking $g = 980$ cm per second squared, calculate: (a) the force in grams between the conductors, and the

direction of motion when in parallel; (b) the force in grams between the conductors, and the direction of motion when in series.

Ans. (a) 0.574 gram attraction; (b) 1.47 gram repulsion.

4. A solenoid of 1000 closely wound turns has a current of 5 amperes. What would be the work in ergs to carry a unit pole completely around the solenoid? If the field is very weak outside, most of the work will be done in carrying the pole down the length $l = 20$ cm of the solenoid. What is the field in the solenoid?

Ans. $W = 6280$ ergs; $H = 314$ oersteds.

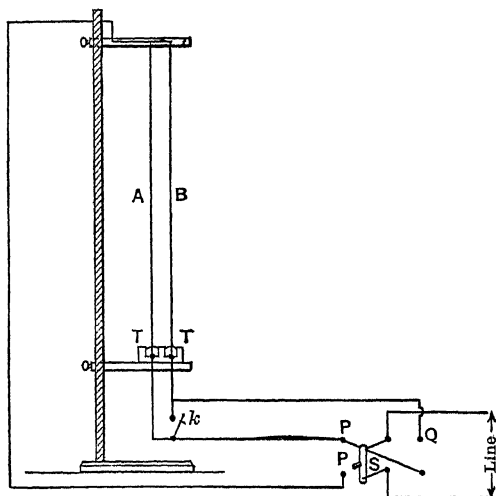


FIG. P43.

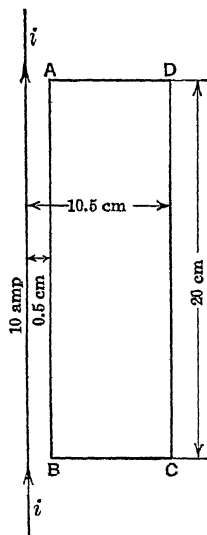


FIG. P44.

5. Given an infinitely long wire i carrying 10 amperes of current upward (see Fig. P44). It lies in the plane of a rectangular area $ABCD$ the side AB of which is 0.5 cm from the wire i and the side CD 10.5 cm from i . The height of the rectangle is 20 cm. Calculate the flux in the area $ABCD$ due to the current in i . (Note: The flux in $ABCD$ varies along AD . Thus HdA must be integrated over appropriate limits, where ϕ is the flux in a small vertical element of area dA , and ϕ varies with the distance from the wire.) What work in ergs would be done by moving a conductor carrying 5 amperes 20 cm long from AB to CD ? *Ans.* 122 lines; 61 ergs.

6. Given an infinitely long wire i carrying 12 amperes of current upward (similar to Fig. P44). It lies in the plane of a rectangular area $ABCD$ the side AB of which is 1 cm from the wire i and the side CD 11 cm from i . The height of the rectangle is 10 cm. Calculate the flux in the area $ABCD$ due to the current in i . (Note: The flux in $ABCD$ varies along AD . Thus ϕdA must be integrated over appropriate limits, where ϕ is the flux in a small vertical element of area dA , and ϕ varies with the distance from the wire.) What work in ergs would be done by moving a conductor carrying 15 amperes 10 cm long from AB to CD ?

Ans. 57.5 lines; 86.3 ergs.

7. The force on a magnetic dipole of moment μ oriented parallel or antiparallel to a divergent magnetic field is given by $f_s = \mu \frac{\partial H}{\partial Z}$, hence the displacement Z in the field direction of such a magnet in moving a distance l with velocity C perpendicular to H is $Z = \pm \frac{\mu}{2m} \frac{\partial H}{\partial Z} \frac{l^2}{C^2}$. Now silver atoms of mass 1.8×10^{-22} gram at 1100°C

have an average thermal velocity C of 5.62×10^4 cm per second. If in the Stern-Gerlach experiment μ is 1 Bohr magneton and the Ag atoms traverse 15 cm in the field perpendicular to Z and parallel to the pole pieces, what value of $\frac{\partial H}{\partial Z}$ is needed to cause a deflection of 1 mm for the beam?

Ans. 5.51×10^4 oersted per centimeter.

8. The radius of the coil in the demonstration experiment of the oscillating vertical coil (see Fig. P45) dipping into mercury is 1.5 cm. The turns of wire are

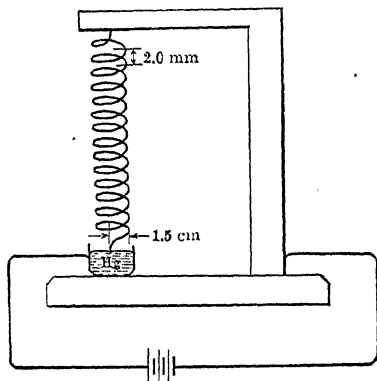


FIG. P45.

2.0 mm apart. If each centimeter of wire weighs 0.015 gram, taking $g = 980$ cm per second squared how much current must be run through the coil to make it oscillate against gravity? (While the contact with the mercury could for the spring extended by gravity at equilibrium be broken theoretically by a magnetic force of contraction of infinitely small amount, in practice, the amplitude of oscillation to start the coil oscillating and to break contact is more nearly several millimeters. This requires that the force of gravity be practically overcome, that is, that each turn be just able to lift the next turn against gravity and so permit the coil to contract by a sufficiently great amount.)

Ans. 12.12 amp.

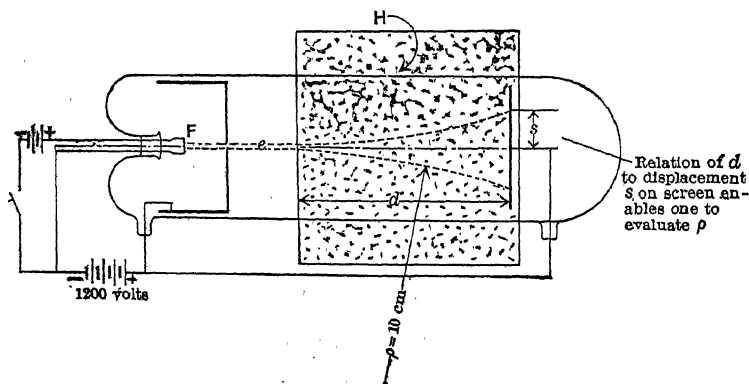


FIG. P46.

9. A cathode-ray beam or stream of electrons e of velocity v (Fig. P46), charge e , and mass m constitutes an electrical current flowing in the opposite sense to the

motion of the electrons. The magnitude of the current is ev . Assume a cathode-ray beam flowing from left to right and a magnetic field H , perpendicular to the plane of the paper in which the current flows, directed into the paper. The force on the beam is Hev , and it can be shown that in the field H the force is always perpendicular to the current ev . The electron or cathode-ray beam will then be bent into a circle of radius ρ . Since for circular motion the centripetal force is $\frac{mv^2}{\rho}$ we can write

$Hev = \frac{mv^2}{\rho}$. If the electrons from a filament F have fallen freely through a potential of V volts before they enter H , they have a velocity v determined by $eV = \frac{1}{2}mv^2$. Given $V = 1200$ volts, $H = 11.64$ oersteds, $\rho = 10$ cm, calculate:

(a) The sense in which the beam will be curved (up or down in figure).

(b) The value of v in terms of $\frac{e}{m}$.

(c) Put the value of v into the equation for ρ and solve for $\frac{e}{m}$.

(d) Then solve for v in centimeters per second.

(e) Given $e = 4.77 \times 10^{-10}$ e.s.u., calculate m , the mass of the electron.

The method outlined above is one in constant use in the study of electrical phenomena and is used to evaluate m the mass of the electron.

Ans. (a) Down; (b) $v = \sqrt{\left(\frac{e}{m}\right) 2V}$, V in e.s.u.; (c) $\frac{e}{m} = \frac{2V}{H^2\rho^2}$, where H and e are in e.m.u., $= 1.770 \times 10^7$ e.m.u. per gram.; (d) $v = 2.06 \times 10^9$ cm per second; (e) $m = 8.98 \times 10^{-28}$ gram.

10. A beam of electrons constitutes a *negative* current of electricity given by ev , where e is the charge and v the velocity of the electrons. In a magnetic field H the force is Hev and is always at right angles to the beam so that the electrons describe a circular path such that Hev equals the centripetal force mv^2/ρ , where m is the mass of the electron and ρ is the radius of curvature of the path. Given an *electron beam going from right to left in the plane of the paper and a field $H = 30$ oersteds perpendicular to the paper and into it*, with

$$e = 4.8 \times 10^{-10} \text{ e.s.u.}$$

$$v = 1 \times 10^9 \text{ cm per second.}$$

$$H = 30 \text{ oersteds.}$$

$$m = 9 \times 10^{-28} \text{ gram.}$$

Calculate ρ in centimeters, and illustrate by diagram how the path will be curved.

$$\text{Ans. } \rho = 1.885 \text{ cm.}$$

11. A rectangular loop of wire 10 cm on a side has a current running through it of 1 ampere in a counterclockwise direction. Calculate the field at the center of this coil. (Hint: At the center the end of each wire subtends an angle with the normal to the wire of value β which can be determined. Using the procedure of Chapter III in deriving Biot and Savart's law, integrate between angles $\pm\beta$.)

$$\text{Ans. } \frac{8\sqrt{2}ia}{l} \text{ for the symbolical solution.}$$

12. A long straight wire of radius a centimeters carries a current of i amperes. What is the magnetic flux ϕ between the surface of the wire and a cylindrical surface coaxial with the wire r centimeters away per length l of conductor? What is the radius of a cylinder that will just contain half the flux about the wire?

$$\text{Ans. } \phi = \frac{2il}{10} \log_e \frac{r}{a}, \sqrt{ra}.$$

13. Two infinitely long parallel wires have radii of 0.2 mm and are 10 cm apart. The current of 5 amperes flows down one and up the other. In an area 10 cm long between them calculate the flux between them, and the field at the center. What is the net total flux if the currents run down in both, and what are the fields between

the wires as a function of the distance as shown in a plot. What work will be done in moving a conductor 10 cm long with 1 ampere of current from one to the other in each case of current flow?

Ans. 124.4 lines, 0.4 oersted, 0 lines, $W = 0$ for opposite flow, $W = 12.44$ ergs same direction of flow.

14. What is the magnetic moment of a plane circular loop of radius 4 cm with a current of 2 amperes flowing in it? Given a magnet of length 1 cm, what would be its pole strength to give the same moment? If the coil had 100 turns, what would be its moment? *Ans.* 10, 10, 10^3 .

15. A cathode-ray beam or stream of electrons e of velocity v (see Fig. P46, problem 9), charge e , and mass m constitutes an electrical current flowing in the opposite sense to the motion of the electrons. The magnitude of the current is ev . Assume a cathode-ray beam flowing from left to right and a magnetic field H , perpendicular to the plane of the paper in which the current flows, directed into the paper. The force on the beam is Hev , and it can be shown that in the field H the force is always perpendicular to the current ev . The electron or cathode-ray beam will then be bent into a circle of radius ρ . Since for circular motion the centripetal force is $\frac{mv^2}{\rho}$ we can write $Hev = \frac{mv^2}{\rho}$. If the electrons from a filament F have fallen freely through a potential of V volts before they enter H , they have a velocity v determined by $eV = \frac{1}{2}mv^2$. Given $V = 1200$ volts, $H = 11.64$ oersteds, $\rho = 10$ cm, calculate:

- The sense in which the beam will be curved (up or down in figure).
- The value of v in terms of e/m .
- Put the value of v into the equation for ρ and solve for e/m .
- Then solve for v in centimeters per second.
- Given $e = 4.77 \times 10^{-10}$ e.s.u., calculate m the mass of the electron.

The method outlined above is one in constant use in the study of electrical phenomena and is used to evaluate m the mass of the electron.

Ans. (a) Down; (b) $4.4 \times 10^5 \sqrt{e/m}$; (c) 1.77×10^7 e.m.u. per gram; (d) 2.06×10^9 cm per second; (e) 8.98×10^{-28} gram.

16. The radius of the unit Bohr electron orbit is 0.58×10^{-8} cm. It has an electron in it with a speed of 2.18×10^8 cm per second. It has a mass $m = 9 \times 10^{-28}$ gram and a charge of 4.8×10^{-10} e.s.u. (Note that magnetic calculations require charge in e.m.u.) Calculate the magnetic moment μ of this unit orbit. This is known as the Bohr magneton. Then calculate the moment of momentum $p = mvr$ and take the ratio $\frac{\mu}{p}$. Prove that this is equal to $\frac{e}{2m}$. When the gyromagnetic

ratio was observed for electrons in iron, $\left(\frac{\mu}{p}\right)_{\text{observed}}$ was found to be $\frac{e}{m}$. Explain.

Ans. $\mu = 1.013 \times 10^{-20}$; $p = 11.38 \times 10^{-28}$.

17. Using the data of problem 16 compute the moment of inertia of the electron in the unit Bohr orbit and its angular velocity. If this orbit is in a field of 10,000 oersteds, calculate its angular velocity of precession. From the velocity of precession and the average distance of the electron of $0.82 \times 0.52 \times 10^{-8}$ from the axis of precession, calculate the magnetic field produced by the precessing electron orbit. Is it large or small compared to the impressed field? Calculate the moment due to precession and compare it with that of the electron in the orbit?

Ans. $I = 3.03 \times 10^{-44}$ g-cm²; $\omega = 3.76 \times 10^{16}$ rad per second; $\omega' = 0.904 \times 10^{11}$ rad per second; $M = 1.32 \times 10^{-26}$; $H = 0.249$ oersted.

18. Since a current i_a of length l perpendicular to a magnetic field H experiences a force $f = i_a l H$, and since a quantity of electricity e passing a point in t seconds can be equated to a current i_a , it is possible to write that $f = i_a l H = \frac{e}{t} l H = e \frac{l}{t} H$, with

$\frac{v}{c} = \frac{1}{\gamma}$ the velocity of the quantity of charge e . From this it can at once be seen that a charge e (an electron or a proton), of velocity v will experience a force perpendicular to v of value $f = Hev$ in a magnetic field perpendicular to v . A force always perpendicular to the direction of motion will cause a charged body of charge e and mass m to take a circular path of radius ρ determined by the fact that the centrifugal force $\frac{mv^2}{\rho} = Hev$.

The cyclotron invented by Professor E. O. Lawrence consists of two halves of a metal pillbox sawed in two across a diameter — the D 's as shown in Fig. 189. These D 's are insulated from each other and are connected to two terminals of a high-frequency oscillator. The D 's are placed in a uniform magnetic field H perpendicular to the faces of the pillbox D 's. Now if at an instant protons, H^+ nuclei, are at D_1 on D_1 , when D_1 is $+$ and D_2 is $-$ they will get the energy eV caused by the potential between D_1 and D_2 and enter D_2 . Here the field H causes them to take a circular path of radius ρ appropriate to their velocity v and the values of H , e , and m . If they arrive at D_2 just as D_2 reaches its maximum $+$ value relative to D_1 they will go to D_1 and again get eV . If they remain in step with the alternating potential V they will gain $2Ve$ for each complete revolution. As a result ρ will continue to increase until the protons reach the limits of the radius ρ_0 of the D 's, at which point they can be deflected out.

The success of the cyclotron results from the fact that despite increasing values of ρ the time $\frac{T}{2}$ for a half circle $\pi\rho$ in the D 's is always constant as long as e , m , and H are constant. Prove this from the equations for v and ρ and be able to evaluate T .

Assume that $H = 10,000$ oersteds, that the limiting radius ρ_0 of the D 's is 70 cm (60-inch cyclotron), that the value of e/m for protons is 9650 e.m.u. per gram, that the energies of the protons in ergs, E , is $\frac{1}{2}mv^2$, and that $e = 1.6 \times 10^{-20}$ e.m.u.

(a) Calculate the energies of the protons in ergs.

(b) Now a proton accelerated by a volt of P.D. in vacuum has an energy of 1.6×10^{-12} erg. Calculate the energy of the protons from the cyclotron in equivalent volts.

(c) What is the time, T , for a complete revolution, and what frequency $\nu = 1/T$ of alternating potential is required?

(d) If the voltage at peak value across the D 's is 20,000 volts, how many complete revolutions did the protons make?

(e) If the proton current was 100 microamperes, what was the power of the proton beam in watts?

Ans. (a) 37.8×10^{-6} erg; (b) 23.6×10^6 volts; (c) $T = 6.52 \times 10^{-8}$ second, $\nu = 1.53 \times 10^7$ cycles per second; (d) 590 revolutions; (e) 2360 watts.

19. Given an electron in the inner orbit of the H atom. This orbit has a radius r on Bohr's theory determined by the relation $mvr = \frac{nh}{2\pi}$, where n is an integer 1, 2, 3, etc., and has the value 1 for the innermost orbit. This gives $r = 0.53 \times 10^{-8}$ cm. The law of centrifugal force acts and requires that $\frac{mv^2}{r} = f = \frac{e^2}{r^2}$, where $\frac{e^2}{r^2}$ is the Coulomb law of force between nucleus and electron, e being the charge on electron and nucleus, and r the radius, v the linear velocity, and m the mass of the electron. Now the velocity $v = r\omega$, where ω is the angular velocity, so that $\frac{\omega}{2\pi} = \nu$,

the frequency of rotation. Assume an electron in the hydrogen orbit the plane of which is normal to a magnetic field H . Depending on the sense of rotation of the electron, the magnetic field acts to push the electron toward or away from the center of the orbit. This added force on the electron in the orbit could under classic laws

change the radius of the orbit. It cannot do this, since the Bohr condition requires r to be determined by $mvr = \frac{nh}{2\pi}$. Hence r remains constant and the electron is speeded up in the orbit to a new velocity v' , which is greater or less than the original velocity depending on whether the force is radially inward or outward. Using the equation $\frac{mv'^2}{r} = \frac{e^2}{r^2} \pm f'$, where f' is the magnetic force, calculate the equation in a field H . From this calculate in an approximate fashion the expression for v' and hence the new frequency ν' in terms of e , H , m , and ν . Given $e = 4.8 \times 10^{-10}$ e.s.u., $m = 9 \times 10^{-28}$, $v = 2.18 \times 10^8$ cm per second, and $H = 20,000$ oersteds, calculate the frequency ν and the change in frequency $(\nu - \nu')$ or $(\nu' - \nu) = \Delta\nu$ of the orbital electron in the field. (Caution: Remember that e must be expressed in electro-magnetic units.) What is the fractional change in ν due to the field? How accurately (i.e., to how many significant figures) must ν be measurable before the quantity $\Delta\nu$ can be measured? Can you see why with the early spectroscopes Faraday was unable to observe the change $\Delta\nu$ in fields of some few thousands of gauss? It was not until the time of Pieter Zeeman in 1896 that this displacement was observed and the Zeeman effect was discovered.

$$\text{Ans. } \nu' - \nu = \Delta\nu = \pm \frac{eH}{4\pi m}; \quad 6.6 \times 10^{15}, \quad 2.8 \times 10^{10}, \quad 4.24 \times 10^{-6};$$

six significant figures; today ν can be measured to eight figures.

PROBLEMS BASED ON CHAPTERS XVIII AND XIX

1. A straight bar of wrought iron 20 cm long the area of which is 4 cm² is placed in a uniform coil where a field of 200 oersteds exists. If the permeability of the iron for this field is 1500:

(a) Find the total flux through the bar.

(b) Find I , the intensity of magnetization.

(c) Assuming the magnetization concentrated at the end surface, find the pole strength of the temporary magnet.

(d) Find the susceptibility of the iron for this field.

Ans. (a) $\phi = 1.20 \times 10^6$ lines; (b) $I = 2.39 \times 10^4$ lines per square centimeter;

(c) $m = 9.55 \times 10^4$ e.m.u.; (d) $\kappa = 119$.

2. A coil of wire 50 cm long with 1000 turns of wire carries a current of 2 amperes. A bar of iron of area 2 cm² and length 10 cm is placed in its center. If the permeability of the iron is 2000, calculate:

(a) The flux through the volume to be occupied by the iron in its absence.

(b) The total flux through the iron bar.

(c) The intensity of magnetization I .

(d) The pole strength of the temporary magnet, assuming the magnetism concentrated at the end surfaces of the iron.

(e) The susceptibility of the iron at this field strength.

Ans. (a) $\phi_{\text{air}} = 32 \pi = 100.6$ lines; (b) $\phi_{\text{Fe}} = 64 \times 10^3 \pi = 2.01 \times 10^6$ lines;

(c) $I = 8.00 \times 10^3$ lines per cm²; (d) $m = 4.00 \times 10^3$ e.m.u.; (e) $\kappa = 159$.

3. An electromagnet of the design shown in Fig. P47 is to be built giving 4000 oersteds across a gap 2 cm long. The area of cross section of the iron is to be 100 cm². The long side is 60 cm long, the short side is 40 cm long (inside dimensions). There are to be two coils, one on each of the 40-cm sides. The coils will not stand more than 4 amperes without overheating. The loss of flux in the gap is 25 per cent, and μ as a function of B is shown from the following data:

B	3506	4000	4434	4980	5340	5801
μ	3100	3200	3110	3000	2900	2750

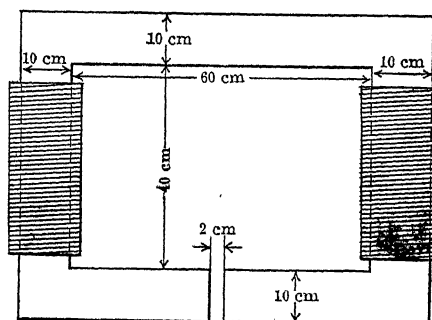


FIG. P47.

Calculate the number of turns of wire required in each coil.

Ans. $n = 940$ turns per coil.

4. Given the electromagnet on a wrought-iron frame shown in Fig. P48. The core has an area of 100 cm^2 . The thickness of the iron is 10 cm . The inner length of the iron is 50 cm . The depth inside the frame is 30 cm . The air gap is 1 cm . Leakage is 20 per cent. The values of μ for this iron relative to B are given as:

B	3506	4000	4440	5000	5340	5801
μ	3300	3200	3110	3000	2900	2750

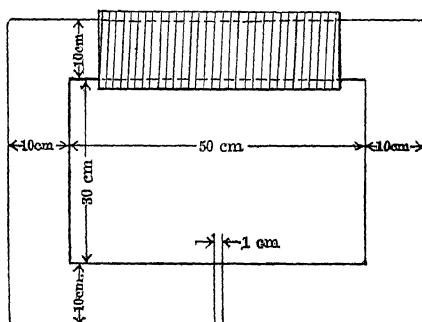


FIG. P48.

If there is to be a single coil which can carry only 3 amperes and the field in the gap is to be equal to 4000 oersteds, what are: (a) the reluctances of the iron and air portions of the circuit; (b) the magnetomotive force; (c) the number of turns needed.

Ans. (a) $Z_{\text{air}} = 10^{-2}$; $Z_{\text{Fe}} = 0.67 \times 10^{-3}$; (b) m.m.f. = $ni = 34.5 \times 10^2$; (c) $N = 1150$ turns.

5. In a hysteresis loop for steel and for soft iron the readings of B are in lines per square centimeter and those of H are in oersteds. The areas of the curves in terms of BH in these units are 100,000 and 500,000 for soft iron and steel, respectively. What is the energy loss in hysteresis per cycle? If these are used in a 60- and in a 500-cycle a-c machine drawing 2 amperes at an average of 100 volts, what is the power loss in watts in each machine, and what percentage of the power consumption does this constitute?

Ans. 0.05 joule for steel, 0.01 joule for iron; at 60 cycles, 3 watts for steel, 0.60 watt for iron; at 500 cycles, 25 watts for steel, 5 watts for iron; at 60 cycles, 1.5 per cent for steel, 0.3 per cent for iron; at 500 cycles, 12.5 per cent for steel, 2.5 per cent for iron.

6. Given the permeability μ for O_2 gas as 1.00000250, the temperature as 300° absolute, the number of molecules of O_2 per cubic centimeter as 3×10^{19} .

Given furthermore that for a paramagnetic substance the average moment of the molecule in the field direction per unit field $\frac{\bar{m}_T}{H} = \frac{\kappa}{n}$, where κ is the volume sus-

ceptibility and n the number of atoms per unit volume, and that $\frac{\bar{m}_T}{H} = \frac{\bar{m}_A^2}{3 kT}$, calculate the value of the molecular magnetic moment \bar{m}_A for the O_2 molecule, given k the average energy of a molecule per degree C as 1.37×10^{-18} erg. If the Bohr magneton is 0.91×10^{-20} absolute unit calculate the number of Bohr magnetons manifested by O_2 . If it is not a whole number remember that \bar{m}_A is the vector sum of a number of unit moments at different angles. *Ans.* 3.14 Bohr magnetons.

7. In the demonstration experiment described on page 276, where it is shown that the lifting power of a magnet is proportional to AB^2 , where A = area and B = induction, the experimental data are as follows. The area of contact for the flat side on each pole piece of the permanent horseshoe magnet is 1 cm^2 . With this, the magnet supports 4 kg. When the rounded side is placed uppermost with 0.25 cm^2 area of contact at each pole piece, the weight supported is 6 kg. Calculate B in each case. Then calculate AB for each case. Since the magnet is permanent it might be expected that the flux AB would be the same for each case, for the flux should be constant. Since they are not the same, explain the discrepancy and calculate the percentage change in flux caused by this factor.

Ans. $B = 0.701 \times 10^4$; $B' = 1.72 \times 10^4$; $AB = 0.701 \times 10^4$; $A'B' = 0.43 \times 10^4$; decrease in total flux = 38.7%.

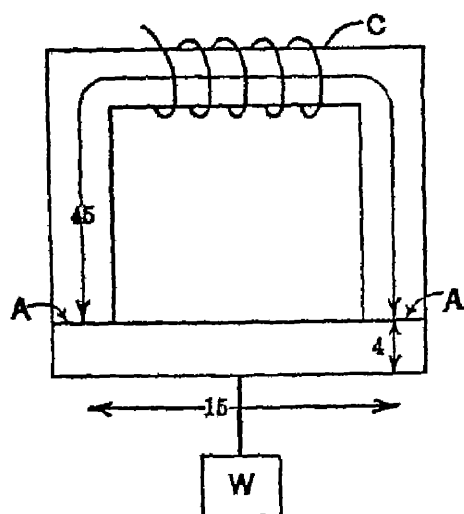


FIG. P49.

8. A horseshoe magnet (Fig. P49) is to be designed to lift a load of 40 kg on a keeper bar. The dimensions are as follows. The length of the horseshoe part is 45 cm. The length of the keeper is 15 cm. The horseshoe has a cross section of $5 \times 5 \text{ cm}$. The keeper has a cross section of $5 \times 4 \text{ cm}$. The air gaps (AA') are each 0.01 cm long. The loss of flux is 10 per cent. The coil C cannot take more than 1 ampere without overheating. The values of B and μ for the iron used are as follows:

B	3506	4000	4440	4935	5340	5801
μ	3300	3200	3110	3010	2900	2750

Then calculate the following data assuming the flux through the keeper is confined to an area $5 \times 5 \text{ cm}$ at the ends of the magnet.

(a) The induction B going through the keeper to hold the weight. (Remember that the weight of 40 kg is supported at two poles A and A' , as well as that the area of cross section of the keeper is less than the area of the entering flux from the horseshoe.)

(b) The fluxes ϕ through the keeper, air gaps, and horseshoe, respectively.

(c) The values of μ for the horseshoe and keeper from B for each.

(d) The reluctances of the horseshoe, the keeper, and the air gaps.

(e) The ampere turns required to lift the weight.

(f) The number of turns required at 1 ampere through the coil.

Ans. (a) $B = 4.44 \times 10^3$ lines per square centimeter through gap and keeper; (b) $\phi_{\text{gap}} = 11.1 \times 10^4$ lines, $\phi_{\text{keeper}} = 8.9 \times 10^4$ lines, $\phi_{\text{shoe}} = 12.3 \times 10^4$ lines; (c) $\mu_{\text{keeper}} = 3110$, $\mu_{\text{shoe}} = 3010$; (d) $Z_{\text{shoe}} = 5.98 \times 10^{-4}$, $Z_{\text{keeper}} = 2.41 \times 10^{-4}$, $Z_{\text{gaps}} = 8.0 \times 10^{-4}$ (both); (e) $n \frac{10}{4\pi} (183.8) = 146$; (f) 146 turns.

9. In Fig. P50 is shown a plunger type of electromagnetic circuit breaker. When the current i in the coil reaches a certain value the plunger P is pulled upward against its weight and the spring S , breaking the contact at C and opening the circuit AC . Given $A_1 = 4 \text{ cm}^2$, A_2 for the frame $= 10 \text{ cm}^2$, l_1 the length of the plunger in the open position $= 10 \text{ cm}$ with a 0.4-cm air gap and having a leakage loss of 20 per cent

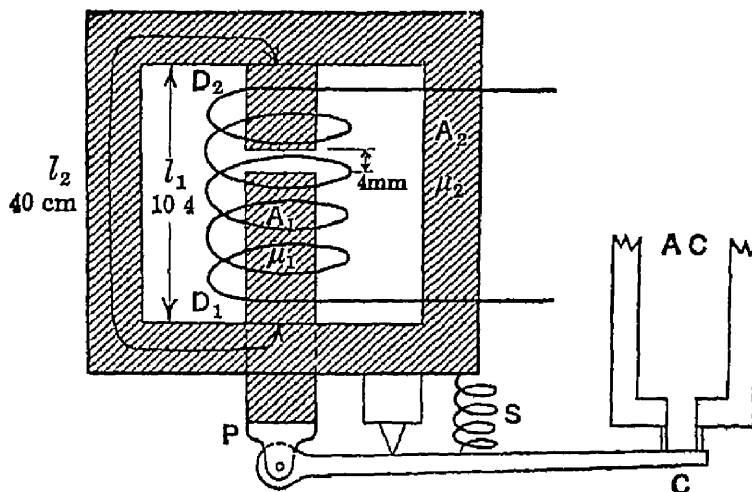


FIG. P50.

at the gap, and $l_2 = 40 \text{ cm}$ for each half of the frame. If the total force needed is 500 grams and the breaker should operate at 1 ampere, calculate the number of turns needed if the values for the $B - \mu$ curve are as follows:

$B \dots$	200	400	800	1200	1600	2000	2200	2400	2800
$\mu \dots$	400	1000	1500	1900	2200	2400	2500	2400	2200

Note: The flux needed to lift the plunger is exerted over an area A_1 across the air gap. All the flux in the metal of A_1 does not cross the air gap and hence contributes little to the lifting. Also the data above require that the $B - \mu$ curve be plotted.
Ans. 707.

10. A horseshoe magnet is to be designed to lift a load of 200 kg on a keeper bar. This load includes the weight of the keeper itself. The horseshoe part has uniform area of cross section of 100 cm^2 and a length of 300 cm. The air gaps at the two ends of the horseshoe are 0.02 cm long, and the keeper is 50 cm long with the side toward the magnet of the same width as the end of the horseshoe magnet but having a cross sectional area of 50 cm^2 . The leakage of flux is 10 per cent. The values of μ and B for the iron may be plotted from the data below.

$B \dots\dots\dots$	2,000	3,000	4,000	5,000	6,000	7,000	8,000
$\mu \dots\dots\dots$	2,800	3,000	2,950	2,875	2,700	2,450	2,200
$B \dots\dots\dots$	9,000	10,000	11,000	12,000	13,000	14,000	15,000
$\mu \dots\dots\dots$	1,950	1,700	1,450	1,200	1,000	900	825

Find:

- The induction B entering the keeper on an area 100 cm^2 at its surface to lift the 200 kg (one-half at each pole).
- The flux ϕ through the keeper and B inside the keeper.
- The flux ϕ' in the horseshoe to allow 10 per cent loss and to let ϕ enter the keeper. The induction B in the horseshoe.
- The values of μ for keeper and horseshoe.
- The reluctances of the horseshoe, the keeper, and the air gaps.
- The ampere-turns needed to force this flux through each and hence the ampere-turns needed for this task.
- If 20 amperes were all that were available how many turns would be required?

Ans. (a) 4960 oersteds; (b) 9920 oersteds; (c) 5500 oersteds; (d) 1720, 2790;
(e) 1.074×10^{-3} , 0.582×10^{-3} , 0.400×10^{-3} ; (f) 857; (g) 43 turns.

11. A horseshoe magnet is to be designed to lift a load of 200 kg on a keeper bar. This load includes the weight of the keeper itself. The horseshoe part has uniform area of cross section of 200 cm^2 and a length of 200 cm. The air gaps at the two ends of the horseshoe are 0.01 cm long, and the keeper is 50 cm long with the side toward the magnet of the same width as the end of the horseshoe magnet but having a cross sectional area of 100 cm^2 . The leakage of flux is 5 per cent. The values of μ and B for the iron may be plotted from the data below.

B	4,000	5,000	6,000	6,500	7,000	7,500
μ	2 880	2,800	2,700	2,560	2,400	2,200
B	8,000	9,000	9,500	10,000	11,000	12,000
μ	1,960	1,580	1,410	1,250	1,000	700

Find:

- The induction B entering the keeper on an area 200 cm^2 at its surface to lift the 200 kg (one-half at each pole).
- The flux ϕ through the keeper and B inside the keeper.
- The flux ϕ' in the horseshoe to allow 5 per cent loss and to let ϕ enter the keeper. The induction B in the horseshoe.
- The values of μ for keeper and horseshoe.
- The reluctances of the horseshoe, the keeper, and the air gaps.
- The ampere-turns needed to force this flux through each and hence the ampere-turns needed for this task.
- If 20 amperes were all that were available, how many turns would be required?

Ans. (a) 3500 lines per square centimeter; (b) 7.0×10^5 , 7×10^5 ; (c) 7.31×10^5 , 3680 lines per square centimeter; (d) keeper 2400, magnet 2880;
(e) 3.47×10^{-4} , 2.08×10^{-4} , 1×10^{-4} ; (f) 376; (g) 18.8.

12. Given the magnetic circuit depicted in Fig. P51.

$$\begin{aligned} A_2 &= 25 \text{ cm}^2. & l_2 &= 120 \text{ cm}. \\ A_1 &= 50 \text{ cm}^2. & l_1 &= 40 \text{ cm}. \\ A_s &= 40 \text{ cm}^2. & l_s &= ? \end{aligned}$$

The $B - \mu$ data for the iron are given in problem 10. Assume no leakage. It is desired to have $\phi_2 = 2 \times 10^6$ lines; ϕ_s is to be $2\phi_2$. Calculate the following data:

- The values of B_2 , B_1 , and B_s , and from the table the values of μ_2 , μ_1 and μ_s .
- The reluctance Z_2 and the reluctance Z_s required to give $\phi_s = 2\phi_2$.
- From Z_s , A_s , and μ_s calculate l_s .
- Calculate Z_1 and the total reluctance Z of the circuit.
- From ϕ_1 and Z calculate ni to give $\phi_2 = 2 \times 10^6$ and $\phi_s = 4 \times 10^6$ lines.

Ans. (a) $\begin{cases} B_2 = 8,000 \text{ gauss}, \mu_2 = 2200. \\ B_s = 10,000 \text{ gauss}, \mu_s = 1700. \\ B_1 = 12,000 \text{ gauss}, \mu_1 = 1200. \end{cases}$
(b) $Z_2 = 2.18 \times 10^{-3}$, $Z_s = 1.09 \times 10^{-3}$.
(c) 74.1 cm.
(d) $Z_1 = 0.67 \times 10^{-3}$.
 $Z = 1.39 \times 10^{-3}$.
(e) 660 ampere turns.

13. Give the magnetic circuit of Fig. P51.

$$\begin{aligned} A_2 &= 100 \text{ cm}^2. & l_2 &= 200 \text{ cm}. \\ A_1 &= 150 \text{ cm}^2. & l_1 &= 60 \text{ cm}. \\ A_s &= 200 \text{ cm}^2. & l_s &= ? \end{aligned}$$

The $B - \mu$ data for the iron are given in problem 11. Assume no leakage. It is desired to have $\phi_2 = 4 \times 10^8$ lines; ϕ_3 is to be $3\phi_2$. Calculate the following data:

- (a) The values of B_2 , B_1 , and B_3 and, from the table the values of μ_2 , μ_1 , and μ_3 .
- (b) The reluctance Z_2 and the reluctance Z_3 required to give $\phi_3 = 3\phi_2$.

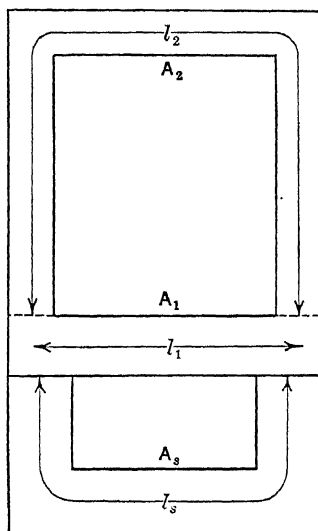


FIG. P51.

- (c) From Z_s , A_s , and μ_s calculate l_s .
- (d) Calculate Z_1 and the total reluctance Z of the circuit.
- (e) From ϕ_1 and Z calculate ni to give $\phi_2 = 4 \times 10^8$ and $\phi_3 = 12 \times 10^8$ lines.

Ans. $\begin{cases} B_1 & 1.065 \times 10^4, & \mu_1 = 1080. \\ B_2 & 4000, & \mu_2 = 2880. \\ B_3 & 6000, & \mu_3 = 2700. \end{cases}$

(b) $Z_2 = 6.95 \times 10^{-4}$, $Z_s = 2.32 \times 10^{-4}$.

(c) $l_s = 125.5$ cm.

(d) $Z_1 = 3.7 \times 10^{-4}$; $Z = 5.44 \times 10^{-4}$.

(e) $ni = 690$ ampere-turns.

14. A Faraday disk 10 cm radius is rotated at 1200 r.p.m. and develops 2 volts e.m.f. What is the strength of the field, assuming it to be uniform over the radius?

Ans. $H = 31,800$ oersteds.

15. In the Barlow's wheel experiment there was a uniform magnetic field of 2000 oersteds over the whole radius of the wheel. The radius of the wheel was 6 cm and 8 amperes were passed through it. Calculate the force on the wheel and the torque which was exerted.

Ans. 9600 dynes; 28,800 dynes \times cm.

16. In the Barlow's wheel experiment there was a uniform magnetic field of 500 oersteds over the whole radius of the wheel. The radius of the wheel was 10 cm, and 10 amperes were passed through it. Calculate the force on the wheel and the torque which was exerted.

Ans. 5.1 g. wt.; 25.5 g. wt. \times cm.

17. A solenoid, 50 cm long and 1 cm internal diameter, has a total of 1000 turns of wire. A secondary of 100 turns is wound around the middle of the solenoid and connected to a ballistic galvanometer. The resistance of the secondary circuit including the galvanometer is 20 ohms. When a current of 2 amperes is made or broken in the solenoid what quantity of electricity is set in motion in the secondary?

Ans. $Q_2 = 1.97 \times 10^{-6}$ coulomb.

18. Suppose that in problem 17 an iron rod just filled the solenoid and that the permeability of the iron for the field was 400. What quantity of electricity would now be set in motion in the secondary when 2 amperes were made, or broken, in the solenoid?

Ans. $Q_2 = 7.90 \times 10^{-4}$ coulomb.

19. A coil of wire 20 cm long with 1000 turns carries a current of $\frac{1}{2}$ ampere. A short iron bar of length 5 cm, area of cross section 2 cm², is placed in its center. If the permeability of iron is 2000, calculate:

- (a) The flux through the volume to be occupied by the iron before it is introduced.
(b) The flux through the iron bar when in the field.

Ans. (a) $\phi = 20\pi = 62.8$ lines; (b) $\phi = 4 \times 10^4 \pi = 12.57 \times 10^4$ lines.

20. The earth inductor shown in the lecture demonstration has 100 turns and a radius of 10 cm. Its resistance together with the galvanometer was 50 ohms. If $4\pi \times 10^{-8}$ coulomb of electricity passed in one-half a revolution, calculate H_T , the total intensity of the earth's field. If the dip was 75°, calculate the horizontal intensity of the earth's field.

Ans. $H_T = 1.00$ oersted; $H_H = 0.26$ oersted.

21. The earth inductor shown in the lecture has fifty turns and a radius of 20 cm. Its resistance with that of the ballistic galvanometer was 100 ohms. If $4.6\pi \times 10^{-8}$ coulomb were observed to flow on one-half a rotation, calculate the total intensity H_T of the earth's field. If the dip was 70° what was the horizontal component?

Ans. $H_T = 0.59$ oersted; $H_H = 0.20$ oersted.

22. In the demonstration experiment a secondary coil of twenty turns is wound about a primary coil of 200 turns having an area of 2 cm² and length 40 cm. A current of 0.5 ampere is suddenly broken in the primary.

(a) How many coulombs passed through a galvanometer the resistance of which with that of the coil was 100 ohms?

(b) Had a piece of iron $\mu = 2000$, and 20 cm long with an area of 1 cm², been in the coil what would the quantity in coulombs have been?

Ans. (a) $Q_2 = 4\pi \times 10^{-9} = 12.57 \times 10^{-9}$ coulomb; (b) $Q_2 = 4\pi \times 10^{-6} = 12.57 \times 10^{-6}$ coulomb.

23. A coil of unknown number of turns n , owing to its self-induction when a current of 0.1 ampere is suddenly broken in it, delivers a quantity of 0.2 coulomb through a galvanometer of resistance of 1000 ohms. The coil is 31.4 cm long, has no iron in it, and has an average area of cross section of 500 cm². Remembering that when the field H due to the current collapses the flux cuts the n turns of the wire to generate the quantity 0.2 coulomb, and taking $\pi = 3.14$, calculate n the number of turns of wire in the coil.

Ans. $n = 10^8$ turns.

24. Given the total intensity of the earth's field as about 0.4 oersted in a region. It is to be studied by means of a ballistic galvanometer and an earth inductor, radius 20 cm and 100 turns. The deflection of the galvanometer is read by telescope and scale; the scale, marked in millimeters, is 1 meter distant from the galvanometer. The galvanometer and coil have 200 ohms resistance. Calculate the figure of merit of the galvanometer to be used in order to measure H_T to 1 per cent if only one-half revolution is to be used, neglecting the damping factor and assuming that the reading can be estimated to millimeters only. The galvanometer period $T = 10$ seconds.

Ans. $k = 3.16 \times 10^{-8}$ amp per millimeter at 1 m.

25. Magnetic fields in solenoids are frequently measured by "flip coils" which are merely small analogues of the earth inductor worked by a spring and ratchet arrangement so that they always make one-half revolution in the same time and can be conveniently automatically released. Such a coil has a radius of 0.5 cm and 1000 turns of wire with a resistance of 2 ohms, whereas the galvanometer has a resistance of 18 ohms. Give the specifications of the sensitivity and figure of merit of a galvanometer to be used at 1 meter with a scale divided in millimeters in order to measure fields of 50 oersteds to 1 per cent. The period of the galvanometer is 10 seconds.

Ans. $k = 2.47 \times 10^{-7}$ amp per millimeter at 1 m; $s = 4.05$ megohms.

26. The hysteresis loop experiment has a primary coil of 400 turns, area 0.25 cm², length 20 cm. The secondary coil has 1000 turns and is connected to a galvanometer of resistance 1000 ohms. The current changes by 0.2 ampere each time the switch

is closed. In order to get 100-mm deflection at 1-meter scale distance, what must the figure of merit k of the galvanometer be if T , the period, is about 10 seconds? If the circuit is broken in 10^{-4} second on the average, what will the instantaneous values of potential in volts and the current in amperes be?

Ans. $k = 7.90 \times 10^{-11}$ amp per mm at 1 m; $E_2 = 0.04 \pi = 0.126$ volt;
 $i = 4 \pi \times 10^{-5} = 1.26 \times 10^{-4}$ amp.

27. In the demonstration experiment where a wire crossed the field of the electromagnet sweeping out an area at the rate of 10 cm^2 in 0.2 second, the wall galvanometer, of 20-second period having a figure of merit $k = 2 \times 10^6$ and resistance 100 ohms, gave a deflection 250 mm at 1-meter distance. How many lines of force were cut in the 10 cm^2 area? What is the value of B across the pole piece of the magnet in which the wire moved? *Ans.* $\Delta\phi = 15.9 \times 10^6$ lines; $B = 1.59 \times 10^6$ gauss.

28. In an experiment, a wire is moved along rails across a field from a powerful electromagnet; the rails were 10 cm apart. When the wire was moved 2 cm in 0.2 second a deflection of 20 cm on a scale at 1-meter distance was obtained when $k = 5 \times 10^{-8}$, $T = 2$ second, ρ was negligible, and R was 20 ohms. What was the number of lines cut, and what was the average value of H in the region cut?

Ans. $\Delta\phi = 6.36 \times 10^3$ lines; $H = 318$ oersteds.

PROBLEMS BASED ON CHAPTERS XX, XXI, AND XXII

1. A secondary coil of 100 turns is wound about a primary coil of 5000 turns, area 4 cm^2 , and length 20 cm. An unknown current is suddenly broken in the primary. The ballistic galvanometer with resistance of 100 ohms including the resistance of the secondary coil indicated that 7.536×10^{-5} coulomb had passed through the secondary during the time of breaking. Calculate: (a) the current flowing in the primary; and (b) the field existing in the primary when the current flowed.

Ans. $i = 6$ amp; $\phi = 7540$ lines.

2. A coil of ten turns, with a mean diameter of 1 cm, is connected to a ballistic galvanometer, the resistance of the complete circuit being 5 ohms. The coil is placed between the poles of an electromagnet initially with its plane normal to the field and then quickly turned through 180° . It is noted that 3.1416×10^{-5} coulomb of electricity pass through the galvanometer. Determine the field between the poles of the magnet.

Ans. $H = 10^3$ oersteds.

3. Five hundred turns of wire are wound on a square frame, the average length of each turn being 160 cm. The frame makes 300 r.p.m. about a horizontal axis, through its center and parallel to an edge. There is a uniform horizontal magnetic field of 10 oersteds at right angles to the axis of rotation. The initial position is taken when the plane of the frame is normal to the field. The wire on the frame is connected through ring connectors with an external circuit, the resistance of the complete circuit being 10 ohms.

(a) Determine the quantity of electricity set in motion for half revolutions, starting from the initial position.

(b) What is the average current during this half revolution? Will this be the average current for an indefinite time the rate of rotation being constant at 300 r.p.m.?

(c) Determine the minimum e.m.f. and the maximum e.m.f. in volts, and the positions of the frame at the instant these values are obtained.

(d) Determine the e.m.f. in volts at the instant the frame makes an angle of 30° with its initial position.

(e) Find the average e.m.f. and the virtual e.m.f.

Ans. (a) 1.6×10^{-2} coulomb; (b) 0.16 amp; yes; (c) min. e.m.f. = 0, $\theta = 0$; max. e.m.f. = 2.515 volts, $\theta = 90^\circ$; (d) $E_{30^\circ} = 1.258$ volts; (e) $E = 1.6$ volts, 1.778 volts.

4. A motor runs on a P.D. of 210 volts and draws 10 amperes of which 9 *actually go to running the motor* and 1 goes to the field and hysteresis losses. It has an internal resistance of 1.11 ohms. Calculate:

(a) The back e.m.f.

(b) The power consumed in useful work if 9 amperes go to useful work and 1 ampere goes to various losses beyond the i^2R loss in the motor.

(c) If the current drawn is 10 amperes while that going to useful work is 9 amperes, calculate first the ideal efficiency and then the real efficiency of the motor.

(d) If it makes $6000/\pi$ r.p.m., what is the force on a pulley of 5-cm radius given $g = 1000$ cm per second squared. (Note: Remember that watts are joules per second.)

(e) If it were run as a dynamo at $6000/\pi$ r.p.m. and were placed across an external resistance of 18.99 ohms, calculate the current drawn and the P.D. that was maintained at the terminals. *Ans.* (a) 200 volts; (b) 1800 watts; (c) 95.3%, 85.8%; (d) 18 kg; (e) 9.95 amp, 189 volts.

5. A motor runs on a P.D. of 105 volts and draws 5 amperes, of which 4.5 actually go to running the motor and 0.5 goes to the field and hysteresis losses. It has an internal resistance of 1.111 ohms. Calculate:

(a) The back e.m.f.

(b) The power consumed in useful work if 4.5 amperes go to useful work and 0.5 ampere goes to various losses beyond the i^2R loss in the motor.

(c) If the current drawn is 5 amperes whereas that going to useful work is 4.5 amperes, calculate first the ideal efficiency and then the real efficiency of the motor.

(d) If it makes $6000/\pi$ r.p.m., what is the force on a pulley of 5-cm radius given $g = 1000$ cm per second squared. (Note: Remember that watts are joules per second.)

(e) If it were run as a dynamo at $6000/\pi$ r.p.m. and were placed across 8.889 ohms, calculate the current drawn and the P.D. that was maintained at the terminals.

Ans. (a) 100 volts; (b) 450 watts; (c) 95.4%, 85.8%; (d) 4.5 kg; (e) 10 amp, 88.89 volts.

6. A motor normally runs at 2400 r.p.m. Its internal resistance is 0.5 ohm, and it draws normally 5 amperes at 200 volts at full speed, giving a force of 10 kg on a pulley of 4-cm radius. A starting box having the following resistances is used: 20, 10, 5, 2.5, 1, 0 ohms. If the motor reaches its equilibrium speed at each resistance before closing the next contact, will the current through the motor exceed 10 amperes at any time? Calculate the useful work in kilowatts and the back e.m.f. at full speed. Next assume that when in *equilibrium* at any contact of the starting box the current drawn is 5 amperes. With this, calculate the back e.m.f. E' at each contact before switching over, and the speed at equilibrium in r.p.m. Finally, assuming that the back e.m.f. is constant during switching, calculate the maximum current drawn at the beginning of each step.

Ans. Useful work = 985 joules per second.

Back e.m.f. = 197.5 volts.

Back e.m.f. before switching: 97.5, 147.5, 172.5, 185, 187.5 volts.

Equilibrium speeds: 1185, 1790, 2095, 2250 r.p.m.

Current at beginning of each step = 9.75, 9.77, 9.54, 9.17, 30 amp.

7. Given a motor running on 100 volts and drawing 2 amperes with light load and an internal resistance of 1 ohm. Assume ideal efficiency. The motor makes 2400 r.p.m. under the conditions noted above. Calculate the back e.m.f. at this speed. If the applied potential is constant, calculate and plot the back e.m.f. as a function of current drawn up to 10 amperes. Then plot the load, i.e., useful work, as a function of i , and from this plot the number of revolutions as a function of load. On the same paper plot the ideal efficiency as a function of load.

Ans. Back e.m.f., light = 98 volts.

8. Two long straight parallel wires form part of a circuit. They have a radius a centimeters each, and their axes are b centimeters apart. Calculate the expression for the inductance per unit length of wires. (Hint: From Biot and Savarts' law, the flux per unit current can be computed, thus giving the self-induction.)

Ans. $L = 4 \log \frac{b}{a}$.

9. Given an air-core coil *radius* 5.65 cm, area of cross section 100 cm², length 30 cm, with $n = 2000$ turns. From the equation for the field H in such a coil and the area A , one can compute $\phi = AH$ in terms of the current i . Since the electromotive force of self-induction $E = n \frac{d\phi}{dt} = n \frac{d\phi}{di} \frac{di}{dt}$, an approximate expression for the self-induction of such an air-core coil can be derived. Derive the expression and evaluate L from it. Nagaoka has given a correction factor K to the equation deduced above, $L = \frac{4\pi A n^2}{l} K$. In this he has computed K as a function of $\frac{2r}{l}$ (which in this case is 0.373 and gives $K = 0.86$). What is the more accurate value of L ? K decreases in value from 1.00 when $\frac{2r}{l} = 0$. Can you explain why K is less than unity for a shorter coil?

Ans. Approximate formula: $L = 0.168$ henry; true value = 0.144 henry.

10. A toroidal solenoid has a mean radius r centimeters, and an area of cross section πa^2 . It is wound with n turns of wire and has a permeability μ . Calculate its self-induction. If $r = 20$ cm, $\pi a^2 = 5$ cm², $n = 2000$, $\mu = 2000$, and the resistance is 10 ohms, calculate the time constant of the inductance. *Ans.* 0.4 second.

11. In winding self-inductances for radio work it is necessary to reduce the resistance of the inductances used. In other words, the ratio $\frac{L}{R}$ is important for a coil, where L = inductance and R is the resistance, and it is necessary to have a certain value L with an R of a certain value or $\frac{L}{R}$ of a certain value. Considering only the single-layer solenoid and using the approximate expression for L without the Nagaoka factor K , determine the value of the ratio $\frac{L}{R}$ in terms of the radius of the wire r , the radius of the coil a , the number of turns, and the specific resistance R_0 . Then given the specific resistance of copper $R_0 = 2 \times 10^{-8}$ ohm \times cm. It is required to construct a single-layer coil of $L = 0.01$ henry with a resistance of 10 ohms. Set up the equations for L and $\frac{L}{R}$ and compute the value of the product ra (r radius wire, a radius of coil) needed. Choose a reasonable value of r and thus get a . From this the specifications for the coil, i.e., the number of turns n , can be calculated.

Ans. $ra = 0.2025$.

Sample answer: $r = 0.04$ cm; $a = 5.00$ cm; $l = 63.2$ cm;

$n = 790$ turns; $R = 10$ ohms; $L = 0.01$ henry.

12. A coil of 10,000 turns is wound on a continuous iron core of 5×5 cm cross section, and 50 cm total length. With 1-ampere current in the coil $\mu = 150$. Calculate the self-induction L of the coil in centimeters and henrys. Had the current been 0.1 ampere, μ would have been higher, perhaps 250. What would L have been? It is seen that in an iron-core coil μ and hence L are functions of the m.m.f. Air-core coils do not suffer from this difficulty. However, since the reluctance is difficult to compute, the self-induction is not calculated so simply. Had the current been broken at the rate of 1 or 0.1 ampere in 2.5×10^{-3} second (i.e., about at the maximum rate of change in a 60-cycle alternating current), what would the e.m.f. of self-induction in volts have been in both cases?

Ans. 94.2×10^9 cm, 94.2 henrys; 157×10^9 cm, 157.0 henrys;
37,650 volts, 6280 volts.

13. Given self-inductions of 0.02, 0.05, and 0.01 henry quite independent of each other and of equal resistance. What is their total self-induction in series? What is it in parallel? A current is changing in the coils in series at the rate of 1 ampere in 10^{-4} second. What will be the e.m.f. of self-induction across each coil and the total e.m.f. of self-induction? If the coils are in parallel and, for some reason, the rate of change of current in the 0.05-henry coil is 10^4 ampere per second, what will be the rate of change in each of the other coils in parallel and the total rate of change of

current in the combination? What will be the e.m.f. of self-induction in each case and the total e.m.f.? *Ans.* Total self-induction in series = 0.08 henry.

Total self-induction in parallel = 0.00588 henry.

E.m.f. in series:

$$E_{0.02} = 200 \text{ volts.} \quad E_{0.1} = 100 \text{ volts.}$$

$$E_{0.05} = 500 \text{ volts.} \quad E_{\text{total}} = 800 \text{ volts.}$$

Rate of change in parallel:

$$0.02 = 2.5 \times 10^4 \text{ amp per second.}$$

$$0.01 = 5 \times 10^4 \text{ amp per second.}$$

$$\text{Total} = 8.5 \times 10^4 \text{ amp per second.}$$

14. If a circuit has negligible resistance, i.e., $R = 0$, set up the equation for the circuit containing self-induction L only when an e.m.f. E is applied. Integrate it, and show how current will grow with time. *Ans.* $i = \frac{E}{L} t$.

15. When a Wheatstone bridge is exactly in balance and the current is suddenly started by closing the battery key, an instantaneous deflection will often be noted when measuring the resistance of a coil of wire. Can you see why the test for balance should be made by closing the galvanometer key only?

16. One of the lecture demonstration electromagnets carried a current of 10 amperes and had an inductance of 2 henrys. The current was suddenly broken and the e.m.f. of self-induction was shorted through the arc path. How much energy in joules was dissipated, and where did it come from? *Ans.* 100 joules.

How does this compare with the energy liberated in 1 second by a strong incandescent light bulb? Can you now understand where the heavy arcs drawn when the magnet current is broken, whose presence led Joseph Henry to the discovery of self-induction, come from?

17. The frequency ν of an oscillating circuit is given by $\nu = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$, where L is the self-induction in henrys and C is the capacity in farads. In an oscillating circuit two inductances L_1 and L_2 of a value of 0.05 henry each were placed in series across a condenser of 0.02 microfarad. They were in such a position that there was considerable interaction between them, though M was unknown. It was assumed from the windings that they were "aiding," i.e., that the current flowed through them in the same sense. A frequency less than 3.5×10^3 cycles per second was anticipated because of mutual inductance. The wave meter gave a frequency of 4000 cycles per second. What was wrong? What was the mutual inductance M of the coils?

$$\text{Ans. } M = 0.0109.$$

18. Given two inductances L_1 and L_2 of values 0.01 and 0.03 henry, respectively. They are connected in series with coils "aiding"; the resultant inductance is 0.06 henry. What is the value of the mutual inductance? Had they been connected with coils "opposed," what would have been the inductance of the combination?

$$\text{Ans. } M = 0.01 \text{ henry; } 0.02 \text{ henry.}$$

19. (Note: In solving this problem do not multiply out or divide by π as the values of π appearing should cancel.) A transformer has a continuous iron core of 15π cm length, $5 \times 4 = 20$ cm² area in cross section, and an average value of $\mu = 2000$. A primary coil has forty turns, and di amperes is the change of current in dt seconds, as will be seen below. The secondary coil has 1×10^4 turns. The transformer is fed by a 100-cycle-per-second alternating current ($N = 100$), i.e., one with $T = 1/100$. For an alternating current $i = i_0 \sin(2\pi t/T)$, whence $di/dt = (2\pi/T)i_0 \cos(2\pi t/T)$, which is a maximum when $\cos(2\pi t/T) = 1$, the time at which sparking occurs. Hence

$$\frac{di}{dt} = \frac{2\pi}{T} i_0.$$

With these data calculate:

(a) The reluctance Z of the transformer.

(b) $\frac{d\phi}{dt}$ in the iron in terms of i_0 .

(c) E in the secondary expressed as volts in terms of i_0 .

(d) The value of i_0 in the primary necessary to give a potential of $10,000 \pi$ volts in the secondary (i.e., that to give a 1-cm spark in air).

(e) The self-induction of primary and secondary coils in henrys.

Ans. (a) $3.75 \pi \times 10^{-4}$; (b) $8.53 \times 10^6 \pi i_0$; (c) $8.53 \times 10^8 \pi i_0$ volts; (d) $i_0 = 1.17$ amp; (e) $L_p = 1.70 \times 10^{-2}$ henry, $L_s = 1.09 \times 10^8$ henry.

20. (Note: In solving this problem do not multiply out or divide by π as the values of π appearing should cancel.) A transformer has a continuous iron core of 10π cm length, $5 \times 4 = 20$ cm² area in cross-section, and an average value of $\mu = 3000$. A primary coil has twenty turns, and di amperes is the change of current in dt seconds, as will be seen below. The secondary coil has 1×10^4 turns. The transformer is fed by a 500-cycle-per-second alternating current ($N = 500$), i.e., one with $T = 1/500$. For an alternating current $i = i_0 \sin(2\pi t/T)$, whence $di/dt = (2\pi/T)i_0 \cos(2\pi t/T)$, which is a maximum when $\cos(2\pi t/T) = 1$, the time at which sparking occurs. Hence

$$\frac{di}{dt} = \frac{2\pi}{T} i_0.$$

With these data calculate:

(a) The reluctance Z of the transformer.

(b) $\frac{d\phi}{dt}$ in the iron in terms of i_0 .

(c) E in the secondary expressed as volts in terms of i_0 .

(d) The value of i_0 in the primary necessary to give a potential of $10,000 \pi$ volts in the secondary (i.e., that to give a 1-cm spark in air).

(e) The self-induction of primary and secondary coils in henrys.

Ans. (a) $4.175 \times 10^{-4} \pi$; (b) $4.8 \pi \times 10^7 i_0$; (c) $4 \pi \times 10^9 i_0$ volts; (d) 2.08 amp; (e) $L_1 = 9.6 \times 10^{-3}$ henry; $L_2 = 1400$ henrys.

21. A toroidal solenoid has a mean radius r centimeters, and an area of cross section πa^2 . It is wound with n turns of wire and has a permeability μ . Calculate its self-induction. If $r = 20$ cm, $\pi a^2 = 5$ cm², $n = 2000$, $\mu = 2000$, and the resistance is 10 ohms, calculate the time constant of the inductance. Ans. 0.4 second.

22. A toroidal solenoid has a reluctance of $Z = 4 \pi \times 10^{-3}$. The primary coil wound about it has $n_1 = 20$ turns and a resistance of 10^{-3} ohm. The secondary coil has $n_2 = 1000$ turns. An alternating potential of frequency $N = \frac{5000}{\pi}$ and period

factor $p = 2\pi N = 10^4$ with a peak value of 100 volts is applied to the primary. Calculate the following quantities:

(a) The self-induction L_1 of the primary, and L_2 of the secondary coils in henrys.

(b) The mutual induction M in henrys.

(c) The impedance z_1 of the primary in equivalent ohms.

(d) The open-circuit e.m.f., E_2 , in volts, in the secondary when $\frac{di_0}{dt}$ has its peak

value $p \frac{E_0}{z}$ amperes.

Ans. (a) $L_1 = 4 \times 10^{-4}$ henry, $L_2 = 1$ henry; (b) $M = 2 \times 10^{-2}$ henry; (c) $z_1 = 4$ ohms; (d) $E_2 = 5000$ volts.

23. A shunt-wound motor runs on 110 volts direct current with a constant field current. It has an armature resistance of 2 ohms. On no load and at full speed it has a back e.m.f. of 104 volts. Calculate:

(a) The starting current.

(b) Power loss in armature at full speed.

(c) Power loss in armature at three-quarters speed.

(d) Power for useful work and other losses at three-quarters speed.

(e) Ideal efficiency at three-quarters speed.

(f) Horsepower in *d*.

Ans. (a) 55 amp; (b) 18 watts; (c) 512 watts; (d) 1248 watts; (e) 71%;

(f) 1.67 hp.

PROBLEMS BASED ON CHAPTER XXIII

1. Given the total intensity of the earth's field as about 0.40 oersted in a region. It is to be studied by means of a ballistic galvanometer and an earth inductor of radius 25 cm and 100 turns. The deflection of the galvanometer is read by telescope and scale, the scale being marked in millimeters and placed 1 meter distant from the galvanometer. The galvanometer and coil have 60 ohms resistance. Calculate the figure of merit of the galvanometer to be used in order to measure H_T to 1 per cent if only one-half revolution is to be used, neglecting the damping factor and assuming that the reading can be estimated to millimeters only. The galvanometer period is 10 seconds.

Ans. 16.5×10^{-9} amp.

2. It is required to have a ballistic galvanometer that will throw a spot of light 500 mm on a scale 1 meter distant when a quantity of 5×10^{-8} coulomb passes through it. The damping on successive swings to the right should not be more than in the ratio of 41 : 40, and the period must exceed 5 seconds for a complete swing. Calculate the figure of merit and sensitivity of the galvanometer in megohms so that in going over the catalogues you would be able to order the proper galvanometer.

Ans. 1.25×10^{-10} amp, 800 megohms.

3. It is required to have a ballistic galvanometer that will throw a spot of light 50 mm on a scale 1 meter distant when a quantity of 2×10^{-7} coulomb passes through it. The damping on successive swings to the right should not be more than in the ratio of 52 : 50, and the period must exceed 30 seconds for a complete swing. Calculate the figure of merit and sensitivity of the galvanometer in megohms so that in going over the catalogues you would be able to order the proper galvanometer.

Ans. 8.22×10^{-10} amp, 122 megohms.

4. A parallel-plate condenser of area 3140 cm² and a plate distance of 0.02 cm achieved by a thin plate of mica of dielectric constant $D = 6$ is charged to a potential of 100 volts. On discharge through a ballistic galvanometer of figure of merit $k = 4.175 \times 10^{-8}$, with a period of 8 seconds, this gives a deflection of 15.7 cm at 1 meter distance corrected for damping.

(a) Calculate C the capacity in e.s.u.

(b) Calculate Q the charge on the condenser.

(c) Calculate C the capacity in absolute e.m.u.

(d) From the ratio of the values of $\frac{C \text{ in e.m.u.}}{C \text{ in e.s.u.}}$ obtain the ratio of the units of

capacity in the e.m. and e.s. system (i.e., $\frac{C \text{ e.m.u.}}{C \text{ e.s.u.}}$). Take $\pi = 3.14$ and note that

this method is one of the methods of obtaining the ratio of the e.s.u. and the e.m.u. *Ans.* (a) 75,000 cm; (b) 25,000 e.s.u.; (c) 8.35×10^{-17} e.m.u.; (d) 9×10^{20} .

5. A high resistance was measured using the throw of a ballistic galvanometer. The condenser had a capacity of 4 microfarads. It gave a deflection when discharged of 50 cm. After recharging and short-circuiting for 100 seconds it gave a throw of 20 cm. Calculate the resistance in ohms and megohms.

Ans. 2.73×10^7 ohms; 27.3 megohms.

6. A capacity of 1 microfarad was charged to 100 volts. It was discharged through a ballistic galvanometer giving a throw of 200 mm on a scale 1 meter distant in a galvanometer of figure of merit 1.47×10^{-6} , corrected for damping. The condenser was charged and then discharged for 50 seconds through a high resistance. It was then discharged through the ballistic galvanometer and gave a throw corrected

for damping of 73.7 mm. What was the value of the unknown resistance in ohms?

Ans. $r = 5 \times 10^7$ ohms.

7. A circuit containing an electromagnet has a self-induction of 2 henrys and a resistance of 20 ohms. An e.m.f. of 100 volts is applied at $t = 0$. Plot the rise of current with time. What is the time constant?

Ans. $T = 0.1$ sec.

8. Given a circuit with 2 henrys self-induction. How long after closing a switch will it take with a resistance of 20 ohms before the current has risen to 50, 90, and 99 per cent of its full value?

Ans. 50%, $t = 0.0693$ second; 90%, $t = 0.2305$ second; 99%, $t = 0.46$ second.

9. A circuit having an inductance of 0.01 henry and a resistance of 100 ohms has a current of 1 ampere flowing in it. What is the time constant? Plot the decrease of current with time if the current is shut off at $t = 0$ and shorted through the 100 ohms.

Ans. $T = 0.0001$ second.

10. A condenser of capacity 1 microfarad is charged to 20,000 volts by a rectifier. At $t = 0$ the power is cut off and the condenser is shorted by a 4-megohm resistance tower. Plot the current through the tower as a function of time. On the same sheet plot the change of quantity of electricity on the condenser with time. What is the time constant? The resistance tower has forty 10^8 -ohm resistors in series, each of which can dissipate 10 watts of energy without undue heating. Calculate the power dissipation for one resistor at 20,000 volts across the tower and see whether this is a safe load. What is the maximum P.D. that the tower can safely carry?

Ans. Power dissipation of one resistor: 2.5 watts; max. P.D. = 40,000 volts.

11. For protection a 1-microfarad Pyranol condenser is permanently shorted by a bank of ten 10-megohm metalized resistors. The circuit is designed to take 20,000 volts. What is the maximum current bled off by the resistor unit? If the current supply is interrupted without discharging the condenser, plot the potential on the condenser as a function of time. How long before it is "safe" to touch the condenser if 200 volts is the limit of safety for a person with reasonably dry hands? What is the time constant of the circuit?

Ans. $i = 2 \times 10^{-4}$ amp; $t = 460$ seconds; $T = 100$ seconds.

12. In spectroscopic studies spark lines are excited by a condensed spark discharge. That is, a condenser of some 2 microfarads is charged to 20,000 volts and discharges across a spark gap in vacuum. The light then passes through a slit and goes to a grating housed in the same tank. This is the famous "vacuum" spectrograph with which Millikan and Bowen extended the spectroscopic studies from 1000 to 40×10^{-8} cm wave length. The spark is operated by a commutator which connected the condensers to a charging line of an "effective" resistance of 2×10^6 ohms. If the spark passes at 19,900 volts, how many times per second will it be possible to discharge the condenser? What is the time constant of the circuit? Plot the current and potential as a function of time?

Ans. $f = 0.47$ discharges per second; $T = 0.4$ second.

13. In a certain experiment where high resistances are needed these are made by drawing an India-ink line on some suitable backing making contact with two metal leads. Such a resistance was made and its value was measured using the time of discharge of a condenser of 1 microfarad. When the condenser was charged to a suitable potential and discharged at once the throw was 20 cm. After short-circuiting the condenser for 20 seconds through the India-ink resistance for the same initial charge the throw was 15 cm on the scale. Calculate the resistance R in ohms. If 10^6 ohms are called 1 megohm calculate the resistance in megohms.

Ans. $r = 7.02 \times 10^7$ ohms; $r = 70.2$ megohms.

14. Capacity is used in power packs for rectifying alternating current at high potentials to "smooth out" the pulsating character of the charge of the system when the filament of the rectifier is negative, and the discharge of the system during the time the filament is positive, on account of leakage in a bleeder resistance or the load. A 10^5 -volt x-ray outfit can be idealized as operating as follows, using a single rectifier. Since this rectifier with filament negative may carry 100 milliamperes at 100,000 volts,

it can be considered as charging the condenser as if through a resistance of 10^6 ohms. It is fed with a 500-cycle alternating current which for simplicity can be considered as making the filament of the rectifier negative for 0.001 second and applying 100,000 volts to the condenser system through an effective megohm of resistance for 0.001 second. The filament then is positive for 0.001 second, and in this time the capacity discharges through an x-ray tube drawing 10 milliamperes. This means that at 100,000 volts, or near it, the effective resistance of the discharge path is 10^7 ohms. Since the charging resistance of the rectifiers is one-tenth that of the tube, the result is that the condenser over some tenths of a second will charge up to very closely 100,000 volts. The condenser will then charge up to near its limit on the negative

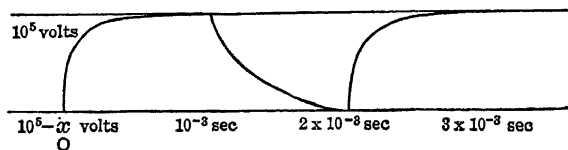


FIG. P52.

filament, and discharge for 10^{-3} second on the positive filament. The result will be a potential fluctuation of the form shown in Fig. P52. Calculate the time constant of the circuit for charge and discharge. Then plot the change in voltage with time, beginning at 100,000 volts on the discharge phase at $t = 0$ to $t = 10^{-3}$, and then the change of voltage with time in the subsequent charge phase from $t = 0$ to $t = 10^{-3}$. What fraction of the total 100,000 volts does the total amplitude of the fluctuation make? That is, calculate the ripple if the capacity is 0.100 microfarad.

Ans. Ripple = 0.001.

Before the advent of Pyranol condensers, it was rare to get a capacity of more than 0.01 microfarad. What would have been the ripple with this condenser using a 50-cycle alternating current?

Ans. Ripple = 0.05.

PROBLEMS BASED ON CHAPTER XXIV

1. Plot the instantaneous value of the current given by the equation $i = i_0 \sin\left(\frac{2\pi t}{T} + \phi\right)$, given the amplitude 2 amperes, the period 1/500 second, and the phase zero, $+45^\circ$ and -90° .

2. Assuming that a very imperfect generator, instead of giving a sine wave AC , gave one which in the interval from 0 to π radians followed a curve the contour of which was the circumference of a semicircle. What would have been the relation between the peak value of the current and the average value of the current read by a meter? To start the solution, it can be shown, from the relations for a circle referred to one end of diameter as origin, i.e., $x = r - r \cos \theta$ and $y = r \sin \theta$, that if y represents the current i , and x marks the time which for a half-cycle goes from $t = x = 0$ to $t = x = 2r$, where r is the radius which is also i_0 , the peak value of the current, that

$$i = \sqrt{i_0^2 - (i_0 - i)^2}.$$

Hence the problem is to find the average value of i^2 from $t = 0$ to $t = 2i_0$. Compare the factor by which the peak current i_0 exceeds the root-mean-square value, in this case with the factor $\sqrt{2}$ holding for a sinusoidal time variation of current. Which is larger? Can you see why? This indicates why for an uncertain wave form it is always dangerous to infer the peak value from the r.m.s. value, or vice versa, using the relation $\bar{i} = \frac{i_0}{\sqrt{2}}$.

Ans. $i = \sqrt{\frac{2}{3}} i_0$; ratio $\frac{2}{\sqrt{3}}$.

3. In radio circuits, in power packs for generating direct current from alternating current, etc., rapid electrical oscillations are prevented from reaching the output lines or from reaching the circuits by means of "choke" coils. These consist of coils of a high self-induction with as low a resistance as possible to enable the direct current to pass. Explain this on the basis of the modification of Ohm's law for the case where self-induction and resistance alone are present when sinusoidal alternating current is impressed on the line. If the resistance is 20 ohms and the inductance is 60 henrys, above what frequencies will the alternating component be reduced to less than 0.1 per cent of its original intensity? Radio-frequency chokes have a resistance of the order of 5 ohms and an inductance of 0.01 henry. At what frequency do they effectively reduce the oscillations to 0.01 per cent?

Ans. 53.1 cycles per second and 796,000 cycles per second.

4. The electromagnet with removable iron core which is in series with the lamp in the demonstration experiment has an inductance of $\frac{1}{\pi}$ henry when the core is inserted. When it is removed the inductance is about $\frac{4}{10\pi}$ henry. The resistance of the lamp and coil is about 80 ohms when the core is in and 100 ohms when it is out. The applied e.m.f. gives a voltmeter reading of 115 volts across the line. The frequency was 60 cycles per second. Calculate:

- The reactance in equivalent ohms.
- The impedance in ohms.
- The average current.
- The angle of phase lag.
- Plot the instantaneous values of E and i as a function of t . Indicate the phase lag.

(f) From (e), calculate and plot the instantaneous values of the product Ei . Plot on the same curve the product of E and i drawn to the same scale as if they were in phase.

(g) Compare the amplitudes of the two curves. Then explain the significance of the effect of the reactance in reducing current. Does it, as does an ohmic resistance R , reduce current by producing a power loss?

(h) Can you see advantage in economy by varying alternating current in a line as needed by varying the reactance instead of R ?

Ans. (a) 120 ohms, 48 ohms; (b) 144.5 ohms, 111 ohms; (c) 0.795 amp, 1.037 amp; (d) 56.4° , 26° .

5. Consider a circuit having self-induction L and resistance R in series and having an e.m.f. $E = E_0 \sin pt$ applied to it. State what will happen and why, when:

(a) R approaches 0 with an appreciable reactance. What will the energy dissipation approach?

(b) R equals the value of the reactance.

(c) R becomes very large compared to the reactance. What will the energy dissipation approach?

(d) When L approaches 0 relative to R .

(e) When the frequency approaches 0.

(f) When the frequency becomes very high, R and L being fixed. On this basis explain the action of the "choke."

(g) The effect of the phase lag when L or frequency N are large compared to R , when R is large compared to L or frequency N .

(h) What effect does a large ratio of L or LN to R have on the current, and on the power loss in heat for the same current?

6. A lamp is placed in series with a condenser. Across the two is applied a 60-cycle e.m.f. of 115 volts. When the capacity is 25 microfarads the lamp resistance is 100 ohms. When the capacity is 10 microfarads the resistance is 80 ohms, and when the capacity is 2 microfarads the resistance is 60 ohms. Calculate:

(a) The capacity reactance in equivalent ohms in each case.

- (b) The impedance in ohms in each case.
 (c) The average current i observed in each case.
 (d) The phase angle.
 (e) Plot the instantaneous values of E and i on the same sheet, indicating the phase change.

(f) On the plot above, plot in also the value of the current if the reactance were zero. Compare to the observed curve and explain.

Ans. (a) 106, 265, 1325 ohms; (b) 146, 277, 1326 ohms; (c) 0.789, 0.415, 0.0868 amp; (d) $46^\circ 45'$, $73^\circ 25'$, $87^\circ 24'$.

7. The resistance of the carbon-filament lamp used for the demonstration of the effect of capacity on a circuit may be assumed constant in what follows and equal to 100 ohms. One hundred and ten volts alternating potential of 60 cycles are applied across the lamp and condenser in series. When the capacity is 10 microfarads and 1 microfarad, calculate:

- (a) The current through the lamp in each case.
 (b) The angle of phase lag in each case in degrees.

Ans. (a) 0.389 amp, 0.0414 amp; (b) 69.3° , 87.85° .

8. A condenser is connected in series with a lamp. The capacity is C , the resistance is R , and an e.m.f. $E = E_0 \sin pt$ is applied to the system. What happens when:

- (a) C approaches 0 irrespective of R ?
 (b) Cp approaches $1/R$?
 (c) C becomes very large compared to $1/R$? What limits the current flow?
 (d) R becomes very large compared to $1/pC$ while Cp is large?
 (e) The frequency is *very* low?
 (f) The frequency is very high? Compare the reaction of a large C and a large L to high values of frequency. Can you see where condenser coupling in circuits where d-c components are not desired has its advantages?
 (g) What happens to the phase when R , frequency N , and C are all large? When R , N , and C are all small?

9. What is the value of:

- (a) An inductance that will reduce the amplitude of a 10^5 -cycle current to 0.1 per cent of a d-c component when $R = 1000$ ohms?
 (b) A capacity which will cut out all frequencies below 10^5 cycles to less than 0.1 per cent when $R = 1000$ ohms?
 (c) The impedance when the inductance in (a) and the condenser in (b) are in series with R ?

Ans. (a) $\frac{5}{\pi}$ henrys = 1.59; (b) $\sim 1.3 \times 10^{-3}$ mf; (c) 10^3 .

10. In a circuit of period factor $p = 2\pi N = 2000$, the self-induction $L = x$ henrys, the capacity is $C = 2$ microfarads ($C = 2.0 \times 10^{-6}$ farad). The resistance $R = 10$ ohms, the current as read by an a-c ammeter was 5 amperes, and the potential as read by an a-c voltmeter was 70.7 volts. The power as read by a wattmeter was 249.9 watts. Assuming that there was a phase lag, calculate the following:

- (a) The apparent watts and the power factor.
 (b) The angle of phase lag and its negative tangent.
 (c) The self-induction $L = x$ in henrys.
 (d) The reactance in equivalent ohms.
 (e) The impedance in equivalent ohms.
 (f) Show that the current observed is consistent with the impedance found.

ϕ	0	13°	23°	35°	45°	53°	60°	73°	90°
$\cos \phi$	1	.974	.921	.819	.707	.600	.500	.292	0.000
$\tan \phi$	0	.231	.425	.700	1.00	1.333	1.732	3.271	∞

The square root of 2 is to be taken as 1.414.

Ans. (a) 353.5, 0.707; (b) 45° , -1 ; (c) 0.130; (d) 10; (e) 14.14.

11. In a circuit of frequency $N = 500$ cycles per second and period factor $p = 2\pi$ 500 the self-induction $L = 1.04/\pi$ henry and the capacity is $C = x/\pi$ microfarads ($C = x/\pi \times 10^{-6}$ farad). The resistance $R = 30$ ohms, the current as read by an a-c ammeter was 2 amperes, and the potential as read by an a-c voltmeter was 100 volts. The power as read by a wattmeter was 120 watts. Assuming there was a phase lag, calculate the following:

- The apparent watts and the power factor.
- The angle of phase lag and its negative tangent.
- The capacity $C = x/\pi$ in microfarads.
- The reactance in equivalent ohms.
- The impedance in equivalent ohms.

Ans. (a) 200, 0.6; (b) $53^\circ 8'$, -1.335 ; (c) $x = 1$; (d) 40 ohms; (e) 50 ohms.

12. In a circuit of frequency $= 5000/\pi$ cycles per second, the self-induction $L = x$ henrys, the capacity $C = 1$ microfarad. The resistance $R = 30$ ohms, the current as read by an a-c ammeter was 2 amperes and the potential as read by an a-c voltmeter was 100 volts. The power as read by a wattmeter was 120 watts. Assuming there was a phase lag calculate the following:

- The apparent watts and the power factor.
- The angle of phase lag and its negative tangent.
- The self-induction $L = x$ in henrys.
- The reactance in equivalent ohms.
- The impedance in equivalent ohms.
- Show that the current observed is consistent with the impedance found.

ϕ	0	13°	23°	37°	45°	53°	60°	73°	90°
$\cos \phi$	1	0.974	0.921	0.800	0.707	0.600	0.500	0.929	0.000
$\tan \phi$	0	0.231	0.425	0.750	1.000	1.333	1.732	3.271	∞

Ans. (a) 200 watts, power factor 0.600; (b) $\phi = 53^\circ$, $-\tan \phi = -1.333$; (c) 0.0140 henry; (d) 40 ohms; (e) $z = 50$ ohms; (f) $i = 2$ amp.

13. A circuit with inductance and resistance only is in series with a 60-cycle generator and gives a root-mean-square current of 10 amperes with a power factor of 0.5. Find:

- The resistance.
- The inductance.
- The rms potential applied if the impedance is 10 ohms.
- What is the heat in calories from the resistance in 1 minute?
- Is there a phase lag or advance? (Hint: Use the vector diagram for this problem.)

Ans. (a) 5 ohms; (b) 0.229 henry; (c) 100 volts; (d) 7.1×10^3 cal.

PROBLEMS BASED ON CHAPTER XXV

1. In a section of a radio circuit an inductance of $0.04/\pi$ henry is in series with a condenser of $0.04/\pi$ microfarad. The resistance is 5 ohms. Three alternating potentials of 100, 10, and 1 volts at frequencies of 50, 10^4 , and 2×10^6 cycles, respectively, are impressed on the circuit. What are the currents at each of the frequencies?

Ans. 100 volts, $i = 4 \times 10^{-4}$ amp; 10 volts, $i = 2.215 \times 10^{-2}$ amp; 1 volt, $i = 6.25 \times 10^{-6}$ amp.

2. An e.m.f. of 110 volts and 500 cycles per second is applied to a circuit having a resistance of 10 ohms, an inductance of $2/\pi$ henry, and a capacity of $1/\pi$ microfarad in series. Calculate:

- The reactance.
- The impedance.
- The current.
- The angle of phase lag or advance, stating which.
- The power factor.

(f) The true power consumption.

(g) The applied or apparent power consumption. Where does the true power consumption go? What happens to the difference between true and apparent power consumption? Prove that the true power consumption is all converted to i^2R loss.

Ans. (a) 1000 ohms; (b) 1000 ohms; (c) 0.11 amp; (d) $89^\circ 26'$ lag;
(e) 0.01; (f) 0.121 watt; (g) 12.1 watts.

3. Plot the relation for the variation of phase angle with frequency for the circuit described in problem 2.

Assuming R , C , and N constant, how do i and ϕ vary with L ?

Assuming R , L , and N constant, how do i and ϕ vary with C ?

4. A toroidal solenoid of 10-cm^2 cross section and 20-cm length, having a permeability $\mu_r = 1000$, has two coils L_1 and L_2 wound close together, L_1 having ten turns and L_2 having 1000 turns, both wound clockwise. Assume no leakage of flux. Calculate:

(a) The self-induction of L_1 and L_2 .

(b) The mutual induction of L_1L_2 .

(c) The e.m.f. induced in L_2 on open circuit when a sinusoidal e.m.f. of 500 cycles and 110 volts is applied to L_1 which has a resistance of 0.001 ohm.

(d) The impedance of L_1 and the current through it.

Ans. (a) $L_1 = 6.28 \times 10^{-4}$ henry, $L_2 = 6.28$ henry; (b) $M = 6.28 \times 10^{-2}$ henry;
(c) 11,000 volts; (d) 55.7 amp.

5. A simple coil of wire has an area of 100 cm^2 and contains 100 turns. It is rotated sixty times per second in a uniform field of 1000 oersteds. If it is short-circuited through a resistance of 1 ohm, calculate the current flowing if the self-induction of the coil is 0.002 henry.

Ans. $i = 30.2 \sin(\omega t - 0.65)$ amp.

6. Given a circuit with a capacity C of 1 microfarad, a self-induction of 1 henry, and a resistance R of 4000 ohms, plot the curve for the discharge of the condenser originally charged to 100 volts. Will it be oscillatory?

7. In making an oscillating circuit, L was chosen as 0.2 henry, C was taken as 0.0014 microfarad. What is the maximum allowable value for the resistance R in order to make the circuit oscillate? It is desirable to have the circuit make ten oscillations at its natural frequency before it is damped to $1/e$ of its value. Assume $T = 2\pi\sqrt{LC}$. What must the limiting value of R be to allow for this? Given $R = 2 \times 10^{-6} \frac{l}{\pi r^2}$ for copper wire and that for the general type of winding of the

single-layer inductance $L = \frac{4\pi^2 An^2}{l 10^9} K$ henry, with K of the order of 0.8, l the length of the wire, n the number of turns, and A the area of the coil, will it be possible to make a coil with $L = 0.2$ henry, with a resistance less than the value required? About what must the wire diameter be, and what would A be?

Ans. $R = 2390$ ohms; $d = 0.23$ cm; $A = 200\text{ cm}^2$.

8. A circuit containing a capacity of 0.002 microfarad, self-induction of 0.01 henry, and a resistance of 1000 ohms is charged to 1000 volts and the circuit is suddenly closed. Calculate:

(a) The period and frequency.

(b) The tangent of the angle of phase lag and its value in degrees.

(c) The damping coefficient.

(d) Plot the charge q as a function of t for 4 cycles.

(e) How long will it take for the oscillation to die out to 0.1 per cent of its initial value.

Ans. (a) $T = 2.88 \times 10^{-6}$ second; $f = 3.47 \times 10^4$ vibrations per second;

(b) $\tan \phi = 0.229$, $\phi = 12^\circ 54'$; (c) 50,000; (e) 1.385×10^{-4} second.

PROBLEMS BASED ON CHAPTERS XXVII AND XXVIII

1. (a) Calculate the value of e/m in electromagnetic units per gram for hydrogen and for cupric ions of copper from this data:

1 faraday = 96,500 coulombs. Atomic weight Cu = 63.57.

Atomic weight H = 1.008. Valence = 2.

(b) In a measurement carriers of electricity are found to have an e/m of 89.5 e.m.u. per gram. List one possible ion which would have this e/m .

(c) Given the Avogadro number (number of molecules in a mole) as 6.02×10^{23} , calculate the charge e on an ion in e.s.u. and e.m.u.

Ans. (a) $e/m_H = 9.58 \times 10^8$ e.m.u. per gram, $e/m_{Cu^{++}} = 3.04 \times 10^2$ e.m.u. per gram; (b) 107.8 silver; (c) 1.60×10^{-20} e.m.u., 4.8×10^{-10} e.s.u.

2. What will be the energy in volts and in ergs of photoelectrons liberated from a nickel surface having a work function of 2.77 volts through the agency of potassium $K_{\alpha 2}$ x-rays of wave length 3.737×10^{-8} cm? What will be their velocity neglecting relativity corrections? Calculate the same for electrons liberated by light from the mercury line of $\lambda = 2.536 \times 10^6$ cm.

Ans. $E = 3.303 \times 10^8$ volts = 5.256×10^{-9} erg; $v = 3.41 \times 10^9$ cm per second;
 $E = 2.10$ volts = 3.34×10^{-12} erg; $v = 8.6 \times 10^7$ cm per second.

3. A gold-leaf electroscope and system has a total capacity of 5 cm. It is charged to 500 volts. In 1 minute and 40 seconds the leaf falls six divisions in the scale of a microscope when ionization due to γ rays from 1 mg of radium falls on it from 2-meter distance. A very feeble source of radium placed 20 cm distant causes a fall of the same leaf of two divisions in 5 minutes. The leakage current due to cosmic rays and surface leakage amounts to a fall of one division in 25 minutes. The scale over the six divisions is linear, and one division amounts to 0.1 volt. Assuming that γ ray ionization properly filtered decreases as the inverse square of the distance, calculate the following data:

(a) The leakage current in amperes.

(b) The current for the 1 mg of Ra corrected for leakage.

(c) The current for the unknown sample corrected for leakage.

(d) The quantity of radium in milligrams in the unknown sample.

(e) If the volume from which the ions were collected was 10 cm^3 calculate the number of ion pairs or equivalent ion pairs produced per cubic centimeter per second in each case. Ans. (a) 3.7×10^{-16} amp; (d) 1.015×10^{-3} mg;

(b) 3.29×10^{-14} amp; (e) 2.092×10^8 pairs per second.

(c) 3.33×10^{-16} amp;

4. Keck and Loeb, *Review of Scientific Instruments*, Vol. 4, p. 486, 1933, state at the end of their paper that it should be possible with a special ion source to separate the isotope of Li of atomic weights 6 and 7 in appreciable amounts. These are needed to interpret the disintegration experiments produced by impacts of deuterons (H of mass 2) and protons H^+ on Li. The results of Keck and Loeb showed that for 345-volt ions of Li^+ the greater proportion had energies lying between 315 and 345 volts. Separation requires that the edge for the most deflected ions of one species from a source of a given width just does not overlap the least deflected ions of the other species. If the source is 1 mm wide, what magnetic field would be used, utilizing the Dempster type of mass spectrograph, to separate Li 7 from 315–345 volts from Li 6 extending over 315–345 volts? Ans. $H = 4100$ oersteds.

5. Radioactive substances decay according to a law $N_t = N_0 e^{-\lambda t}$, where t is the time in seconds, λ is the constant characteristic of a given change, N_t is the number of atoms at a time t , and N_0 is the initial number of atoms at $t = 0$. The rate of growth of the resultant substance is $N_t = N_\infty (1 - e^{-\lambda t})$, where N_∞ is the ultimate number of particles after all the parent substance has transformed. λ for radium is 4.4×10^{-4} (years $^{-1}$). If radium changes at this rate to emanation and emanation undergoes change to radium A, B, C, and D at rates far greater than the rate of change of radium, the amount of radium transformed in a year will have undergone practically complete change to D in this time. (a) In the changes involved from

Ra to RaD how many α particles are given off? (b) starting with 1 gram of radium having $\frac{6.06 \times 10^{23}}{226}$ atoms how many atoms undergo change in the course of a year?

(c) If each of these gives the same number of α rays, compute how many mm³ of He at N.T.P. (number of atoms in 1 cm³ = 2.705×10^{19} at N.T.P.) are formed from a gram of Ra in a year? (d) Check this value against that computed from the fact that 1 gram of radium gives out 1.48×10^{11} α particles per second, in its change to RaD.

Ans. (a) 4 α particles; (c) He₁ = 174 mm³;
(b) 4.71×10^{18} atoms per year; (d) He₂ = 174 mm³.

6. Scintillation counts show that 1 gram of radium in equilibrium produces 1.36×10^{11} α particles per second. Rutherford let the fraction of α particles from 10 mg of radium that entered a cone of 1 cm in diameter at 10-cm distance fall on a Faraday cylinder in vacuum connected to an electrometer, the capacity of the system being 100 cm. It was observed that the electrometer indicated a change of 2.43 volts in 16 minutes and 40 seconds. Calculate the charge on an α particle. If the electron has a charge of 4.77×10^{-10} e.s.u., what multiple of this charge does the α particle carry?

Ans. $\alpha = 9.53 \times 10^{-10}$ e.s.u.; $\alpha = 2 e$.

7. In the thorium series of disintegrations, given the following sequence of changes:

Th α Meso Th 1 β Meso Th 2 β Radio Th α Th X α Th Em α Th A α Th B β Th C β Th C' α Th D. If thorium falls in Group IV (the lead group) and in the seventh period having an atomic weight 232, deduce the chemical behavior of the transformation products down the chain and deduce the atomic weight of Th D as well as its chemical behavior.

Th D at wt = 208, isotopic with Pb.

8. Given the atomic number of rubidium as 37. Construct a rough diagram of the atom showing the disposition of electrons in levels, and giving approximate dimensions of the various quantities entering in. Do the same for iodine, I, atomic number 53. On this basis interpret the electrochemical behavior of Rb and I.

9. Calculate the wave lengths of the following radiations from the potential through which an electron must fall to generate them. Given $h = 6.56 \times 10^{-27}$ erg sec, and the electronic charge as 4.77×10^{-10} e.s.u.

2537 A mercury line.....	4.9 volts
K α line of carbon	288 volts
K α line of tungsten.....	57,200 volts
K α line of uranium.....	99,500 volts
Hard x-rays.....	750,000 volts
γ rays RaC, very hard.....	1.79×10^6 volts

Ans. 2.53×10^{-6} cm; 1.243×10^{-9} cm;
 4.293×10^{-7} cm; 1.649×10^{-10} cm;
 2.164×10^{-9} cm; 6.915×10^{-11} cm.

10. A measurement of the charge on the α rays was made as follows. From virtually a point source of 2 mg of radium in vacuum the α particles were allowed to impinge on the circular opening of a shielded Faraday collecting cage of radius 2.5 cm (radius of opening), the source being 10 cm away. The cylinder had a capacity with electrometer system of 40 cm, and an increase of positive potential of 13.68 volts was noted in 30 minutes. The same source was next placed in a long evacuated tube 2 meters long and the scintillations counted by microscope in a field of 1 mm² area at the other end of the tube as 81 per minute. Calculate: (a) the number of α particles falling into the Faraday cage per second; (b) the charge given the cage per second in e.s.u. (c) Finally calculate the charge carried by an α particle. (How does it compare with that of an electron?)

Ans. (a) 1.06×10^6 per second; (b) 1.013×10^{-3} e.s.u.; (c) 9.56×10^{-10} e.s.u.

11. What will be the energy in volts and in ergs of photoelectrons liberated from a platinum surface having a work function of 6.27 volts through the agency of potas-

sium $K_{\alpha 2}$ x-rays of wave length 3.737×10^{-8} cm. What will be their velocity, neglecting relativity corrections? Calculate the same for electrons liberated by light from the mercury line of $\lambda = 2.536 \times 10^{-8}$ cm from copper of work function 3.85 volts.

$$\text{Ans. } E = 5.26 \times 10^{-9} \text{ erg} = 3310 \text{ electron volts;}$$

$$v = 3.42 \times 10^9 \text{ cm per second;}$$

$$E_{Hg} = 1.63 \times 10^{-12} \text{ erg} = 1.025 \text{ electron volts;}$$

$$v = 6.014 \times 10^7 \text{ cm per second.}$$

13. A series of radioactive disintegrations starts with an element 1 of atomic weight 235, Group VI of the periodic table, and undergoes the following series of transformations, the elements being labeled 1, 2, 3, etc.

$$1 - \alpha \rightarrow 2 - \beta \rightarrow 3 - \alpha \rightarrow 4 - \beta \rightarrow 5 - \alpha \rightarrow$$

$$6 - \alpha \rightarrow 7 - \alpha \rightarrow 8 - \alpha \rightarrow 9 - \beta \rightarrow 10 - \beta \rightarrow 11.$$

Determine the group in the table and atomic weight of each element. What are elements 2, 7, and 11? Indicate all isotopes.

NAME INDEX

- Abraham, 392
 Alvarez, Luiz, 475
 Amontons, 6
 Ampère, A. M., 7, 8, 52ff, 53, 56, 392, 395, 418
 Anderson, Carl D., 10, 339, 433
 Anderson, John A., 219
 Arago, 19, 180
 Arrhenius, Svante, 127
 d'Arsonval, 103, 588
 Aston, F. W., 437, 438
 Avogadro, 118

 Baeyer, O. von, 537
 Balmer, 444
 Barkhausen, 25, 26
 Barkla, C. G., 450
 Barlow, 233, 234, 619
 Barnett, S. J., 264
 Becquerel, H., 523
 Bennett, W. H., 501, 502
 Bertrand, 179
 Biot, 56, 611
 Birge, R. T., 392, 417, 418, 431
 Bitter, Francis, 25
 Blondlot, 523
 Bohr, Niels, 7, 9, 120, 122, 123, 253, 256, 257, 442ff, 443, 444, 445, 446, 447, 449, 455, 469
 Boltzmann, Ludwig, 4, 409
 Bowen, Ira S., 451, 627
 Boys, C. V., 167, 168
 Bragg, W. H., 451, 454
 Bragg, W. L., 454
 Branly, 524
 Braun, 196, 350
 Brickwedde, H., 465
 de Broglie, L. V., 5, 7, 120, 432, 445, 463, 596
 de Broglie, M., 451, 509

 Carrara, N., 459
 Cavendish, 6, 8, 176
 Chadwick, Sir James, 10, 440, 462

 Charles the Second, 3
 Child, C. D., 532
 Clausius, 4, 6, 7
 Coates, Wesley, 473
 Coblentz, W. W., 108
 Cockcroft, J. D., 10, 453, 461, 472
 Colpitts, 547
 Compton, A. H., 5, 9, 446, 485, 456ff, 459
 Compton, K. T., 508
 Condon, E. U., 10, 175
 Coolidge, W. D., 452, 453
 Coulomb, Charles A., 6, 8, 28, 29, 35, 175ff, 203, 409, 420, 439, 442, 460, 484, 596
 Crookes, Sir William, 496, 500, 551
 Curie, Pierre, 256
 Curie-Joliot, Irène and Frédéric, 10, 466

 Daniell, 141, 145, 591
 d'Arsonval, 103, 588
 Darwin, C. G., 441
 Davisson, C. J., 5, 9, 446, 528
 Davy, Sir Humphry, 6, 8
 Debye, Peter, 135, 136, 208, 212, 215, 409
 De Forest, Lee, 537
 Dempster, A. J., 437, 438
 Dirac, P. A. M., 5, 7, 9, 446, 512, 514, 528, 530
 Doan, 459
 Dorsey, N. E., 392
 Drude, 409, 527
 Duane, 410
 DuFay, 8, 51, 523
 Dulong, 511
 Dunnington, F. G., 599
 Dushman, S., 527, 528

 Edison, T. A., 147, 149, 524
 Einstein, A., 5, 7, 9, 264, 427, 464, 507, 508
 Ellis, C. D., 451
 Elster, 504, 507, 510, 513

- Fajans, 437
 Faraday, M., 6, 7, 8, 47, 118, 172ff, 179,
 189, 199, 203, 204, 233, 278, 299, 398,
 408, 416, 418, 428, 433, 496, 500
 Farnsworth, Philo T., 519
 Feddersen, 409
 Fermi, Enrico, 512, 514, 528, 530
 Fleming, 504, 524
 Fourier, 4
 Franck, James, 5, 9, 447, 448
 Franklin, Benjamin, 51, 64, 69, 171
 Fresnel, 7
 Friederich, 454

 Galilei, Galileo, 1, 7, 8
 Galvani, 51ff
 Gamow, G., 10
 Gauss, K. F., 4, 8, 30, 34, 35, 68, 199,
 204, 390, 391, 411, 417, 418
 Gay-Lussac, 19
 Geiger, H., 551
 Geitel, 504, 507, 510, 523
 Gerlach, W., 257, 258, 259, 569
 Germer, 5, 9, 446, 528
 Gibbs, Willard, 4
 Gilbert, William, 1, 8, 14, 15, 18, 19,
 169, 484
 Goldstein, Eugen, 434
 Grassot, 324, 325, 344
 Gray, 8, 51
 Green, 4, 8
 Grover, F. W., 304, 338
 Guericke, Otto von, 8
 Guerne, 10

 Haas, Arthur Erich, 264
 Hahn, Otto, 10, 469
 Hamilton, 4
 Hansen, W. W., 555
 Hartley, 546
 Heaviside, 419
 Heisenberg, Werner, 5, 7, 9, 446
 Helmholtz, 4, 60, 118, 226, 230, 322
 Henry, Joseph, 8, 299, 624
 Hertz, G., 5, 9, 447, 448
 Hertz, Heinrich, 7, 9, 409, 410, 451, 503,
 554
 Hing, Yih, 13
 Hittorf, W., 133
 Hopper, 431

 Hückel, 136
 Hudson, G. G., 501
 Hughes, A. L., 509
 Hull, A. W., 544
 Huygens, 7

 Joffé, A. F., 115
 Joule, James Prescott, 68, 74, 156, 593,
 594

 Kallman, 448
 Kamerlingh, Onnes, H., 81
 Kapitza, 277
 Keck, Paul, 633
 Kelvin, Lord (W. Thomson), 4, 6, 8, 108,
 157, 158, 159, 160, 164, 313, 325, 333,
 335, 392, 409
 Kennedy, 10
 Kerst, D. W., 10, 482
 Kip, A. F., 501
 Kirchhoff, G., 4, 8, 90ff, 354, 370, 376,
 383, 409, 582
 Knipping, 454
 Kohlrausch, 390, 391
 Kossel, W., 124

 Laby, T. H., 431
 Ladenburg, R., 508
 Lagrange, 4
 Landé, 256
 Langevin, Paul, 256, 257, 549
 Langmuir, Irving, 532
 Laplace, 4
 v. Laue, Max, 9, 451, 454
 Lawrence, E. O., 10, 472, 475, 518, 519,
 599, 613
 Lenard, Philipp, 505, 507, 508, 537
 Lenz, 8, 250, 278
 Lewis, Gilbert N., 124
 Linford, Leon, 519
 Loeb, L. B., 493, 501, 633
 Lorentz, H. A., 409, 419, 433, 527
 Lyman, 451

 MacLean, 410
 Marsden, 551
 Maxwell, James Clerk, 4, 6, 7, 9, 34, 47,
 99, 179, 206, 215, 339, 390, 393ff,
 396ff, 403, 408, 442, 445, 450, 451, 514,
 515, 527, 529

- Mayer, 4
 McClelland, 524
 McMillan, E. M., 10, 479, 481
 Meek, J. M., 493
 Meissner, 260
 Meitner, Lisa, 451
 Mendeleev, 78, 119, 437
 Mercier, 410
 Michelson, A. A., 5, 9, 31, 425, 459
 Millikan, R. A., 5, 79, 108, 118, 119, 424, 428ff, 431, 451, 454, 501, 509, 627
 Mitchell, A. Crichton, 13
 Mohler, F. L., 518
 Morley, 5, 9, 425
 Moseley, H. G. J., 119, 440, 454ff
 Mueller, 551
 Murphy, 465

 Nagaoka, 623
 Neckham, Alexander, 13
 Newton, Sir Isaac, 3, 4, 7, 31, 32, 427
 Nichols, 451, 554

 O'Day, M. D., 501
 Oersted, 8, 19, 52
 Ohm, G. S., 8, 73, 83, 90, 373, 489, 592

 Onsager, 136
 Ostwald, Wilhelm, 138
 Owens, 339

 Page, Newell C., 576, 581, 587, 598
 Paschen, 451
 Pauli, W., Jr., 5, 120, 122, 256, 260, 471, 513
 Peltier, 154, 155, 164, 593, 594
 Peregrinus, Petrus, 13, 15
 Petit, 511
 Pierce, G. W., 550
 Planck, 5, 31, 120, 432, 443, 444
 Plato, 12
 Pliny, 12
 Pohl, 115, 510, 518
 Poincaré, H., 410
 Poisson, 4, 532
 Preece, 524
 Priestley, 523
 Pringsheim, P., 517

 Quincke, 330

 Raether, H., 493
 Ramsauer, 518
 Regener, 551
 Richardson, O. W., 508, 525ff, 526, 529
 Riecke, 527
 Righi, 504
 Robinson, H., 451, 459
 Roentgen, W. K., 7, 420, 452
 Rosa, E. B., 304, 392, 501
 Rosen, 448
 Rowland, H. A., 266, 392
 Rubens, S. M., 62, 322, 451
 Rumford, Count, 6
 Rutherford, Lord, 5, 7, 9, 119, 439ff, 459, 460, 508, 551, 634
 Rydberg, 445, 455

 Saha, M. N., 447, 498
 St. Elmo, 501
 Sanders, F. G., 589
 Saunders, 410
 Savart, 56, 611
 Scheffers, 259
 Schottky, W., 519, 532
 Schroedinger, E., 5, 7, 9, 446
 Schumann, 451
 Seaborg, Glenn, 10
 Seebeck, 154
 Siemens, Werner V., 9
 Sloan, David, 453
 Snell, 408
 Soddy, F. G., 437, 460
 Sommerfeld, A., 9, 155, 445, 510, 511ff, 516ff, 529
 Stark, J., 9
 Starling, S. G., 217
 Stern, Otto, 10, 257, 258, 569
 Stewart, T. Dale, 433
 Stokes, 135
 Stoletow, 504
 Stoner, E. C., 123
 Strassman, 10
 Swann, 598

 Tear, 451, 554
 Thales, 12
 Thibaud, 451
 Thomson, G. P., 5, 9, 446
 Thomson, J. J., 5, 8, 118, 119, 420ff, 428, 433, 434ff, 438, 447, 450, 457, 505, 524, 531, 590

- Thomson, William, *see* Kelvin, Lord
Toepler, 173
Tolmon, R. C., 433
Townsend, J. S. E., 490, 491, 492
Trichel, G. W., 501
Trowbridge, Augustus, 410
- Urey, Harold, 465
- Van de Graaff, R. J., 10, 173ff, 331, 453, 461, 472, 598
Van der Bijl, 538, 540, 542
Varian, R. H., 555
Varian, S. F., 555
Veksler, 10, 479
Vinal, G. W., 147
Volta, Alessandro, 51ff, 71, 116, 141
- Wagner, 215
Walton, E. T. S., 9, 453, 460, 472, 523
Watson, 523
Weber, Wilhelm, 8, 390, 391
Webster, D. L., 555
Weiss, Pierre, 256
Wentzel, 510, 517, 518
Wheatstone, 8, 92ff, 333, 335, 624
Wien, Wilhelm, 434
Williamson, 518
Wilson, C. T. R., 439, 457, 551
Wilson, H. A., 428, 526
Wylie, Alexander, 13
- Young, Thomas, 7
- Zeeman, Pieter, 9, 408, 433, 439, 445
Zworykin, 519

SUBJECT INDEX

This index comprises three kinds of references: (1) chapter and section titles and other important topics are shown by "ff" after the page number that indicates the beginning of the topic; (2) relatively important topics defined or discussed are designated by page numbers in italic type; (3) casual mention or incidental discussion of important topics is indicated by page numbers in ordinary type.

- Absolute current balance, 313, 325, 328
- Absolute current measurement, 325
- Absolute current standards, 325
- Absolute determination, of capacity, 335, 336, 338
 - of current, 63, 313, 325, 326, 328
 - of pole strength and magnetic fields, 40ff
 - of potential, 71, 185, 328, 332
 - of quantity static, 178
 - of resistance, 333, 335
 - of self-induction, 338, 339
- Absolute dynamometer, 313, 325
- Absolute electromagnetic measurement
 - of current, 63, 313, 325, 328
- Absolute electromagnetic system, 54ff, 411, 412, 413, 417
- Absolute electrometer, 194ff, 329, 332
- Absolute electrostatic system, 175ff, 411, 412, 413, 417
- Absolute e.m.u., of capacity, 200, 201, 412, 413
 - of potential, 69, 412, 413
 - of quantity, 56, 412, 413
 - of resistance, 73
 - of self-induction, 302, 412, 413
- Absolute e.s.u., of capacity, 201, 412, 413
 - of current, 180, 412, 413
 - of potential, 184, 412, 413
 - of quantity, 179, 412, 413
 - of resistance, 412, 413
 - of self-induction, 302, 412, 413
- Absolute ohm, determination of, 333, 335
- Absolute static currents, measurement
 - of, 326, 328
- Absorption coefficients, in energy determination, 332
- Absorption coefficients, of x-rays, 450, 451, 452
- Achievement of high-energy particles, 472ff
- Acid cell, 141ff
- Air-core coil, coefficient of self-induction for, 303
- Air gaps in magnetic circuits, 274
- Alnico, 245
- Alpha particles, 439
 - anomalous scattering of, 441
 - diffraction of, 441
 - nuclear transformations by, 442
 - orbits about nuclei, 440
 - problems on, 634ff
 - scattering laws of, verified, 440
 - scattering of, 439ff
 - method of study of, 441
- Alpha-particle tracks, 439, 440
- Alpha-particle waves, 441
- Alpha rays, 460
- Alternating current, angular velocity in, 366
 - effect of capacity on, 375ff
 - effect of self-induction and capacity combined, 375ff, 378ff
 - effect of self-induction on, 363ff
 - failure of Ohm's law in, 373, 378
 - flow of, through capacity, 375
 - frequency of, 364, 366
 - importance of, 363ff
 - period factor in, 366
 - period of, 364, 366
 - phase in, 364, 366
 - power measured by wattmeter, 340
 - small high-frequency, measurement of, 325, 327

- Alternating current, transients in, 371
 vector diagram in, 374, 378
 Alternating-current ammeters, 110
 Alternating-current bridge, comparison
 of capacities, 336, 338
 comparison of inductances, 339
 Alternating-current definitions, 363ff
 Alternating-current dynamos, 291
 Alternating-current instruments, values
 of currents given by, 366ff
 Alternating-current measurement, 327,
 328
 Alternating-current rectifiers, 327, 328
 Alternating-current studies, oscillogra-
 phic, 353
 Alternating-current voltmeters, 110, 329,
 332
 Alternating currents, problems on, 628ff
 Alternating currents I, 363ff
 Alternating currents II, 375ff
 Alternating e.m.f., effect of capacity on,
 375ff
 effect of self-induction on, 370ff
 Alternating potential, sinusoidal, illus-
 trated, 365
 Alternating potential measurement, 330,
 332
 Alternating potentials measured by
 quadrant electrometer, 197
 Alternative definition of unit current, 55
 Ammeter, a-c, 110
 and voltmeter, problems on, 588ff
 d-c, 109ff, 326, 328
 requires calibration, 110
 Ammeter shunts, 111ff, 112
 Ampere, 55, 412, 413
 Ampère's equation, 53, 392
 Ampère's law, 53
 applications of, 56ff
 generalization of, 395ff
 Ampère's molecular magnets, 253ff
 Ampère's rule, 53
 Ampere-turns, 267, 269, 271, 273, 275
 Amplification by three-electrode tube,
 541, 542
 Amplifying factor, 540
 Amplitude, total, in alternating e.m.f.,
 366
 Anderson's bridge for inductance meas-
 urement, 339
 Angle, of phase advance, 378, 379
 Angle, of phase lag, 373, 379
 evaluated from power factor, 381
 Angular velocity in alternating current
 or potential, 366
 Annihilation radiation, 426
 Anode, 116
 Anode fall of potential, 497
 Anode glow, 496
 Anomalous scattering of alpha particles,
 441
 Antenna or aerial, 544
 Anticathodes, x-ray, 450
 Apparent watts, 340, 381
 Apparent work function, 516, 518, 528
 Application, of Ampère's law, 56ff
 of Coulomb's law to special cases, 35ff
 of electrolysis, 136
 of electromagnetic induction to second-
 ary-primary coil system, 285ff, 286
 Approximate e.m.f. of cell, 143ff
 Arc discharge, 495, 497
 Arc rectifiers, 499
 Arcs, Poulsen and Chaffee, 544
 Artificial radioactivity, 466ff
 Astatic galvanometers, 108, 326, 328
 Aston's mass spectrograph, 437
 Atom, Bohr, 120ff, 442ff
 and atomic spectra, 445
 emission of light from, 444
 dynamic, 442
 ionized, 448
 nuclear, electron arrangement in, 120ff
 of Rutherford, 439
 Thomson, and nuclear, 438ff
 Atom model of Thomson, 433
 Atomic and molecular magnetism in
 solids, gases, and metals, 253ff
 Atomic Bohr orbits, energy, radius, and
 velocity in, 443
 Atomic bomb, 470
 Atomic dimensions, 119
 Atomic gyroscopes, precession of, 249
 Atomic light emission, duration of, 444
 Atomic magnetic moment, 255
 Atomic magnets, quantization of, 253ff
 Atomic mass number, 464
 Atomic nature of electricity, 118, 427ff
 Atomic nucleus, 119, 439, 440, 441, 442,
 460ff, 464ff, 466ff, 467ff
 Atomic number, 119, 123, 433, 438, 439,
 440, 443, 449, 455, 456, 464, 465

- Atomic orbits, elliptical, 119, 121, 255, 445
- Atomic period, 9
- Atomic spectra and Bohr atom, 445
- Atomic structure, 120, 420ff
and conduction, 119ff
problems on, 633ff
- Atomic weight, isotopic, 467
- Atomic weights, physical and chemical scales, 416
table of Mendeleev, 119, 437
- Atoms, excitation and ionization in, 446ff
magnetic behavior of, 245ff
number of electrons in, 123, 433, 438, 439, 440, 442, 449, 464
photoelectric excitation of, 447
- Attracted disk electrometer, 194, 329, 332
- Audio-frequency transformer, 544
- Audion detector, 537
- Auxiliary constants, table of, 415ff
- Average energy to create ion pair, 485
- Average power consumption, 380
- Avogadro number, 428
- Avogadro's rule, 118
- Azimuthal quantum number, 120ff, 255, 445
- B*, evaluation of, by secondary-primary system, 285ff, 286
- B-H* curves, 240, 241
- B-μ* curve, 241
- Back electromotive force, 295ff
- Ballistic galvanometer, 284, 341ff, 342, 343
problems on, 626ff
- Barkhausen jumps, 25, 26, 243
- Barlow's wheel, 234
problems on, 619ff
- Batteries, 136ff
calculations on, 591ff
- Bell, electric, first, problems on, 607ff
- Beta decay process, 471
- Beta decay reactions, 466, 469
- Beta rays, 434, 460
- Betatron, 453, 461, 482ff
in radiography, 458
- Binding energy, nuclear, 467
- Biot and Savart's law, 56ff, 233, 611
problem on, 622
- Bismuth spiral, 323
- Bohr atom, 120, 442ff
emission of light by, 444
frequency of light radiated from, 445
- Bohr criterion for stable orbits, 443
- Bohr equation for frequency in hydrogen-like spectra, 455
- Bohr magneton, 253, 256, 257, 444
- Bohr orbit, 120, 443, 444
problems on, 596ff, 612, 613
unit, 121, 444, 449
- Bohr stable orbits explained by de Broglie wave length, 445
- Bohr-Stoner table of electron assignment, 123
- Boiling-point lowering, 132
- Boys radio micrometer, 168
- Braun electrostatic voltmeters, 196
- Braun tube oscillograph, 350
- Bridge methods, of capacity measurement, 336, 338
of mutual inductance measurement, 339
of resistance measurement, 333, 335
- Buncher, 555
- Calculation of thermal e.m.f.'s, 157ff, 161, 162, 164
- Calibration of ammeters and voltmeters, 110
- Calorimetric measurement of potential, 71
- Calutron, 438
- Canal rays, 434ff
- Capacity, absolute measurement of, 335, 338
and resistance transients, 354ff, 358
and self-induction, effect of, on alternating current, 375ff, 378ff
by comparison methods, 336, 338
by frequency of oscillation, 337, 338
causes phase advance, 362, 376, 379
defined, units, dimensions, ratios, all systems, 412, 413
definition of, 199ff, 412, 413
effect on alternating current, 375ff
e.m. unit of, 201, 412, 413
e.s. unit of, 201, 412, 413
flow of alternating current through, 375
impedance, 375, 378

- Capacity, in alternating current, problems on, 628ff
 in parallel, 221
 in series, 221
 measurement of, 335ff, 338
 of condenser with mixed dielectrics, 216
 of cylindrical condenser, 217
 of parallel-plate condenser, 216
 of spherical condenser, 202
 practical unit of, 201, 412, 413
 problems on, 601ff
 ratios of units of, 201, 412, 413
 reactance, 378
 small, determination of, 337, 338
 stimulates current flow, 301
 transient phenomena in discharge of, 360
 transients in charge of, 358
 units of, 200ff, 412, 413
- Capacity bridge, 336, 338
- Capacity comparison by a-c bridges, 336, 338
- Capillary electrometer, 330, 332
- Catcher, 555
- Cathode, 116
- Cathode fall of potential, normal and abnormal, 495
- Cathode glow, 495
- Cathode-ray beams, problems on, 610ff, 612
- Cathode-ray oscillograph, 337, 338, 350ff
- Cathode rays, 420
 e/m for, 421ff
 magnetic and electric deflection of, 422
 problems on, 574
 velocity of, 423, 425
- Cautions in electroplating, 146
- Cavity resonators, 554
- Cell, acid, 141
 approximate e.m.f. of, 143ff
 concentration, 140
 Daniell, 141
 dry, 148
 Edison, 147ff
 energetics of, 140, 141, 143, 144ff
 entropy correction for, 145
 lead storage, 147ff
 polarization, 143
 standard Weston, 149, 329, 332
 temperature coefficient of, 149
- Cells, in parallel, 151
 in series, 150
 most efficient arrangement of, 150ff
 problems on, 591ff
 reversibility of, 146, 147
 storage, 147ff
- Center of magnetism, 29
- Centimeter, of capacity, 200, 412, 413
 self-induction, 302, 412, 413
- C.g.s. system, 31
- Chain reaction, 469
- Change of charge, ionic, 448
- Character, of electrical units and their dimensions, 390ff
 of excitation and ionization in atoms, 446
 of nuclear reactions, 470ff
- Characteristic curves of three-electrode tube, 542, 543
- Characteristic or fluorescent x-rays, 456
- Characteristic or photoelectric absorption of x-rays, 458
- Charge, electric, 181
 nuclear, 119, 123, 439, 440, 443, 455, 456, 464, 465
 number of lines of force from, 191, 211, 393, 492
 of the electron, 427ff
 surface distribution of, 192
- Charge density, surface, 182, 189, 191ff, 397, 417
 and its consequences, 192ff
 volume, 393, 397, 400, 416, 532
- Charges and fields, electric, 169ff
 problems on, 595ff
- Charging a capacity, transients in, 358
- Charging by induction, 171, 173
- Chemical scale of atomic weights, 416
- Choke coils, 536
- Circuit, magnetic, 266ff
 oscillating, period of, 382, 388
 resonant or tuned, 410
 time constant of, 356, 361, 386, 388
- Circuits, solution of, by Maxwell's method or by equivalent Y, 99ff
- Circular current, moment of, 230
- Cloud tracks of C.T.R. Wilson, 439, 493, 551
- Cockcroft and Walton accelerator, 461, 472
- Coefficient, of coupling, 309

- Coefficient, of mutual induction, 305ff, 339
 of self-induction, 301ff, 339, 412, 413
 equations for, 301
 for air-core coil, 304
 units of, 302, 412, 413
 Thomson, 158, 164, 165
- Coercive force, 239, 244
- Coil, carrying current, moment of, 230, 250
 circular, field at center of, 58
 with current, field of, 59
 flip, 283
 induction, 317
 with current equivalent to bar magnet, 17, 230
- Colpitts oscillating circuit, 547
- Combination of resistances, 82ff
- Commutator, 292
- Comparison, of diamagnetic and paramagnetic effects, 252
 of potentials, 84ff, 329, 332
- Compass, discovery of, 12
 magnetic, 16
 importance of, 45
- Compass deviation, 46
- Compass error, 46
- Compass variation, 46
- Competing nuclear reactions, 466
- Compton effect, 456, 457, 458
- Compton scattering, 458
- Compton shift on scattering x-rays, 457
- Concentration cell, 140ff
- Concentric cylinders, field strength between, 217
- Concept, of capacity, 200, 412, 413
 of current, electromagnetically, 55, 412, 413
 electrostatically, 179, 412, 413
 of magnetomotive force, 266ff
 of potential, 64ff, 183ff, 412, 413
 of potential difference in e.s. system, 183ff, 412, 413
 of quantity, electromagnetically, 56, 412, 413
 electrostatically, 179, 412, 413
 of resistance, 73ff, 412, 413
 of self-induction, 299, 412, 413
- Concepts, auxiliary, table of, 415ff
- electrical, fundamental or derived, 299, 321, 411, 412, 413
- Concepts, electrical, important, 410ff, 412, 413
 six basic, of electricity, 299, 321, 411, 412, 413
- Condenser, cylindrical, capacity of, 217
 energy of charge of, 218ff, 219, 220
 flow of alternating current through, 375
 oscillatory discharge of, illustrated, 388
 parallel-plate, and cylindrical, capacities of, 216ff
 field in, 190
 spherical, capacity of, 202
 transients in charge and discharge of, 358
 with resistance and self-induction, derivation of equation for oscillatory discharge of, 382ff
- Condenser charge, transient, 358
- Condenser discharge, non-oscillatory, 386
 oscillatory nature of, 388, 409
 problems on, 626ff
 transient, 360
- Condensers, in series, for potential measurement, 222, 331, 332
 potential distribution between, 222
 problems on, 602ff
 use of, 218
- Conducting surfaces are equipotentials, 188
- Conduction, and atomic structure, 119ff
 in gases, 484ff
 types of, 77, 115
- Conductivity, electrical, 77, 115, 132, 415, 484
 of flames, 484
 of solutions, 132
 produced in gases, 484
 specific, 78, 132, 416
 units of, 78, 416
- Conductor, magnetic, 237
 work done in moving in a magnetic field, 226ff
- Conductors, 78, 115, 169
 liquid, 115ff
- Consequences of induction, 172ff
- Consequent poles, 22
- Conservation of mass and energy, 426, 464, 465, 466, 467
- Considerations of relativity, 425ff
- Constant, dielectric, 203ff, 204, 390, 416, 417

- Contact difference of potential, 504, 506
 Control grid, 544
 Controlled quantitative investigation, 1ff
 Conventional representation, of electric fields, 181, 188
 of fields of force, 47ff, 181, 188
 of magnetic fields, 47ff
 Conversion of mass to energy, 426, 467
 Coolidge x-ray tube, 452
 Copper cell, 141ff
 Cores, laminated, 316
 Corona, pulses, 501
 Coronas, 495, 499ff
 Corpuscles, 424
 Cosmic-ray energies, 332
 Cosmic rays, 434, 450, 451, 484
 Coulomb, 56, 412, 413
 Coulomb's law, 28ff, 175ff, 203, 238, 417, 439, 440, 442, 460
 application of, 35ff
 validity of, about nucleus, 178, 440, 441, 461
 Coupling coefficient, 309
 Crookes dark space, 496, 500
 Cross, thermal, 168
 Cross section for nuclear reactions, 463
 C.T.R. Wilson cloud tracks, 439, 493, 551
 Cumulative ionization, 448, 490ff, 492
 Curium, 466
 Curl, 396
 Current, absolute static and small, measurement of, 326, 328
 alternating, effect of capacity on, 375ff
 importance of, 363ff
 measurement of, 327, 328
 peak value of, 364, 366
 problems on, 628ff
 alternating I, 363ff
 alternating II, 375ff
 defined, units, dimensions, ratios, all systems, 412, 413
 definition of unit, 54, 55, 103, 180, 412, 413
 electric, 51ff
 problems of fields of, 570ff
 sources of, 51
 energy in electromagnetic field of, 310ff, 312
 force produced by, 53
 forces between, problems on, 608ff
 in e.s. system, 180ff
 Current, in field, direction of motion of, 231
 induced electric, 278ff
 magnetic field of, 52ff
 measurement of, 325ff
 by electrode position, 326, 328
 legal standard of, 326, 328
 practical unit of, 55
 rise and fall in circuit with self-induction, 357, 358
 root mean square, 366
 small, measurement of, 180, 326
 transient, from charge and discharge of condenser, 360
 unit, alternative definition, 55, 226
 electromagnetic, definition of, 54, 103, 412, 413
 electrostatic, definition of, 180ff, 412, 413
 value of, given by a-c instruments, 366ff
 work to create or annihilate, 310
 Current balance, absolute, 313, 325, 328
 Current curves for charge and discharge of condenser, 361
 Current density, 132, 396
 Current flow, direction of, 64, 69, 70
 stimulated by capacity, 361
 Current period, 8
 Current-potential characteristics of thermionic emission, 531ff, 534
 Current transients, oscillographic study of, 353
 Currents, and fields, interaction between, 230
 eddy, 314ff
 parallel, fields about, 232
 force between, 233
 Curve of magnetization and hysteresis, 238ff
 Cut-off field of magnetron, 557
 "Cutting" lines of force gives e.m.f., 280, 281, 282
 Cyclotron, 461, 475ff
 problem on, 613
 Cylindrical condenser, capacity of, 217
 standard, 336, 338
 Damped galvanometer, 341, 346ff
 Damped oscillations, 388

- Damping, electrical, 315, 341, 346, 347, 388
 Damping factor, 347
 Daniell cell, 141ff, 145
 Dark currents, 495
 D'Arsonval galvanometer, 103ff, 326, 328, 588ff
 problems on, 588ff
 De Broglie wave equation, 445
 De Broglie wave length, 432, 445, 463
 De Broglie waves, 432, 441, 445, 463
 Declination, magnetic, 46
 Decrement, 347
 Definition, of capacity, 199ff, 412
 of charge density, 182, 191, 393, 397, 415, 532
 of conductivity, 77, 132, 415
 of current, and work done in moving a conductor in a field, 226ff
 in e.m. system, 53, 54, 55, 412
 in e.s. system, 180ff, 412
 of dielectric constant, 176, 203, 391, 415, 418
 of electric field strength, 181ff, 415
 of electric resistance, 73ff
 of e.m.f., 86ff, 88, 278, 398
 of e.m.u. of potential, 69, 282, 412
 of e.m.u. of quantity, 56, 412
 of e.s.u. of potential, 184, 412
 of e.s.u. of quantity, 179, 412
 of flux, 49, 267, 415
 of magnetic field strength, 29, 415
 of magnetic induction, 237, 415
 of magnetic moment, 39, 415
 of magnetic pole strength, 29, 415
 of permeability, 29, 238, 415
 of reluctance, 266ff
 of self-induction, 301, 412
 Definitions, alternating current, 363ff
 Degaussing, 572, 573
 Degaussing coils, problems on, 572
 Degenerate electrons, 514, 516, 530
 Degree of dissociation, 132
 Demagnetization, 26, 273ff
 by reversals, 242ff
 Demagnetizing field, 26, 273, 286
 Dempster mass spectrograph, 424, 438
 Density, current, 396
 of charge, surface, 182, 189, 191ff, 397, 417
 of charge, volume, 393, 397, 400, 416, 532
 Deperming, 45, 243
 Derivation, of equation for ballistic galvanometer, 342ff
 of equation for oscillatory discharge, 382ff
 of equation for simple dynamo, 289ff
 of space-charge equation, 532ff
 of wave equation, 400ff
 Derived concepts, 74, 199, 321, 411, 412, 413
 Determination, of absolute ohm, 333, 335
 of nuclear constituents, 460
 Deuteron, 462, 465
 Development, of electricity and magnetism, 8
 of physical science, 1ff
 Diagram of energy levels, for Bohr atom, 444
 for Na, 446
 Diamagnetic action, quantitative analysis of, 249ff
 Diamagnetic and paramagnetic effects, comparison of, 252
 Diamagnetic body in uniform field, 248
 Diamagnetism, 245ff, 246, 248, 249, 252
 gives average electron orbit radius, 252
 Dielectric, inner field of, 211
 Dielectric absorption, 213
 Dielectric action, nature of, 205
 "true," 205
 Dielectric constant, 127, 176, 203ff, 204, 206, 208, 212, 322, 391, 397, 408, 416, 417
 and index of refraction, 206, 208, 408
 and refractive index, table of, 409
 and temperature, 208, 209, 211, 212
 dimensions of, 391
 dissociation in solution through, 127
 measurement of, 204
 Dielectric constants, table of, 204
 Dielectric hysteresis, 203ff, 213, 214, 215
 Dielectric induction, 210, 397, 416
 Dielectric polarization, 130, 203ff, 211, 212, 213, 214, 215
 Dielectric relaxation times, 208, 213, 214
 Diffraction, of α particles, 441
 of x-rays, 452, 454ff
 Dimensions, and character of electrical units, 390

- Dimensions, of electrical concepts, 410ff,
 412, 413
 of electrons, nuclei and atoms, 119,
 178, 440, 441, 461
 of e.m.u. of self-induction, 302, 412
 of e.s.u. of capacity, 200, 412
 of magnetic pole strength, 34ff
 of nuclei, 119, 178, 441, 461
 of specific resistance, 77
 physical, 30, 33
 table of, 412, 413
 Dip, magnetic, 46
 Dipole, electrical, 206, 208, 209, 212, 416
 magnetic, moment of, 39, 415
 Dipole moment, electrical, 125, 206, 208,
 209, 211ff, 212, 416
 determination of, 211ff, 212
 Dipole moments, problems on, 608
 Direct-current ammeters, 109ff
 Direct-current dynamos, 292
 Direct-current power by voltmeter-am-
 meter measurement, 340
 Direct-current voltmeters, 109ff
 Direction, of current flow, 64, 69, 70
 of easy magnetization, 23
 of induced current, 278
 of motion of current in magnetic field,
 231
 Discharge, electrical, from sharp points,
 193
 nonoscillatory, illustrated, 386
 of condenser, oscillatory, illustrated,
 388
 transient, 360
 oscillatory, derivation of equation of,
 382ff
 through gases, 420ff
 Discovery, of compass, 12
 of magnetism, 11
 of thermionic effect, 523ff
 scientific, nature of, 6
 Displacement current, Maxwell's, 396ff
 Displacement polarizability, 206, 207,
 212
 Displacement polarization, 213
 Dissociation, electrolytic, 127
 in solution, mechanics of, 127, 128
 Divergence, 395
 Divergent field, paramagnetic and dia-
 magnetic bodies in, 247, 248
 Domains, magnetic, 22ff, 243, 265
 Drift velocity of electrons, 487
 Dry cells, 149
 Dualistic nature of light and matter,
 457
 Dulong and Petit's law, 511
 Duration of atomic light emission, 444
 Dynamic atom, 442
 Dynamic stability, 124
 Dynamo, efficiency of, 293ff
 e.m.f. and p.d. from, 293
 energy losses in, 294
 equation derived, 290ff
 simple, problems on, 621ff
 Dynamometer, absolute, 313, 325, 328
 Dynamos, a-c and d-c, 291
 and motors, reciprocal relations of,
 295ff
 simple, 290ff

 e/m , for cathode rays, 421ff
 for photoelectrons, 504
 for positive ions, 437
 for thermionic particles, 524
 problems on, 553ff
 e/m determinations, problems on, 610ff
 612
 Earth inductor, 283ff, 323, 325, 619ff
 principle of, 283ff
 problems on, 619ff
 Earth's magnetic field, 45ff
 horizontal component of, 46
 intensity of, 46, 284
 Earth's magnetic poles, 15
 Easy magnetization, direction of, 23
 Eddy currents, 243, 314ff
 effect of, in demagnetizing, 243
 Edison cell, 147
 Effect, of capacity and self-induction
 combined, 375ff, 378ff
 of capacity on alternating current,
 375ff
 of self-induction, on alternating cur-
 rent, 363ff
 on alternating e.m.f., 370ff
 Efficiency, of dynamo, 293ff, 294, 621ff
 real and ideal, 294
 dynamos and motors, problems on,
 621ff
 of motors, 295ff
 of storage cells, 148
 of transformer, 318

- Efficient arrangement of cells, 150ff
 calculation for, 153
- Einstein law, conservation of mass and energy, 426, 460, 464, 465, 466
- Einstein photoelectric law, 507ff
- Elastic electron impacts, 447
- Electric and magnetic measurements, 321ff
- Electric and magnetic units, systems of, 417ff
- Electric bell, first, problems on, 607ff
- Electric capacity, definitions of, 199ff, 412
 units and ratios of, 200, 201, 412, 413
- Electric charge, number of lines from, 182
- Electric charges and fields, 169ff
- Electric circuit and potential difference, 64ff
- Electric concepts, dimensions of, 412, 413
 fundamental, 299, 321, 410ff, 412, 413
 important, 410ff
- Electric conductivity, 77, 115, 132, 415, 416, 484
- Electric conductors and insulators, 78, 170
- Electric current, 51ff
 and resistance, 76ff
 flow conventions, 64, 69, 70
 force of, on magnet pole, 59
 practical unit of, 55ff, 412
 problems on fields of, 570ff
 sources of, 51
 unit, electromagnetic, 54, 55, 412
 alternative definition, 55, 412
 electrostatic, 180, 412
 practical, 55, 412
- Electric currents, induced, 378ff
- Electric damping, 346
- Electric dipole, 125, 206, 208, 209, 212, 416
- Electric dipole moment, 125, 206, 208, 209, 211ff, 212, 416
- Electric discharge from sharp points, 193
- Electric field, conventional representation of, 181
- Electric field strength, 181ff, 416
- Electric inertia, 300, 350, 362, 373, 381.
 See also Self-induction
- Electric measuring instruments, 103ff
- Electric neutrality of atoms, 124
- Electric oscillations, 375, 381ff, 383ff
- Electric pinwheel, 193
- Electric potential, absolute, 185
- Electric potential difference, 68, 412, 413
- Electric power, consumption of, 72, 340, 381, 417
 units of, 72, 417
- Electric quantity, practical unit of, 56, 412, 413
- Electric resistance, concept of, 73ff, 412
- Electric screening, 172
- Electric surface charge density, 177, 181, 182, 189, 191ff, 397, 417
 and its applications, 191ff
- Electric susceptibility, 416
- Electric systems, tables of dimensions of, 418
- Electric units, dimensions and character of, 390ff
 important, 410, 412, 413
 relations of, 410, 412, 413
- Electric volume charge density, 393, 416, 532
- Electric whirl, 193
- Electric wind, 193.
- Electricity, atomic nature of, 118, 427ff
 development of, 8
 one and two fluid theories of, 171ff
 six basic concepts of, 299, 321, 411, 412, 413
 static I, 169ff
 static II, 183ff
 static III, 199ff
- Electrification, resinous and vitreous, 64, 69, 169
- Electrochemical equivalent, 145, 326
- Electrode, 116
 gas, 139
 inert, in acid cell, 142
- Electrodeposition, problems on, 575ff
- Electrodynamics, Maxwellian, 4, 392ff
- Electrolysis, 115ff, 116, 117, 129, 136, 590ff
 applications of, 136
 through glass, 129
 problems on, 590ff
- Electrolyte, influence of, on sign of potential, 139
- Electrolytes, strong, 127
- Electrolytic dissociation, 127
- Electromagnet, forces of, 270, 614ff
 problems on, 614ff
 structure of, 275

- Electromagnetic and electrostatic units,
ratio of, 390ff, 411, 412, 413
- Electromagnetic definition, of current,
54, 55, 282, 412
of potential, 69, 412
of quantity, 56, 412
- Electromagnetic field of a current, energy
of, 310ff, 312
- Electromagnetic induction, 278ff, 279,
285, 286, 288, 398, 620ff
and electron theory, 279ff
and secondary-primary coil system,
285ff, 286
applied to Faraday's disk, 288
generalization of Faraday's laws of,
398ff
problems on, 620ff
- Electromagnetic potential standard, 149,
328, 329, 332
- Electromagnetic system, 54ff, 321, 391,
411, 412, 417
- Electromagnetic unit, of capacity, 201,
412, 413
of current, 54, 412, 413
of potential, 69, 412, 413
alternative definition, 282
of quantity, 55, 412, 413
of resistance, 73, 412, 413
of self-induction, 302, 412, 413
dimensions of, 302, 412
- Electromagnetic units, 54, 55, 69, 73,
201, 282, 302, 412, 413
- Electromagnetic waves, 390ff, 392ff, 407,
410, 450, 451, 554
diagrammatic representation of, 407
nature, origin, and length of, 450, 451
short, 554
spectrum of, 450, 451
table of determinations of, 410
- Electromagnetics, 226ff
problems on, 608ff
- Electrometer, 194, 197, 329, 332
absolute, 194, 329, 332
quadrant, 197, 329
- Electromotive force, alternating, effect
of capacity on, 375ff
effect of self-induction on, 370ff
and p.d. from dynamo, 293
back, 295
by cutting lines of force, 280, 281,
282
- Electromotive force, definition of, 86ff,
88, 278, 398
determination of, 84ff, 329
evaluation of, 84ff, 89, 329
frequency of, in alternating current,
364, 366
from Faraday's disk, 288
from simple dynamo, 292
induced, direction of, 278
magnitude of, 280, 281
instantaneous value of, 366
with self-induction, 374
of cell, approximate, 143ff
problems on, 591ff
of self-induction, 374
peak value in a-c, 364, 366
period of, in a-c, 364, 366
rectified, 292, 293, 499, 522, 534ff
thermal, calculation of, 157ff, 161, 162,
164
- Electron, 119, 424ff
charge of, 179, 427ff
alternative method for measure-
ment, 431
early determination of, 428
de Broglie wave length of, 432
degenerate, 514, 516, 530
energy of, in Bohr orbits, 443
fast, ionization of gases by, 485
field emission of, 502, 519
free, cause of metallic state, 76, 155,
260, 264, 433, 511
in metals, Sommerfeld concept of,
155, 260, 511, 527, 529
in atoms, problems on, 574
in metals, centrifuging of, 433
in static fields, problems on, 596ff
ionization by collision by, 489
magnetic moment of, 432
mechanical moment of momentum of,
432
positive, 434
properties of, 431ff
secondary emission of, by electron
bombardment, 522
size of, 119, 432
spin moment of, 432
unitary character of, 430
valence, 123
- Electron arrangement in nuclear atoms,
120ff, 123

- Electron assignment of Bohr and Stoner, 123
- Electron beams, deflected in magnetic fields, problems on, 610ff, 612
- Electron behavior, problems on, 533ff
- Electron energy, 425
- Electron energy distribution in gases, 488
- Electron energy levels, in Bohr atom, 443
in isolated or crowded atoms, 261
- Electron exchange forces, 264
- Electron impact, elastic, 447
exciting, 447
- Electron impact ionizing, 448
- Electron-ion recombination, 449, 487
- Electron levels, occupation of, 121, 449, 455
- Electron liberation by positive ion impact on surfaces, 491, 497
- Electron mass entirely in magnetic field, 432
- Electron mobility, 487
- Electron multipliers, 522
- Electron not free in nuclei, 434, 466
- Electron octets, 124
- Electron orbit, 121, 248, 253, 443, 445
average radius of, from diamagnetism, 252
Bohr, radius, velocity, and energy in, 443
elliptical, 119, 121, 255, 445
in magnetic field, 481
magnetic moments of, 253
shapes of, 121
- Electron orbits in magnetron, 558
- Electron pair production, 426, 434
- Electron pairs, atomic and molecular, 124
- Electron phase wave, 432
- Electron photoconductors, 79
- Electron shells, *K* and *L*, 122, 449, 455
- Electron theory, and electromagnetic induction, 279
of metals, 76, 115, 126, 155, 260, 433, 511
- Electron velocities, 424
in fields in gases, 487
- Electron velocity distribution in photoelectric effect, 505
- Electron volts, 331, 424, 505, 506
- Electronic conductivity, 76, 115, 126, 155, 260, 433, 511
- Electronic gyroscope, 248
- Electrons, number of, in atoms, 119, 123, 433, 438, 439, 440, 442, 449, 455, 456, 464, 465
- Electroplating, cautions in, 146
- Electroscope, gold leaf, 169, 196
- Electrophorus, 173
- Electrostatic definition, of current, 180ff, 412
of potential, 183, 184, 412
of quantity, 179, 412
- Electrostatic deflection of cathode rays, 422
- Electrostatic field, energy in, 218ff, 220
of force, 183, 416
- Electrostatic fields and forces, problems on, 595ff
- Electrostatic focussing, 474, 478
- Electrostatic induction, 170ff, 172, 176
uses of, 173
- Electrostatic potential divider, 223
- Electrostatic potential multiplier, 224
- Electrostatic potentials, problems on, 597ff
- Electrostatic system, 179, 183, 321, 411, 412, 413, 417ff
- Electrostatic unit, of capacity, 200, 412, 413
dimensions of, 200, 412, 413
of current, 180, 412, 413
of p.d., 184, 412, 413
of quantity, 179, 412, 413
of resistance, 412, 413
of self-induction, 302, 412, 413
- Electrostatic units, 179, 180, 184, 200, 302, 412, 413
- Electrotechnical period, 8
- Ellipsoid, measurement of permeability with, 324, 325
- Elliptical electron orbits, 445
- Emission of light by Bohr atom, 444
- Energy, and mass, conservation of, 426, 460, 464, 465, 466
in electromagnetic field, 312
in electromagnetic field of a current, 310ff, 312
in electrostatic field, 218ff, 220
nuclear, 465, 468ff
of charge of condensers, 218ff, 219, 220
of electron in Bohr orbits, 443
of electrons, 425
uncertainty in evaluation of, 6, 433

- Energy distribution of electrons in fields
in gases, 488
- Energy level diagram, Bohr, 443, 444
for Na, 446
- Energy levels of electrons and paramagnetism, 262
- Energy loss, and self-induction, 314ff
in dielectric hysteresis, 203ff, 213, 214, 215
in dynamo, 294
in magnetic hysteresis, 244
in transformers, 319
- Energy-mass relations, 426ff, 466ff
- Entities, six basic, of electricity, 299, 321, 411, 412, 413
- Entropy correction for cells, 145
problems on, 591ff
- Equation, of continuity, 393
wave, derivation of, 400ff
- Equilibrium in solutions, 137
- Equipotential surfaces, 187, 188
- Equipotentials, and lines of force, 187, 188
for two isolated charges, 188
- Equivalents, gram atom, 116, 118
- Exchange forces, 464
electronic, 264
- Excitation, atomic, duration of, 444
in atoms, 446ff
photoelectric, 447
thermal, 447
- Excitation potentials, 447
- Exciting impact, 447
- Exclusion principle of Pauli, 6, 120, 512, 513
- Existence of Ampère's molecular magnets, 253ff
- "Exploding" wires, 219
- External resistance, 87, 150, 293
- Fall of potential, across condensers in series, 223
along a wire, 83
- Farad, 201
- Faraday, 118, 416, 428
- Faraday constant, 416
problems on, 599ff
- Faraday dark space, 496, 500
- Faraday disk, 234, 288
and electromagnetic induction, 288
e.m.f. from, 288
- Faraday disk, problem on, 619
- Faraday electromagnetic induction experiment, 278
- Faraday ice-pail experiment, 172ff
- Faraday laws, of electrolysis, 115ff, 116, 118
of electromagnetic induction, generalization of, 398ff
- Fast neutron reactions, 463, 469
- Fathometers, 549
- Fermi-Dirac statistics, 513, 514, 515, 528, 530
- Ferromagnetism, 18, 245ff, 246, 253ff, 264ff
gyromagnetic ratio in, 264
- "Fictitious" dielectric action, 205
- Field, and currents, interactions between, 230ff
and magnet, interactions, problems on, 561ff
at center of circular coil with current, 58
demagnetizing, 273
electric, 169ff, 415
electrostatic, energy in, 218ff, 220
in long solenoid, 227
line integral of, 268, 395
magnetic, calculations on, 561ff
concept of, 28, 415
energy stored in, 310ff, 312
idealized, 20
torque on magnet in, 39
of electric currents, problems on, 570ff
of force, electrostatic, 183
of Helmholtz coils, 61
of infinitely long straight current, 56ff
space charge, at electrodes, 489
uniform electric, achievement of, 189
- Field emission of electrons, 502, 519
- Field strength, and potential difference, 190, 416
at conducting charged surface, 189
at distance from axis of concentric cylinders, 217
electric, 181ff, 189, 190, 191, 416
practical unit of, 191, 416
magnetic, definition of, 29, 415
- Fields and forces, electrostatic, problems on, 595ff
- Figure of merit of galvanometer, 105ff
- First electric bell, problems on, 607ff

- Fission, 462, 468ff, 469, 470
 Fission pile, 470
 Flames, conductivity of, 484
 Flip coil, 283, 323, 325
 problems on, 620ff
 Fluid theories of electricity, 171ff
 Fluorescent lamps, 497
 Fluorescent radiation, 447
 Fluorescent scattering of x-rays, 458
 Fluorescent screens, 351
 Flux, in coils, problems on, 611ff
 magnetic, 48, 266, 267, 268ff, 415, 611ff
 rate of change of, gives e.m.f., 281
 unit magnetic, 415
 Flux cut times current gives work, 227
 Fluxgate magnetometer, 43, 323, 325
 Fluxmeter, 343ff
 General Electric, 345
 Grassot, 324, 325, 343
 Focussing, magnetic and electric 474 478
 Forbidden transitions, 444
 Force, between parallel currents, 233
 electric, lines of, 181
 field of, electrostatic, 183
 lines of, 47, 181, 232
 magnetic, lines of, 47
 on surface in electrical field, 195
 tube of, 48
 Formation of negative ions, 487
 Forms of gaseous discharge, 495
 Free electrons, gyromagnetic ratio of, 254
 Freezing-point lowering, 132
 "Freon," 453
 Frequency, of alternating current, 364, 366
 of light emitted from Bohr atom, 445
 of oscillating circuit, 382, 388
 Frequency-modulated cyclotron, 479ff, 481
 Fundamental concepts, 74, 321, 411, 412, 413
 Fundamental particles, 471ff
 Fundamental units, 321
 Furnace, induction, 315

 Galvanometer, 103, 105, 107, 284, 326, 328, 329, 341ff, 346ff, 588
 ballistic, 284, 341ff
 figure of merit of, 105
 high-resistance, for potential measurement, 329, 332
 Galvanometer, Kelvin astatic, 108
 moving-coil or d'Arsonval, 103ff
 period of, 107, 284, 341
 problems on, 588ff
 responsiveness of, limitations to, 107, 108, 588
 sensitivity of, 106
 tangent, 62, 325
 thermo-, 168
 Galvanometer constant, 105ff, 342
 Galvanometer damping, 341, 346ff
 Galvanometer formula, 104, 344
 Galvanometers, types of, 108ff
 Gamma-ray emission, 469
 Gamma-rays, 450, 451, 459, 460, 465, 469, 509
 origin of, 465
 Gas, as an insulator, 484
 conduction in, 484ff
 discharge through, 420ff
 electron velocities in, 487
 ionization currents in, 488
 "natural" ionization of, 484
 photoelectric effect in, 518
 rendered conducting, 484
 Gas electrode, 139
 Gas-filled thermionic rectifiers, 537
 Gaseous discharge, forms of, 495
 Gaseous ions, 485ff
 Gases, effect of, on photoelectric currents, 506
 Gauss, 30, 415
 Gaussian system of units, 34, 204, 391, 411, 417, 418
 Geiger-Mueller counters, 550ff, 552
 General Electric fluxmeter, 345
 General Electric million-volt x-ray tube, 453
 Generalization, of Ampère's law, 395ff
 of Faraday's laws of electromagnetic induction, 398ff
 of Ohm's law, 90ff
 Glass, electrolysis through, 129
 Glow discharges, 495ff
 Gold-leaf electroscope, 169, 196, 329, 332
 Gram atom, 116
 Gram-atom equivalent, 416
 Grassot fluxmeters, 324, 325, 343ff
 Grid, 538
 Grid bias, 539, 540, 542, 543, 546, 547
 Grid leak, 546, 547

- Grid potential, 539
 Grids, screen, control, and suppressor, 544
 Gyromagnetic ratio, 254, 262, 264, 432
 electronic, 254
 in iron, 264
 of paramagnetic metals, 263
 orbital, 254
 Gyroscope, electronic, 248

h, Planck's, 253, 331, 432, 433, 443, 508, 509, 513
 dimensions of, 33
 Hard x-rays, 450, 451
 Hartley oscillating circuit, 546, 548
 HCl molecule, description of, 209
 Heat, effect of, on magnetism, 19
 of chemical reaction, 140, 141, 143, 144
 Heat waves, 450, 451
 Heating effect of current, resistance measurement by, 334, 335
 Heaviside-Lorentz system, 419
 Heisenberg's uncertainty principle, 6, 433
 Helmholtz coils, 60, 226, 322, 325
 Henry, 302, 412, 413
 Hertzian waves, 409, 451, 554
 Heussler alloys, 18
 High-energy particles, achievement of, 472ff
 measurement of, 331, 332
 High-frequency currents, small a-c, measurement of, 327, 328
 High-potential measurement, 330, 332
 High-resistance measurement, 334, 335, 362ff
 High-resistance towers, 330, 332
 Historical, 1ff
 History of discovery of electric currents, 51ff
 Hittorf's transport ratio, 133
 Homopolar compounds, 79, 124, 210
 Hydration of ions, 136
 Hydrogen, circular orbit energy level diagram for, 444
 Hypnic, 245
 Hysteresis, dielectric, 203ff, 213, 214, 215
 Hysteresis curve, magnetic, 25, 237ff, 240, 274
 Hysteresis loops, magnetic, 25, 240
 significance of, 244
 with air gaps, 274

 Ice-pail experiment, 172ff, 176, 177
 and inverse square law, 177
 modified, 176
 Ideal efficiency, of dynamo, 294
 of motor, 297
 Idiostatic connection of quadrant electrometer, 197
 Impact ionization by electrons, 448
 Impacts, elastic electron, 447
 electron, exciting, 447
 Impedance, 319, 373, 378, 379
 effect of capacity on, 375ff
 Importance of alternating current, 363ff
 Important electrical concepts, 410ff
 Inclination, magnetic, 46
 Index of refraction, 206, 208, 408, 458
 and dielectric constant, 206, 208, 408
 negative, 458
 Induced current, 278ff
 direction of, 278
 Induced e.m.f., magnitude of, 280, 281
 Induced magnetism, 18
 Inductance, and resistance, transients, 354ff
 defined, 300ff, 304, 412, 413
 in alternating current, problems on, 628ff
 mutual, 304, 308, 313, 339
 units of, 302ff, 412, 413
 Inductance calculations, problems on, 623ff
 Inductance measurement by Anderson, Maxwell, or Owen's bridges, 339
 Inductances, comparison of, by a-c bridges, 339
 interacting in series aiding and opposing, 308
 in parallel, noninteracting, 307
 in series, noninteracting, 306
 Induction, dielectric, 416
 electrostatic, 170ff, 172, 176
 charging by, 171, 173
 consequences of, 172ff
 uses of, 172
 magnetic, 236, 238, 258ff, 276, 285, 395, 415
 measured by secondary-primary coil system, 285ff, 286
 measurement of, 238
 mutual, 304ff
 of new poles in iron, 236

- Induction, self, 299ff. *See also* Inductance; Self-induction
 coefficient of, 301ff, 412, 413. *See also* Inductance
 Induction coil, 317
 Induction furnace, 315
 Inductive reactance, 373
 Inductive winding, 303
 Inductometer, 309
 Inductor, earth, principle of, 283ff
 Inert electrode in acid cell, 142
 Inert gases, nonmagnetic, 256
 Inertia, electrical, 300. *See also* Self-induction
 Influence of Sommerfeld-Pauli theory on photoelectric emission, 516
 Infra-red waves, 450, 451
 Initial magnetization curve, 240
 Inner field correction, magnetic, 251
 Inner field of dielectric, 211
 Instantaneous value, of e.m.f., 366
 of e.m.f. due to self-induction, 374
 of time in alternating currents, 366
 Instrumental period, 8
 Insulators, 78, 170, 205, 484
 and dielectrics, 205
 gaseous, 484
 Integral equation, 358, 376
 Intensity, of magnetization, 237, 415
 of photoelectric emission, 509ff, 519, 520
 Interacting inductances in series, aiding and opposing, 308
 Interactions between fields and conductors carrying currents, 230ff
 Internal resistance, 86, 151, 293
 evaluation of, 89
 International ohm, 333, 335
 International standard, of current, 326, 328
 of potential, 329, 332
 International standards, 321
 Inverse square law, electric, 177ff. *See also* Coulomb's law
 proof of, 177ff, 441
 magnetic, 29, 415
 nuclear, 441
 Ion-ion recombination, 449, 487
 Ion recombination, 487ff
 Ionic change of charge, 448
 Ionic conductors, 79, 115
 Ionic mobility in solutions, 131
 Ionic space charges, in gases, 488ff
 in solutions, 134
 Ionization, atomic, 447
 by collision, 489ff, 551
 multiplication by, 551
 by electron impact, 448
 by positive ions in gases, 491
 cumulative, 448, 490ff, 492
 in atoms, 446ff
 of gases, 484
 photoelectric, 447
 Ionization currents in gases, 488
 Ionization potentials, 448
 Ionized atom, 448
 Ions, electrolytic, 172
 gaseous, 485ff
 average energy to create, 485
 mobility of, 486
 in solution, hydration of, 136
 mobilities of, 133
 size of, 135
 motion of, in magnetic field, 477
 positive and negative, 125
 relative solution tensions of, 138
 Isobars, 466
 Isoclinic magnetic lines, 46
 Isodynamic magnetic lines, 46
 Isogonic magnetic lines, 46
 Isolated pole, nonexistent, 17
 Isotopes, 437, 465ff, 466, 470
 stable, 470
 and unstable, 437, 465, 466
 Isotopic atomic weight, 470
 Joule's law of heating, 74
 and Peltier effect, 156
 K and L x-radiation, mechanism of, 449
 K-capture, 466, 471
 K electron shell, 122, 499
 K x-ray series, 451, 455, 456
 Kallman-Rosen effect, 448
 Kelvin absolute current balance, 313, 325, 328
 Kelvin astatic galvanometers, 320, 328
 Kelvin bridge, 333
 Kenotron rectifier, 452, 535
 Kirchhoff's law equations, solution of, 95ff

- Kirchhoff's laws, 90ff, 91, 95, 99, 295,
 344, 354, 358, 370, 376, 383, 583ff
 by Maxwell's method, 99ff
 problems on, 583ff
 Kirchhoff's second law, 91, 344, 354, 358,
 370, 376, 383, 583ff
 Klystron, 554, 555, 556
 principle and diagram of, 555, 556

L electron shell, 122, 449
L x-ray series, 451, 455, 456
 Laby-Hopper method of measuring elec-
 tron charge, 431
 Laminated cores, 315
 Landé splitting factor, 255, 257
 Larmor precession, 251
 Law, Ampère's, 53ff
 applications of, 56
 generalization of, 395ff
 Biot and Savart's, 56, 233
 Coulomb's, 175ff
 Lenz's, 250, 278ff, 295
 of thermionic emission, 525
 Ohm's, 73
 Laws, for resistances in series and parallel,
 82
 of Kirchhoff, 90, 91
 Lead storage cell, 147ff
 Leakage, magnetic, 273ff, 274
 Legal ohm, 333, 335
 Legal standard, of current, 326, 328
 of potential, 329, 332
 Lenz's law, 250, 278ff, 295
 Levels, x-ray, 449ff
 Leyden jar, 199
 Liberation of electrons by light, 505ff
 Lifting power of magnets, 275ff
 Light, electron liberation by, 505ff. *See*
 also Photoelectric effect
 Light emission from Bohr atom, 444
 Lightning surge generator, 225
 Line integral of the field, 268, 395
 Linear accelerator, 461, 473
 Lines of force, 47, 49, 181, 187, 188, 232,
 415
 and equipotentials, 187, 188
 cutting of, gives e.m.f., 280, 281, 282
 electric, 181
 number of, emerging from poles, 49
 from charge, 182
 Liquid conductors I, 115ff
 Liquid conductors II, 136ff
 Lissajous figures, 353
 Logarithmic decrement, 347
 Long-wave-length limit in photo effect,
 507, 508, 511, 517
 Lyman series, 451

 Machines, static, 173
 Magnet, computation of field about, 37
 definition of, 14
 effect of heat on, 19
 equality of poles of, 17
 period of oscillation in field, 42, 44
 torque of, in uniform field, 39
 Magnetic and electrical systems, units of,
 417ff
 Magnetic behavior of atoms, 245ff
 Magnetic block moment, 237
 Magnetic circuit, 266ff
 air gaps in, 274
 problems on, 614ff
 Magnetic compass, 16
 importance of, 45
 Magnetic conductor, 236, 237, 268
 Magnetic declination, 46
 Magnetic deflection of cathode rays, 422
 Magnetic dip, 46
 Magnetic domains, 22ff
 Magnetic field, at center of circular coil,
 58
 definition of, 28, 415
 divergent, measured by bismuth spiral,
 323, 325
 energy of, 310ff, 312
 in solenoid, 226ff, 227
 manner of regarding iron in, 236ff, 266
 motion of charged particles in, 477
 of current, 52ff
 discovery of, 52
 of earth, 16
 of infinitely long current, 56
 of ships and degaussing, problem on,
 572
 rotation of plane of polarization by,
 408
 uniform, 60, 226
 work done in moving a current in,
 226
 Magnetic field strength, 28, 415
 absolute evaluation of, 40ff, 324
 defined, 29, 415

- Magnetic fields, and currents, interactions of, 230
 conventional representation of, 47
 electron orbits in, 481
 idealized, 20
 measured by earth inductor and flip coils, 283ff, 324, 325
 produced by poles, 35
 stray, 274
 use of oscillating magnets in measuring, 44, 324
- Magnetic flux, 48, 266, 267, 268ff
 in coils, problems on, 611ff
- Magnetic focussing, 474, 478
- Magnetic force, lines of, 47, 232
- Magnetic forces, quantitative treatment of, 28ff
- Magnetic hysteresis loop, significance of, 244
- Magnetic inclination, 46
- Magnetic induction, 18, 236, 268ff, 276, 395, 415
 measured by primary-secondary coil system, 285ff, 286, 324, 325
 measurement of, 238, 285, 324, 325
- Magnetic induction accelerator, 481ff
- Magnetic inner field correction, 251
- Magnetic keeper, 26, 274
- Magnetic leakage, 273ff, 274
- Magnetic measurements, 321ff, 324
- Magnetic "memory," 242
- Magnetic mine sweeping, problem on, 573
- Magnetic mines, 43, 44, 243, 565, 566, 572, 573
- Magnetic moment, 39, 250, 253, 255, 259, 415, 432
 atomic, 255
 of electron, 432
 of electronic orbit, 250, 253
- Magnetic moments, quantization of, 259
- Magnetic permeability, 29, 237, 239, 247, 268ff, 286ff, 321, 324, 325, 391, 415, 417
 dimensions of, 391
 measurement of, 239, 286ff, 324, 325
- Magnetic pole, defined, 14
 force of current on, 53
 number of lines emerging from, 49
- Magnetic pole strength, defined, 29ff, 415
 dimensions of, 34
- Magnetic poles, induction of, in iron, 18, 226
 sign of forces between, 15
- Magnetic potential, 227, 266, 267, 415
- Magnetic problems on Bohr orbits, 612, 613
- Magnetic properties of materials, 236ff, 246
- Magnetic quantum number, 121, 255
- Magnetic retentive power, 18. *See also* Hysteresis
- Magnetic retentivity, 18, 239, 241
- Magnetic saturation, 19, 238, 239, 244
- Magnetic screening, 18
- Magnetic shaking, 243
- Magnetic shell, 275
- Magnetic shunt, 271ff
 problems on, 617ff
- Magnetic signatures of ships, problems on, 565, 566
- Magnetic specific inductive capacity, 417
- Magnetic spin moment, electronic, 254
- Magnetic susceptibility, 237, 415
- Magnetism, center of, 21, 29
 development of, 8
 discovery of, 11
 of earth, 15
 "permanent," 18
 phenomena of, 11ff
 qualitative phenomena of, 14
 residual, 239
- Magnetization, changes in length during, 19
 direction of easy, 23
 intensity of, 237, 415
 loss of, 26. *See also* Demagnetization
 spontaneous, 22, 264
- Magnetization curve, 25, 238ff
 of polycrystalline solids, 25, 238ff. *See also* Hysteresis loop
- Magnetization curves of single crystals, 25
- Magnetometer, 43, 238, 322, 324, 325
 fluxgate, 43, 323, 325
- Magnetomotive force, 267
 concept of, 266ff
- Magneton, 253ff, 256, 444
 Bohr, 444
 Weiss and Bohr, 256
- Magnetostriction, 19, 548, 550
- Magnetostriction oscillators, 548ff, 550

- Magnetron, 554, 557ff
 electron orbits in, 558
 schematic and diagrammatic representation, 559
- Magnets, atomic, quantization of, 121, 257, 258
 interactions of, problems on, 561ff
 lifting power of, 270, 275ff
 matching of, in galvanometers, 108
 molecular, of Ampère, 253ff
 production of, 19
- Magnitude of induced e.m.f., 280, 281
- Manner of regarding iron in magnetic field, 236ff
- Mass, and energy, conservation of, 426, 466ff
 change of, relativistic, 426, 471, 474, 479, 481, 483
- Mass spectrograph, 424, 434, 435, 436, 437, 633
 of Aston, 437
 of Dempster, 424, 438
 of Thomson, 435, 436
 problems on, 633ff
- Maxwell, 48, 415
- Maxwell's bridge for inductance measurement, 339
- Maxwell's displacement current, 396ff
- Maxwell's distribution law, 512ff, 513, 514, 515, 516, 527, 529
- Maxwell's electrodynamics, 4, 392ff
- Maxwell's equations, 390ff, 392ff
- Maxwell's solution of Kirchhoff's laws, 99ff
- Mean free path, 463
- Measurement, electric and magnetic, 321ff
 magnetic, using earth inductor or flip coils, 283ff
 of alternating currents, 327, 328, 366ff
 of capacity, 335ff, 338
 of current, 62, 103ff, 119, 180, 313, 325ff, 328, 366
 in absolute e.m.u., 63, 313, 325
 of dielectric constant, 204
 of high resistances, 362ff
 of magnetic induction, 238, 286ff, 323
 of permeabilities, 324, 325
 of potential, 71, 84, 109, 194, 197, 328ff
 of power, 72, 340
- Measurement, of resistance, 89, 94, 333ff, 335, 362
 of self-induction, 338ff, 339
 of small currents, 180
 of very small or high-frequency alternating current, 327, 328
 of x-ray wave lengths by gratings, 458ff
- Measuring instruments, electrical, 103ff
- Mechanical moment of momentum, of electron, 254, 259, 432, 508
 orbital, 250, 254
- Mechanics, Newtonian, 31
 wave, 5, 446
- Megohm, 106
- Megohm sensitivity, 106
- Mendeleev's periodic table, 119, 261, 437, 455
- Mesh in circuit, defined, 91
- Mesotron or meson, 471
- Metallic conduction, 78, 116, 126, 155, 172, 259, 279, 433, 511ff, 527, 529
 electron flow in, 172
 electron theory of, 155, 511ff
- Metallic electrons, Sommerfeld-Pauli concept of, 511, 527, 529
- Metallic paramagnetism, 260ff
- Metallic state, 126
- Metastable states, 444
- Method of equivalent Y, 99ff
- Method of substitution in resistance measurement, 334, 335
- Mho, 78, 416
- Michelson-Morley experiment, 5, 425
- Microammeters, 326, 328
- Microfarad, 201, 412, 413
- Micromho, 544
- Microphones, piezoelectric, 550
- Milliammeters, 326, 328
- Millikan's oil-drop method, 428ff
- Million electron volts, 331, 425
- Mine sweeping, magnetic, problem on, 573
- M.k.s. system of units, 31
- Mobility, of electron, 487
 of gaseous ions, 486
 of ions in solutions, 131, 133
- Mobility constant, 486
- Moderators, 464
- Molecular susceptibility, 251

- Molecules, polar, 79, 124, 208, 209, 210, 409
 and nonpolar, structure of, 209, 210
 structure of, 210
- Moment, magnetic, 39, 415
 of coil carrying a current, 230
 of electronic orbit, 121, 250, 253, 254
 of momentum, 121ff, 250, 253ff, 254, 432, 443, 508
- Moments, dipole, determination of, 211ff, 212
- Momentum, of electron in orbit, quantized, 121, 253, 254, 443
 uncertainty in evaluation of, 6, 433
- Moseley's laws, 454ff
- Motor, d-c, problems on, 622ff
 simple, 235ff, 290ff, 295ff
- Motor rule, 231
- Motors, and dynamos, reciprocal relations of, 295ff
 efficiency of, 295ff
 power consumption of, 296ff
 real and ideal efficiency of, 297
 torque of, 295ff
- Moving-coil galvanometers, 103ff, 326, 328
- Mu metal, 245
- Multigrid tubes, 544
- Multipliers, phototube, 522
 potential, 225
- Multiply ionized atoms, 450
- Mutual conductance, 541
- Mutual inductance, measurement of, 339
- Mutual induction, 304ff, 339
 coefficient of, 305ff
- Mutual interaction of coils, 306
- Na, energy level diagram for, 446
- Nature of resistance and its variation, 76ff
- Needle points, discharge from, 193
- Negative corona, 495, 500ff
- Negative electron from nuclei, 466
- Negative glow, 490, 500
- Negative ion, formation of, 487
 gaseous, 485
- Negative ion space charges, 489
- Negative point Geiger counter, 551
- Negative solution ions, 125
- Negative space charge, 531
- Negative thermionic emission, 524
- Neon isotopes, 437
- Neon signs, 497
- Neutral point, of magnetic fields, 20
 of thermocouple, 163
- Neutrino, 471ff
- Neutron, 462ff, 464, 469
- Neutron reactions, fast, resonance, and thermal, 463, 469
- Neutron reflectors, 463
- Newtonian mechanics, 4, 31
- Newton's laws, 3
- Newton's *Principia*, 3
- Nitrogen nucleus, transmutation of, by α particles, 461
- Noninductive winding, 303
- Noninteracting inductances in series and parallel, 306, 307
- Nonpolar compounds, 79, 124, 210
- Nonpolar molecules, structure of, 210
- Nonpolar substances, 79
- Normal cathode fall of potential, 495
- Normal magnetization curve, 240
- Notations of nuclear physics, 465
- Note on solution of Kirchhoff's law equations, 95ff
- Nuclear atom, 120ff, 438ff
 electron arrangement in, 120ff
 of Rutherford, 439
- Nuclear binding energy, 467
- Nuclear charge, 439, 440, 443, 455
 effective, 443
- Nuclear constituents, determination of, 460
- Nuclear constitution, 464ff
- Nuclear dimensions, 119, 441, 461
- Nuclear energy, 465, 468ff
- Nuclear energy sources, 468ff
- Nuclear period, 10
- Nuclear reactions, 461, 462, 463, 465, 466, 469, 470
 character of, 470ff
 competing, 466
 cross section for, 463
- Nuclear stability, 466ff, 467
- Nuclear transformations, 461, 463, 465, 466, 469, 470
 by α particles, 442
- Nuclei do not contain free electrons or α particles, 434, 461, 466
- Nucleus, atomic, 119, 439

- Number, of electrons in atoms, 123, 433, 438, 440, 442, 449, 455, 456
 of lines of force emerging from pole, 49
 of lines of force from charge, 191
- Octet theory of valence, 124
- Oersted, 30, 415
- Ohm, 73, 333, 335, 412, 413
 absolute determination of, 333, 335
 legal or international, 333, 335
- Ohm's law, 73, 76, 79, 82, 86, 90, 132, 373, 378, 488, 489
 failure of, 79, 373, 378, 488, 489
 generalization of, 90ff
 problems on, 574ff
- One and two fluid theories of electricity, 171ff
- Orbit shapes, 121]
- Orbital mechanical moment of momentum, 250, 254, 432, 508
- Oscillating circuit, period of, 382, 388
 problems on, 631ff
- Oscillations, damped, 346, 388
 electrical, 375, 381ff, 383ff
 of magnets in field, problems on, 566ff
- Oscillators, piezoelectric and magnetostrictive, 548ff
 thermionic, 545ff
 tuned, grid type, 548
- Oscillatory discharge, derivation of equation for, 382ff
 of condenser, illustrated, 388
- Oscillatory nature of condenser discharge, 375, 381, 388
- Oscillograph, mechanical, 350
- Oscilloscope, cathode-ray, 350, 352
 single-sweep, 352
- Osmotic pressure, 132
- Other ultimate particles, 471
- Owen's bridge for inductance measurements, 339
- Pair production, 426, 434, 457
- Parallel capacities, 221
- Parallel connection of cells, 151
- Parallel currents, fields about, 232
 force between, 233
- Parallel inductances, 306
- Parallel-plate condenser, capacity of, 216ff
 field in, 190
- Parallel resistances, 82
- Paramagnetic body, in divergent field, 247
 in uniform field, 247
- Paramagnetic moment, 257
- Paramagnetism, 245ff, 246, 253ff, 257, 260ff
 metallic, 260
 temperature variation of, 257
- Pauli exclusion principle, 6, 120, 256, 260, 512, 513
- Peak value of e.m.f. on alternating current, 364, 366
- Peltier coefficient, 156
- Peltier effect, 154ff
 problems on, 592ff
- Period, atomic, 9
 current, 8
 electrotechnical, 8
 instrumental, 3, 8
 nuclear, 10
 of alternating current, 364, 366
 of galvanometer, 107, 284, 342ff
 of magnet oscillating in field, 42, 44
 of oscillating circuit, 382, 388
 quantitative, 8
 quantum, 9
 systematic, 4, 9
 wave mechanical, 9
- Period factor in alternating current, 366
- Periodic table, 119, 261, 437, 455
- Permalloy, 245
- "Permanent" magnetism, 18
- Permeability, magnetic, 29, 237, 247, 268ff, 321, 324, 325, 391, 415, 417
 dimensions of, 391
 measurement of, 239, 286ff, 324, 325
 with diamagnetic and paramagnetic bodies, 247
- Permeameter, 324, 325
- Phase advance, 362, 364, 366, 379
 angle of, 378
 by capacity, 362, 376, 379
- Phase angle, definition of, 364, 366
 from power measurement, 340
 in alternating current, 340, 364, 366, 372, 373, 377, 379, 388
 in oscillatory discharge of condenser, 388
 in sinusoidal alternating potential, illustrated, 366

- Phase lag, 362, 364, 366, 373, 379
 by self-induction, 362, 373, 379
 in oscillatory discharge of condenser, 388
 in sinusoidal alternating potential, illustrated, 366
- Phase stability, 474, 476, 478, 479, 480
 principle of, 479, 480
- Phase wave, electronic, 432. *See also* De Broglie waves
- Phenomena of magnetism, 11ff
- Photocells, 522
 circuits for use of, 521
 types of, 520ff
- Photoconductivity, 79, 115
- Photoelectric currents, effect of gas on, 506
- Photoelectric effect, 503ff
 advantages in use of, 520
 in gases, 518
 long wave length limit in, 507, 508, 511, 517
 proportional to light intensity, 507, 509ff, 519, 520
 selective, 510, 517, 519, 520
 velocity distribution in, 518
- Photoelectric emission, and Sommerfeld-Pauli theory, 516ff
 and temperature, 516
 effect of strong fields on, 519
 intensity of, 509ff, 519, 520
- Photoelectric excitation, 447
- Photoelectric ionization, 447, 449, 493
 by x-rays, 449
 in gas, 447, 493
- Photoelectric law of Einstein, 507ff
- Photoelectric work function, 507, 508, 511, 516, 517
- Photoelectric work functions, table of, 517
- Photon, 447, 449, 457
- Phototube multipliers, 522
- Phototubes, infra red, 520
- Photronic cells, 522
- Physical scale of atomic weights, 416
- Physical science, development of, 1ff
- Piezoelectric effect, 549
- Piezoelectric microphones, 550
- Piezoelectric oscillators, 337, 338, 548ff, 549
- Pinwheel, electrical, 193
- Planck radiation law, 444
- Planck's constant, *see* Planck's h
- Planck's h , 33, 253, 331, 432, 433, 443, 508, 509, 513
 dimensions of, 33
- Plate potential, 539
- Plate resistance, 541
- Plot of binding energy vs. atomic mass number, 467
- Plutonium, 469
- Point in circuit, defined, 91
- Poisson's equation, 395, 532
- Polar compounds, 79, 124, 208
- Polar molecules, 79, 124, 208, 209, 210, 409
 structure of, 209, 210
- Polarizability, dielectric, 206ff, 212, 416
 displacement, 206ff, 207, 212, 213, 408
- Polarization, dielectric, 203ff, 211, 212, 213, 214, 215, 408, 416
 displacement, 213, 408
 of cells, 143
 rotation of plane of, by magnetic field, 408
- Pole, magnetic, defined, 14
 unit, defined, 28ff, 415
- Pole strength, absolute evaluation of, 40ff, 324
 magnetic, defined, 28ff, 415
- Poles, consequent, 22
 induction of, in iron, 236
 magnetic, interaction of, problems on, 561ff
 sign of forces between, 15
 of magnet equal, 17
 of magnet in pairs, 17
- Position, uncertainty in establishment of, 6, 433
- Positive column, 496, 500
- Positive corona, 495, 499
- Positive electron, 434, 466
 from nuclei, 466]
- Positive ion impact on surfaces, electron liberation by, 491
- Positive ion space charges, 489
- Positive ions, e/m for, 437
 gaseous, 485
 ionization by, in gases, 491
- Positive rays, 434ff, 435
 deflection of, by magnetic and electric fields, 435ff

- Positive solution ions, 125
 Positive thermionic emission, 524
 Positive wire corona, 552
 Positrons, 434, 466
 Potential, absolute, 185
 absolute e.m.u. of, 69, 282, 412, 413
 absolute measurement of, 71, 194, 328, 332
 and electromotive force in circuits, 86ff, 150, 194
 calorimetric, measurement of, 71
 comparison of, 84ff
 concept of, 64ff, 183ff, 199
 contact, 504, 506
 defined, units, dimensions, ratios, all systems, 412, 413
 electric, 183ff
 e.m.u. of, alternatively defined, 282
 defined, 69, 282, 412, 413
 electrostatic, 183ff
 problems on, 597ff
 is a scalar, 184, 185
 magnetic, 227, 266, 267, 415
 practical unit of, 70, 282, 412, 413
 problems on, 574ff
 Potential difference, and field strength, 190, 416
 from a dynamo, 293
 in electric circuit, 64ff
 in e.s. system, concept of, 183ff
 independent of path, 186
 magnetic, 226, 266, 415
 sign conventions in, 69, 184
 Potential distribution, across condensers in series, 222
 with space charge, 533
 Potential divider, capacitive, 223
 resistive, 84, 85. *See also* Potentiometer
 Potential fall, across dielectric, 216
 along a wire, 83
 Potential measurement, 328ff
 by high-energy particles, 331, 332
 by sphere gaps, 330, 332
 high, 330, 332
 by condensers in series, 331, 332
 Potential multiplier, capacitive, 223, 225
 Potential root mean square, 369
 Potential transients, oscillographic study of, 353
 Potentials, ionization and excitation, 447, 448
 static and alternating, measurement of, 330, 332
 Potentiometer, 84ff, 166, 329, 332
 Power, electrical, defined, 72
 thermoelectric, 157, 160
 to useful work in motor, 295ff
 Power consumption, 72, 114, 296, 318, 340, 380, 417
 average, 380
 of motors, 296ff
 of transformers, 318
 problems on, 575ff
 true, 114, 381
 Power factor, 340, 379ff, 381
 gives phase angle, 381
 Power measurement, 114, 340, 382
 Practical unit, of capacity, 201ff, 412, 413
 of current, 55ff, 412, 413
 of field strength, 191, 416
 of potential, 70, 282, 412, 413
 of quantity, 56, 412, 413
 of resistance, 73, 412, 413
 of self-induction, 302, 412, 413
 Practical units, 412, 413, 415, 416
 Precession of atomic gyroscopes, 249
 Primary-secondary combination for permeability and induction studies, 324, 325
 Principal quantum number, 120ff, 443, 449
 Principle of earth inductor, 283ff
 Prony brake, 296
 Proof of inverse square law, 177
 Proof plane, 192
 Properties of electrons, 431ff
 Protons, 464, 466
 Pyranol condensers, 213
 Pyrex glass, dielectric polarization in, 130
 Quadrant electrometer, 197, 329, 332
 Qualitative phenomena of magnetism, 14
 Quantitative analysis of diamagnetic action, 249ff
 Quantitative period, 8
 Quantitative treatment, of magnetic forces, 28ff
 of static electricity, 176ff

- Quantity, by ballistic galvanometer, 284, 342
 curves for charge and discharge of condenser, 361
 defined, units, dimensions, ratios, all systems, 412, 413
 electromagnetic, definition of, 56, 412, 413
 electrostatic, 179, 412, 413
 practical unit of, 56, 412, 413
- Quantization, of atomic magnets, 253ff
 of atomic orbits, Bohr, 121, 259, 443
 of magnetic moments, 259
- Quantum action, unit of, 33. *See also* Planck's h
- Quantum number, 120ff, 253ff, 260ff, 443, 511ff
 azimuthal, 120, 253ff
 magnetic, 121, 253ff
 principal, 120, 253, 443
 spin, 121, 253ff, 260ff, 443, 513
- Quantum period, 9
- Quantum restrictions, 120, 253ff, 443, 511ff
- Quantum statistics, 512
- Quantum theory, 5, 120, 253ff, 443, 511
- Quartz and Rochelle salt crystals, piezoelectric, 549
- Radar, 450, 474, 553ff
- Radiating impacts, 447
- Radiation, atomic, duration of, 444
 resonance and fluorescent, 447
- Radiation law of Planck, 444
- Radii of Bohr orbits, 443
- Radio and radar waves, 450, 451
- Radio reception, 544
- Radio transformer, 544
- Radioactive transformation series, 460, 466
- Radioactivity, 460, 466, 533ff
 artificial, 466ff
 problems on, 533ff
- Radiography by x-rays, 458
- Radiomicrometer of Boys, 168
- Range of ammeters and voltmeters, extension of, 111ff, 112, 113
- Rate of scientific advance, 7
- Ratio, between units of capacity, 201, 412, 413
- Ratio, between units of self-induction, 302, 412, 413
 of c.s.u. and e.m.u., 390ff, 392, 411, 412, 413, 417
 of units, 390ff
 problems on, 626ff
 table of, 412, 413
 table of evaluations of, 392
- Rays, alpha, beta, gamma, 460
 canal, 434
 positive, 434ff, 435
- Reactance, capacity, 378
 inductive, 373
- Real efficiency, of dynamo, 294
 of motor, 297
- Reciprocal ohm, 78, 132, 416
- Reciprocal relations of dynamos and motors, 295ff
- Recombination, ion and electron, 449, 487
- Rectified e.m.f. from dynamo, 292
- Rectifier, a-c, 327, 328
 arc, 499
 thermionic, 534
- Reduced mobility, 486
- Refraction of x-rays, 458ff
- Refractive index and dielectric constant 208, 409
- Relativistic change of mass, 426, 474, 479, 481, 483
- Relativistic mass, 471
- Relativity, 5, 425ff, 426, 471, 474, 479, 481, 483
- Reluctance, definition of, 266ff
 value of, 267
- Remanence, 239, 244
- Rendering gases conducting, 484
- Residual magnetism, 239
- Resinous electrification, 64, 70, 169
- Resistance, and currents, 76ff
 and temperature, 80ff
 by method of substitution, 334, 335
 concept of, 73ff, 412, 413
 defined, units, dimensions, ratios, a systems, 412, 413
 high, measurement of, 334, 335, 362
 in parallel, 82
 in series, 82
 internal, 86, 150, 293, 295
 nature and variation of, 70ff
 of solutions, 132

- Resistance, problems on, 574ff
 range in values of, 78ff
 specific, 77
- Resistance measurement, 91, 333ff, 335
- Resistance networks, problems on, 578ff
- Resistance thermometry, 82
- Resistance towers, high, 330, 332
- Resonance in radio reception, 544
- Resonance neutron reactions, 463, 469
- Resonance radiation, 447
- Resonant circuits in thermionic oscillator, 546
- Resonant or tuned circuits, 410
- Retentivity, magnetic, 18, 239, 241
- Reversibility of cells, 147
- Richardson theory of thermionic emission, 527, 528, 529
- Right-hand rule, 53, 231
- Root mean square, potential, 369
- Root mean square averages, 363ff, 366, 369
- Root mean square current, 366ff
- Rotation of plane of polarization by magnetic field, 408
- Rubens coils, 62, 322, 325
- Rule, Ampère's, 53
 right-hand, 53, 231
- Rutherford's nuclear atom, 439
- Rydberg constant, 445, 455
- Saha equation, 447
- Saha equilibrium, 498
- Saturation, magnetic, 19, 238, 239, 244
- Saturation thermionic current, 526
- Saw-tooth wave, 352
- Scattering of α particles, 439
 anomalous, 441
- Scientific advance, rate of, 7
- Scientific discovery, nature of, 6
- Scintillation counting, 440, 441, 551
- Screen grid, 544
- Screening, electrical, 172
 magnetic, 18
- Search coils, 324, 325
- Secondary coil method of measuring induction, 238, 285ff, 286
- Seebeck effect, 154ff
 problems on, 592ff
- Selective photoelectric effect, 510, 517, 519, 520
- Self-induction, 299ff
 absolute measurement of, 338, 339
 an electrical inertia, 356, 362, 373, 381
 and capacity, effect of, on alternating current, 375
 and energy losses, 314ff
 and transients, 354ff
 causes phase lag, 362, 373, 379
 centimeter of, 302
 coefficient of, 301ff
 problems on, 623ff
 concept of, 299
 defined, units, dimensions, ratios, all systems, 412, 413
 effect of, on alternating e.m.f., 370ff
 measurement of, 338ff, 339
 nature of, 303ff
 ratios of units of, 302, 412, 413
 units of, 302, 412, 413
- Semi-conductors, 79, 522
- Sensitivity of galvanometer, 106
- Series capacities, 221
- Series connection of cells, 64, 91, 150
- Series interacting inductances aiding and opposing, 308
- Series-parallel connection of cells, 151
- Series resistance, 82
 for voltmeters, 111ff, 113
- Shaking, magnetic, 243
- Shell, magnetic, 275
- Ship's magnetic signatures, problems on, 565, 566
- Short electromagnetic waves, 554
- Shunt, magnetic, 271ff
 problems on, 617ff
- Shunts, 111ff, 112, 326, 327, 328
 for ammeters, 111ff, 112
 problems on, 588
- Sign conventions of potential difference, 69, 184
- Silver voltameter, 326, 328
- Simple dynamos and motors, 290ff
- Simple motor, 235, 295
- Size, of an electron, 432
 of nucleus, 461
 of solution ions, 135
- Sloan x-ray tube, 453, 454
- Small capacities, determination of, 337, 338
- Small currents, static, measurement of, 326, 328

- Snell's law, 408, 458
 Soft x-rays, 450, 451
 Solenoid, magnetic field of, 227
 with current, field in, 226ff
 Solution, conductivity of, 132
 equilibrium in, 137
 ionic space charges in, 134
 Solution ions, size of, 135
 Solution of wave equation, 402ff
 Solution pressure, 137
 Solution tension, 137ff
 relative, 138, 142
 Sommerfeld electron theory of metals, 156, 511ff, 516, 527, 529, 530
 Sommerfeld-Pauli concept, of free electrons and thermionic emission, 527, 529, 530
 of free electrons in metals, 511ff, 527, 529
 Sommerfeld-Pauli theory and photoelectric emission, 516
 Space charge, ionic, in gases, 489
 in solution, 134
 negative, 531, 532, 533
 Space-charge equation, derivation of, 532ff
 Spark breakdown, 492ff
 condition for, 492, 493
 streamer mechanism of, 493
 Sparks, vacuum, 501
 Specific conductivity, 78, 132, 237, 268, 416
 of solutions, 132
 Specific inductive capacity, 203, 204, 416, 417
 Specific resistance, 77, 132, 237, 268
 of solutions, 132
 Sphere gap measurement of potential, 330, 332
 Spherical condenser, capacity of, 202
 Spin moment of electron, 432
 Spin quantum number, 121
 Spontaneous magnetization, 23, 264
 St. Elmo's fire, 501
 Stability, nuclear, 466, 467
 phase, 474, 476, 478, 479, 480
 Stability chart of nuclei, 467, 468
 Stable isotopes, 437, 465
 Stable orbits, Bohr criterion, 443
 Standard cell, potential evaluation of, 329
 Standard Weston cell, 149, 329, 332
 Standards, International, 321
 Starting boxes, 298
 Static electricity I, 169ff
 Static electricity II, 183ff
 Static electricity III, 199ff
 Static generating voltmeters, 331, 332
 Static machines, 173
 Static period, 8
 Static potentials, 183, 194, 330, 332
 Static voltmeters, 329, 332
 Statistics, Fermi-Dirac, 513, 514, 515, 528, 530
 Stern-Gerlach experiment, 257ff, 569, 609
 problem on, 609
 Stokes' law, 135, 430
 Storage cells, 146ff
 efficiency of, 148
 sulfating of, 148
 Stray magnetic fields, 274
 Streamer, corona, 499, 500
 Streamer mechanism of spark breakdown, 493, 500
 Strong electrolytes, 127
 Structure of electromagnets, 275
 Submarine detection, supersonic, 549
 Successive transformations, theory of, 460
 Sulfating of storage cells, 148
 Superconductivity, 81
 Supersonic submarine detection, 549
 Supersonic vibrations, 549
 Suppressor grid, 544
 Surface, force on, in electrical field, 195
 Surface density of charge, 182, 191, 397, 417
 Surface distribution of charge, 192
 Surface equipotential, 187ff
 Susceptibility, electrical, 416
 magnetic, 237, 251, 415
 molecular, 251
 Synchrocyclotron, 461, 479ff, 481
 Synchrotron, 461, 479ff, 480, 481
 Systematic period, 4, 9
 Systems of electrical and magnetic units, 417ff
 Table, of auxiliary concepts, 415ff
 of dimensions in electrical systems, 418
 of electromagnetic wave lengths, 451
 of electromagnetic waves, 451

- Table, of electron arrangement in atoms,
123
of evaluation of ratio of units, 392
of magnetic properties of materials,
246
of photoelectric and thermionic work
functions, 517, 530
of solution tensions, 138
of thermoelectric powers, 160
of units and dimensions, 412, 413
 explanation of, 414
of values of dielectric constant and re-
fractive index, 409
of values of dielectric constants, 204
Tables of determinations of velocities of
electromagnetic waves, 410
Tangent galvanometer, 62ff, 325, 328
Target area for molecular collisions, 463
Television, systems of Farnsworth and
Zworykin, 519
Temperature, and resistance, 80ff
 effect of, on dielectric constant, 208,
 209, 211, 212
 on magnetism, 19, 25, 252, 257ff,
 263, 265
Temperature coefficient of resistance, 80
Temperature excitation, 447
Temperature ionization, 498
Temperature variation of paramagnet-
ism, 257
Theory of successive transformations,
460
Thermal cross, 168
Thermal e.m.f.'s, calculation of, 157ff,
161, 162, 164
Thermal excitation, 447
Thermal ionization, 498
Thermal neutron reactions, 464, 469
Thermionic amplifier, 541, 542
Thermionic current, equation for, 534
Thermionic effect, 503, 523ff
 positive and negative, 524
 values of currents, 527
Thermionic emissions, arrangement for
 study of, 525
 current-potential characteristics of,
 531ff, 534
 laws of, 525
 theories of, 527, 528, 529
Thermionic oscillator, 545ff
 resonating circuits in, 546
Thermionic rectifier, 534ff
 diagram of potentials and currents
 from, 536
Thermionic vacuum tube, 534
Thermionic work function, 511, 516, 517,
527, 528, 530
Thermocouple, heavy currents from, 166
 neutral point of, 163
 problems on, 593ff
 uses of, 165ff
Thermoelectric effects, 154ff
 problems on, 592ff
Thermoelectric force and temperature,
162
Thermoelectric power, 157ff, 160
Thermoelectric powers, table of, 161
Thermoelectricity, 154ff
Thermogalvanometer, 168, 327, 328, 547,
548
Thermopile, 166
Thermopile vacuum, 167
Thermoscope, problems on, 593
Thomson astatic galvanometer, 108
Thomson atom, 433, 438ff
Thomson coefficients, 158, 164, 165
Thomson effect, 158ff, 164, 165
Thomson mass spectrograph, 435, 436'
Three-electrode tube, 537ff
 as thermionic oscillator, 544ff
 characteristic curves of, 542, 543
Throw of ballistic galvanometer, 343
Time, of excitation, 444
 uncertainty in evaluation of, 6, 433
Timeconstant of circuit, 356, 361, 386, 388
 problems on, 626ff
Toepler machine, 173
Tolman-Stewart experiment, 433
Toroidal solenoid, 229
Torque, of motors, 295ff
 on magnet in uniform field, 39
Total intensity of earth's field, 46, 283
Townsend's first coefficient, 490
Townsend's second coefficient, 491
Tracks of α particles, 439, 440
Traction, magnetic, 276
Traction methods of measuring induc-
tion, 238
Transconductance, 541
Transformations, successive, 460
Transformer, 271, 318
 voltage multiplication in, 318ff

- Transformers, step-down and step-up, 319
- Transient, caused by inductance and capacity, 354ff
in alternating currents, 371, 376
- Transient currents from charge and discharge of condenser, 360
- Transient phenomena, 341ff, 349ff
problems on, 627ff
- Transition, electron, forbidden, 444
- Transition elements, 122, 123, 124
- Transmutation of nuclei, 461, 463, 465, 466, 469, 470
- Transport ratio, 133
- True power consumption, 114, 340, 381ff
- True watts, 340, 381ff
- Tube counters, 550ff
- Tube of force, 48
- Tubes, x-ray, 452
- Tuning radio, 544
- Two isolated charges, equipotentials for, 188
- Types, of conduction, 115
of galvanometers, 108ff
- Ultimate particles, other, 471ff
- Ultra-high-frequency electric wave generation, 553ff
- Ultra-violet and extreme ultra-violet light waves, 450, 451
- Ultra-violet light in photoelectric effect, 503
- Uncertainty principle of Heisenberg, 6, 433
- Unification of physical science, 3
- Uniform electrostatic field produced by parallel-plate condenser, 190
- Uniform field, paramagnetic and diamagnetic bodies in, 247, 248
- Uniform fields, achievement of, 189
- Uniform magnetic field, 61, 226, 322
- Unit, absolute e.m., of potential, 69, 412, 413
absolute e.s., of potential, 184, 412, 413
electromagnetic, of quantity, 56
e.s., of potential difference, defined, 184
of capacity, 200, 412, 413
practical, 201
of conductivity, 77, 416
of current, e.m., 54ff
- Unit, of current, e.s., 180, 412, 413
practical, 55ff
of electric charge: the electron, 118, 430
of electrical power consumption, 72, 416
of magnetic induction, 238, 415
of potential, practical, 70, 412, 413
practical, of electric current, 55, 412, 413
of field strength, 191, 416
- Unit Bohr orbit, 121, 444, 449
- Unit charge, absolute, 56, 179, 412, 413
- Unit current, alternative definition of, 55, 412, 413
definition of, 54, 55, 103, 180, 412, 413
- Unit electrical field, 181, 191, 416
- Unit e.s. quantity, 179
- Unit flux, magnetic, 49, 415
- Unit magnetic field, 29, 415
- Unit magnetic moment of Bohr, 253
- Unit magnetic pole, 29, 415
- Unit pole, defined, 28ff
- Units, and dimensions, 30
c.g.s. system of, 30ff
electrical interrelations of, 410ff, 412, 413
electromagnetic and electrostatic, ratio of, 390ff
fundamental and derived, 73, 299, 321, 410
Gaussian system of, 34, 204, 411, 417, 418
important electrical, 410, 412, 413
m.k.s. system of, 31
of capacity, 200ff, 412, 413
ratio of, 201, 412, 413
of conductivity, 78
of resistance, 73, 412, 413
of self-induction, 302, 412, 413
ratio of, 390ff, 410ff, 412, 413
table of evaluation for, 392
ratios of, table of, 412, 413
systems of electrical and magnetic, 417ff
table of, 412, 413
- Unstable isotopes, 465, 466
- Uses, of condensers, 218
of electrostatic induction, 173
of thermocouples, 165ff
- Vacuum sparks, 501ff

- Vacuum thermopile, 167
 Vacuum-tube voltmeter, 330, 332
 Valence electrons, 123
 Value of reluctance, 267
 Values of currents given by a-c instruments, 366ff
 Van de Graaff generators, 173, 331, 453, 472
 Variometer, 309
 Vector diagram, a-c, 374, 378
 Velocity, of cathode rays, 423, 425
 of electrons, 424
 in Bohr orbits, 443
 in fields in gases, 487
 of e.m. waves, table of determinations, 410
 of solution ions, 131
 Velocity distribution in photoelectric effect, 518
 Velocity modulated beam, 555
 Virtual current, 374
 Visible light waves, 450, 451
 Vitreous electrification, 70, 169
 Volt, 71
 legal or international, 329, 332
 Voltage amplification constant, 540, 541
 Voltage multiplication in transformer, 318ff
 Voltaic piles in series, 64
 Voltaic replacement series of Ostwald, 138
 Voltmeter, silver, 119, 326, 328
 Voltmeter, vacuum-tube, 330, 332
 Voltmeters, 109ff, 110, 329, 332
 a-c, 110, 329, 332
 generating static, 331, 332
 require calibration, 110
 series resistance for, 111ff, 113
 problems on, 589ff
 Volts, electron, 424, 505
 Volume density of charge, 393, 394, 397, 416

 Water molecule, description of, 209
 Watt, 72, 381, 417
 Wattmeter, 113ff
 for a-c power measurement, 340
 Watts, "true," 114, 340
 and apparent, 340
 Wave equation, derivation of, 400ff
 of de Broglie, 445

 Wave equation, solution of, 402ff
 Wave guides, 554
 Wave lengths, of electromagnetic waves, 450, 451
 of x-rays, 454, 455
 by grating, 458
 Wave mechanics, 5, 120, 446, 457
 Wave mechanics period, 9
 Waves, α particle of de Broglie, 441
 electromagnetic, 390ff, 392ff
 diagrammatic representation of, 407
 spectrum of, 450, 451
 Hertzian, 409
 Weber, 49
 Weiss magneton, 256
 Weston cell, standard, 149, 329, 332
 Wheatstone bridge, 92, 94, 333, 335
 for self-induction measurement, 339
 White x-rays, 450, 456
 Wiedemann-Franz law, 511, 515
 Wilson, C. T. R., cloud tracks, 439, 493, 551
 Wimshurst machine, 173
 Wind, electrical, 193
 Windings, inductive and noninductive, 303
 Work, and potential difference, 64
 done in moving a conductor in a field, 226ff
 to carry pole around current, 266
 to move charge in field independent of path, 186
 to start or annihilate a current, 310
 Work function, 507, 508, 511, 516, 517, 527, 528, 530

 X-radiation characteristic, mechanism of, 449
 X-radiography, 458
 X-ray, grating, wave lengths of, 459ff
 X-ray levels, 449ff
 X-ray series, K and L , 451, 455, 456
 X-ray tubes, 452ff
 X-ray wave lengths, 454, 455, 459
 X-rays, 449ff
 absorption coefficients of, 450, 451, 452
 diffraction of, 454
 discovery of, 420
 from decelerated electrons, 450
 ionization of gases by, 485

X-rays, problems on, 533ff
refraction of, 458ff
soft and hard, 450, 451
totally reflected, 458
white and fluorescent, 450

Yoke to remove demagnetizing field, 26,
274, 287

Zeeman effect, 408, 433, 439, 445
problems on, 614

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